Intertemporal Decision Making: a Mental Load Perspective

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Abstract. In this paper intertemporal decision making is analysed from a framework that defines the differences in value for decision options at present and future time points in terms of the extra amount of mental burden or work load that is expected to occur when a future option is chosen. It is shown how existing hyperbolic and exponential discounting models for intertemporal decision making both fit in this more general framework. Furthermore, a specific computational model for work load is adopted to derive a new discounting model. It is analysed how this intertemporal decision model relates to the existing hyperbolic and exponential intertemporal decision models. Finally, it is shown how this model relates to a transformation between subjective and objective time experience.

1 Introduction

In realistic environments, decision making often involves a choice between decision options that each provides or promises some benefit, but at a different point in time. For example, in [5], Ch 7, pp. 193-220, following [2] and [7], Dennett presents a perspective on the evolution of moral agency, describing how by evolution cognitive capabilities have developed to estimate the value of an option in the future in comparison to the value of an option in the present. Such capabilities enable not to choose for self-interest in the present, but for other agents' interests, under the expectation that at some future point in time more benefits will be obtained in return. More in general, such capabilities enable to resist temptations in the present, and are helpful, for example, in coping with risks for addiction (e.g., [7]).

In intertemporal decision making it is assumed that in order to be comparable, the value estimated for a certain option has to be higher when the considered time point of realising the option is more in the future. This implies that value is not preserved over time, but can increase due to the passing of time. A central issue in intertemporal decision making is the specific discounting relation between values over time, as used by humans. Well-known approaches to intertemporal decision making consider hyperbolic or exponential discounting models (e.g., [1], [2], [9], [16], [17]). Support for both an exponential model and a hyperbolic model can be found. An interesting challenge is how such models can be explained on the basis of assumptions on more specific mental characteristics and processes involved. For example, they have been related to assumptions on subjective perception of time that essentially differs from objective time (e.g., [14], [20], [23], [25]). Also relations to underlying neural processes have been explored (e.g., [10], [13], [21], [24]).

In this paper the assumption is analysed that the difference between value for an option in the present and in the future is due to expected extra effort or mental load that is accompanying the future option. This load can be seen, for example, as the mental burden to keep the issue in mind, to worry about it, and to suppress impatience during the time interval involved. There are two advantages of this perspective. One is that computational models are available in physiological and cognitive sciences (e.g., [3], [4], [26], [27]) that describe mental load. A second advantage is that according to this perspective value is not increasing just due to passing of time, but is explained in terms of extra work invested, in line with basic intuitions in economics, for example, behind the well-known Labour Theory of Value (cf. [6], [22]; see also [18], [28]).

The paper is organised as follows. First in Section 2 a general framework is introduced that describes temporal discounting relations in terms of mental load, and it is shown how the well-known hyperbolic and exponential discounting models can be derived from the general framework using certain assumptions on the mental load over time. In Section 3 an available dynamical model on work load is used to derive a new discounting model, called the hyperbolic-exponential discounting model. In Section 4 this new model is analysed in some detail and it is shown, for example, how it relates to hyperbolic and exponential discounting. Section 5 shows how the hyperbolic-exponential model also can be related to a suitable assumption on subjective perceived time. It is shown how the transformation between objective and subjective time needed for this can be approximated by logarithmic functions and power functions as used in the literature.

2 Relating Temporal Discounting to Expected Mental Work Load

In this section, in Section 2.1 a general setup to relate intertemporal decision making to expected mental load is presented. Next, in Section 2.2 it is described how the well-known hyperbolic temporal discounting model fits in this general setup, and in Section 2.3 the same is done for the exponential temporal discounting model.

2.1 General Setup

As for decision making in general, to model intertemporal decision making often a valuation-based approach is followed: each decision option is valued and in principle the option with highest value is chosen. This perspective is supported by recent neurological research; e.g., [10], [13], [14], [21], [24]. For intertemporal comparison between values of options an element usually taken into account is a certain discounting rate. For example, when someone has to pay money to you and asks for a one year delay of this payment, you may ask some interest percentage in addition. Many of the experiments in intertemporal decision making address such situations with subjects having to choose between getting an amount of money in the present and getting a different amount at a given future time point. Here the comparison is between two options which are both expressed in terms of money. In the general setting addressed here an option 1 is compared to an option 2, where option 1 is an option realised in the present, and option 2 is an option realised in the future, after some time duration t. The value assigned to option 1 in the present is compared to the value assigned to option 2 after t. It is this dependence of the time duration of this option that is addressed here. In an abstract form this is represented as follows. Take $V(0)$ the value of the option (for a given agent) in the present, and $V(t)$ the value for the (other) option after time duration t.[1] To be comparable to the value $V(0)$ for the option in the present, the value $V(t)$ of the option for a future time point has to be higher; e.g., [1], [2], [5], [9], [14], [17]. In this paper the assumption is explored that a higher value of $V(t)$ compared to $V(0)$ is due to the expected extra amount of mental energy $E(t)$ needed for the future option during the time interval [0, t] before the option has actually been obtained at time t; this can be formulated as follows:

$$V(\mathrm{t}) = V(0) + E(t) \quad (1)$$

The intuition behind this is that the mental work invested adds value to a product, in line with, for example, the Labour Theory of Value in economics (e.g., [6], [18], [22], [28]). The extra energy $E(t)$ can be considered as related to the burden of having to wait, and suppressing the impatience to have the option. This extra energy is assumed proportional to the initial value $V(0)$:

$$E(t) = W(t)\, V(0) \quad (2)$$

where $W(t)$ (work) is the expected extra energy spent per unit of V during the time interval from 0 to t. Using this in (1) the following *general temporal discounting relation* is obtained, depending on the expected extra work $W(t)$ accompanying the choice for the option at time t.

$$V(\mathrm{t}) = V(0) + W(t)V(0) = (1 + W(t))\, V(0) \quad (3a)$$

$$\frac{V(t)}{V(0)} = 1 + W(t) \quad (3b)$$

$$\frac{V(0)}{V(t)} = \frac{1}{1 + W(t)} \quad (3c)$$

The dependency expressed in (3c) is depicted in Fig. 1: a hyperbolic dependency of the fraction $V(0)/V$ on W. The *discounting rate* is defined as

$$\frac{\boldsymbol{d}V}{\boldsymbol{d}t} / V$$

Using (3a) and (3c), this can be expressed in $W(t)$ as follows:

$$\frac{\boldsymbol{d}V(t)}{\boldsymbol{d}t} = V(0)\ \frac{\boldsymbol{d}W(t)}{\boldsymbol{d}t}$$

$$\frac{\boldsymbol{d}V(t)}{\boldsymbol{d}t} / V(t) = \frac{V(0)}{V(t)} \frac{\boldsymbol{d}W(t)}{\boldsymbol{d}t} = \frac{1}{1 + W(t)} \frac{\boldsymbol{d}W(t)}{\boldsymbol{d}t} \quad (4)$$

The relations between $V(t)$ and the expected extra work $W(t)$ provide a way to derive temporal discounting models from models for expected work depending on the duration t of the interval. Conversely, (3) can also be used to derive a function $W(t)$

[1] Note that to explicitly indicate the two options the notations $V_1(0)$ and $V_2(t)$ could be used. For the sake of simplicity these indices are left out.

from any given temporal discounting model: any discounting model $V(t)$ can be related to an expected work function $W(t)$, by the following *inverse relation*:

$$W(t) \ = \frac{V(t)}{V(0)} \ - \ 1$$

In principle to obtain $W(t)$ any mathematical function can be considered and based on that function a temporal discounting model and discounting rate can be derived by (3) and (4). It is reasonable to assume at least that such a function is monotonically increasing. However, not every monotonically increasing function may be considered plausible. Below different ways to obtain $W(t)$ will be considered. First it will be analysed which work functions $W(t)$ fit to the well-known classical hyperbolic and exponential discounting models.

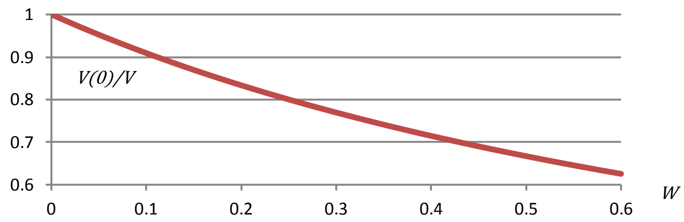

Fig. 1. Hyperbolic relation for *V(0)/V* vs. work *W* (3c)

2.2 Deriving the Hyperbolic Discounting Model

From the perspective of the expected extra work $W(t)$, a simplest model occurs when it is assumed that the power P (the expected extra energy spent per time unit and per unit of V) is constant over the time interval, and based on that the amount of extra work is proportional to the length of the time interval:

$$W(t) \ = \ P\,t \tag{5}$$

By taking the linear dependencies (3b) and (5) together a linear form follows:

$$\frac{V(t)}{V(0)} = 1 + P\,t \tag{6}$$

Using the format of (3c), this provides a derivation of the well-known *hyperbolic model* for temporal discounting:

$$V(0) \ = \frac{V(t)}{1 + P\,t} \tag{7}$$

$$\frac{V(0)}{V(t)} \ = \frac{1}{1 + P\,t} \tag{8}$$

Note that from (4) and (5) it immediately follows that the discounting rate for this case is

$$\frac{P}{1 \ + \ P\,t}$$

which starts at P for $t = 0$ and is decreasing to 0 for increasing t.

2.3 Deriving the Exponential Discounting Model

Another well-known model for temporal discounting is the exponential model:

$$\frac{V(t)}{V(0)} = e^{\alpha t} \qquad (9)$$

From (3b) it can be derived that this exponential model occurs when it is assumed that the expected extra work $W(t)$ has an exponential relation to t:

$$W(t) = e^{\alpha t} - 1 \qquad (10)$$

For this case from (4) and (10) it follows that the discounting rate is α, which does not depend on t. Note that the derivation of (10) from (9) by itself does not provide a justification of why expression (10) for the extra work $W(t)$ would be reasonable. It suggests that either expected power P is not constant but increasing over time, or, if P is constant, from the start on the level will be higher when longer time intervals are involved. In Section 3 an existing model for estimating the amount of extra work will be adopted to derive what may be a reasonable expression for $W(t)$.

3 A Temporal Discounting Model based on a Work Load Model

A more specific model is obtained if more detailed assumptions are made for the amount of extra work $W(t)$ spent. In the physiological literature on work load usually a critical power level CP is assumed which is the power that can be sustained without becoming (more) exhausted. Moreover, this critical power may be slightly affected by using power above this level. In [26] a computational model for work load was used in order to address the question how (physiological) effort can best be distributed over a certain time interval. Below this is adopted as a biologically inspired model for mental work load to address the issue of deciding between two temporally different options. The following equations (adopted from [26]) describe this model. Dynamics of critical power is described by a linear dependency of the change in critical power on the effort spent above the critical power, with proportion factor γ:

$$\frac{dCP}{dt} = -\gamma P \qquad (11)$$

Here P is a (non-constant) function of time t indicating the extra (non-sustainable) power spent at time t; it is assumed that only the expected non-sustainable energy spent based on extra resources is taken into account. The expected extra work W spent over time is described by

$$\frac{dW}{dt} = P \qquad (12)$$

The following relation for the derivatives of CP and W follows from (11) and (12):

$$\frac{dCP}{dt} = -\gamma \frac{dW}{dt} \qquad (13)$$

Moreover, if it is assumed that the total expected power $P + CP$ spent (both the non-sustainable power P and sustainable power CP) is kept constant over time, it holds

$$\frac{\boldsymbol{d}CP}{\boldsymbol{d}t} = - \frac{\boldsymbol{d}P}{\boldsymbol{d}t}$$

From this, (12) and (13) it follows

$$\frac{\boldsymbol{d}P}{\boldsymbol{d}t} = \gamma \frac{\boldsymbol{d}W}{\boldsymbol{d}t} = \gamma P \tag{14}$$

Relation (14) is a simple first-order differential equation in P which has as solution $P(t) = P(0)\, e^{\gamma t}$. From this and (12) it follows that

$$\frac{\boldsymbol{d}W}{\boldsymbol{d}t} = P(0)\, e^{\gamma t} \tag{15}$$

Using (15), for $\gamma \neq 0$ an explicit analytic solution for the function $W(t)$ can be found by integration, thereby using $W(0) = 0$:

$$W(t) = W(0) + P(0)\,(e^{\gamma t} - 1)/\gamma = \frac{P(0)}{\gamma}\,(e^{\gamma t} - 1) \tag{16}$$

Note that when it is assumed $P(0) = \gamma = \alpha$, this exactly provides relation (10) found in Section 2.3 for the exponential model. Using (16) in (3b) the following *hyperbolic-exponential model* for temporal discounting is obtained:

$$\frac{V(t)}{V(0)} = 1 + \frac{P(0)}{\gamma}(e^{\gamma t} - 1) = \frac{P(0)}{\gamma} e^{\gamma t} + \frac{\gamma - P(0)}{\gamma} = \frac{P(0)}{\gamma}\left(e^{\gamma t} + \frac{\gamma - P(0)}{P(0)}\right) \tag{17}$$

The hyperbolic-exponential model (17) can be written in a different form as follows:

$$\frac{V(t)}{V(0)} = 1 + P(0)\, t\, \frac{e^{\gamma t} - 1}{\gamma t} \tag{18}$$

$$\frac{V(0)}{V(t)} = \frac{1}{1 + P(0)\, t\, \frac{e^{\gamma t} - 1}{\gamma t}} \tag{19}$$

This form shows that the difference with the hyperbolic model is in the exponential factor

$$\frac{e^{\gamma t} - 1}{\gamma t}$$

which is close to 1 for smaller values of γt, as is shown, e.g., in Fig. 2 in Section 4, where a more precise analysis will be presented of how this hyperbolic-exponential model relates to both the hyperbolic and the exponential model. From (4), (15) and (16) it follows that the discounting rate for this case is obtained as follows:

$$\frac{\boldsymbol{d}V(t)}{\boldsymbol{d}t} / V(t) = \frac{\boldsymbol{d}W(t)}{\boldsymbol{d}t} / (1 + W(t)) = \gamma / \left(\frac{\gamma - P(0)}{P(0)}\, e^{-\gamma t} + 1\right)$$

Note that for $t = 0$ this discounting rate is $P(0)$ and for larger time durations from 0 to t the rate converges to γ (upward when $P(0) < \gamma$ and downward when $P(0) > \gamma$).

4 Comparative Analysis

To be able to compare the hyperbolic-exponential model given by (18), (19) to existing models, first the expression $\frac{e^{\gamma t}-1}{\gamma t}$ as a function of γt is analysed. In Fig. 2 a graph is shown for this function. The Taylor series is:

$$\frac{e^{\gamma t}-1}{\gamma t} = \frac{(\sum_{k=0}^{\infty} \frac{1}{k!}(\gamma t)^k - 1)}{\gamma t} = 1 + \sum_{k=2}^{\infty} \frac{1}{k!} (\gamma t)^{k-1} \qquad (20)$$

From this it follows that

$$\frac{e^{\gamma t}-1}{\gamma t} \text{ is monotonically increasing in } \gamma t \text{ with 1 as limit for } \gamma t \text{ at 0} \qquad (21)$$

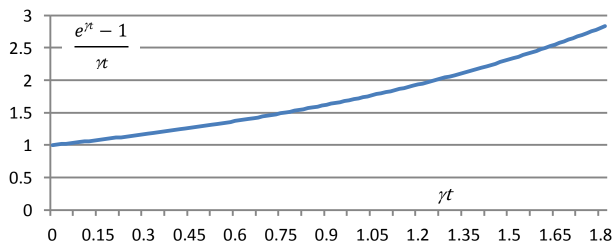

Fig. 2. Graph of the function $\frac{e^{\gamma t}-1}{\gamma t}$ against $\gamma t > 0$

From (21) it follows that for shorter time durations t and/or smaller values of γ the hyperbolic-exponential model (18), (19) is approximated by a hyperbolic model; cf. (6), (7), (8) with $P = P(0)$:

$$\frac{V(t)}{V(0)} = 1 + P(0)\, t \qquad (22)$$

$$\frac{V(0)}{V(t)} = \frac{1}{1 + P(0)\, t} \qquad (23)$$

This is illustrated, for the shorter time durations t in Fig. 3. Moreover, for γ very small (for example, 10^{-8}) the hyperbolic-exponential model is approximated by the hyperbolic model for all t with high accuracy, with difference estimated by (20) as

$$P(0)t\,(\frac{e^{\gamma t}-1}{\gamma t} - 1) = P(0)\, t\,(1 + {}^1/_2\, \gamma t\ + \ldots\ - 1)\ = {}^1/_2\, P(0)\, \gamma t^2$$

based on which is proportional to γ. In particular, note that $\gamma = 0$ provides an exact match with the hyperbolic model. Using

$$\lim_{\gamma t \to \infty} \frac{e^{\gamma t}-1}{e^{\gamma t}} = 1$$

and (17) it can be shown that for longer time durations t and/or larger values of γ the hyperbolic-exponential model is approximated by the following exponential model:

$$\frac{V(t)}{V(0)} = \frac{P(0)}{\gamma}\ e^{\gamma t} \qquad (24)$$

or

$$\frac{V(0)}{V(t)} = \frac{\gamma}{P(0)}\ e^{-\gamma t} \qquad (25)$$

This is illustrated for the longer time durations t in Fig. 3. Note that in general $P(0) \neq \gamma$ so that this approximation cannot be used for shorter time durations. However, if $P(0) = \gamma$, then the model is exponential.

The above analysis shows that the introduced hyperbolic-exponential model provides one unified model that can be approximated both by a hyperbolic pattern for shorter time durations and/or smaller values of γ and by an exponential pattern for longer time durations and/or larger values of γ. For the first approximation for smaller t, the constant in the hyperbolic model usually indicated by k is equal to $P(0)$. Note that this still leaves freedom in the value of parameter γ for the exponential approximation for larger t. In Fig. 3 the differences between the hyperbolic and hyperbolic-exponential are displayed for some examples settings. Both the hyperbolic-exponential and the hyperbolic curve are shown for $\gamma = 0.005$, and $P = P(0) = 0.05$, with time on the horizontal axis.

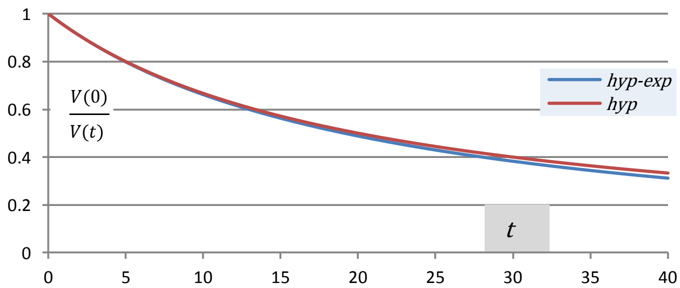

Fig. 3. Comparing hyperbolic and hyperbolic-exponential models: $\gamma = 0.005$, $P = P(0) = 0.05$

The graph shows the forms of (8) and (19). The graph shows that indeed for shorter time durations they coincide, whereas for longer time durations they diverge. In general, the smaller γ the more the two curves coincide. There is also another way in which the two curves can be related to each other: by taking P different from $P(0)$. In Fig. 4 it is shown how the two curves coincide when $P = 0.053$ and $P(0) = 0.05$. The upper graph shows the forms of (8) and (19). In the second graph the absolute differences are depicted.

5 Relating the Model to Subjective Experience of Time

In the recent literature another way of taking mental processing into account is by assuming perception of subjective time which is different from objective time (for example, advocated in [14], [20], [23], [25]). The different time measures can be related according to a time transformation function

$$\tau: t \rightarrow u = \tau(t)$$

where t is the objective time and u the subjective (perceived) time. Such a time transformation explains a form of human discounting different from the exponential form, whereas internally an exponential model is used:

$$\frac{V(t)}{V(0)} = e^{\alpha\,\tau(t)}$$

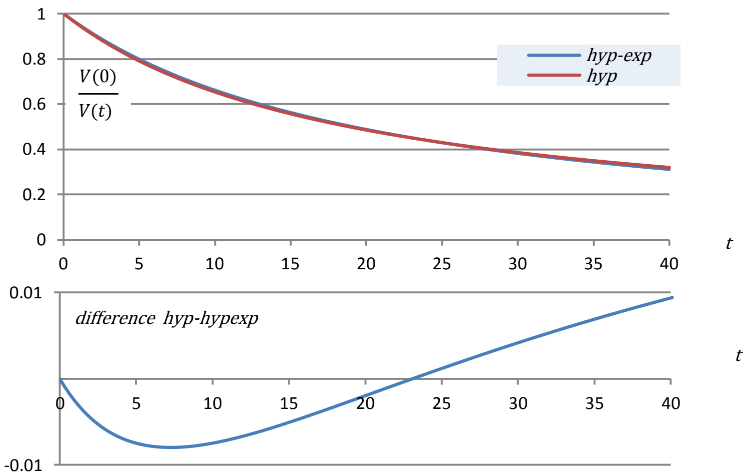

Fig. 4. Comparison of hyperbolic and hyperbolic-exponential models: $\gamma = 0.005$, $P = 0.053$, $P(0) = 0.05$.

The hyperbolic-exponential model can also be related to an exponential form according to this principle, when an appropriate time transformation is chosen. This time transformation can be found using (17):

$$\frac{V(t)}{V(0)} = 1 + \frac{P(0)}{\gamma}(e^{\gamma t} - 1) = e^{\alpha u} \qquad \alpha u = \ln(1 + \frac{P(0)}{\gamma}(e^{\gamma t} - 1))$$

Therefore

$$\tau(t) = u = \frac{1}{\alpha} \ln(1 + \frac{P(0)}{\gamma}(e^{\gamma t} - 1)) \qquad (26)$$

This shows how the hyperbolic-exponential model can be obtained by assuming a subjective internal time perception different from the objective time. In Figure 6 it is shown how the transformation τ from objective time to objective time compares to the first and second order approximations, and to other transformations put forward in the literature: the logarithmic transformation (e.g., [23]), and a transformation defined by a power function (e.g., [25]):

$$u = \eta \ln(1+\beta t) \qquad\qquad u = \lambda\, t^{\kappa}$$

This figure shows that the time transformation τ related to the hyperbolic-exponential model can be approximated very well by a logarithmic function and also reasonably well by a power function.

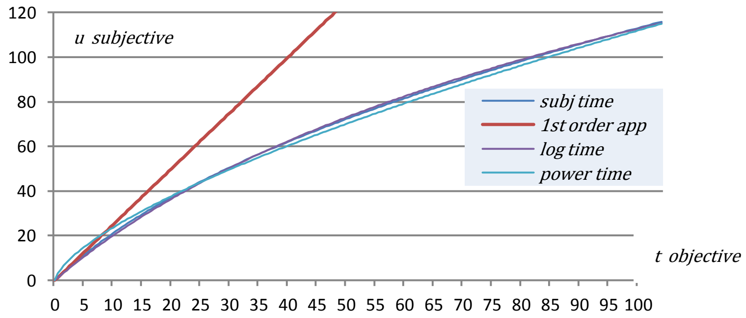

Figure 6. Time transformation τ from objective time t to subjective time u compared to 1st and 2nd-order, logarithmic and power function approximations: $\gamma = 0.01$, $\alpha = 0.02$, $P(0) = 0.05$, $\beta = 0.025$, $\eta = 90$, $\lambda = 5$, $\kappa = 0.675$.

6 Discussion

The framework for intertemporal decision making at the core of this paper defines the differences in value for options at present and at future time points in terms of the expected extra amount of mental burden, worries, or work load accompanying the choice for a future option. Conceptually this provides an approach in which an attributed higher value for the option in the future is in balance with a lower value for the option in the present plus an expected amount of extra work needed to actually obtain the extra value in the future. Thus value becomes higher due to extra work added, in line with intuitions in economics, at the basis of, for example, the Labour Theory of Value (e.g., [6], [18], [22], [28]). It has been shown how the existing hyperbolic and exponential discounting models for intertemporal decision making fit in this framework for specific patterns for the amounts of work depending on the duration. Furthermore, based on elements from an existing biologically inspired computational model for work load (cf. [26], [27], [3], [4]) a new hyperbolic-exponential discounting model has been derived.

This hyperbolic-exponential discounting model makes use of the concept critical power which indicates the level of power that can be provided without increasing fatigue (e.g., [8], [11], [12], [15], [19], [26], [27]). Moreover, the assumption was made that the considered power spent is kept constant over time. The model has a parameter γ for the extent to which critical power is affected by using power above the critical power. It has been shown that this model is approximated by the well-known hyperbolic model for shorter time durations or a lower value of the parameter γ (for $\gamma = 0$ by an exact match) and that it is approximated by the exponential model for longer time durations or a higher value of the parameter γ. Furthermore, it has been explored how this hyperbolic-exponential model relates to an approach assuming a difference between subjective and objective time (following, for example, [14], [20], [23], [25]) and a specific transformation between subjective and objective time, which can be approximated well by a time transformation based on a logarithmic function (e.g., as considered in [23]) or a power function (e.g., as considered in [25]).

Thus the introduced hyperbolic-exponential discounting model for intertemporal decision making provides a form of unification of a number of existing approaches. Moreover, it provides an explanation of the increase of value over time in terms of expected extra mental load, which relates well to elementary economic intuitions: you have to work to obtain more value. Alternative approaches to explain the difference in value at different points in time often relate this to the risk of losing the future option. However, in principle this leads to an exponential discounting model, which often has been rejected by empirical data acquired from experiments which favour a hyperbolic model. The approach to relate this difference in value to the expected burden implied by a choice for the future option relates in a quite natural manner to non-exponential or not fully exponential discounting models.

The proposed intertemporal decision model can be used, for example, as a basis for virtual agents that have to cope with temptations, for example in the context of addictions, or in a social context where they are to show moral agency by sometimes deciding for options against their present self-interest; cf. [7], [5], pp. 193-220.

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