BANK BALANCE SHEET OPTIMIZATION

By
H.J.A. Hellemens

A Thesis submitted to the Faculty of Science
in partial fulfillment of the requirement for the degree of
MASTER OF SCIENCE
(Business Analytics)

Thesis supervisors:
dr. Sandjai Bhulai
dr. Diederik Fokkema
prof. dr. André Ran

VU University Amsterdam
Amsterdam
December 2012
# CONTENTS

LIST OF TABLES ................................................................. iii
LIST OF FIGURES ............................................................... iv
PREFACE ........................................................................... vi
ABSTRACT ........................................................................ vii

1. Introduction ................................................................. 1
  1.1 Business Analytics .................................................... 1
  1.2 Internship at Ernst & Young ....................................... 1
  1.3 Background ............................................................. 2
  1.4 The problem ........................................................... 5
  1.5 Report outline ......................................................... 6

2. Banking concepts ......................................................... 7
  2.1 Banks ..................................................................... 7
    2.1.1 Banking business ................................................. 7
    2.1.2 Banking and trading books ................................... 8
    2.1.3 Financial statements ........................................... 9
      2.1.3.1 Balance sheet ................................................ 9
      2.1.3.2 Profit and loss report ..................................... 9
    2.1.4 Performance measures for banks ......................... 10
  2.2 Basel regulation ....................................................... 11
    2.2.1 Background ....................................................... 12
    2.2.2 Basel I ............................................................. 12
      2.2.2.1 Framework .................................................. 13
      2.2.2.2 Amendment for market risk ......................... 14
    2.2.3 Basel II ........................................................... 20
      2.2.3.1 Basel II framework ....................................... 21
    2.2.4 Basel 2.5 .......................................................... 25
      2.2.4.1 Revisions to the market risk framework .......... 25
    2.2.5 Basel III .......................................................... 28
      2.2.5.1 The capital base ............................................ 29
      2.2.5.2 Stronger risk coverage .................................. 32
      2.2.5.3 Liquidity ..................................................... 32
6. Results ........................................................................................................... 70
  6.1 Test cases ................................................................................................. 70
    6.1.1 Test case 1: Large bank ................................................................. 73
    6.1.2 Test case 2: Retail bank ................................................................. 76
    6.1.3 Test case 3: Investment bank ....................................................... 78
  6.2 Comparison of the test three cases ......................................................... 81

7. Conclusion ................................................................................................... 83
  7.1 Bank balance sheet optimization model ................................................... 83
  7.2 Further research ...................................................................................... 84

BIBLIOGRAPHY ............................................................................................... 86

APPENDICES

A. Starting values ............................................................................................ 88
  A.1 Test case 1: large bank ......................................................................... 88
  A.2 Test case 2: retail bank ......................................................................... 88
  A.3 Test case 3: investment bank ............................................................... 89

B. Optimization results .................................................................................... 91
  B.1 Optimal portfolio weights ..................................................................... 91
## LIST OF TABLES

2.1 Risk weight per asset category in Basel I ............................ 14  
2.2 Capital Tiers in Basel I ............................................. 14  
2.3 Simplified approach for options under Basel I ....................... 18  
2.4 Risk components within the IRB approach under Basel II ............ 22  
2.5 Transitional implementation of capital base for Basel III ............ 30  
2.6 Minimum capital conservation ratios under Basel III .................. 31  
4.1 Basel III constraints .................................................. 50  
4.2 Stock of high-quality liquid assets ................................... 54  
4.3 Factors of cash inflows and outflows prescribed by Basel III ........ 54  
4.4 Factors of required stable funding (asset instruments) ............... 56  
4.5 Factors of available stable funding (liability & equity instruments) 56  
6.1 Constants in the model for the three different test cases .............. 72  
6.2 Balance sheet statistics for large bank including LCR and NSFR .... 74  
6.3 Balance sheet statistics for large bank excluding LCR and NSFR .... 75  
6.4 Balance sheet statistics for retail bank including LCR and NSFR .... 76  
6.5 Balance sheet statistics for retail bank excluding LCR and NSFR .... 77  
6.6 Balance sheet statistics for investment bank including LCR and NSFR 79  
6.7 Balance sheet statistics for investment bank excluding LCR and NSFR 80  
B.1 Optimal portfolio weights for the large bank including LCR and NSFR 92  
B.2 Optimal portfolio weights for the large bank excluding LCR and NSFR 93  
B.3 Optimal portfolio weights for the retail bank including LCR and NSFR 94  
B.4 Optimal portfolio weights for the retail bank excluding LCR and NSFR 95  
B.5 Optimal portfolio weights for the investment bank including LCR and NSFR 96  
B.6 Optimal portfolio weights for the investment bank excluding LCR and NSFR 97
# LIST OF FIGURES

1.1 Liquidity spirals ............................................................... 5

2.1 Comparison of mean RoE between investment an commercial banks .... 11

2.2 Organigram for the calculation of the minimal capital requirement under Basel I ................................................................. 15

2.3 Overview Basel II framework .................................................. 21

2.4 Organigram for the calculation of the minimal capital requirement under Basel II ................................................................. 22

3.1 The efficient frontier ............................................................. 38

3.2 Visual interpretation of the difference between VaR and CVaR ....... 40

4.1 Stylized balance sheet ......................................................... 44

5.1 Average yield on 10 year government bonds per credit rating bucket ... 63

5.2 Conditional correlation between S&P 500 and four other assets ....... 66

5.3 Efficient frontier of mean-CVaR optimization for February 2010 ....... 67

5.4 Returns on the asset instruments from January 2003 to October 2012 .... 68

5.5 Returns on the liability instruments from January 2003 to October 2012 .... 69

6.1 Initial weights of the portfolio instruments for the three different banks .... 71

6.2 Efficient frontier for the large bank including LCR and NSFR ....... 74

6.3 Efficient frontiers of the two optimizations for the large bank ....... 75

6.4 Efficient frontier for the retail bank including LCR and NSFR ....... 77

6.5 Efficient frontiers of the two optimizations for the retail bank ....... 78

6.6 Efficient frontier for the investment bank including LCR and NSFR .... 79

6.7 Efficient frontiers of the two optimizations for the investment bank .... 80

6.8 Histogram of the retained earnings for the large bank ................. 81

6.9 Histogram of the retained earnings for the retail bank ................. 81

6.10 Histogram of the retained earnings for the investment bank ........... 82
PREFACE

The final part of the Master Business Analytics at the VU University Amsterdam consists of writing a thesis during an internship. My internship took place at the Financial Services Risk department of Ernst & Young in Amsterdam, where I had the opportunity to get to know the daily activities of the company and dedicate my time to this thesis.

In the first place, I would like to thank my thesis supervisor, Sandjai Bhulai, for his support, constructive comments and guidance during the internship. Secondly, I would like to thank my supervisor at Ernst & Young, Diederik Fokkema. Our discussions helped to tackle many problems encountered during the internship and were crucial in developing the final model. Thirdly, I would also like to acknowledge André Ran, who accepted to take on the task of second reader. In the last place, I would like to thank all of my colleagues at Ernst & Young whom supported me during the internship. The opportunity to share my thoughts with colleagues led to considerable input for this thesis.

Hans Hellemans - Amsterdam, December 2012
ABSTRACT

In the aftermath of the financial crisis, a new regulatory framework for banks, Basel III, was introduced. Banks are seeking for tools and models to be in compliance with Basel III, while still maximizing their profits. In the last decades, banking business has become much more complex and encompasses a wider range of activities with varying degrees of complexity. This thesis aimed at designing a model in compliance with Basel III that is capable of taking the different complex banking activities into account.

The proposed model is a single-period portfolio optimization problem. A stylized balance sheet is introduced, such that it is possible for all types of banks to reflect its actual balance sheet. Two different types of optimization objectives are presented for the model. The first objective it to maximize the expected retained earnings, while disregarding the risk of that portfolio allocation. Secondly, the conditional expected loss at a certain confidence level is minimized, while meeting a minimum amount of required retained earnings. Consequently, when both optimization objectives are combined, it is possible to generate an efficient frontier. This gives the banks the opportunity to choose a portfolio allocation that fits its risk appetite.

The model developed during this research project shows to be capable of optimizing a stylized balance sheet for any type of banks. As such this model can be used by every type of bank as a top-down strategic balance sheet management tool to obtain an optimal balance sheet allocation.
CHAPTER 1
Introduction

1.1 Business Analytics

The last part of the Master program Business Analytics (BA)\(^1\) consists of a six month internship. Business Analytics is a multidisciplinary programme, encompassing mathematics, computer science and business management, aimed to improve business processes. The goal of the internship is to apply all these three disciplines on a problem defined in accordance with the company where the internship takes place. The internship provides students the opportunity to apply the theoretical knowledge from the Master and at the same time gain working experience. This thesis is written during an internship at Ernst & Young from May 2012 to November 2012.

1.2 Internship at Ernst & Young

Ernst & Young is a global leader in assurance, tax, transaction and advisory services. Because of the globalization, clients are becoming more demanding and require integrated, cross-border service, while still demanding high quality wherever they do business around the world. Ernst & Young is able to provide these services due to their global structure. The global structure of connecting their business units around the world, enables them to respond faster than their competitors.

The research for this thesis took place at the Financial Services Risk (FS Risk) department in Amsterdam. FS Risk is an industry-focused business unit providing a broad range of integrated services which leverage deep financial services industry experience with strong functional capability and product knowledge. They help clients mitigate risk, while at the same time improving and sustaining their business performance. In today’s complex business environment, that means understanding the relationship between risk, the regulatory environment and performance improvement. Then Ernst & Young applies its knowledge to help clients achieve their business objectives.

Within the FS Risk department, the Quantitative Advisory Services (QAS) team is a separate team that is concerned with financial and risk modeling for clients in the

---

\(^1\) As of the school year 2011-2012, the official name of the Master Business Mathematics & Informatics was changed to Business Analytics.
financial services industry. This thesis is most affiliated with the line of work of the QAS team.

1.3 Background

The financial market turmoil in 2007 and 2008 has led to one of the most severe financial crisis since the Great Depression in the 1930s and has had large repercussions on the real economy since. In the aftermath of this crisis, bank regulators presented two sets of new rules, Basel 2.5 and Basel III, in order to strengthen the resilience of the banking sector. The objective of the reforms is to improve the banking sector’s ability to absorb shocks arising from financial and economic stress, whatever the source, thus reducing the risk of spillover from the financial sector to the real economy (BCBS 2010a).

Two trends in the banking sector laid the foundation for the financial crisis. First, instead of holding loans on the banks’ balance sheet, banks moved to an “originate and distribute model”. In this model banks repackaged and tranched their assets (e.g., diversified portfolio of mortgages and other type of loans, corporate bonds, and other assets like credit card receivables) into structured asset-backed securities (ABS) and sold them to various other financial investors, thereby off-loading the risk (securitization). However, most of the credit risk never left the banking system, since banks, including sophisticated investment banks, were among the most active buyers of these structured products (Duffie 2008). The underlying motive for the banks to move to the new model was regulatory arbitrage, as the regulatory capital required to keep the securitized debt instruments was much smaller than the regulatory capital that would be required to hold the assets themselves. This is because the assets are kept on the banking book, whereas the securitized debt instruments are kept on the trading book. Buyers of these securitized debt instruments are also able to purchase insurance contracts, so-called credit default swaps (CDS), which protect the holder of the contract against the default of the asset. As an illustration of the magnitude of securitization, estimates of the gross notional amount of outstanding CDS in 2007 range from $45 trillion to $62 trillion (Brunnermeier 2009).

Banks were traditionally exposed to a maturity mismatch, as long-term mortgages and loans are financed by demand deposits that can be withdrawn at short notice. In order to circumvent this maturity mismatch, commercial banks transferred it to so-called off-balance-sheet investment vehicles\(^2\). These vehicles raise funds by selling short-term and

\(^2\)A special purpose vehicle (SPV) is a legal entity established to perform certain projects or activities of a parent company, and the associated assets and their funding is organized on a separate balance -
medium-term asset-backed commercial paper (e.g., by a pool of mortgages or loans as collateral) with an average maturity of three months and one year respectively. The strategy of borrowing with short-term paper and investing in long-term assets exposed the banks to funding liquidity\(^3\) (i.e., when investors stop buying short-term asset-backed commercial paper, the vehicles are unable to roll over short-term debt to the bank). A credit line, called a “liquidity backstop“, was granted to the vehicle by the bank to guarantee funding liquidity. This way banks are still exposed to the liquidity risk from the maturity mismatch, while it does not appear on the banks’ balance sheet. Also the investment banks were increasingly relying on short-term funding, as they rolled over a large part of their funding on a daily basis by short-term repurchase agreements, or “repos“\(^4\).

In summary, leading up to the crisis, banks were increasingly exposing themselves to structured financial products and vehicles to enhance profits by evading regulation. Investment in structured financial products seemed to be part of a culture of excessive risk taking that had overtaken banks (Rajan 2005, Kashyab et al. 2008). A key contributing factor according to (Kashyab et al. 2008) is that, over short time periods, it is very hard, especially in the case of new products, to tell whether a financial manager is generating true excess returns adjusting for risk, or whether the current returns are simply compensation for a risk that has not yet shown itself but will eventually materialize.

Consequently, these developments in the financial markets led to an overflow of cheap credit and falling lending standards. These two developments in turn triggered an explosive increase in subprime mortgages of all kinds\(^5\). As the housing prices in the United States had not yet experienced a nationwide decrease since the World War II, mortgages were granted under the premise that background checks were not necessary, because the increased value of the house was expected to be sufficient to refinance the mortgages for the borrowers. However, when interest rates rose and housing prices did drop, many homeowners were unable to meet the obligations of the bank, as a result of which housing prices plumped (Graafland & van de Ven 2011). This in turn triggered the start of the liquidity crisis in 2007, as the number of mortgage defaults rapidly increased.

Estimations of the subprime losses in 2007-2008 are in the range of several hundred billion

---

\(^3\)Funding liquidity describes the ease with which expert investors and arbitrageurs can obtain funding from (possibly less informed) financiers (Brunnermeier & Pedersen 2008).

\(^4\)In a repo contract, a firm borrows funds by selling a collateral asset today and commits to repurchase identical assets on a specified date at a specified price.

\(^5\)Mortgage brokers offered teaser rates, no-documentation mortgages, piggyback mortgages (a combination of two mortgages that eliminates the need for a down payment, and NINJA (“no income, no job or assets”) loans.
dollars, which is equivalent to about a 1 percent change in the US stock market - which often occurs on a daily basis (Adrian & Shin 2008). However, since they were primarily borne by levered financial institutions with significant maturity mismatch, spiral effects amplified the crisis such that, for example, the overall stock market losses amounted to more than 8 trillion dollars (Brunnermeier & Pedersen 2008).

Investors financed their trades through collateralized borrowing from financiers who set their margins to keep within their Value-at-Risk (VaR) levels. As the financiers are able to re-evaluate and set the margins in each period accordingly, the investors are faced with funding liquidity risk due to the risk of a rise in the margins or losses on existing positions. Destabilizing margins force the investors to de-leverage their positions in times of stress (i.e., pro-cyclical behavior). Since the increase of mortgage defaults made it more difficult for banks to roll over short-term commercial paper for their positions in mortgage-backed securities (MBS) due to the increase in margins on these instruments. This continued until the moment that those banks hit their capital constraints over the life of those trades. At this time they reduced their positions and market liquidity declined, which tightened the banks’ funding constraints even further. The prices are then no longer driven by their fundamental value, but rather by funding liquidity considerations. This effect is also known as a margin spiral and forces banks to de-leverage during downturns.

Evidence of the margin spiral for investment banks was presented by (Adrian & Shin 2009). The effects of the margin spiral is further reinforced by the loss spiral, as shown in Figure 1.1. The loss spiral arises when a bank holds an initial position that is negatively correlated with customers’ demand shock. In this case, a funding shock increases market illiquidity, leading to losses on their initial position, forcing the banks to sell more, causing a further price drop, and so on (Brunnermeier & Pedersen 2008). Eventually this led to the dry-up of funding liquidity of banks and unfolded the credit crisis. Consequently, governments had to step in with unprecedented injections of capital and liquidity.

The crisis exposed some fundamental flaws in regulation on the banks’ trading book. Over time, the structure of the trading books and the nature of the financial sector changed dramatically. As a short-term solution the BIS\(^6\) introduced a set of revisions to the market risk framework in July 2009, referred to as Basel 2.5 rules. Including a requirement for banks to hold capital against default risk as well as migration risk, for unsecuritized credit products. In additional response to the crisis is the introduction of a stressed Value-at-Risk requirement.

\(^6\)Bank for International Settlement, serves central banks in their pursuit of monetary and financial stability, to foster international cooperation in those areas and to act as a bank for central banks.
1.4 The problem

In the aftermath of the financial crisis, the BIS launched a program to revise the Basel II regulatory framework. The new regulatory framework, Basel III, was introduced November 2010 and consists of two new frameworks. First, strengthening the regulatory capital framework, building on the Basel II framework. Second, a new liquidity framework by developing two minimum standards for funding liquidity. Both frameworks will be gradually implemented from 2013 to 2019.

The new regulation will have a great impact on banks. Banks are seeking for tools and models to be in compliance with Basel 2.5 and Basel III, while still maximizing their profits. Thus, how to optimally compose its balance sheet considering the current balance sheet and the new regulation. Most large banks engage both in commercial and investment banking. Therefore the balance sheet in the model will consist of banking as well as trading business. This makes it possible for all kinds of banks to optimize its balance sheet using the model.

A model is proposed, where the balance sheet is considered a portfolio of asset, liability and equity balance sheet items. Whereas the asset instruments produce a positive return, the liability instruments generate a negative return. The constraint that the sum of all asset instruments equals the sum of the liability instruments plus the shareholders’ equity, makes that it is possible to represent a balance sheet. Basel 2.5 and Basel III regulation is taken into account by translating them as constraints in the portfolio optimization model.
1.5 Report outline

This section describes the structure of the report. Chapter 2 discusses some fundamental banking concepts and the history of the Basel regulation. The chapter starts with describing the different kind of banking businesses, followed by an introduction to financial statements. The history of the Basel regulation includes all the different Basel Accords and the motives behind the new regulation as well as the differences with the previous regulation. Chapter 3 gives an introduction to portfolio optimization theories used in this thesis. Then, Chapter 4 is dedicated to defining all instruments included in the model and also defines the final optimization model. Chapter 5 is concerned with describing the data that is used for the returns on the portfolio instruments and also the source of the data is given. As some of the data had to be generated, the generation process is also described in this chapter. Subsequently, Chapter 6 describes the results of three different types of banks. It follows from the results that the model is capable of modeling each type of bank. In the last place, Chapter 7 concludes this thesis and will suggest further research.
CHAPTER 2
Banking concepts

2.1 Banks

The word ‘bank’ originates from the Italian word ‘banco’, which is translated as a ‘bench’ or a ‘counter’. This term stemmed from the Renaissance, where Florentine bankers made their transactions on benches covered by a green tablecloth. The original role of banks was to act as an intermediary in order to connect customers with capital deficits (lenders) to customers with capital surpluses (borrowers) by providing them with loans and deposits respectively. Interest income is generated from loans, while deposits produces interest expenses. As the interest charged on loans is greater than the interest paid on deposits, banks are able to make profits, at least when this difference covers administrative costs and loan losses (i.e., when the borrower fails to oblige to the agreed payments of interest and principal).

2.1.1 Banking business

Nowadays, banking business has become much more complex and encompasses a wider range of activities with varying degrees of complexity. A distinction can be made between two main types of banking activities, namely commercial and investment banking. Most large banks engage in both commercial and investment banking. Commercial banking is concerned with, among other things, the deposit-taking and lending activities. The main activity of an investment bank is assisting companies or governments in raising debt and equity financing with new issues of securities. In addition, investment banks also provide companies with advice on mergers, acquisitions and other corporate finance decisions. Most of the large banks are also involved in securities trading.

Commercial banking can be further classified as ‘retail banking’ or ‘wholesale banking’. Where retail banking involves taking relatively small deposits from private individuals and small businesses and also making relatively small loans to them in return. Wholesale banking is concerned with providing these banking services on a larger scale to medium and large corporate clients, fund managers and other financial institutions. The spread between the cost of funds and the lending rate is usually smaller for wholesale banking. This is due to the fact that the expected loan losses and administrative cost are less than for retail banking.
Most large banks are involved with securities trading, providing brokerage services and making a market in individual securities by providing bid and offer spreads. The counterparties in their trading activities are typically other banks, corporations and fund managers. Banks trade for mainly three reasons:

1. To meet the needs of their counterparties (e.g., selling a foreign currency option to a corporate client in order to reduce the foreign exchange risk for that client).

2. To reduce the banks own risks (i.e., hedging the risk to which the bank is exposed after selling a security to a counterparty).

3. For proprietary trading purposes, which refers to taking a speculative position in the hope of making a profit.

### 2.1.2 Banking and trading books

For capital management purposes, banks make a distinction between their activities. Banking activities are split between either ‘banking book activities’ or ‘trading book activities’. The primary difference between the two books is the intention for the acquisition of the assets on each book. For the banking book the assets are intended to be held for the long term, while the assets on the trading book are held for very short terms. Also the regulatory capital requirements are different for each book (see Basel).

Traditional banking activity, such as loans made to corporations or individuals, are recorded in the banking book. The values of the assets and liabilities on the book are calculated following the accrual concept, which accrues cashflows as they occur. As long as the borrower is up-to-date on principal and interest payments on a loan, the loan is recorded on the bank’s book. Assets and liabilities on the banking book are exposed to credit risk and liquidity risk.

As its name implies, the trading book consists of the market-making activity and the proprietary trading. The value of the assets and liabilities in the trading book are marked to market daily. This means that the values of the book reflects the current changes in the market prices. The risk exposures on the trading book are market risk, credit risk and liquidity risk. Trading book activity mostly involves derivative instruments. These can be further categorized as equity, interest rate, foreign exchange or commodity derivatives.
2.1.3 Financial statements

The most important financial statement of a bank comprises the balance sheet and the profit & loss (P&L) report. Typically, these statements are released at the end of the fiscal year. The objective of the financial statements is to provide information about the banks’ financial position and performance over the last fiscal year.

2.1.3.1 Balance sheet

A bank’s balance sheet is considered to be a snapshot of the financial condition, as it states the financial situation at a single point in time of the fiscal year. It comprises the assets and liabilities as determined by the accounting rules. For large banks, the balance sheet is often complex and can be considerably different among other large banks.

2.1.3.2 Profit and loss report

The income statement for a bank is the P&L report, which includes all revenues and expenses during a specific period. It contains the following items of profits and losses:

Net interest income

Traditionally, this is the primary source of income for banks. Interest income is generated by lending activities and interest expense arises from funding. The key variable in generating interest income is the cost of funding, as the interest paid on deposits is far below the interest on wholesale market interest rate.

Net trading income

This source of income is generated by proprietary trading of bank and the profits from the proprietary trading desk is highly volatile. Proprietary trading could be defined as using the banks’ own capital to take positions in various instruments, often for speculative or arbitrage purposes. The positions on the proprietary trading book are kept secret, as trading desks of other banks or hedge funds can use this information to buy the same trades or intentionally take opposite positions against bad positions. Large investment banks make most of their annual profits through the proprietary trading desk.

Net fee and commission income

Fee and commission revenue is generated by providing financial services to customers. It is an attractive source of income, as it represents a stable revenue for banks. However, the size of fee and commission income strongly differs among
banks and it is does not directly follow from the asset and liability positions, which makes it hard to model.

Provisions
As banks expect that a certain percentage of their assets will suffer loss, therefore they set aside provisions each year to cover write-downs on these assets. A write-down occurs when an asset has an overstated book value (i.e., the book value exceeds the future incoming cash flows). The write-down reduces the book value of an asset to the fair value.

Operating expenses
These include remuneration costs and other operating expenses such as building & infrastructure costs and software costs. This source of expenses can be considered stable over years.

Dividend and tax policy
Finally, tax should be paid over net profit, i.e., the sum of all revenues and expenses mentioned above. The amount of taxation depends on the location where the bank is established. Depending on the dividend policy of the bank, the retained earnings is calculated (i.e., the net profit that will be reinvested on the balance sheet).

2.1.4 Performance measures for banks
Traditional performance measures include the Return on Equity (RoE), Return on Assets (RoA) and net interest margin. These are similar to measures applied in other industries. The most common risk measure for a bank’s performance is the Return on Equity (RoE). It is emphasized in (Bank 2010) that the RoE is a short-term indicator and must be interpreted as a snapshot of the current health of institutions.

RoE is the most popular measure, because among other things, it is easily available for analysts and gives a direct assessment of the financial return of a shareholders’ investment. It is calculated as follows

\[
\text{RoE} = \frac{\text{Retained earnings}}{\text{Total equity}}
\]

As for all profit-seeking organizations, the main goal of a bank is to preserve and create wealth for its owners. This means that the RoE must be at least equal but preferably larger than the cost of equity. Generally, investment banks aspire a higher RoE than commercial banks, which in finance always translates to taking more risk. This is easily
observable in the comparison between the mean RoE for investments banks and commercial banks in Figure 2.1. It is clear that the RoE for investment banks is more volatile due to the uncertainty of their trading income, whereas the commercial banks rely for the most part on interest income and have more stable RoE measures over time.

Figure 2.1: Comparison of mean RoE between investment an commercial banks

Notes: Investment banks include: Credit Suisse, Deutsche Bank, Morgan Stanley, UBS and Goldman Sachs. Commercial banks include: RBS, HSBC, Bank of America, Barclays, Banco Santander and UniCredit.

Source: Bloomberg

2.2 Basel regulation

Banks are of fundamental importance to the economy. A strong and resilient banking system is the foundation for sustainable economic growth, as banks are at the center of credit intermediation process between savers and investors. Moreover, banks provide critical services to consumers, small and medium-sized enterprises, large corporate firms and governments who rely on them to conduct their daily business, both at a domestic and international level (BCBS 2010a). Therefore strict regulation is required to ensure that banks keep enough capital reserves available to absorb shocks arising from financial and economic stress in order to minimize the probability of a bank failing.

The international standards for bank regulation are issued by the Bank of International Settlements (BIS) and are also known as the Basel capital accords, from the city in Switzerland where the BIS is founded. The BIS was established in 1930. At the time it was set up in the context of the Young Plan, which dealt with issues of the reparation payments imposed on Germany by the Treaty of Versailles following the First World
War. This original role is where the bank derived its name from. When the reparations issues faded, the BIS mission changed to:

“Serving central banks in their pursuit of monetary stability and financial stability, to foster international cooperation in those areas and to act as a bank for central banks“.

2.2.1 Background

On June 26, 1974, German regulators forced Herstatt Bank, a privately owned bank in West Germany, into liquidation. However, that same day a number of banks had released payments of Deutsche Marks to Herstatt Bank in exchange for US Dollars that were to be delivered in New York. Due to the time-zone differences, Herstatt Bank ceased operations between the times of the respective payments. Consequently, the counterparties in New York did not receive their US Dollar payments. This event illustrated settlement risk in international finance and led to serious disturbances in the international currency and banking markets. Eventually, this was the reason for the central bank governors of the Group of Ten\(^7\) countries to establish the Basel Committee on Banking Supervision (BCBS) by the at the end of 1974.

The main purpose of the Committee was to reduce or rather prevent the possibility of transfer of cross border contagion arising from the failure of internationally active banks. At the time that the Committee was founded there was little to none coordination between the central banks of the more developed countries. Therefore in the beginning its primary purpose was the exchange of information on the financial condition of internationally active banks.

Over the years the Committee became the core body for influencing banking supervisory standards worldwide. The Committee does not possess any formal supranational supervisory authority. Thus, its conclusions do not have, and were never intended to have, legal force. Rather, it formulates broad supervisory standards and guidelines and recommends statements of best practice in the expectation that individual authorities will take steps to implement these in their own national system.

2.2.2 Basel I

In the early 1980s, the Committee became concerned about the worsening of capital ratios of the most important international banks. Especially, in the combination that at

\(^7\)The Group of Ten (G-10) is made up of eleven industrial countries (Belgium, Canada, France, Germany, Italy, Japan, the Netherlands, Sweden, Switzerland, the United Kingdom and the United States) which consult and co-operate on economic, monetary and financial matters
the time international risks, notably between heavily indebted countries, were growing. In order to stop the deteriorating of the capital standards the Committee worked towards convergence in the measurement of the minimal capital adequacy.

The United States (US) and the United Kingdom (UK) were concerned that higher capital requirements compared to the requirement for Japanese banks would affect the global competitiveness negatively. In fact, it has been alleged that it was primarily the consensus reached on global standards between the US Federal Reserve Bank and the Bank of England that was forced upon the rest of the G-10 members of the BCBS.

In July 1988, the BCBS published the Basel Capital Accord BCBS (1988) after taking comments and suggestions from the financial sector into account on a consultative paper (published in December 1987). The accord is commonly referred to as the Basel I agreement.

### 2.2.2.1 Framework

The framework was provided with two fundamental objectives, stated by the Committee as firstly, that the new framework should serve to strengthen the soundness and stability of the international banking system; and secondly that the framework should be fair and have a high degree of consistency in its application to banks in different countries with a view to diminishing an existing source of competitive inequality among international banks.\(^8\)

The Committee viewed that the most important risk the banks management needed to guard was credit risk. Credit risk is defined as the risk of loss arising from the default by a creditor or a counterparty. Therefore, the central focus of the initial framework was on credit risk. Other kinds of risks, like market risk would be captured in later supplements to the accord.

The agreement required internationally active banks to hold capital equal to at least 8% of a basket of assets measured in different ways according to their riskiness. A portfolio approach was taken to measure the riskiness. The banks assets were divided into four risk buckets according to the debtor category. In Table 2.1 the risk weights per asset categories are displayed.

The Risk-Weighted Assets (RWA) are these risk weights multiplied by their respective

---

\(^8\)BIS (1988; Page 1, Paragraph 3)
Table 2.1: Risk weight per asset category in Basel I

<table>
<thead>
<tr>
<th>Risk bucket</th>
<th>Loans to</th>
<th>Risk weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Sovereign</td>
<td>OECD-governments*</td>
<td>0%</td>
</tr>
<tr>
<td>2. Bank</td>
<td>Banks and securities firms in OECD countries</td>
<td>20%</td>
</tr>
<tr>
<td>3. Mortgage</td>
<td>Residential mortgage</td>
<td>50%</td>
</tr>
<tr>
<td>4. Corporate</td>
<td>Corporate and consumers</td>
<td>100%</td>
</tr>
</tbody>
</table>

* Governments from countries that are full members of the Organization for Economic Co-ordination and Development, established on 14 December 1960

exposures, as is shown in Equation 2.1 below.

\[
\text{Risk Weighted Asset} = \text{Asset exposure} \cdot \text{Risk weight}
\]  

(2.1)

Banks have to hold at least 8% capital against the RWA, also denoted as the core capital asset ratio. The required capital was divided into two Tiers consisting of different elements, displayed in Figure 2.2.

Table 2.2: Capital Tiers in Basel I

<table>
<thead>
<tr>
<th>Capital</th>
<th>Contains</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tier 1</td>
<td>Shareholders’ equity</td>
</tr>
<tr>
<td></td>
<td>Retained earnings</td>
</tr>
<tr>
<td>Tier 2</td>
<td>Additional internal and external resources available</td>
</tr>
</tbody>
</table>

Additionally, at least half of its measured capital should be in the form of Tier 1 capital.

2.2.2.2 Amendment for market risk

The Committee issued a proposal in April 1995 to apply capital charges for market risk in order to provoke feedback in the form of comments and recommendations by the financial sector. After these were taken into account, the BCBS released an amendment to the Basel I Accord to incorporate market risks (BCBS 1996). The objective from this amendment, as stated by the Committee, is to provide an explicit capital cushion for the price risks to which banks are exposed, particularly those arising from their trading activities\(^9\).

\(^9\)BIS (1996b; Page 1; Paragraph 2)
Market risk is defined as the risk of losses in on and off-balance-sheet positions arising from movements in market prices\textsuperscript{10}. The capital charge for market risk comes on top of the capital requirement for credit risk. The principal form of eligible capital to cover market risk consists of Tier 1 and Tier 2 capital as defined in the principal 1988 Accord. However, banks may also, at the discretion of their national authority, employ a third Tier of capital (Tier 3), consisting of short-term subordinated debt for the sole purpose of meeting a proportion of the capital requirement for market risk.

In measuring market risk, a choice between two broad methodologies will be permitted, namely the standardized approach and the internal models approach. The organigram in Figure 2.2 gives an overview of the possible approaches to calculate the minimal capital requirement under Basel I.

\textbf{Figure 2.2: Organigram for the calculation of the minimal capital requirement under Basel I}

Banks are expected to manage the market risk in their trading book in such a way that the capital requirements are being met on a continuous basis, i.e., at the close of each business day. The banks’ overall minimum capital requirement will be the sum of:

- The credit risk requirement, excluding debt and equity securities in the trading book and all positions in commodities, but including the credit counterparty risk on all over-the-counter (OTC) derivatives.

- The market risk requirement.

In order to ensure the consistency in the calculation of the capital requirement for credit risk and market risk, the measure of market risk is multiplied by 12.5 (i.e., the reciprocal

\textsuperscript{10}BIS (1996a; Page 1, Paragraph 1)
of the minimum capital ratio of 8%) and adding this resulting number to the sum of the RWA for credit risk. The ratio will then be calculated as the sum of the two, using as the numerator only eligible capital:

\[
\text{Capital Ratio} = \frac{\text{Total Capital}}{\text{Credit Risk} + \text{Market Risk}} \geq 8\% \tag{2.2}
\]

**Standardized approach**

To measure the risks in a standardized manner. The capital charge under this methodology will be the sum of the measures for the following risks: interest rate risk, equity position risk, foreign exchange risk, commodities risk and the price risk in options of all kinds. These risks and the corresponding capital charges will be discussed below.

**Interest rate risk**

Subjected to positions in debt securities and other interest rate related instruments in the trading book. The minimum capital requirement is expressed in terms of two separately calculated charges. The first term is the “specific risk” of each instrument, which is designed to protect against adverse movement in the price of an individual security owing to factors to the individual issuer. And the other is created to capture the risk of loss arising in the market interest rates (termed “general market risk“) on the whole portfolio (i.e., long and short positions in different securities or instruments can be offset).

- Specific risk charge: depends on the external credit assessment of the individual issuer (e.g., government 0%, public sector entities 0.25% to 1.60% and a 8% risk charge for private-sector borrowers).

- General market risk charge: a choice between two principal methods (based on maturity/duration) is permitted. In each method, the capital charge is sum of the following:
  - The net short or long position in the whole trading book
  - A small proportion of matched positions in each time-band (“vertical disallowance“)
  - A larger proportion of matched positions in different time-bands (“horizontal disallowance“)

**Equity position risk**

Applies to long and short positions in all instruments that exhibit market behavior
similar to equities in the trading book. The minimum capital requirement will again be calculated in two terms, namely:

- **Specific risk charge**: 8% for the banks' gross equity positions (i.e., absolute sum of all long and short equity positions), unless the portfolio is both liquid and well-diversified, in which case the charge is 4%.

- **General market risk**: 8% for the bank’s net equity position in the equity market (i.e., the difference between the sum of the longs and the sum of the shorts). Additional a capital charge of 2% will apply to the net long or short position in an index contract comprising a well-diversified portfolio of equity. This capital charge is intended to cover factors such as execution risk\(^{11}\).

**Foreign exchange risk**

Covers the risk of holding or taking positions in foreign currencies. Firstly, the bank’s net open position in each currency position and gold should be calculated. Secondly, the sum of the net short positions or the sum of the net long positions, whichever is the greater, plus the net position in gold amounts for the bank’s overall net open position. Finally, the capital charge will be 8% of the overall net position.

**Commodities risk**

Commodities are often more complex, volatile and less liquid than that associated with currencies or interest rates. These market characteristics can make price transparency and the effective hedging of commodities risk more difficult. A commodity is defined as a physical product which is or can be traded on a secondary market, e.g., agricultural products, minerals (including oil) and precious metals (excluding gold)\(^{12}\). There are two alternatives for measuring commodities position risk:

1. **Simplified approach**: the capital charge will be equal 15% of the net position, long or short, in each commodity. An additional capital charge equivalent to 3% of the bank’s gross position (i.e., long or short) for each commodity will be subjected in order to protect the bank against basic risk, interest rate risk and forward gap risk.

2. **Maturity ladder approach**: a bank will calculate its either only long or only short commodities positions based on maturity, to which a capital charge of 15% will apply.

---

\(^{11}\)Execution risk is the risk that a transaction will not be executed within the range of recent market prices or within the stop order limits that have been set.

\(^{12}\)BIS (1996; Page 27, paragraph 1)
Treatment of options

Depending on the diversity of a bank’s activity in options, they are obliged to use one of three approaches. Banks which solely use purchased options will be permissible to use the simplified approach, described in Table 2.3.

<table>
<thead>
<tr>
<th>Position</th>
<th>Capital charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Long cash and long put or short</td>
<td>The market value of the underlying security multiplied by the sum of specific</td>
</tr>
<tr>
<td>cash and long call</td>
<td>and general market risk charges for the underlying less the amount the option</td>
</tr>
<tr>
<td></td>
<td>is in the money (if any) bounded at zero</td>
</tr>
<tr>
<td>Long call or long put</td>
<td>The lesser of, the market value of the underlying security multiplied by the</td>
</tr>
<tr>
<td></td>
<td>sum of specific and general market risk charges, or the market value of the</td>
</tr>
<tr>
<td></td>
<td>option</td>
</tr>
</tbody>
</table>

Bank that also write options will be expected to use one of the intermediate approaches, like the Delta-plus method. In this method the delta-weighted option positions are calculated by multiplying the market value of the underlying by the delta\(^{13}\). The capital charge on the delta-weighted positions is calculated using the same method as used for their underlying instrument. However, since delta does not fully cover the risks associated with option positions, in addition banks will be required to measure gamma\(^{14}\) and vega\(^{15}\) as well. Banks should calculate the gamma en vega for each option position separately. The total gamma capital charge will be the sum of the absolute value of the net negative gamma impacts (calculated for each underlying separately). The total capital charge for vega risk will be the absolute sum of the vegas for all options on the same underlying.

Internal models approach

Alternatively, banks can decide to measure the risks using internal risk management models. Provided that these models are compliant to the following criteria:

1. Explicit approval of the bank’s supervisory authority
2. Guidelines for stress testing
3. Validation procedures for external oversight of the use of models

\(^{13}\)Delta measures the rate of change of option value with respect to changes in the price of the underlying

\(^{14}\)Gamma measures the rate of change of delta

\(^{15}\)Vega measures the sensitivity of the value of an option with respect to a change in volatility
4. Rules for banks which use a mixture of models and the standardized approach

5. Guidelines for specifying an appropriate set of market risk factors

6. Qualitative standards concerning the adequacy of the risk management system
   - Independent risk control unit responsible for the design/implementation of the banks risk management system.
   - The unit should conduct a regular back-testing program
   - The unit should conduct the initial and on-going validation of the internal models
   - A routine and rigorous program of stress testing
   - Boards of directors and senior management should actively involved in risk management
   - An independent review of the risk measurement system should be carried out regularly in the banks own internal auditing process

7. Quantitative standards (setting out the use of common minimum statistical parameters for measuring risk)
   - Value-at-Risk must be computed on a daily basis
   - In calculating the VaR, a 99th percentile, one-tailed confidence interval is to be used
   - In calculating VaR, an instantaneous price shock equivalent to a 10 day movement in prices is to be used, i.e., the minimum holding period will be 10 trading days
   - The choice of the historical observation period (sampling period) for calculating VaR will be constrained to a minimum length of one year
   - Banks should update their data sets quarterly
   - No particular type of model prescribed, banks are free to use models based, for example, on variance-covariance matrices, historical simulations, or Monte Carlo simulations
   - Banks must meet, on a daily basis, a capital requirement expressed as the higher of:
     - Its previous day VaR
An average of the daily VaR measures on each of the preceding sixty business days, multiplied by a multiplication factor. The multiplication factor will be set by the supervisory authority based on the quality of the banks risk management, subject to an absolute minimum of 3.

- Banks using internal models will also be subjected to a capital charge for specific risk (interest rate related instruments and equity securities) as described under the standardized approach.

2.2.3 Basel II

In June 1999, the Basel Committee published a consultative paper (BCBS 1999) proposing to introduce a new capital adequacy framework to replace Basel I. In this paper the Committee points out that the two fundamental merits of Basel I were accomplished. Namely, the committee claims that it helped to strengthen the soundness and stability of the international banking system and it enhanced competitive equality among internationally active banks\(^\text{16}\). Despite these accomplishments, the committee also acknowledged that it also has its weaknesses\(^\text{17}\):

- Due to the developments in the evolving financial world during the ten years after the introduction of Basel I the calculation of the banks capital ratio, may not always be a good indicator of its financial condition. The credit risk exposure is not adequately differentiated between different borrowers (i.e., the capital requirement does not depend on whether the credit rating of the borrower is a AAA or CCC).

- The ability of banks to arbitrage their regulatory capital requirement, for example, through forms of securitization, which can lead to a shift in a banks portfolio to lower quality asset classes.

- Finally, for some types of transactions, it does not provide the proper incentives for credit risk mitigation techniques.

In acknowledgement of these weaknesses the Committee proposed a revised capital adequacy framework in 1999, called Basel II. Initially this second Basel capital Accord was planned to start in 2003, but because of critique from the financial community it has been postponed. After the consultative paper of 1999 there have been three more consultative documents and the Committee conducted also three Quantitative Impact Studies (QIS). The consultative documents served for the purpose of engaging in a dialogue with

\(^{16}\)BIS (1999; Page 8, paragraph 5)

\(^{17}\)BIS (1999; Page 8, paragraph 6, 7 and 8)
the financial sector. In June 2004, the committee published the final version of Basel II (BCBS 2004), which was scheduled to be implemented by the end of 2006.

2.2.3.1 Basel II framework

The Basel II framework is built upon three pillars. The first pillar covers the minimum capital requirements, the second pillar concerns a supervisory review process and the third pillar concerns market discipline. These three pillars should be mutually enforcing and lead to soundness and stability in the financial sector. The three pillars are illustrated in Figure 2.3 and will be briefly discussed below.

![Figure 2.3: Overview Basel II framework](image)

**Pillar 1: minimal capital requirement**

In addition to the risk categories already treated in Basel I a new type of risk will be included, called operational risk. The committee defines operational risk as the risk of direct or indirect loss resulting from inadequate or failed internal processes, people and systems, or from external events. The calculation of required capital is not much different than under Basel I:

$$\text{Capital Ratio} = \frac{\text{Total Capital}}{\text{Credit Risk} + \text{Operational Risk} + \text{Market Risk}} \geq 8\% \quad (2.3)$$

Further credit risk will be split into basic credit risk, credit risk mitigation and securitization risk. This leads to the organigram displayed in Figure 2.4 of the possible approaches to calculate the minimal capital requirement under Basel II.

**Basic credit risk** was envisaged by the committee to be improved by introducing a standardized approach and an internal ratings based (IRB) approach.

---

18BIS (2004, Page 144, Paragraph 644)
Figure 2.4: Organigram for the calculation of the minimal capital requirement under Basel II

- Standardized approach: in this method a risk weight is assigned to each debtor category and for different credit ratings within a debtor category. For example, an exposure to an AAA rated sovereign would receive a lower risk weight than an exposure to a BB+ rated sovereign. The credit ratings are assigned by external credit rating agencies. The capital charge is then calculated in the same manner as under Basel I, namely by multiplying the RWA’s by 8%.

- IRB approach: under this approach instead of relying on the external credit ratings, the banks base rate their exposures for risk weight purposes with an internal ratings system. This internal model takes four risk components into account, displayed in Table 2.4.

Table 2.4: Risk components within the IRB approach under Basel II

<table>
<thead>
<tr>
<th>Component</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>PD</td>
<td>Probability of Default</td>
</tr>
<tr>
<td>LGD</td>
<td>Loss Given Default</td>
</tr>
<tr>
<td>EAD</td>
<td>Exposure At Default</td>
</tr>
<tr>
<td>M</td>
<td>Maturity</td>
</tr>
</tbody>
</table>

The risk components are inputs in a formula to calculate the capital require-
ment (K). The formula is different for other types of assets. What the formula aims to attain is a confidence level of 99.9% or in other words, a bank in only expected to use up its capital with a probability of 0.001% in one year.

Securitization risk: was included in the Basel II framework to refrain that the framework would remain vulnerable to capital arbitrage, as some securitization exposures have enabled banks under Basel I to avoid maintaining capital commensurate with the risks to which they are exposed. To address this concern, Basel II requires banks to look at the economic substance of a securitization transaction when determining the appropriate capital requirement\textsuperscript{19}.

Operational risk should be captured by one of three methods depending on how sophisticated a bank is.

1. Basis indicator approach: The least advanced approach and is designed for less sophisticated banks. The required capital is simply 15% of the bank’s three-year average gross income.

2. Standardized approach: Meant for the somewhat more sophisticated banks. This approach makes a difference between business lines. Each business line receives some sort of risk weight (beta). There are 8 business lines in total, which all receive a different risk weights. For example, corporate finance has a beta of 18% and the beta for retail banking is only 12%. The capital requirement for each business line can be calculated by multiplying the beta by its three-year average gross income.

3. Advanced measurement approach (AMA): Intended for banks that use the IRB approach for credit risk. The AMA framework must be approved by the national supervisor and include the use of the following data elements:

   - Internal loss data
   - External loss data
   - Scenario analysis
   - Business environment and internal control factors

Market risk is captured in the same manner as under Basel I. The amendment to incorporate market risk for Basel I (BCBS 1996) was adopted fully and remained unchanged.

\textsuperscript{19}BIS (2003; Page 7; Paragraph 35)
**Pillar 2: supervisory review**

The second pillar is intended to support the first pillar for certain types of risks that remain uncovered, like:

- The dimension of the risks is not considered (e.g., large exposures and credit concentrations)
- Risks not taken into account (e.g., interest rate risk in the banking book, business risk and strategic risk)
- External factors to the bank (e.g., business cycles)

It is also meant to ensure that banks are in compliance with both the quantitative and qualitative requirements of the first pillar. And finally to encourage banks to develop and use better risk management techniques in monitoring and managing their risks.

Four principles lie at the heart of Pillar 2:

1. Bank’s own assessment of capital adequacy: an internal capital assessment process for assessing their overall capital targets that are commensurate with the banks risk profile and a strategy for maintaining their capital levels.
2. Supervisory review process: to review and evaluate bank’s internal capital adequacy assessment and strategies.
3. Capital above regulatory minima: supervisors expect banks to operate above the minimum regulatory capital ratios.
4. Supervisory intervention: to prevent capital from falling below the minimum levels.

**Pillar 3: market discipline**

The Committee aims to encourage market discipline by developing a set of disclosure requirements which will allow market participants to assess key pieces of information on the scope of application, capital, risk exposures, risk assessment processes, and hence the capital adequacy of the institution\(^{20}\). The committee believes that market discipline is significantly enhanced due to the disclosures of the banks. As market participants have a better idea about risk-taking of a certain institution, which enables them to demand an appropriate risk premium.

\(^{20}\)BIS (2004; Page 226; Paragraph 809)
2.2.4 Basel 2.5

At the time of the introduction of the amendment for market risk in 1996, the trading books of most banks consisted of relatively simple risk positions, which are trading in liquid markets. Over time, the structure of the trading books and the nature of the financial sector changed dramatically. Especially with the large growth in traded credit exposures. This led to concerns within the Committee that the capital requirements were becoming inadequate. Hence, in 2005 the committee agreed on reforms to the capital requirement in the trading book. Including a requirement for banks to hold capital against default risk that was incremental to the capital in the VaR models.

However, the financial crisis that began in mid-2007 revealed that these reforms would not suffice. As a short-term solution the BCBS introduced a set of revisions to the market risk framework in July 2009 (BCBS 2009b), referred to as Basel 2.5 rules. The banks were proposed to comply with the revised requirements by 31 December 2010.

2.2.4.1 Revisions to the market risk framework

In May 2012, the BCBS published a fundamental review of the trading book (BCBS 2012). This identified the flaws in both the internal models-based and the standardized approach of the amendment for market risk in 1996. This section will first describe the shortcomings of the two approaches, after which the revisions to both approaches are set forth.

Shortcomings to the standardized approach

- Lack of risk sensitivity: the current models fail to identify the riskiness of different portfolios. This leads to the same capital charges for products that share very different risk characteristics.
- Limited recognition of hedging and diversification
- Inadequate for complex or innovative products

Shortcomings to the internal models approach

The metric used to calculate the capital requirement was the 10-day VaR computed at the 99th percentile, one-tailed interval. This metric did not adequately or failed to meet the following prudential objectives:

- Inability to capture credit risk: this concern had already been identified with the introduction of the market risk amendment in 1996. However, the rapid
growth in the market for traded credit exposures since the early 2000’s made this an important point to take into account in the new revisions.

- Inability to capture market liquidity risk: the crisis revealed that banks were often unable to exit or hedge certain trading positions due to market illiquidity. Moreover, the sharp increase in liquidity premia led to banks incurring substantial mark-to-market losses.

- Incentives to take on tail risk: when not looking beyond the 99th percentile, the VaR fails to capture “tail risks”. Increasing exposures to super-senior securitization tranches are an example of the build-up of exposures to tail risk events not adequately captured by the regulatory framework.

- Inadequate to capture basis risk: the entire framework was based on estimates of correlations derived from historical data based on normal market conditions. Hedging benefits hold under normal market conditions, but in times of crisis these correlations do not hold.

- A bank-specific notion of risk: an individual bank can judge that it is able to hedge or exit a specific exposure at any time without affecting market prices. But when most other banks also have a similar exposures in a market (e.g., the super-senior tranches of securitization exposures), in times of stress that market quickly becomes illiquid.

The reliance on the VaR metric to calculate the capital requirement also increased the pro-cyclicality of the market. As it allowed banks to take on more risks in good times and amplified the shocks in bad market times. Due to the fact that in good times, as asset prices rise and volatility drops, the capital charges based on the VaR metric is low. But as the crisis intensified, asset prices dropped and volatility rose, VaR-based capital charges also increased dramatically. This led to market illiquidity as all banks wanted to exit their positions at the same time.

Revisions to the standardized approach

**Interest rate risk:** the Committee agreed that modelling methodologies used by banks do not adequately capture the risks of securitization products. Therefore the capital charges will be based on the banking book risk weights for these exposures. With a limitation exception for certain correlation trading activities. In this case the specific risk charge for the correlation trading portfolio will be the larger of the computed total specific risk capital charge for the net long or net short positions in securitization instruments in the trading book.
**Equity position risk:** the capital charge for both specific risk and for the general market risk will be each 8% (instead of 4% for well-diversified and liquid portfolios under Basel II).

**Revisions to the internal models approach**

**Quantitative standards:** in addition to the regular VaR, banks must calculate a “stressed Value-at-Risk“ (sVaR) measure. This measure is intended to to replicate a VaR calculation if the relevant market factors were experiencing a period of stress; and should therefore be based on the 10-day, 99th percentile, one-tailed confidence interval VaR measure of the current portfolio, with model inputs calibrated to historical data from a continuous 12-month period of significant financial stress. For example, the 12-month period relating to the significant losses in 2007/2008 would adequately reflect such a period. The sVaR should be calculated at least weekly. Each bank must meet, on a daily basis, a capital requirement expressed as the sum of:

- The higher of (multiplied by a multiplication factor (\(m_c\))):
  - The previous day Value-at-Risk number (\(VaR_t-1\))
  - An average of the daily Value-at-Risk measures on each of the preceding sixty business day (\(VaR_{avg}\))

- The higher of (multiplied by a multiplication factor (\(m_s\))):
  - The latest available stressed Value-at-Risk number (\(sVaR_t-1\))
  - An average of available Value-at-Risk measures of the preceding sixty business day (\(sVaR_{avg}\))

Thus, the capital requirement (C) is calculated as follows:

\[
C = m_c \cdot \max(VaR_{t-1}, VaR_{avg}) + m_s \cdot \max(sVaR_{t-1}, sVaR_{avg})
\]

(2.4)

The multiplication factors will be set by the supervisory authority based on the quality of the banks risk management, subject to an absolute minimum of 3.

Another change will be that the banks must update their data sets monthly (instead of quarterly).

**Introduction of the Incremental Risk Charge (IRC):** The banks must have an approach in place to capture in its regulatory capital default risk and migration risk in positions subject to a capital charge for specific interest rate risk, with the
exception of securitization exposures and n-th-to-default credit derivatives that are incremental to the risks captured by the VaR-based calculation. No specific approach for capturing the incremental risks is prescribed. The committee provided guidelines BCBS (2009a) to specify the positions and risks to be covered by this incremental capital charge. Key supervisory parameters for computing IRC:

- Soundness standard comparable to IRB: 99.9% soundness standard over a one-year capital horizon. A banks’ IRC model must measure losses due to default and migration at 99.9% confidence interval over a capital horizon of one year, taking into account the liquidity horizon applicable to individual trading positions.
- Constant level of risk over one-year capital horizon.
- Liquidity horizon: represents the time required to sell the position or to hedge all material risks covered by the IRC model in a stressed market.
- Correlation: the IRC charge includes the impact of correlations between default and migration events among obligors.
- Concentration: IRC model must appropriately reflect issuer and market concentrations.
- Risk mitigation and diversification effects: within the IRC model, exposure amount may only be netted when long and short positions refer to the same financial instrument. Intra-obligor hedges and inter-obligor hedges may not be recognized through netting of exposure amounts.
- Optionality: banks models should include the nonlinear impact of options and other positions with material nonlinear behavior with respect to price changes.

2.2.5 Basel III

Basically, the main reasons that the financial crisis, which began in 2007, became so severe were:

- Excessive built up of on and off-balance sheet leverage.
- Gradual erosion of the level and the quality of the capital base.
- Banks were holding insufficient liquidity buffers.
Due to these reasons the banking system was unable to absorb the resulting systemic trading and credit losses. Moreover, the interconnectedness of the systemic institutions and the pro-cyclical deleveraging process amplified the crisis even further. This was the reason that financial institutions lost confidence in the solvency and liquidity of each other. Eventually the financial crisis spread to the real economy, resulting in massive contraction of liquidity and credit availability. Consequently, governments had to step in with unprecedented injections of capital and liquidity.

In the aftermath of the financial crisis, the Committee launched a program to revise the Basel II regulatory framework. At their summit in Seoul in November 2010, the G-20 countries endorsed the need for this new regulatory framework and agreed on the revised framework for more resilient banks and banking systems (BCBS 2010a) and on an international framework for liquidity risk (BCBS 2010b). Both frameworks will be gradually implemented from 2013 to 2019.

The revised framework for more resilient banks and banking systems will expand on the three pillars of the Basel II regulatory framework. Strengthening of the new global framework is based upon five objectives:

1. Increase the quantity and quality of the capital base
2. Stronger risk coverage, notably counter-party risk
3. Supplementing the risk-based capital requirement with a leverage ratio
4. Reducing pro-cyclicality and promoting counter-cyclical buffers
5. Addressing systemic risk and interconnectedness

2.2.5.1 The capital base

The committee views that it is critical that banks’ risk exposures are backed up by a high quality capital base\(^{21}\). The crisis demonstrated that is of utmost importance for banks to have a high quality capital base in order to absorb credit losses. Inconsistency in the definition of capital across jurisdictions made it difficult for the market to compare the quality of capital between financial institutions. Therefore under Basel III the definition of the capital base will be tightened.

\(^{21}\)BIS (2010a; Page 2; Paragraph 8)
Increase the quantity and quality of the capital base

A key element of this new definition will be a greater focus on common equity, which is the highest quality component of capital of a bank. In addition, Tier 2 capital instruments will be harmonised and Tier 3 capital for market risk under Basel II will be eliminated. The capital base under Basel III will consist of the following elements:

- **Tier 1 capital**
  - Common Equity Tier 1 (CET1)
  - Additional Tier 1 capital

- **Tier 2 capital**

Also the quantity of the capital base will be increased in order to further strengthen the capital base. New standards for increased capital base will be transitionally implemented to ensure that the banking sector can meet the higher capital requirements, while still supporting lending to the economy. Transitional implementations of the capital base are shown in Table 2.5. The total capital requirement remains unchanged at the existing level of 8.0% and so does not need to be phased in. The difference between the total capital requirement of 8.0% and the total Tier 1 requirement can be met with Tier 2 capital.

### Table 2.5: Transitional implementation of capital base for Basel III

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Year</th>
<th>Requirement</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Common Equity Tier 1</strong></td>
<td>2012</td>
<td>2.0%</td>
</tr>
<tr>
<td>Risk Weighted Assets</td>
<td>2013</td>
<td>3.5%</td>
</tr>
<tr>
<td></td>
<td>2014</td>
<td>4.0%</td>
</tr>
<tr>
<td></td>
<td>2015-2019</td>
<td>4.5%</td>
</tr>
<tr>
<td><strong>Tier 1</strong></td>
<td>2012</td>
<td>4.0%</td>
</tr>
<tr>
<td>Risk Weighted Assets</td>
<td>2013</td>
<td>4.5%</td>
</tr>
<tr>
<td></td>
<td>2014</td>
<td>5.5%</td>
</tr>
<tr>
<td></td>
<td>2015-2019</td>
<td>6.0%</td>
</tr>
</tbody>
</table>

**Capital conservation buffer**

Under the new framework, banks will be required to build up capital buffers outside periods of stress, which can be drawn down as losses are incurred. Banks should hold a capital conservation buffer of 2.5% above the regulatory minimum capital requirement. The conservation buffer will be comprised of Common Equity Tier 1 only. In Table 2.6 the minimum capital conservation ratios for various levels of the Common Equity Tier
1 capital ratios are displayed. For example, a bank with a CET1 ratio in the range of 5.750% - 6.375% is required to conserve 60% of its earnings in the subsequent financial year. This buffer will be phased in between 2016 en 2019. It will begin at 0.625% of the RWA’s on 1 January 2016 and will increase with 0.625% each subsequent year to reach the final level of 2.5% of RWA’s on 1 January 2019.

<table>
<thead>
<tr>
<th>Common Equity Tier 1 ratio</th>
<th>Minimal capital conservation ratios</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.500% - 5.125%</td>
<td>100%</td>
</tr>
<tr>
<td>5.125% - 5.750%</td>
<td>80%</td>
</tr>
<tr>
<td>5.750% - 6.375%</td>
<td>60%</td>
</tr>
<tr>
<td>6.375% - 7.000%</td>
<td>40%</td>
</tr>
<tr>
<td>&gt; 7.000%</td>
<td>0%</td>
</tr>
</tbody>
</table>

**Countercyclical buffer**

Another addition to the new framework will be a countercyclical buffer. This buffer is aimed to ensure that the capital requirements take account of the macro-financial environment in which the bank operates. When the built-up of system-wide risk is observed by the national jurisdiction, a buffer of capital should protect against potential future losses. The buffer for banks active in multiple jurisdictions will be a weighted average of these different buffers. This countercyclical buffer will will vary between 0.0% and 2.5% of the risk weighted assets. Banks have 12 months to comply with the buffer after the nation jurisdiction made the decision. The countercyclical buffer will be introduced parallel with the conservation buffer between 2016 and 2019. This mean that the maximum countercyclical buffer will be 0.625% on 1 January 2016 and will also increase with 0.625% each subsequent year.

**Leverage ratio**

Excessive built-up of on and off-balance sheet leverage was another underlying of the crisis. During the most severe part of the crisis, the banking sector was forced by the market to deleverage, which led to a decline in bank capital and less credit availability. The leverage ratio is set at 3%, i.e., a bank’s total assets (both on and of-balance sheet assets) should not be 33.3 times the bank’s CET1. National supervisory will start monitoring the leverage ratio on 1 January 2013. From 1 January 2015, the banks will be required to disclose their leverage ratio. In the first half of 2017 a final adjustment is carried out to the definition and calibration of the leverage ratio in order to migrate it into the first pillar on 1 January 2018.
2.2.5.2 Stronger risk coverage

Counterparty risk for derivative related instruments were another key factor that amplified the crisis and was not included in the Basel II framework. In the new framework, banks must add a capital charge to cover mark-market counterparty risk losses (Credit Value Adjustments (CVA)) for Over-The-Counter (OTC) derivatives.

2.2.5.3 Liquidity

The objective of the new framework for liquidity risk is to improve the banking sector’s ability to absorb shock arising from financial an economic stress, whatever the source, thus reducing the risk of the spillover effect from the financial sector to the real economy\(^\text{22}\).

To strengthen the liquidity framework, the Committee has developed two minimum standards for funding liquidity. Both designed to achieve two separate but complementary objectives. The two minimum standards are\(^\text{23}\):

1. **Liquidity Coverage Ratio (LCR)**: to promote short-term resilience of bank’s liquidity risk profile by ensuring that is has sufficient high quality assets to survive significant stress scenarios for one month.

2. **Net Stable Funding Ratio (NSFR)**: to promote resilience over a longer time horizon by creating additional incentives for banks to fund their activities with more stable sources of funding on an ongoing basis.

**Liquidity Coverage Ratio**

The BCBS defines this standard as to ensure that a bank maintains an adequate level of unencumbered, high-quality liquid assets that can be converted into cash to meet its liquidity needs for a 30 calendar day time horizon under a significantly severe liquidity stress scenario specified by supervisors. At a minimum, the stock of liquid assets should enable the bank to survive until Day 30 of the stress scenario, by which time it is assumed that appropriate corrective actions can be taken by management and/or supervisors, and/or the bank can be resolved in an orderly way\(^\text{24}\). This can be defined by the following

---

\(^{22}\)BIS (2010b; Page 1; Paragraph 1)

\(^{23}\)BIS (2010b; Page 1; Paragraph 4)

\(^{24}\)BIS (2010b; Page 3; Paragraph 15)
Equation 2.5:

\[
\frac{\text{Stock of high quality liquid assets}}{\text{Total net cash outflows over the next 30 calendar days}} \geq 100\% \tag{2.5}
\]

The numerator in Equation 2.5 for LCR “Stock of high quality liquid assets” needs to suffice for the following requirements:

- **Fundamental characteristics**
  - Low credit and market risk
  - Ease and certainty of valuation
  - Low correlation with risky assets
  - Listed on a developed and recognized exchange market

- **Market-related characteristics**
  - Active and sizable market
  - Presence of committed market makers
  - Low market concentration
  - Flight to quality

In Equation 2.5, the denominator is defined as the total expected cash outflows minus the expected cash inflows in the specified stress scenario for the subsequent 30 calendar days\textsuperscript{25}.

**Net Stable Funding Ratio**

The BCBS defines the NSFR as to establish a minimum acceptable amount of stable funding based on the liquidity characteristics of an institution’s assets and activities over a one year horizon\textsuperscript{26}. This can be captured by Equation 2.6.

\[
\frac{\text{Available amount of stable funding}}{\text{Required amount of stable funding}} \geq 100\% \tag{2.6}
\]

Available stable funding (AFS) is defined as the portion of those types and amounts of equity and liability financing expected to be reliable sources of funds over a one-year time

\textsuperscript{25}BIS (2010b; Page 12; Paragraph 50)
\textsuperscript{26}BIS (2010b; Page 25; Paragraph 119)
horizon under conditions of extended stress. The required amount of stable funding is a function of the liquidity characteristics of various types of assets held, off-balance sheet contingent exposures incurred and/or the activities pursued by the institution\textsuperscript{27}.

\textsuperscript{27}BIS (2010b; Page 26; Paragraph 122)
CHAPTER 3
Portfolio optimization

Portfolio optimization is concerned with allocating capital among a portfolio of assets in order to maximize the return and minimize the risk. In theory, the portfolio is exposed to reduced risk from the diversification effect of holding multiple assets. The goal for the investor is to construct a portfolio that best reflects his preferences, such as a desired return or bearing a maximum level of risk.

The process of portfolio optimization can be divided into two stages. First, a market analysis is performed in order to obtain a view about the future market movements. Secondly, this analysis is used to create a portfolio. This chapter is concerned with the second stage of the process.

In portfolio theory, the first mathematical model for portfolio management was proposed by Markowitz (see Markowitz 1952). This model laid the foundation for the Capital Asset Pricing Model by Sharpe (see Sharpe 1964) and is still considered to be a benchmark for new portfolio theories. H. Markowitz and W. Sharpe both received a Nobel prize of economics in 1990 for their pioneering work in the theory of financial economics.

The remainder of this chapter is organized as follows. First some necessary terminology is introduced that is required for the understanding of the basic portfolio optimization problems by Markowitz derived subsequently. Then, a portfolio optimization problem is derived using the risk measure prescribed by Basel regulation.

3.1 Terminology

Consider a portfolio consisting of \( n \) assets. The weights to each asset is given by a decision vector \( \mathbf{x} = (x_1, \ldots, x_n)^T \), with \( x_i \) being the position in asset \( i \) and

\[
\sum_{i=1}^{n} x_i = 1
\]

The returns on the assets are denoted by a random vector \( \mathbf{y} = (y_1, \ldots, y_n)^T \), such that \( y_i \) represents the return on asset \( i \). Consequently, the expected rates of returns are defined.
as $E[y_1], \ldots, E[y_n]$ and the expected portfolio return equals

$$\sum_{i=1}^{n} x_i E[y_i]$$

In probability theory and statistics, the variance of the return on asset $i$ (i.e., the variance of a random variable) is given by

$$\sigma_i^2 = E[(y_i - \bar{y}_i)^2] = E[(y_i^2 - 2y_i\bar{y}_i + \bar{y}_i^2)] = E[y_i^2] - 2\bar{y}_i E[y_i] + \bar{y}_i^2 = E[y_i^2] - \bar{y}_i^2$$

and the covariance between asset $i$ and $j$ is defined as

$$\sigma_{ij} = E[(y_i - \bar{y}_i)(y_j - \bar{y}_j)]$$

Subsequently, the variance of a portfolio consisting of $n$ assets is given by

$$\sigma_p^2 = E[(y_p - \bar{y}_p)^2] = E\left[\left(\sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \bar{y}_i\right)^2\right] = E\left[\left(\sum_{i=1}^{n} x_i (y_i - \bar{y}_i)\right)^2\right] = E\left[\left(\sum_{i=1}^{n} x_i (y_i - \bar{y}_i)\right)\left(\sum_{i=1}^{n} x_i (y_i - \bar{y}_i)\right)\right] = E\left[\sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j (y_i - \bar{y}_i) (y_j - \bar{y}_j)\right] = \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \sigma_{ij}$$

where $i \neq j$. Or equivalently, the portfolio variance can also be written in matrix notation as

$$\sigma_p^2 = \vec{x}^T \Sigma \vec{x}$$

with $\Sigma \in \mathbb{R}^{n \times n}$ representing the covariance matrix.
3.2 Mean-variance portfolio optimization

In the Markowitz model, mean-variance portfolio optimization, return is quantified as the expected value of the random portfolio return, while risk is specified as the variance of this return. An efficient portfolio is then defined as a combination of assets that maximizes the returns for a certain level of risk (see equation 3.1).

\[
\begin{align*}
\text{maximize} & \quad \sum_{i=1}^{n} x_i E[y_i] \\
\text{subject to} & \quad \sum_{i=1}^{n} x_i = 1 \\
& \quad \bar{x}^T \Sigma \bar{x} \leq \tilde{\sigma}
\end{align*}
\]

(3.1)

Here \(\tilde{\sigma}\) represents the maximum level of risk. Or alternatively, a combination of assets that minimizes the risk for a certain level of return (see equation 3.2).

\[
\begin{align*}
\text{minimize} & \quad \bar{x}^T \Sigma \bar{x} \\
\text{subject to} & \quad \sum_{i=1}^{n} x_i = 1 \\
& \quad \sum_{i=1}^{n} x_i E[y_i] \geq R
\end{align*}
\]

(3.2)

Here \(R\) denotes the minimum level of required return. In addition, other constraints could be imposed to both optimization problems. For example, disallowing short selling (i.e., \(x_i \geq 0 \ \forall i\)) or upper and lower bounds on the decision vector \(\bar{x}\).

3.2.1 Efficient frontier

In the previous section, the efficient portfolio in equation 3.2 is defined for one specific required return. To obtain the efficient frontier, a curve that shows all efficient portfolios in a risk-return framework, all possible efficient portfolios need to be calculated.

The first step in obtaining all efficient portfolios is solving equation 3.2, while ignoring the return constraint. The solution gives the lower bound on the expected return, \(R_{\text{min}}\). Subsequently, equation 3.1 is solved without the risk constraint in order to get the upper bound on the expected return, \(R_{\text{max}}\). Finally, the efficient frontier is obtained by solving equation 3.2 for a certain number of required returns on the interval \([R_{\text{min}}, R_{\text{max}}]\). Figure 3.1 shows an example of such an efficient frontier.
3.3 Value-at-Risk

Current regulation for the financial sector prescribes the use of percentiles of loss distributions, the Value-at-Risk (VaR), in order to measure the potential risk. According to (Jorion 2001), VaR measures the worst expected loss over a given horizon under normal market conditions at a given level of confidence.

**Definition.** Given a confidence level $\alpha \in (0, 1)$, the Value-at-Risk of a portfolio at $\alpha$ over the time period $t$ is the lowest amount $\zeta$ such that, with probability $\alpha$, the loss will not exceed $\zeta$ over time period $t$.

Mathematically, VaR is defined as

$$VaR = \zeta_\alpha (y) = \min \{ y \mid P (y \leq \zeta) \geq \alpha \}$$

Here $y$ is a random variable. Banks are not prescribed to use any particular model to calculate VaR. The three most common ways of calculating VaR are:

1. Variance-covariance matrices
2. Historical simulation
3. Monte Carlo simulation

3.3.1 Shortcomings of Value-at-Risk

Although VaR is a very popular measure of risk, it has some undesirable mathematical characteristics (Rockafellar & Uryasev 2000). In (Artzner et al. 1999) VaR is rejected as
a coherent measure of risk, because of the following shortcomings:

- Lack of subadditivity of risks, even independent risks, which creates aggregation problems (i.e., VaR of a portfolio with two instruments may be greater than the sum of the independent VaRs of these instruments).

- VaR does not encourage and, indeed, sometimes prohibits diversification.

In addition, (Mausser & Rosen 1999) show the VaR measure also lacks convexity when it is calculated using scenarios. Thus, it can have multiple local extrema, which makes optimizing rather difficult.

### 3.4 Conditional Value-at-Risk

An alternative measure of risk is Conditional Value-at-Risk (CVaR). This is the conditional expected loss under the condition that it exceeds VaR.

**Definition.** Given a confidence level $\alpha \in (0, 1)$, the Conditional Value-at-Risk of a portfolio at $\alpha$ over the time period $t$ is the conditional expectation of the losses above the Value-at-Risk at $\alpha$ over a time interval $t$.

The definitions of VaR and CVaR ensure that the $\alpha$-VaR is never more than the $\alpha$-CVaR, so portfolios with low CVaR must have low VaR as well (Rockafellar & Uryasev 2000). A visual interpretation of VaR and CVaR is given in Figure 3.2. The mathematical definition of CVaR equals

$$CVaR = E[y \mid y \geq \zeta_\alpha(y)] = E[y \mid y \geq VaR]$$

Where $y$ is a random variable.
In contrast to VaR, CVaR is proven to be a coherent risk measure (Pflug 2000). More importantly, it was shown by (Rockafellar & Uryasev 2000) that it can be optimized by linear programming and optimizing algorithms. This means that portfolios with large numbers of instruments and scenarios can be handled. Besides according to (Uryasev 2000), minimization of CVaR leads to near optimal solutions in VaR terms because CVaR is always greater than or equal to VaR.

### 3.4.1 Application to portfolio optimization

This section outlines the approach suggested by (Rockafellar & Uryasev 2000). First, for each \( \vec{x} \), the loss function is defined as the negative of the portfolio return, thus

\[
f (\vec{x}, \vec{y}) = - [x_i y_i + \ldots + x_n y_n] = -\vec{x}^T \vec{y}
\]  

(3.3)

For sake of simplicity it is assumed that \( \vec{y} \) follows a continuous distribution and denotes its density by \( p(\vec{y}) \)\(^{28}\). For each specific decision vector \( \vec{x} \), the loss \( f (\vec{x}, \vec{y}) \) is a random variable having a distribution induced by the probability distribution of \( \vec{y} \).

The probability that the loss function \( f (\vec{x}, \vec{y}) \) does not exceed threshold \( \zeta \) is then given by

\[
\Psi(\vec{x}, \zeta) = \int_{f(\vec{x}, \vec{y}) \leq \zeta} p(\vec{y})d\vec{y}
\]

(3.4)

As a function of \( \zeta \) for fixed \( \vec{x} \), \( \Psi(\vec{x}, \zeta) \) represents the cumulative distribution function

\(^{28}\) An analytical expression for \( p(\vec{y}) \) is not required for the implementation of the approach. It is enough to have an algorithm which generates random samples from \( p(\vec{y}) \) (Rockafellar & Uryasev 2000).
for the loss associated with the fixed decision vector $\vec{x}$. $\Psi(\vec{x}, \zeta)$. The function $\Psi(\vec{x}, \zeta)$ is convex and nondecreasing w.r.t. $\zeta$ (see Rockafellar & Uryasev 2000).

Now $\alpha$-VaR ($\zeta_\alpha(\vec{x})$) and $\alpha$-CVaR ($\phi_\alpha(\vec{x})$) can now be defined as

$$\zeta_\alpha(\vec{x}) = \min \{ \zeta \in \mathbb{R} : \Psi(\vec{x}, \zeta) \geq \alpha \} \quad (3.5)$$

and

$$\phi_\alpha(\vec{x}) = \frac{1}{1 - \alpha} \int_{f(\vec{x}, \vec{y}) \geq \zeta_\alpha(\vec{x})} f(\vec{x}, \vec{y}) p(\vec{y}) d\vec{y} \quad (3.6)$$

Here $\zeta_\alpha(\vec{x})$ comes out as the left endpoint of the nonempty interval consisting of the values $\zeta$, such that $\Psi(\vec{x}, \zeta) = \alpha$. Therefore the probability that $f(\vec{x}, \vec{y}) \geq \zeta_\alpha(\vec{x})$ is equal to $1 - \alpha$. Thus, $\phi_\alpha(\vec{x})$ is the conditional expectation of the loss associated with decision vector $\vec{x}$ relative to the loss being equal to or greater than $\zeta_\alpha(\vec{x})$.

Now as long as the decision vector $\vec{x}$ belongs to the feasible set $X \subset \mathbb{R}^n$, the minimization of the excess loss function $\phi_\alpha(\vec{x})$ can be reduced to minimization of the function

$$F_\alpha (\vec{x}, \zeta) = \zeta + \frac{1}{1 - \alpha} \int_{\vec{y} \in \mathbb{R}^n} \left[ -\vec{x}^T \vec{y} - \zeta \right]^+ P (\vec{y}) d\vec{y} \quad (3.7)$$

An important observation at this point is that $F_\alpha (\vec{x}, \zeta)$ is convex as a function of $(\vec{x}, \zeta)$, not just $\zeta$. The integral in equation 3.7 can be approximated by sampling scenarios from the probability distribution of $\vec{y}$. If the sampling process produces a collection of vectors $\vec{y}_1, \vec{y}_2, \ldots, \vec{y}_J$, then the corresponding approximation $\tilde{F}_\alpha (\vec{x}, \zeta)$ equals

$$\tilde{F}_\alpha (\vec{x}, \zeta) = \zeta + \frac{1}{J (1 - \alpha)} \sum_{j=1}^J \left[ -\vec{x}^T \vec{y}_j - \zeta \right]^+ \quad (3.8)$$

If the loss function $f (\vec{x}, \zeta)$ and the feasible set $X$ are both convex, the convex optimization problem

$$\minimize_{\vec{x} \in X, \zeta \in \mathbb{R}} \tilde{F}_\alpha (\vec{x}, \zeta) \quad (3.9)$$

can be solved in order to find the optimal $\vec{x}^*$ (portfolio weights), the corresponding $\zeta^*$ (VaR), and the optimal $\tilde{F}_\alpha (\vec{x}^*, \zeta^*)$ (CVaR). Moreover, the optimization problem can be further reduced to a linear programming problem when the loss function $f (\vec{x}, \vec{y}_j)$ is linear in $\vec{x}$ and the feasible set $X$ is constraint by linear (in)equalitys only. This gives

---

29 For proof (see Rockafellar & Uryasev 2000)
the following linear programming problem

\[
\begin{align*}
\text{minimize} & \quad \zeta + \frac{1}{J(1 - \alpha)} \sum_{j=1}^{J} z_j \\
\text{subject to} & \quad \bar{x} \in X \\
& \quad z_j \geq f(\bar{x}, \bar{y}_j) - \zeta \\
& \quad z_j \geq 0
\end{align*}
\]

(3.10)

3.4.2 Mean-CVaR portfolio optimization

The linear formulation of the mean-CVaR portfolio is highly beneficial, as it enables to optimize large portfolio. In (Krokhmal et al. 2002) it is proven that CVaR can also enter the model as an linear equation in order to maximize the return. The CVaR minimization model described in equation 3.10 can be formulated as the mean-CVaR optimization problem

\[
\begin{align*}
\text{minimize} & \quad \zeta + \frac{1}{J(1 - \alpha)} \sum_{j=1}^{J} z_j \\
\text{subject to} & \quad \sum_{i=1}^{n} x_i = 1 \\
& \quad \sum_{i=1}^{n} x_i E[y_i] \geq R \\
& \quad z_j \geq -\bar{x}^T \bar{y}_j - \zeta \\
& \quad z_j \geq 0
\end{align*}
\]

(3.11)
CHAPTER 4
Model description

A portfolio optimization technique is applied to optimize the banks’ balance sheet, as it will be viewed as a portfolio consisting of investment instruments (i.e., balance sheet items). Assets on the balance sheet produce a positive return, whereas the liabilities produce a negative return, which is translated into the P&L account as income and expense respectively. Subsequently, taking other revenues and expenses (e.g., fee and commission income and staff costs) into account and applying the tax and dividend policy, the retained earnings can be calculated. The retained earnings is the most important measure of a bank's performance and will therefore serve as the objective to be maximized in the model.

This chapter will define and outline the balance sheet optimization model. First all the variables of the model components will be defined. Followed by the calculation of the retained earnings on the P&L account. Subsequently, the Basel 2.5 and Basel III regulation is translated into constraints in the model. Finally, the full problem formulation of the model is given.

4.1 Components of the model

Nowadays, the balance sheet of a large bank consists of many types of assets and liabilities. To take all of these instruments into account would make the model too complex and probably intractable. Therefore the presented model will optimize a stylized balance sheet, which is presented in Figure 4.1. This stylized balance sheet can be considered a good representation for a balance sheet of a bank involved in commercial banking as well as investment banking. In addition, the model is flexible in adding or replacing instruments in order to have a better fit to the banks’ real balance sheet. The presented stylized balance sheet is based on the publicly released financial statements of three large Dutch banks over the last five years.

4.1.1 Balance sheet items

Let us consider a portfolio consisting of \( n, \) \((i = 1, \ldots, n)\) different investment instruments of the balance sheet of a bank, among which there are asset and liability instruments.
The stylized balance sheet consists of 7 asset instruments and 4 liability instruments, thus a total of \( n = 11 \). Let the vector \( \vec{x}^0 = (x_1^0, \ldots, x_n^0)^T \), to be chosen from a certain subset \( X \subset \mathbb{R} \), denote the positions, i.e., a proportion of the total size of the balance sheet, of each instrument in the initial portfolio. Consequently, let decision vector \( \vec{x} = (x_1, \ldots, x_n)^T \) denote the positions in the optimal portfolio (i.e., optimal balance sheet). Furthermore, each instrument has a certain return, which is denoted by the return vector \( \vec{y} = (y_1, \ldots, y_n)^T \), such that \( y_i \) denotes the return on instrument \( i \).

The balance sheet is in balance when the sum of all the positions in the asset instruments equals the sum of the liability & equity instruments. Furthermore, it is not allowed to go short on positions on the balance sheet items, thus

\[
x_i \geq 0, \quad i = 1, \ldots, n
\]

### 4.1.1.1 Asset instruments

The model includes seven different asset instruments and are referenced by the letter \( i \), which is given by

\[
i = \begin{cases} 
\text{Cash and cash equivalents} & \text{if } i = 1 \\
\text{Loans and advances to banks} & \text{if } i = 2 \\
\text{Retail loans} & \text{if } i = 3 \\
\text{Corporate loans} & \text{if } i = 4 \\
\text{Retail mortgages} & \text{if } i = 5 \\
\text{Government bonds} & \text{if } i = 6 \\
\text{Financial assets held for trading} & \text{if } i = 7
\end{cases}
\]
Furthermore, an asset instrument could be divided into a sub class. Three different sub classes are taken into account, namely a set of maturities $M = \{1, 2, 3\}$, a set of credit ratings $C = \{1, 2, 3, 4\}$ and a set of portfolio riskiness $P = \{1, 2, 3, 4, 5\}$. Where $m \in M$, $c \in C$ and $p \in P$ respectively represent

\[
m = \begin{cases} 
\text{Less than 1 year} & \text{if } m = 1 \\
\text{Between 1 and 5 years} & \text{if } m = 2 \\
\text{Over 5 years} & \text{if } m = 3 
\end{cases}
\]
\[
c = \begin{cases} 
\text{AAA to AA}^- & \text{if } c = 1 \\
\text{A}^+ \text{ to A}^- & \text{if } c = 2 \\
\text{BBB}^+ \text{ to BBB}^- & \text{if } c = 3 \\
\text{BB}^+ \text{ to B}^- & \text{if } c = 4 
\end{cases}
\]
\[
p = \begin{cases} 
\text{Absolute risk averse} & \text{if } p = 1 \\
\text{Relative risk averse} & \text{if } p = 2 \\
\text{Risk neutral} & \text{if } p = 3 \\
\text{Absolute risk seeking} & \text{if } p = 4 \\
\text{Relative risk seeking} & \text{if } p = 5 
\end{cases}
\]

The first part of the decision vector $\vec{x}$, the optimal positions in the asset instruments, can be given by

\[
\begin{pmatrix} x_1 & x_2 & x_{3,m} & x_{4,m} & x_{5,m} & x_{6,c} & x_{7,p} \end{pmatrix}^T
\]

Subsequently, the first part of the returns vector $\vec{y}$ is given by

\[
\begin{pmatrix} y_1 & y_2 & y_{3,m} & y_{4,m} & y_{5,m} & y_{6,c} & y_{7,p} \end{pmatrix}^T
\]

Lastly, the sum of all the positions in the asset instruments is as follows

\[
x_1 + x_2 + \sum_{i=3}^{5} \sum_{m \in M} x_{i,m} + \sum_{c \in C} x_{6,c} + \sum_{p \in P} x_{7,p} = 1 \quad (4.1)
\]

### 4.1.1.2 Liability instruments

Liability instruments in the model are referenced in the same manner as the asset instruments, so by the letter $i$. This is supplemented by

\[
i = \begin{cases} 
\text{Saving deposits} & \text{if } i = 8 \\
\text{Demand deposits} & \text{if } i = 9 \\
\text{Debt securities} & \text{if } i = 10 \\
\text{Wholesale funding} & \text{if } i = 11 
\end{cases}
\]
Likewise, the liability instruments are also divided into three sub classes. Let us define a set of deposit maturities $D = \{1, 2\}$, a set of deposit holder type $H = \{1, 2\}$ and a set of wholesale funding $F = \{1, 2, 3\}$. Such that $d \in D$, $h \in H$ and $f \in F$ respectively represent

$$d = \begin{cases} 
\text{Short-term} & \text{if } d = 1 \\
\text{Long-term} & \text{if } d = 2 
\end{cases}$$

$$h = \begin{cases} 
\text{Retail} & \text{if } h = 1 \\
\text{Corporate} & \text{if } h = 2 
\end{cases}$$

$$f = \begin{cases} 
\text{Overnight interbank lending} & \text{if } f = 1 \\
\text{REPO market} & \text{if } f = 2 \\
\text{Central bank lending rate} & \text{if } f = 3 
\end{cases}$$

The decision vector $\vec{x}$ is supplemented by the positions in the liability instruments, thus by

$$\begin{pmatrix} 
x_{8,d} & x_{9,h} & x_{10} & x_{11,f} 
\end{pmatrix}^T$$

In the same manner the return vector $\vec{y}$ is also supplemented with the returns on the liability instruments. The second part of that vector is defined as

$$\begin{pmatrix} 
y_{8,d} & y_{9,h} & y_{10} & y_{11,f} 
\end{pmatrix}^T$$

### 4.1.1.3 Equity instruments

Finally, only the positions in the equity instruments are the left to define. The shareholders’ equity is divided into three categories depending on the quality of the equity, in accordance with the Basel III definition of capital components. Furthermore, the retained earnings must be taken into account, which is is part of the highest quality component of equity. Thus, in total four types of equity instruments are included in the model. Let us define vector $\vec{e} = (e_1, ... e_4)$ as the weights of the equity instruments in the portfolio, such that $e_k$ represents the weight of equity instrument $k$. Where $k$ is given by

$$k = \begin{cases} 
\text{Common equity Tier 1} & \text{if } k = 1 \\
\text{Additional equity Tier 1} & \text{if } k = 2 \\
\text{Tier 2 equity} & \text{if } k = 3 \\
\text{Retained earnings} & \text{if } k = 4 
\end{cases}$$

In this model it is assumed that the equity instruments with $k = \{1, 2, 3\}$ do not make any return and are taken as a constant. However, the retained earnings on the other
hand are to be calculated through the profit & loss account, which will be discussed later in this chapter.

Hence, the sum of the positions of all liability & equity instruments should also be equal to 1, as the sum of all the asset instruments also add up to 1. Thus,

\[ \sum_{k=1}^{4} c_k + \sum_{d \in D} x_{8,d} + \sum_{h \in H} x_{9,h} + x_{10} + \sum_{f \in F} x_{11,f} = 1 \] (4.2)

### 4.1.2 Bounds on positions

Besides the constraints enforced by the new Basel III regulation, also constraints on the change in individual positions (liquidity constraints) or bounds on positions are inserted in the model. Liquidity constraints on the individual instruments are defined in the following manner

\[ u_i^- \leq x_i^0 - x_i \leq u_i^+ \quad i = 1, ..., n \] (4.3)

here \( u_i^- \) and \( u_i^+ \) respectively denote the maximum downward and upward position change for each balance sheet instrument \( i \).

Additionally, the individual positions themselves can be bounded by adding

\[ x_i^- \leq x_i \leq x_i^+ \quad i = 1, ..., n \] (4.4)

here \( x_i^- \) and \( x_i^+ \) respectively represent the lower and upper bound on balance sheet instrument \( i \).

### 4.1.3 Profit & loss account

Given the definition of the variables described above, it is now possible to deduct the P&L account. The first source of revenue of the stylized balance sheet is the net interest income (i.e., interest income minus interest expense). The interest income is produced by the interest-bearing asset instruments, while interest expense is generated by the interest-bearing liability instruments. In addition, trading income or loss is generated by the financial assets held for trading. Lastly, there are other sources of revenues or expenses for a bank that can not be be deducted from the model in the same manner as the other sources of revenue. The next section will briefly discuss these revenues and expenses.
4.1.3.1 Net interest income

As stated above the interest income is produced by interest-bearing asset instruments. Which include the asset instruments with $i = \{1, 2, 3, 4, 5, 6\}$. Thus the interest income is calculated by

$$
\sum_{i=1}^{2} x_i y_i + \sum_{i=3}^{5} \sum_{m \in M} x_{i,m} y_{i,m} + \sum_{c \in C} x_{6,c} y_{6,c}
$$

On the other hand the interest expense is generated by the liability instruments with $i = \{8, 9, 10, 11\}$ and is given by

$$
\sum_{d \in D} x_{8,d} y_{8,d} + \sum_{h \in H} x_{9,h} y_{9,h} + x_{10} y_{10} + \sum_{f \in F} x_{11,f} y_{11,f}
$$

Hence, the net interest income equals

$$
\sum_{i=1}^{2} x_i y_i + \sum_{i=3}^{5} \sum_{m \in M} x_{i,m} y_{i,m} + \sum_{c \in C} x_{6,c} y_{6,c} + \sum_{d \in D} x_{8,d} y_{8,d} + \sum_{h \in H} x_{9,h} y_{9,h} + x_{10} y_{10} + \sum_{f \in F} x_{11,f} y_{11,f}
$$

(4.5)

4.1.3.2 Net trading income

Trading income is only generated by the financial assets held for trading instrument ($i = 7$). Consequently, the net trading income is given by the profit or loss generated by instrument, thus

$$
\sum_{p \in P} x_{7,p} y_{7,p}
$$

(4.6)

4.1.3.3 Write-downs

As explained in Chapter 2, a write-down reduces the book value of an asset to the fair value. The cause for a write-down is a suffered loss on a specific asset. Some assets are more exposed to losses than other assets, therefore the write-downs on the asset instruments are denoted by the write-down vector $\vec{l} = (l_1, \ldots, l_n)$, where $l_i$ represents the write-down percentage on asset instrument $i$. Only the asset instruments with $i = \{2, 3, 4, 5\}$ are exposed to write-downs, therefore the write-down percentage on all other instruments in the model equals zero. The write-down vector on the eligible instruments
is given by

$$\begin{pmatrix} l_1 \\ \hat{L}_2 \\ \hat{L}_3 \\ \hat{L}_4 \end{pmatrix} = \begin{pmatrix} 2\% \\ 3\% \\ 3\% \end{pmatrix}$$

These percentages are approximated from the annual reports of three large Dutch banks. Hence, the loss produced by the write-downs is given by

$$x_2 l_2 + \sum_{i=3}^{5} \sum_{h \in H} x_{i,h} l_{i,h}$$

(4.7)

4.1.3.4 Other revenues and expenses

The P&L account is completed with other revenues and expenses. These do not directly follow from the variables defined in the chapter earlier. Therefore, these revenues or expenses are taken as a constant, to be determined by the bank using the model. The following revenues and expenses are taken into account:

- $a_1$ = Fee and commission income
- $a_2$ = Staff expenses
- $a_3$ = Other operating expenses

and are taken as a proportion of the total balance sheet size.

4.1.3.5 Retained earnings

Besides all the income and expense sources, the retained earnings depend on the dividend payout and on the tax rate. First the result before tax is calculated by taking the sum of all income and expenses. After which the taxation is subtracted from the net profit before taxes. Finally, depending on the dividend policy of the bank, the retained earnings is calculated. The calculation of the net profit before tax ($N$) is given by

$$N = \frac{2}{\text{Net interest income}} \left( \sum_{i=1}^{2} x_{i} y_{i} + \sum_{i=3}^{5} \sum_{m \in M} x_{i,m} y_{i,m} + \sum_{c \in C} x_{6,c} y_{6,c} + \sum_{d \in D} x_{8,d} y_{8,d} + \sum_{h \in H} x_{9,h} y_{9,h} + x_{10} y_{10} + \sum_{f \in F} x_{11,f} y_{11,f} \right)$$

$$+ \frac{5}{\text{Trading income}} \left( \sum_{p=1}^{5} x_{7,p} y_{7,p} - x_{2} l_{2} - \sum_{i=3}^{5} \sum_{d=1}^{3} x_{i,d} l_{i,d} + \sum_{o=1}^{3} a_{o} \right)$$

$$+ \frac{3}{\text{Write-downs}}$$

$$+ \frac{3}{\text{Other}}$$
Subsequently, the calculation of the retained earnings, $e_4$, is given by

$$e_4 = \left( (N(1 - \tau))(1 - \delta) \right)$$  \hspace{1cm} (4.8)

where $\tau$ represents the tax rate and $\delta$ denotes the dividend payout percentage.

### 4.1.4 Basel III constraints

Chapter 2 described the new regulation, Basel 2.5 and Basel III, imposed by the BCBS. In this section the regulation is translated into constraints in the optimization model. Table 4.1 gives an overview of the prescribed constraints and the transition period for the implementation.

<table>
<thead>
<tr>
<th>Table 4.1: Basel III constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>-------</td>
</tr>
<tr>
<td><strong>Capital constraints</strong></td>
</tr>
<tr>
<td>CET1 ratio</td>
</tr>
<tr>
<td>Tier 1 ratio</td>
</tr>
<tr>
<td>Capital ratio</td>
</tr>
<tr>
<td><strong>Liquidity constraints</strong></td>
</tr>
<tr>
<td>LCR</td>
</tr>
<tr>
<td>NSFR</td>
</tr>
<tr>
<td><strong>Leverage constraint</strong></td>
</tr>
<tr>
<td>Leverage ratio</td>
</tr>
</tbody>
</table>

### 4.1.4.1 Capital ratios

The crisis exposed that it is critical that banks have a high quality capital base in order to absorb credit losses and writedowns. As the crisis showed that most of these losses came out of the retained earnings of the banks. The quality and quantity of the capital ratios are incremented.

Three risk types are taken into account to calculate the capital requirement, namely credit risk, market risk and operational risk. The asset instruments in the model are exposed to credit risk and market risk. For credit risk the standardized approach is used in order to calculate capital requirement, while the internal models approach is followed for market risk. The basic indicator approach is used to calculate the capital requirement for operational risk.
Credit risk

In the standardized approach for credit risk, the capital requirement is calculated using risk weights. To obtain the RWA of all asset instruments exposed to credit risk, the risk weights are multiplied by the positions to each asset instrument. The asset instruments with \( i = \{1, 2, 3, 4, 5, 6\} \) are exposed to credit risk. Let us denote the risk weights by vector \( \vec{w} = (w_1, \ldots, w_n) \), such that \( w_i \) corresponds to asset instrument \( i \). The risk weights are set to\(^{30}\)

\[
\begin{pmatrix}
0\% \\
20\% \\
75\% \\
65\% \\
35\%
\end{pmatrix} =
\begin{pmatrix}
w_{5,1} \\
w_{6,2} \\
w_{6,3} \\
w_{6,4}
\end{pmatrix} =
\begin{pmatrix}
0\% \\
20\% \\
50\% \\
100\%
\end{pmatrix}
\]

Subsequently, the risk weights to all liability instruments are zero. As the positions in each asset instrument is already defined, the calculation of the credit risk of the portfolio is given by

\[
\sum_{i=1}^{2} x_i w_i + \sum_{i=3}^{5} \sum_{m \in M} x_{i,m} w_i + \sum_{c \in C} x_{6,c} w_{6,c}
\]

Market risk

Besides asset instruments exposed to credit risk, asset instrument \( x_{7,p} \) (financial assets held for trading) bears market risk. Under the internal models approach in Basel 2.5, the market risk is calculated by the 10-day 99% Value-at-Risk plus the 10-day 99% stressed Value-at-Risk multiplied by a multiplication factor. As previous chapter showed that the risk measure CVaR is a more conservative risk measure than VaR and is by definition always greater than or equal to the VaR measure of a portfolio, the capital requirement for market risk will be the sum of the 10-day 99% CVaR measures of the five specified portfolios multiplied by the sum of the multiplication factors of the normal \( (m_c) \) and stressed period \( (m_s) \). These values can be considered risk weights for market risk and part of the vector \( \vec{w} \), such that \( w_{7,p} \) denotes the 10-day 99%-CVaR measure of portfolio \( p \) multiplied by \( (m_c + m_s) \). Consequently, the market risk capital requirement is calculated by

\[
\sum_{p \in P} x_{7,p} w_{7,p}
\]

\(^{30}\)See (BCBS 2004, Part 2, section II: Credit Risk The Standardised Approach)
Operational risk

In contrast to credit risk and market risk, the basis indicator approach requires data from previous years, because the required capital is 15% of the banks’ three-year average gross income. However, as the model optimizes the coming year only, the operational risk will simply be 15% of the generated gross income. Thus the capital requirement for operational risk is given by

\[ 0.15N \]

4.1.4.2 Ratios

As the total RWA is the sum of the exposure to credit risk, market risk and operational risk, it is given by

\[ RWA = \sum_{i=1}^{2} x_i w_i + \sum_{i=3}^{5} \sum_{m \in M} x_{i,m} w_i + \sum_{c \in C} x_{6,c} w_{6,c} + \sum_{p \in P} x_{7,p} w_{7,p} + 0.15N \]

The three capital ratios prescribed by Basel III can now be defined by

\[ \frac{CET1R}{Total\ RWA} = \frac{Common\ equity\ tier\ 1}{Credit\ risk + Market\ risk + Operational\ risk} \]

\[ = \frac{e_1 + e_4}{\sum_{i=1}^{2} x_i w_i + \sum_{i=3}^{5} \sum_{m \in M} x_{i,m} w_i + \sum_{c \in C} x_{6,c} w_{6,c} + \sum_{p \in P} x_{7,p} w_{7,p} + 0.15N} \]  

\[ \frac{TCR}{Total\ RWA} = \frac{Total\ capital}{Credit\ risk + Market\ risk + Operational\ risk} \]

\[ = \frac{e_1 + e_3 + e_4}{\sum_{i=1}^{2} x_i w_i + \sum_{i=3}^{5} \sum_{m \in M} x_{i,m} w_i + \sum_{c \in C} x_{6,c} w_{6,c} + \sum_{p \in P} x_{7,p} w_{7,p} + 0.15N} \]
4.1.4.3 Leverage ratio

Leverage rate is introduced to serve as a backstop to the risk-based capital measures. The excessive build-up of on- and of-balance sheet leverage in banking system was one of the underlying factors of the credit crisis of 2007-2008. Under the previous regulation, Basel II, it was possible to be highly leverages, but still have strong capital ratios. The leverage ratio puts a restriction on the balance sheet size compared to core equity of a bank. And is defined as

\[ LR = \frac{\text{Tier 1 capital}}{\text{Balance sheet size}} \]

\[ = \frac{\text{Tier 1 capital}}{\text{Total assets}} \]

\[ = \frac{e_1 + e_2 + e_4}{1} = e_1 + e_2 + e_4 \]

4.1.4.4 Liquidity ratios

Despite the adequate capital levels of many banks at the start of the crisis, losses still arose because liquidity was not sufficiently taken into account. Two standards are developed in order to strengthen the liquidity framework. First the Liquidity Coverage Ratio (LCR) is introduced to ensure that banks have sufficient high quality assets to survive significant stress for a period of a month. In addition the Net Stable Funding Ratio (NSFR) is introduced to provide sustainability of maturity in the structure of the asset and liability composition.

LCR

In Basel III two types of highly liquid assets are defined. To what extend an asset instrument is highly liquid is determined by a factor (percentage). Table 4.2 displays the assets instrument that are highly liquid and the corresponding factor. All portfolio instruments excluded from the table have a factor that is equal to 0%.
Table 4.2: Stock of high-quality liquid assets

<table>
<thead>
<tr>
<th>Asset instrument</th>
<th>Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level 1 assets</td>
<td></td>
</tr>
<tr>
<td>$x_1$ Cash and cash equivalents</td>
<td>100%</td>
</tr>
<tr>
<td>$x_{6,1}$ Government bonds (AAA to AA−)</td>
<td>100%</td>
</tr>
<tr>
<td>Level 2 assets</td>
<td></td>
</tr>
<tr>
<td>$x_{6,2}$ Government bonds (A+ to A−)</td>
<td>85%</td>
</tr>
</tbody>
</table>

*Source:* See (BCBS 2010b, Annex 1)

Hence that the total exposure to stock of highly liquid assets is

$$x_1 + x_{6,1} + 0.85x_{6,2}$$ (4.13)

This should be greater than the total net cash outflows over the next 30 calendar days. Which is equal to the sum of the cash inflows (asset instruments) subtracted by the sum of the cash outflows (liability instruments). The factors for the cash inflows and outflows are given in Table 4.3. Only the instruments displayed in this table are related to the liquidity coverage ratio. Let us denote vector $\vec{b} = (b_1, \ldots, b_n)$, such that $b_i$ represents the cash flow factor for instrument $i$. The cash flow factors for instruments excluded from Table 4.3 are set to zero.

Table 4.3: Factors of cash inflows and outflows prescribed by Basel III

<table>
<thead>
<tr>
<th>Cash inflows</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_2$ Loans and advances to banks</td>
<td>$\frac{1}{12} \cdot 100%$</td>
</tr>
<tr>
<td>$x_{3,1}$ Retail loans with maturity ≤ 1 year</td>
<td>$\frac{1}{12} \cdot 50%$</td>
</tr>
<tr>
<td>$x_{4,1}$ Corporate loans with maturity ≤ 1 year</td>
<td>$\frac{1}{12} \cdot 50%$</td>
</tr>
<tr>
<td>$x_{5,1}$ Retail mortgages with maturity ≤ 1 year</td>
<td>$\frac{1}{12} \cdot 50%$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cash outflows</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{8,1}$ Saving deposits: short-term</td>
<td>5%</td>
</tr>
<tr>
<td>$x_9$ Demand deposits</td>
<td>10%</td>
</tr>
<tr>
<td>$x_{10}$ Debt securities</td>
<td>100%</td>
</tr>
<tr>
<td>$x_{11,1}$ Overnight interbank lending</td>
<td>75%</td>
</tr>
<tr>
<td>$x_{11,2}$ REPO market</td>
<td>100%</td>
</tr>
</tbody>
</table>

*Source:* See (BCBS 2010b, Annex 1)

*Note:* As the LCR only takes the next 30 calendar days into account but the maturity of the regarding asset instruments is one year, the factor is multiplied by $\frac{1}{12}$.
Consequently, the total net cash inflows is given by

$$x_2 b_2 + \sum_{i=3}^{5} x_{i,1} b_{i,1}$$

(4.14)

and the net cash outflows by

$$x_{8,1} b_{8,1} + \sum_{h \in H} x_{9,h} b_{9,h} + x_{10} b_{10} + \sum_{f \in F} x_{11,f} b_{11,f}$$

(4.15)

Combining equation 4.2, equation 4.14 and equation 4.15 gives the liquidity coverage ratio (LCR).

$$LCR = \frac{\text{Highly liquid assets}}{\text{Net cash outflows} - \text{Net cash inflows}}$$

(4.16)

$$= \frac{x_1 + x_{6,1} + 0.85x_{6,2}}{x_{8,1} b_{8,1} + \sum_{h \in H} x_{9,h} b_{9,h} + x_{10} b_{10} + \sum_{f \in F} x_{11,f} b_{11,f} - x_2 b_2 - \sum_{i=3}^{5} x_{i,1} b_{i,1}}$$

NSFR

The NSFR is introduced in order to promote medium and long-term funding. It is defined as the available stable funding divided by the required stable funding. Table 4.4 lists the required stable funding factor of a bank’s assets instruments. Factors for the available stable funding of the equity and liability instruments are displayed in Table 4.5. We now define vector $\vec{s} = (s_1, \ldots, s_n)$ in such a way that it contains all required and available stable funding factors. Thus, $s_i$ denotes the stable funding factor of instrument $i$. 
Table 4.4: Factors of required stable funding (asset instruments)

<table>
<thead>
<tr>
<th>Asset instrument</th>
<th>Required factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$ Cash and cash equivalents</td>
<td>5%</td>
</tr>
<tr>
<td>$x_2$ Loans and advances to banks</td>
<td>65%</td>
</tr>
<tr>
<td>$x_{3,1}$ Retail loans with maturity $\leq$ 1 year</td>
<td>85%</td>
</tr>
<tr>
<td>$x_{3,2} &amp; x_{3,3}$ Retail loans with maturity $&gt; 1$ year</td>
<td>65%</td>
</tr>
<tr>
<td>$x_{4,1}$ Corporate loans with maturity $\leq$ 1 year</td>
<td>85%</td>
</tr>
<tr>
<td>$x_{4,2} &amp; x_{4,3}$ Corporate loans with maturity $&gt; 1$ year</td>
<td>65%</td>
</tr>
<tr>
<td>$x_{5,1}$ Retail mortgages with maturity $\leq$ 1 year</td>
<td>85%</td>
</tr>
<tr>
<td>$x_{5,2} &amp; x_{5,3}$ Retail mortgages with maturity $&gt; 1$ year</td>
<td>65%</td>
</tr>
<tr>
<td>$x_{6,1}$ Government bonds (AAA tot AA$^-$)</td>
<td>20%</td>
</tr>
<tr>
<td>$x_{6,2}$ Government bonds (A$^+$ tot A$^-$)</td>
<td>20%</td>
</tr>
<tr>
<td>$x_7$ Financial assets held for trading</td>
<td>50%</td>
</tr>
<tr>
<td>All other assets instruments</td>
<td>100%</td>
</tr>
</tbody>
</table>

*Source: See (BCBS 2010b, Annex 2)*

It leads that the total required stable funding is given by

$$x_2 s_2 + \sum_{i=3}^{5} \sum_{m \in M} x_{i,m} s_{i,m} + \sum_{c \in C} x_{6,c} s_{6,c} + \sum_{p \in P} x_{7,p} s_{7,p}$$

(4.17)

Table 4.5: Factors of available stable funding (liability & equity instruments)

<table>
<thead>
<tr>
<th>Liability &amp; equity instruments</th>
<th>Required factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_k$ Shareholders’ equity</td>
<td>100%</td>
</tr>
<tr>
<td>$x_{8,1}$ Saving deposits (short-term)</td>
<td>80%</td>
</tr>
<tr>
<td>$x_{8,2}$ Saving deposits (long-term)</td>
<td>100%</td>
</tr>
<tr>
<td>$x_{9,1}$ Demand deposits (retail)</td>
<td>80%</td>
</tr>
<tr>
<td>$x_{9,2}$ Demand deposits (corporate)</td>
<td>90%</td>
</tr>
<tr>
<td>$x_{11,1}$ Overnight interbank lending</td>
<td>50%</td>
</tr>
<tr>
<td>$x_{11,3}$ Central bank lending rate</td>
<td>50%</td>
</tr>
<tr>
<td>All other liability instruments</td>
<td>0%</td>
</tr>
</tbody>
</table>

*Source: See (BCBS 2010b, Annex 2)*
Consequently, the total amount of available stable funding equals

\[
\sum_{k=1}^{4} e_k + \sum_{d \in D} x_{8,d}s_{8,d} + \sum_{h \in H} x_{9,h}s_{9,h} + \sum_{f \in F} x_{11,f}s_{11,f}
\]  

(4.18)

This results in the following constraint for the NSFR:

\[
\text{NSFR} = \frac{\text{Available amount of stable funding}}{\text{Required amount of stable funding}}
\]  

(4.19)

\[
= \frac{\sum_{k=1}^{4} e_k + \sum_{d \in D} x_{8,d}s_{8,d} + \sum_{h \in H} x_{9,h}s_{9,h} + \sum_{f \in F} x_{11,f}s_{11,f}}{x_2s_2 + \sum_{i=3}^{5} \sum_{m \in M} x_{i,m}s_{i,m} + \sum_{c \in C} x_{6,c}s_{6,c} + \sum_{p \in P} x_{7,p}s_{7,p}}
\]

4.1.5 Single period optimization problem

Portfolio optimization problems are usually based on assumptions about the future returns on specific investment instruments. Most commonly, the returns are inferred by financial analysts by combining information about the historical returns and by predictions or expectations about the future movement of the economy. Likewise, the returns in this portfolio optimization are based on historical data, such that a feasible set of returns \(Y\) is generated. Next chapter will describe the generation process of this scenario set in more detail.

Mean portfolio optimization problem

The mean portfolio optimization problem maximizes the mean retained earnings over all scenarios in scenario set \(Y\), such that the optimal decision vector \(\vec{x}\) is compliant with the Basel regulation in all scenarios. Consider a certain starting balance sheet composition of a bank \((\vec{x}_0)\), the weights of the banks’ equity instruments \((e_k)\) and a set of scenarios such that \(y \in Y\), the mean portfolio optimization problem is given by Equation 4.20.

However, the solution to the mean portfolio optimization problem does not provide any information about the worst cases scenarios of the scenario set \(Y\). Consider the case where the optimal decision vector \(\vec{x}\) produces high retained earnings in most scenarios, but on the other hand performs poorly in some scenarios which leads to extreme losses. In this plausible case the average retained earnings can be optima, but the excessive losses
are filtered out as the average over all scenarios is optimized. Due to the constraints imposed by the Basel regulation this effect will be reduced, though it is a shortcoming to the optimization problem.

\[
\begin{align*}
\text{maximize} & \quad \bar{e}_4 \\
\text{subject to} & \quad x_1 + x_2 + \sum_{i=3}^{5} \sum_{m \in M} x_{i,m} + \sum_{c \in C} x_{6,c} + \sum_{p \in P} x_{7,p} = 1 \\
& \quad \sum_{k=1}^{4} e_k + \sum_{d \in D} x_{8,d} + \sum_{h \in H} x_{9,h} + x_{10} + \sum_{f \in F} x_{11,f} = 1 \\
& \quad u_i^+ \leq x_i^0 - x_i \leq u_i^-, \quad i = 1, \ldots, n \\
& \quad x_i^- \leq x_i \leq x_i^+, \quad i = 1, \ldots, n \\
& \quad x_i^- \geq 0, \quad i = 1, \ldots, n \\
\end{align*}
\]

\[
\begin{align*}
\frac{e_1 + e_4}{2} & \geq CETIR \\
\frac{2}{2} \sum_{i=1}^{5} x_i w_i + \sum_{i=3}^{5} \sum_{m \in M} x_{i,m} w_i + \sum_{c \in C} x_{6,c} w_{6,c} + \sum_{p \in P} x_{7,p} w_{7,p} + 0.15N & \geq TIR \\
\frac{e_1 + e_2 + e_4}{2} & \geq TCR \\
\frac{2}{2} \sum_{i=1}^{5} x_i w_i + \sum_{i=3}^{5} \sum_{m \in M} x_{i,m} w_i + \sum_{c \in C} x_{6,c} w_{6,c} + \sum_{p \in P} x_{7,p} w_{7,p} + 0.15N & \geq LCR \\
& = x_1 + x_{6,1} + 0.85 x_{6,2} \\
& \geq NSFR \\
& \geq LR \\
\end{align*}
\]

**CVaR portfolio optimization problem**

Alternatively, a CVaR portfolio optimization approach is considered to minimize the conditional expectation of the extreme losses. Thus this optimization problem gives a more conservative and robust solution than the mean portfolio optimization problem. Let \( f(\bar{x}, \bar{y}) \) be the loss.
associated with the decision vector $\vec{x}$, such that $f(\vec{x}, \vec{y}) = -e_4$. Now combining the theory of Chapter 3 and the constraints derived in this chapter gives the following CVaR portfolio optimization problem

\[
\begin{align*}
\text{minimize} & \quad \zeta + \frac{1}{J (1 - \alpha)} \sum_{j=1}^{J} z_j \\
\text{subject to} & \quad e_4 \geq R \\
& \quad z_j \geq f(\vec{x}, \vec{y}_j) - \zeta \\
& \quad z_j \geq 0 \\
& \quad x_1 + x_2 + \sum_{i=3}^{5} \sum_{m \in M} x_{i,m} + \sum_{c \in C} x_{6,c} + \sum_{p \in P} x_{7,p} = 1 \\
& \quad \sum_{k=1}^{4} e_k + \sum_{d \in D} x_{8,d} + \sum_{h \in H} x_{9,h} + x_{10} + \sum_{f \in F} x_{11,f} = 1 \\
& \quad u_i^- \leq x_i^0 - x_i \leq u_i^+, \quad i = 1, \ldots, n \\
& \quad x_i^- \leq x_i \leq x_i^+, \quad i = 1, \ldots, n \\
& \quad x_i^- \geq 0, \quad i = 1, \ldots, n \\
& \quad \frac{2}{e_1 + e_4} \sum_{i=1}^{2} x_i w_1 + \sum_{i=3}^{5} \sum_{m \in M} x_{i,m} w_i + \sum_{c \in C} x_{6,c} w_{6,c} + \sum_{p \in P} x_{7,p} w_{7,p} + 0.15N \geq CET1R \\
& \quad \frac{2}{e_1 + e_2 + e_4} \sum_{i=1}^{2} x_i w_1 + \sum_{i=3}^{5} \sum_{m \in M} x_{i,m} w_i + \sum_{c \in C} x_{6,c} w_{6,c} + \sum_{p \in P} x_{7,p} w_{7,p} + 0.15N \geq T1R \\
& \quad \frac{2}{e_1 + e_2 + e_3 + e_4} \sum_{i=1}^{2} x_i w_1 + \sum_{i=3}^{5} \sum_{m \in M} x_{i,m} w_i + \sum_{c \in C} x_{6,c} w_{6,c} + \sum_{p \in P} x_{7,p} w_{7,p} + 0.15N \geq TCR \\
& \quad x_1 + x_{6,1} + 0.85x_{6,2} \geq LCR \\
& \quad x_{8,1} b_{8,1} + \sum_{h \in H} x_{9,h} b_{9,h} + x_{10} b_{10} + \sum_{f \in F} x_{11,f} b_{11,f} - x_2 b_2 - \sum_{i=3}^{5} x_{i,1} b_{i,1} \\
& \quad \sum_{k=1}^{4} e_k + \sum_{d \in D} x_{8,d} s_{8,d} + \sum_{h \in H} x_{9,h} s_{9,h} + \sum_{f \in F} x_{11,f} s_{11,f} \\
& \quad e_1 + e_2 + e_4 \geq LR
\end{align*}
\]
Implementation

The two models from are implemented in R, software for statistical computing, and optimized using the NLOPT package by Johnson (2011). This package includes numerous optimization algorithms that can be easily used on non-linear optimization problems. For the models in this research, the sequential quadratic programming (SQP) algorithm for non-linearly constrained gradient-based optimization (supporting both inequality and equality constraints), based on the implementation by Kraft (1994), is used.
CHAPTER 5
Scenario set description

This chapter describes scenario set $Y$, which is used in the model described in the previous chapter. Most commonly, the returns are based on historical returns. Likewise the returns in the scenario set are also based on historical data, such that a feasible set of returns $Y$ is generated. The outline of the chapter is as follows. First the definition of the historical data will be discussed, after which the used data and its source will be described and finally the generation of the trading portfolio data will be outlined.

5.1 Historical data

In order to obtain a robust and optimal portfolio allocation (balance sheet) under uncertainty about the movement in the financial markets, the used scenarios should include all possible events. Thus, periods where the prevailing market trend is upward moving (i.e., bull markets), downward moving (i.e., bear markets) and a period where uncertainty remains about the prevailing market trend. A historical period that includes all these events is the period from January 2003 to October 2012. The period between the dot-com bust and the credit crisis from 2003 to 2007 can be considered a bull market. Followed by the credit crisis from 2007 to 2009, which can be considered a bear market. Finally, the period from 2009 until 2012 can be considered a period with uncertainty about the prevailing market trend.

It is a great advantage that the available data for this period is abundant. On the other hand, as a stylized balance sheet is used for the model, assumptions had to be made for the returns or interest rates on certain instruments in the model. When no return or interest rate was specifically available for an instrument, a reasonable approximation was made in order to reflect the return or interest rate best.

5.1.1 Data used

As the stylized balance sheet is based on the financial statements of three large Dutch banks, it is assumed that the bank in the model is also based in the Netherlands. All instruments on the balance sheet, except for the shareholder’s equity, have a return. These returns are therefore based upon Dutch or European interest rates. The following enumeration lists all instruments in chronological order of $i$ with the corresponding return used in the scenario set.

1. Cash and cash equivalents
European Central Bank (ECB) deposit rate. This is the rate at which financial institutions can make overnight deposits at the ECB.

2. Loans and advances to banks
   6-months EURIBOR\(^{31}\) (Euro interbank offered rate). As the EURIBOR itself is already an average interest rate of a panel of prime banks in the Eurozone, it can be considered an appropriate approximation.

3. Retail loans
   The weighted average interest rate on retail loans of Dutch monetary financial institutions for all three maturities.

4. Corporate loans
   The weighted average interest rate on corporate loans of Dutch monetary financial institutions for all three maturities.

5. Retail mortgages
   The weighted average interest rate on retail mortgages of Dutch monetary financial institutions for all three maturities.

6. Government bonds
   For a group of countries\(^{32}\) the credit rating history over the period 2003 until 2012 is considered. Subsequently, for each month in this period, the average yield on the 10 year government bond of the countries in the specific bucket was calculated (e.g., in March 2010, the credit rating of China (A\(^{+}\)), Czech Republic (A\(^{+}\)), Malta (A\(^{+}\)), Poland (A\(^{+}\)) and Slovakia (A\(^{-}\)) were in the credit bucket A\(^{+}\) to A\(^{-}\), thus the approximation for this bucket is the average of the yields of these countries). This process has been repeated for each month in the period from 2003 to 2012. These four approximations of the average yield of the four credit rating buckets are displayed in figure 5.1.

7. Financial assets held for trading
   Five efficient portfolios (i.e., portfolios on the efficient frontier) with different risk profiles are considered, where risk is measured with CVaR. As banks do not reveal any information about the proprietary trading (financial assets held for trading), a portfolio consisting of equities and commodities is optimized using the mean-CVaR portfolio optimization model described in Chapter 3. Ten different equity indexes are included in

\(^{31}\)The rate at which a prime bank is willing to lend funds in euro to another prime bank. The EURIBOR is calculated daily for interbank deposits with a maturity of one week and one to 12 months as the average of the daily offer rates of a representative panel of prime banks, rounded to three decimal places.

\(^{32}\)Including the following countries: Austria, Belgium, Brazil, Bulgaria, China, Cyprus, Czech Republic, Denmark, Finland, France, Germany, Greece, Hungary, India, Indonesia, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, Mexico, Netherlands, Poland, Portugal, Romania, Russia, Slovakia, Slovenia, Spain, Sweden, United Kingdom and the United States
the trading portfolio: S&P 500 (US), NASDAQ (US), Bovespa (Brazil), Bolsa (Mexico), DAX (Germany), AEX (Netherlands), Micex (Russia), Hang Seng (Hong Kong), Sensex (India) and Kospi (South Korea). Thus, indices from all over the world are considered in the trading portfolio. As a measure for the commodities, the S&P GSCI commodities indices are taken into account: S&P GSCI Energy, S&P GSCI Industrial metals, S&P GSCI Precious metals, S&P GSCI Agriculture and S&P GSCI Livestock.

8. Saving deposits

The weighted average interest rate on saving deposits of Dutch monetary financial institutions for both retail and corporate clients.

9. Demand deposits

The weighted average interest rate on demand deposits of Dutch monetary financial institutions for both short-term and long-term maturities.

10. Debt securities

The average interest rate on commercial paper issued by Dutch monetary financial institutions.

11. Wholesale funding

The S&P GSCI is widely recognized as a leading measure of general price movements and inflation in the world economy. It provides investors with a reliable and publicly available benchmark for investment performance in the commodity markets, and is designed to be a “tradable“ index. The index is calculated primarily on a world production-weighted basis and is comprised of the principal physical commodities that are the subject of active, liquid futures markets.
• The overnight interbank lending rate in the Eurozone is the EONIA\textsuperscript{34}.
• ECB marginal lending facility serves as a source of funding for European financial institutions.
• Repurchase agreements (REPOs) are collateralized lending transactions. The British Bankers Association (BBA) introduced a new benchmark for euro denominated REPO rates. The 6 months Repo rate will serve as a benchmark.

5.1.2 Data source

In the previous, the approximation of the the returns for each balance sheet item is discussed. Consequently, this section will describe the data sources of these returns. As the data sources for many balance sheet items are the same, the data sources will be given with reference to all corresponding $i$.

Firstly, the interest rate on the instruments with $i = \{1, 2, 11\}$ can be obtained at the official website of the ECB. Also the yields on the 10 year government bonds ($i = 6$) for the Eurozone countries were collected from this website. Monthly annualized returns were retrieved from this data source, which comes down to 118 data points per instrument.

Weighted average interest rates of Dutch monetary financial institutions are provided on the website of the Dutch Central Bank (DNB). Thus, the returns on instruments with $i = \{3, 4, 5, 9, 10\}$ are gathered from this website. These are again 118 monthly annualized returns per instrument.

Yields on the 10 year government bonds on the non-Eurozone countries are obtained from Bloomberg\textsuperscript{35}, and are likewise monthly annualized returns. Historical daily returns (i.e., 2456 data points) on the equity and commodity indices ($i = 7$) are also acquired from Bloomberg.

Finally, the credit rating history is based on the long-term sovereign rating published by Fitch Ratings\textsuperscript{36}, updated as from 24 Augustus 2012. Notations of the credit ratings by Fitch Ratings follow the methodology prescribed by Basel regulation.

\textsuperscript{34}A measure of the effective interest rate prevailing in the euro interbank overnight market. It is calculated as a weighted average of the interest rates on unsecured overnight lending transactions denominated in euro, as reported by a panel of contributing banks.

\textsuperscript{35}Bloomberg L.P. is an American multinational mass media corporation, which delivers a vast array of global financial information (e.g., data, analytics, news, communications, and charts)

\textsuperscript{36}Fitch Ratings is a global rating agency dedicated to providing value beyond the rating through independent and prospective credit opinions, research and data
5.2 Scenario generation

As mentioned in the previous sections, all the interest rates (i.e., returns on the interest-bearing asset and liability instruments) constitutes of annualized monthly data. However, daily returns are considered for the assets held for trading because for each month in the data period, five efficient portfolios will be estimated using the mean-CVaR portfolio optimization model defined in Chapter 3. The assumption is made that the markets trading these assets are highly liquid, so each month a completely different portfolio than the previous month can be created without transaction costs.

These efficient portfolios are based upon daily returns of the specific month, but around 20 returns (i.e., the number of business in one month) is too scarce to optimize. Therefore returns will be generated based on the returns of the particular month. One of the most important requirements is that the returns remain correlated. The difficulty in estimating correlation between different assets is that the correlation is not constant nor stable over time. In fact, numerous empirical studies have confirmed that correlation increases in times of stress (i.e., high volatility). Correlation measured over the whole historical data may miss changes in risk unless the method is carefully designed to update estimates rapidly.

5.2.1 Trading portfolio optimization

This section will discuss the generation of the five efficient portfolios for each month in the data period. The first step is to obtain the conditional correlation matrix of the 15 assets in the trading portfolio. An econometric model capable of measuring correlations over time is the dynamic conditional correlation (DCC) GARCH (Generalized Autoregressive Heteroskedasticity) model by Engle (2002). This model first estimates the univariate volatility for each individual asset, then constructs standardized residuals (i.e., returns divided by conditional standard deviations) and finally estimates the correlations between the standardized residuals. The calculations are executed in the software program R, which is a language and environment for statistical computing and graphics. Two available packages, RUGARCH by Ghahanos (2012) and CCGARCH by Nakatani (2010), are used to calculate the correlation matrix for each month using the DCC GARCH model. As an indication to show that the correlation is in fact not constant over time, the correlation between the S&P 500 and respectively the NASDAQ, AEX, Hang Seng, S&P GSCI industrial metals and S&P GSCI precious metals are displayed in Figure 5.2.
It is clearly visible that the correlation between the S&P 500 and the four other assets moves around a certain level, but shifts slightly up or down according to market movements. Note that the correlation is more volatile in times of stress (2008) in accordance with the theory.

Subsequently, the next step is to use the generated weekly correlation matrices to generate correlated returns for each month. This step requires the assumption that the returns are Normally distributed in order to be able to generate enough returns to optimize over. For each month the mean and the standard deviation of the asset returns of the previous month are used to generate 1000 returns from the Normal distribution for each asset. However, these returns are not correlated as the draws from the Normal distribution are independent. A common method to create correlated returns is to multiply the Cholesky decomposition of the correlation matrix by the generated uncorrelated returns. The Cholesky decomposition is constructed from the correlation matrix of the last week from the previous month is used. This process is repeated for each month in the period from January 2003 to October 2012.

The last step is to optimize the trading portfolio using the mean-CVaR optimization model over the generated correlated returns of each month in order to obtain the efficient frontier. In order to calculate the efficient frontier, first the portfolio with the least amount of risk (CVaR) is calculated, which serves as the benchmark of the efficient portfolio for the absolute risk averse investor. Secondly, the portfolio with the maximum possible return is calculated, which serves as the benchmark for the efficient portfolio for the absolute risk seeking investor. Finally, a sequence of 18 required returns between the two portfolios is obtained. For each required return in the sequence, the efficient portfolio with the minimal CVaR given the required return is calculated. These 20 portfolio solutions now represent the efficient frontier. To illustrate
this optimization process, the calculated efficient frontier for February 2010 is displayed in Figure 5.3.

Figure 5.3: Efficient frontier of mean-CVaR optimization for February 2010

Here the return on the y-axis is the annualized return and the risk on the x-axis stands the risk weight (i.e. the 10-day 99% CVaR measure multiplied by the sum of the multiplication factors $m_s$ and $m_c$). In addition to the plotted efficient frontier, the portfolios of investing in the individual assets are plotted. It is clearly visible that investing in a portfolio on the efficient frontier is more favorable than investing in an individual asset, as the return can be the same but the risk weight is always smaller through effect of risk diversification.

5.2.2 Scenario set $Y$

Consequently, all 118 scenarios in the scenario set $Y$ consist of returns on all asset and liability instruments and the five risk weight measures corresponding to the five efficient portfolios. Indicatively, the returns on the asset instruments $i = \{1, 2, 3, 4, 5\}$ are displayed in Figure 5.4.
it follows that all interest rates on asset instruments dropped significantly after the credit crisis. In addition, the highest return is generated by retail loans and over time the return on retail mortgages is the most stable, due to the long maturity.

As an indication of the returns on the liability instruments $i = \{8, 9, 10, 11\}$, these interest rates over the whole data period are displayed in Figure 5.5. The trend of declining interest rates after the credit crisis also applies to the liability instruments. Until 2009, demand deposits were the cheapest form of funding for financial institutions, as the interest rate was by far the lowest among all liability instruments. However, in the period after 2009, the interest rate on overnight interbank lending, debt securities and the REPO rate declined sharply, such that the three liability instruments in addition with demand deposits became the cheapest from of funding for financial institutions.

Figure 5.4: Returns on the asset instruments from January 2003 to October 2012
Figure 5.5: Returns on the liability instruments from January 2003 to October 2012
CHAPTER 6

Results

This chapter discusses the results of this research project. Using the models described in Chapter 4 and the scenario set defined in Chapter 5, three different types of banks will be considered. The first bank will be a large bank involved in both commercial banking and in investment banking. Secondly, the balance sheet of primary a retail bank will be optimized. Lastly, a bank primary focused on investment banking will be optimized. This gives a good indication of the risks and returns for the different types of banks.

To be able to produce comparable results, it is necessary to choose similar default values for the positions in shareholders’ equity among the three test cases. Other constants do change among the different types of banks and will be discussed later this chapter.

The outline of the remainder of this chapter is as follows. In the first place, the reasoning behind the initial portfolio weights and the other constants in the model will be discussed. Secondly, the results of the optimization for the three different types of banks are given. In the last place, the difference between the three types of banks is briefly discussed.

6.1 Test cases

As mentioned earlier, three different balance sheets for banks involved in different banking activities will be optimized to obtain a good overview of the effects of these banking activities on return and risk. The starting balance sheets will be based on the annual reports of three large banks in the Netherlands and will be adjusted such that more emphasize is placed on the different banking activities.

The starting weights of the portfolio instruments for the different types of banks are displayed in Figure 6.1. It follows from this figure that the large bank is involved in both interest bearing instruments as well as the trading instruments, whereas the focus for the retail bank and investment bank lies mainly on the interest bearing instruments or the trading instruments.

Subsequently, the starting weights for the liability instruments, also displayed in Figure 6.1, highlights the difference between the three banks. A retail bank is expected to have more exposure to saving deposits and demand deposits, whereas the investment bank will be more involved in debt securities and wholesale funding. The initial weights to the liability instruments for the large bank will be more diversified.
Figure 6.1: Initial weights of the portfolio instruments for the three different banks
An interested reader can find the exact weights to all portfolio instruments as well as the upper and lower bounds for the three different types of banks in Appendix A. It should be noted that these weights remain an approximation and can easily be adjusted by a bank using this model.

Different banking activities among the three different banks have an impact on the constants for other revenues and expenses that are used in the model. For instance, investment banking activity generates higher fee and commission income (e.g. raising debt and equity financing for corporations and governments and advice on mergers and acquisitions), whereas commercial banks are less active in this field and generate lesser of this kind of income. However, on the other hand the staff expenses are relatively lower for retail banks, as investment banks give out large bonuses to their employees. The constants for the other revenues and expenses are given in Table 6.1. Also the positions in the shareholders’ equity are given in this table for the different types of banks. But in contrast to the other revenues and expenses these are considered equal among the three different banks to obtain comparable results.

<table>
<thead>
<tr>
<th>Table 6.1: Constants in the model for the three different test cases</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Shareholders’ equity</strong></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td><img src="" alt="Table" /></td>
</tr>
<tr>
<td><strong>Other revenues and expenses</strong></td>
</tr>
<tr>
<td><img src="" alt="Table" /></td>
</tr>
<tr>
<td><strong>Tax and dividend policy</strong></td>
</tr>
<tr>
<td><img src="" alt="Table" /></td>
</tr>
</tbody>
</table>

As the Basel III regulation will be transitionally implemented, the liquidity ratios do not yet apply to financial institutions in 2013. Therefore two optimizations will be executed per test case, namely one already taking the LCR and NSFR constraints into account and the other optimization will discard the two constraints. Thus, the second optimization can be considered more conservative.

Each test case will be build up in the same manner. For each of the two optimizations in the
test case, first the solution to the mean portfolio optimization problem is calculated. This gives the highest possible mean retained earnings over all scenarios, \( R_{\text{max}} \). Subsequently, the 90% CVaR portfolio optimization problem without constraints on required retained earnings will be calculated in order to obtain the lower bound on the mean retained earnings, \( R_{\text{min}} \). Finally a sequence of required mean retained earnings (\( R \)) on the interval \([R_{\text{min}}, R_{\text{max}}]\) is created. Then for each required mean retained earnings value in the sequence, the 90% CVaR portfolio optimization is solved. This gives the efficient frontier of the optimal balance sheet with the RoE on the y-axis and the 90%-CVaR on the x-axis.

6.1.1 Test case 1: Large bank

The large bank is involved in both commercial and investment banking. Therefore the retained earnings are generated by fee and commission income, interest income and trading income. Consequently, it could be assumed beforehand that the income is more diversified than the other two types of banks and thus the 90% conditional Value-at-Risk will presumably be the lowest among the three banks.

The results of the optimization, including the liquidity ratio constraints, are given in Table 6.2. It follows that the initial portfolio is far from optimal, as the retained earnings are almost twice as high and the Basel ratios also improve significantly. However, the difference of the in the capital ratios among the optimized portfolios is relatively small. This is because higher retained earnings are accompanied with taking on more risk (i.e., higher RWA), which is the denominator for the capital ratios. This makes that even though the retained earnings do increase, the capital ratio only increases slightly. The most interesting fact from Table 6.2 is the negative CVaR measures. This means that the conditional expected loss is negative, i.e., the expectation of the retained earnings over the worst scenarios is positive. Another result is that the NSFR decreases for the portfolios with higher risk taking.
Table 6.2: Balance sheet statistics for large bank including LCR and NSFR

<table>
<thead>
<tr>
<th></th>
<th>$x^0$</th>
<th>$p = 1$</th>
<th>$p = 2$</th>
<th>$p = 3$</th>
<th>$p = 4$</th>
<th>$p = 5$</th>
<th>$p = 6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{e}_4$</td>
<td>0.00809</td>
<td>0.00811</td>
<td>0.00835</td>
<td>0.00859</td>
<td>0.00882</td>
<td>0.00906</td>
<td>0.00918</td>
</tr>
<tr>
<td>90% CVaR</td>
<td>-0.00093</td>
<td>-0.00673</td>
<td>-0.00647</td>
<td>-0.00594</td>
<td>-0.00518</td>
<td>-0.00402</td>
<td>-0.00171</td>
</tr>
<tr>
<td>RoE</td>
<td>9.28%</td>
<td>16.86%</td>
<td>17.27%</td>
<td>17.67%</td>
<td>18.07%</td>
<td>18.47%</td>
<td>18.67%</td>
</tr>
<tr>
<td>CET1R</td>
<td>4.68%</td>
<td>5.44%</td>
<td>5.69%</td>
<td>5.68%</td>
<td>5.69%</td>
<td>5.71%</td>
<td>5.70%</td>
</tr>
<tr>
<td>T1R</td>
<td>7.75%</td>
<td>8.39%</td>
<td>8.73%</td>
<td>8.70%</td>
<td>8.67%</td>
<td>8.67%</td>
<td>8.65%</td>
</tr>
<tr>
<td>TCR</td>
<td>10.81%</td>
<td>11.33%</td>
<td>11.78%</td>
<td>11.71%</td>
<td>11.66%</td>
<td>11.63%</td>
<td>11.60%</td>
</tr>
<tr>
<td>RWA</td>
<td>0.40772</td>
<td>0.42458</td>
<td>0.41040</td>
<td>0.41491</td>
<td>0.41883</td>
<td>0.42175</td>
<td>0.42401</td>
</tr>
<tr>
<td>LCR</td>
<td>31.55%</td>
<td>100.00%</td>
<td>100.00%</td>
<td>100.00%</td>
<td>100.00%</td>
<td>100.00%</td>
<td>100.00%</td>
</tr>
<tr>
<td>NSFR</td>
<td>91.43%</td>
<td>114.67%</td>
<td>117.54%</td>
<td>116.38%</td>
<td>114.71%</td>
<td>112.44%</td>
<td>108.27%</td>
</tr>
<tr>
<td>LR</td>
<td>3.16%</td>
<td>3.56%</td>
<td>3.58%</td>
<td>3.61%</td>
<td>3.63%</td>
<td>3.66%</td>
<td>3.67%</td>
</tr>
</tbody>
</table>

Note: here $p = 1$ denotes the portfolio with $R_{min}$ and $p = 6$ represents solution to the mean portfolio optimization problem. The portfolios $p = \{2, 3, 4, 5\}$ have an increasing required mean retained earnings ($R$) on the interval $[R_{min}, R_{max}]$.

The efficient frontier of the optimization of the large bank including the LCR and NSFR is displayed in Figure 6.6. It clearly shows higher return on equity is accompanied with higher risk. As all portfolios on the efficient frontier are optimal by definition, the chosen portfolio (i.e., balance sheet composition) depends on the risk appetite of the large bank.

Figure 6.2: Efficient frontier for the large bank including LCR and NSFR

Note: see note at Table 6.2.

Secondly, an optimization without taking the LCR and the NSFR constraints into account is executed. As these liquidity ratios are not yet required in 2013 by Basel, a bank could prefer...
to make higher retained earnings over being already compliant with these new liquidity ratios. Table 6.3 gives the results for this optimization problem.

**Table 6.3: Balance sheet statistics for large bank excluding LCR and NSFR**

<table>
<thead>
<tr>
<th></th>
<th>$x^0$</th>
<th>$p = 1$</th>
<th>$p = 2$</th>
<th>$p = 3$</th>
<th>$p = 4$</th>
<th>$p = 5$</th>
<th>$p = 6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{e}_4$</td>
<td>0.00409</td>
<td>0.00819</td>
<td>0.00846</td>
<td>0.00873</td>
<td>0.00900</td>
<td>0.00928</td>
<td>0.00941</td>
</tr>
<tr>
<td>90% CVaR</td>
<td>-0.00093</td>
<td>-0.00693</td>
<td>-0.00675</td>
<td>-0.00632</td>
<td>-0.00561</td>
<td>-0.00438</td>
<td>-0.00193</td>
</tr>
<tr>
<td>RoE</td>
<td>9.28%</td>
<td>17.00%</td>
<td>17.46%</td>
<td>17.92%</td>
<td>18.38%</td>
<td>18.82%</td>
<td>19.05%</td>
</tr>
<tr>
<td>CET1R</td>
<td>4.68%</td>
<td>4.87%</td>
<td>5.26%</td>
<td>5.31%</td>
<td>5.32%</td>
<td>5.34%</td>
<td>5.34%</td>
</tr>
<tr>
<td>T1R</td>
<td>7.75%</td>
<td>7.49%</td>
<td>8.06%</td>
<td>8.11%</td>
<td>8.08%</td>
<td>8.09%</td>
<td>8.07%</td>
</tr>
<tr>
<td>TCR</td>
<td>10.81%</td>
<td>10.12%</td>
<td>10.86%</td>
<td>10.91%</td>
<td>10.85%</td>
<td>10.84%</td>
<td>10.81%</td>
</tr>
<tr>
<td>RWA</td>
<td>0.40772</td>
<td>0.47634</td>
<td>0.44614</td>
<td>0.44678</td>
<td>0.45158</td>
<td>0.45476</td>
<td>0.45714</td>
</tr>
<tr>
<td>LCR</td>
<td>31.55%</td>
<td>57.65%</td>
<td>56.62%</td>
<td>56.19%</td>
<td>56.31%</td>
<td>56.44%</td>
<td>56.50%</td>
</tr>
<tr>
<td>NSFR</td>
<td>91.43%</td>
<td>105.55%</td>
<td>108.64%</td>
<td>109.29%</td>
<td>107.81%</td>
<td>105.70%</td>
<td>101.79%</td>
</tr>
<tr>
<td>LR</td>
<td>3.16%</td>
<td>3.57%</td>
<td>3.60%</td>
<td>3.62%</td>
<td>3.65%</td>
<td>3.68%</td>
<td>3.69%</td>
</tr>
</tbody>
</table>

*Note: see note at Table 6.2.*

It appears that leaving the LCR and NSFR constraint out of the model results in higher retained earnings, however the CVaR measure increases and the capital ratios drop slightly. Nonetheless, these ratios are still adequate, such that the optimal portfolios are still in compliance with Basel III in 2013. The difference in retained earnings between the two portfolio optimizations is better visible in Figure 6.3. Depending on the risk appetite and the choice of whether or not to include the liquidity ratios, the optimal balance sheet composition for a large bank lies on one of the two efficient frontiers.

**Figure 6.3: Efficient frontiers of the two optimizations for the large bank**
6.1.2 Test case 2: Retail bank

A retail bank is mainly focused on commercial banking, but is not refrained from investment banking activities. Therefore it is assumed that at least 80% of their total balance sheets is invested in interest bearing asset instruments. Consequently, the constraint in Equation 6.1 has to be added to the model.

\[
x_1 + x_2 + \sum_{i=3}^{5} \sum_{m \in M} x_{i,m} + \sum_{c \in C} x_{6,c} \geq 80\%
\]  

(6.1)

The results of the optimization, including the LCR and NSFR constraint, are given in Table 6.4. Like the optimization for the large bank, it follows that the initial portfolio weights are far from optimal. In the optimal portfolios, more weight is placed on asset instruments with higher risk weights according to the higher RWA. Consequently, this generates significant higher retained earnings, while the Basel ratios do not drop. Just as the results for the large bank, the most striking result is the negative CVaR measures. Even the CVaR measure for the solution to the mean portfolio optimization problem \((p = 6)\) is lower than the CVaR measure of the initial portfolio. Thus, aside from significant higher retained earnings and the compliance to the Basel constraints, the risk is reduced on top of that.

Table 6.4: Balance sheet statistics for retail bank including LCR and NSFR

<table>
<thead>
<tr>
<th></th>
<th>(x^0)</th>
<th>(p = 1)</th>
<th>(p = 2)</th>
<th>(p = 3)</th>
<th>(p = 4)</th>
<th>(p = 5)</th>
<th>(p = 6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\bar{e}_4)</td>
<td>0.00442</td>
<td>0.00985</td>
<td>0.01004</td>
<td>0.01024</td>
<td>0.01044</td>
<td>0.01064</td>
<td>0.01074</td>
</tr>
<tr>
<td>90% CVaR</td>
<td>-0.00358</td>
<td>-0.00832</td>
<td>-0.00810</td>
<td>-0.00765</td>
<td>-0.00687</td>
<td>-0.00598</td>
<td>-0.00425</td>
</tr>
<tr>
<td>RoE</td>
<td>9.95%</td>
<td>19.75%</td>
<td>20.07%</td>
<td>20.39%</td>
<td>20.70%</td>
<td>21.01%</td>
<td>21.16%</td>
</tr>
<tr>
<td>CET1R</td>
<td>4.52%</td>
<td>4.85%</td>
<td>4.85%</td>
<td>4.86%</td>
<td>4.96%</td>
<td>5.09%</td>
<td>4.92%</td>
</tr>
<tr>
<td>T1R</td>
<td>7.43%</td>
<td>7.28%</td>
<td>7.28%</td>
<td>7.26%</td>
<td>7.40%</td>
<td>7.57%</td>
<td>7.31%</td>
</tr>
<tr>
<td>TCR</td>
<td>10.34%</td>
<td>9.72%</td>
<td>9.70%</td>
<td>9.67%</td>
<td>9.84%</td>
<td>10.05%</td>
<td>9.70%</td>
</tr>
<tr>
<td>RWA</td>
<td>0.42971</td>
<td>0.51269</td>
<td>0.51591</td>
<td>0.51968</td>
<td>0.51246</td>
<td>0.50384</td>
<td>0.52328</td>
</tr>
<tr>
<td>LCR</td>
<td>31.13%</td>
<td>100.00%</td>
<td>100.00%</td>
<td>100.00%</td>
<td>100.00%</td>
<td>100.00%</td>
<td>100.00%</td>
</tr>
<tr>
<td>NSFR</td>
<td>100.76%</td>
<td>113.22%</td>
<td>113.51%</td>
<td>112.29%</td>
<td>110.83%</td>
<td>109.29%</td>
<td>106.13%</td>
</tr>
<tr>
<td>LR</td>
<td>3.19%</td>
<td>3.73%</td>
<td>3.75%</td>
<td>3.77%</td>
<td>3.79%</td>
<td>3.81%</td>
<td>3.82%</td>
</tr>
</tbody>
</table>

*Note:* see note at Table 6.2.

Figure 6.4 displays the efficient frontier of the optimization for the retail bank. Again the bank using the model has to decide, depending on its risk appetite, which portfolio on the efficient frontier fits their risk profile best.
Again also the optimization process without the liquidity constraints is executed. The results for this optimization are displayed in Table 6.5. Compared to the optimization including the liquidity constraints, there is little improvement in the new results. Likewise to the other optimization, the CVaR measures are negative. An interesting fact from Table 6.5 is that the NSFR constraint is met in most cases, although the constraint was not included in the optimization.

Table 6.5: Balance sheet statistics for retail bank excluding LCR and NSFR

<table>
<thead>
<tr>
<th></th>
<th>$x^0$</th>
<th>$p = 1$</th>
<th>$p = 2$</th>
<th>$p = 3$</th>
<th>$p = 4$</th>
<th>$p = 5$</th>
<th>$p = 6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{e}_4$</td>
<td>0.00442</td>
<td>0.00991</td>
<td>0.01017</td>
<td>0.01043</td>
<td>0.01069</td>
<td>0.01095</td>
<td>0.01108</td>
</tr>
<tr>
<td>90% CVaR</td>
<td>-0.00358</td>
<td>-0.00859</td>
<td>-0.00847</td>
<td>-0.00797</td>
<td>-0.00708</td>
<td>-0.00576</td>
<td>-0.00461</td>
</tr>
<tr>
<td>RoE</td>
<td>9.95%</td>
<td>19.86%</td>
<td>20.28%</td>
<td>20.69%</td>
<td>21.09%</td>
<td>21.50%</td>
<td>21.70%</td>
</tr>
<tr>
<td>CET1R</td>
<td>4.52%</td>
<td>4.52%</td>
<td>4.60%</td>
<td>4.61%</td>
<td>4.63%</td>
<td>4.65%</td>
<td>4.65%</td>
</tr>
<tr>
<td>T1R</td>
<td>7.43%</td>
<td>6.79%</td>
<td>6.89%</td>
<td>6.88%</td>
<td>6.88%</td>
<td>6.89%</td>
<td>6.88%</td>
</tr>
<tr>
<td>TCR</td>
<td>10.34%</td>
<td>9.06%</td>
<td>9.18%</td>
<td>9.15%</td>
<td>9.14%</td>
<td>9.13%</td>
<td>9.10%</td>
</tr>
<tr>
<td>RWA</td>
<td>0.42971</td>
<td>0.55076</td>
<td>0.54665</td>
<td>0.55114</td>
<td>0.55482</td>
<td>0.55786</td>
<td>0.56112</td>
</tr>
<tr>
<td>LCR</td>
<td>31.13%</td>
<td>52.42%</td>
<td>52.14%</td>
<td>52.00%</td>
<td>52.12%</td>
<td>52.25%</td>
<td>47.12%</td>
</tr>
<tr>
<td>NSFR</td>
<td>100.76%</td>
<td>106.19%</td>
<td>106.37%</td>
<td>106.01%</td>
<td>104.43%</td>
<td>102.35%</td>
<td>99.94%</td>
</tr>
<tr>
<td>LR</td>
<td>3.19%</td>
<td>3.74%</td>
<td>3.77%</td>
<td>3.79%</td>
<td>3.82%</td>
<td>3.85%</td>
<td>3.86%</td>
</tr>
</tbody>
</table>

Note: see note at Table 6.2.

A comparison between the two optimizations is best visible in Figure 6.5. Just as for the large bank, the efficient frontier of the optimization without the liquidity constraints lies slightly left and above the efficient frontier of the optimization including the LCR and NSFR.
6.1.3 Test case 3: Investment bank

Besides investment banking activities, an investment bank is also involved in traditional banking business. Therefore, again an extra constraint is presented in Equation 6.2. This restriction ensures that the trading portfolio of the investment bank is equal or greater than 40% of the total balance sheet size. The trading book can be considered the most volatile investment instrument on the balance sheet, therefore it is assumed beforehand that the results for the CVaR measures will be significantly higher than for the other types of banks.

\[
\sum_{p \in P} x_{\tau,p} \geq 40\% \quad (6.2)
\]

The results of the optimization including the liquidity constraints are displayed in Table 6.6. In contrast to the other two banking types, the CVaR measures for all optimized portfolios is positive. Another interesting result is that it is impossible for an investment bank to comply to the NSFR constraint. As expected, the mean retained earnings are the highest for optimized portfolios of the investment bank among the three types of banks. In return the risks (i.e., CVaR measures) are also significantly higher.
Table 6.6: Balance sheet statistics for investment bank including LCR and NSFR

<table>
<thead>
<tr>
<th></th>
<th>$x^0$</th>
<th>$p = 1$</th>
<th>$p = 2$</th>
<th>$p = 3$</th>
<th>$p = 4$</th>
<th>$p = 5$</th>
<th>$p = 6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{e}_4$</td>
<td>0.00621</td>
<td>0.01222</td>
<td>0.01224</td>
<td>0.01226</td>
<td>0.01228</td>
<td>0.01230</td>
<td>0.01230</td>
</tr>
<tr>
<td>90% CVaR</td>
<td>0.00917</td>
<td>0.00491</td>
<td>0.00513</td>
<td>0.00539</td>
<td>0.00565</td>
<td>0.00592</td>
<td>0.00605</td>
</tr>
<tr>
<td>RoE</td>
<td>13.44%</td>
<td>23.41%</td>
<td>23.43%</td>
<td>23.46%</td>
<td>23.49%</td>
<td>23.51%</td>
<td>23.52%</td>
</tr>
<tr>
<td>CET1R</td>
<td>5.08%</td>
<td>5.70%</td>
<td>5.72%</td>
<td>5.75%</td>
<td>5.78%</td>
<td>5.80%</td>
<td>5.82%</td>
</tr>
<tr>
<td>T1R</td>
<td>8.07%</td>
<td>8.32%</td>
<td>8.34%</td>
<td>8.38%</td>
<td>8.42%</td>
<td>8.46%</td>
<td>8.48%</td>
</tr>
<tr>
<td>TCR</td>
<td>11.06%</td>
<td>10.93%</td>
<td>10.97%</td>
<td>11.02%</td>
<td>11.07%</td>
<td>11.12%</td>
<td>11.14%</td>
</tr>
<tr>
<td>RWA</td>
<td>0.41789</td>
<td>0.47942</td>
<td>0.47808</td>
<td>0.47608</td>
<td>0.47408</td>
<td>0.47208</td>
<td>0.47108</td>
</tr>
<tr>
<td>LCR</td>
<td>17.32%</td>
<td>115.81%</td>
<td>115.91%</td>
<td>116.31%</td>
<td>116.72%</td>
<td>117.12%</td>
<td>117.32%</td>
</tr>
<tr>
<td>NSFR</td>
<td>57.58%</td>
<td>96.49%</td>
<td>96.48%</td>
<td>96.48%</td>
<td>96.48%</td>
<td>96.47%</td>
<td>96.47%</td>
</tr>
<tr>
<td>LR</td>
<td>3.37%</td>
<td>3.97%</td>
<td>3.97%</td>
<td>3.98%</td>
<td>3.98%</td>
<td>3.98%</td>
<td>3.98%</td>
</tr>
</tbody>
</table>

Note: see note at Table 6.2.

The efficient frontier of the optimization including the liquidity constraints is displayed in Figure 6.6. Compared to the other two types banks, the efficient frontier is a more straight line. Also the difference in retained earnings and CVaR measures is smaller for the optimized portfolios of the investment bank.

![Figure 6.6: Efficient frontier for the investment bank including LCR and NSFR](image)

Note: see note at Table 6.6.

Again the optimization is repeated without the liquidity constraints. These results are displayed in Table 6.7. It follows that excluding the LCR and NSFR improves the statistics of the portfolios considerably. However, considering that the LCR will be introduced as of 2013, it
appears to be unwise not to take the LCR constraint into account. Because the gains of the higher retained earnings will probably not outweigh the losses inflicted by decomposing the balance sheet next year to comply to the LCR.

Table 6.7: Balance sheet statistics for investment bank excluding LCR and NSFR

<table>
<thead>
<tr>
<th></th>
<th>$x^0$</th>
<th>$p = 1$</th>
<th>$p = 2$</th>
<th>$p = 3$</th>
<th>$p = 4$</th>
<th>$p = 5$</th>
<th>$p = 6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{e}_4$</td>
<td>0.00621</td>
<td>0.01392</td>
<td>0.01414</td>
<td>0.01436</td>
<td>0.01458</td>
<td>0.01480</td>
<td>0.01490</td>
</tr>
<tr>
<td>90% CVaR</td>
<td>0.00917</td>
<td>0.00316</td>
<td>0.00381</td>
<td>0.00473</td>
<td>0.00601</td>
<td>0.01054</td>
<td>0.01432</td>
</tr>
<tr>
<td>RoE</td>
<td>13.44%</td>
<td>25.81%</td>
<td>26.11%</td>
<td>26.41%</td>
<td>26.71%</td>
<td>27.00%</td>
<td>27.15%</td>
</tr>
<tr>
<td>CET1R</td>
<td>5.08%</td>
<td>5.03%</td>
<td>5.00%</td>
<td>5.04%</td>
<td>5.18%</td>
<td>5.52%</td>
<td>5.60%</td>
</tr>
<tr>
<td>T1R</td>
<td>8.07%</td>
<td>7.20%</td>
<td>7.15%</td>
<td>7.18%</td>
<td>7.37%</td>
<td>7.83%</td>
<td>7.94%</td>
</tr>
<tr>
<td>TCR</td>
<td>11.06%</td>
<td>9.37%</td>
<td>9.29%</td>
<td>9.33%</td>
<td>9.55%</td>
<td>10.15%</td>
<td>10.28%</td>
</tr>
<tr>
<td>RWA</td>
<td>0.41789</td>
<td>0.57743</td>
<td>0.58475</td>
<td>0.58473</td>
<td>0.57329</td>
<td>0.54204</td>
<td>0.53617</td>
</tr>
<tr>
<td>LCR</td>
<td>17.32%</td>
<td>20.96%</td>
<td>14.40%</td>
<td>9.89%</td>
<td>9.90%</td>
<td>9.77%</td>
<td>9.78%</td>
</tr>
<tr>
<td>NSFR</td>
<td>57.58%</td>
<td>77.42%</td>
<td>76.05%</td>
<td>74.74%</td>
<td>73.70%</td>
<td>72.48%</td>
<td>72.48%</td>
</tr>
<tr>
<td>LR</td>
<td>3.37%</td>
<td>4.14%</td>
<td>4.16%</td>
<td>4.19%</td>
<td>4.21%</td>
<td>4.23%</td>
<td>4.24%</td>
</tr>
</tbody>
</table>

*Note: see note at Table 6.2.*

Figure 6.7 displays the efficient frontiers for both optimizations. It follows from this figure that not taking the liquidity constraints into account, considerably increases the ability of the bank to apply its risk profile better. However, for the same reasons as mentioned above, it appears unwise to exclude the LCR constraint from the model.

Figure 6.7: Efficient frontiers of the two optimizations for the investment bank
The exact portfolio weights of each optimization for all three test cases can be found in Appendix B.

6.2 Comparison of the test three cases

The difference between the three types of banks is already emphasized by the trade-off between risk and return (i.e., RoE versus the CVaR measure). However, the difference is even more obvious when comparing the histograms of the retained earnings for each type of bank. The histogram of the retained earnings for the portfolio with the least amount of risk ($p = 1$) for the large bank, retail bank, and investment bank are respectively displayed in Table 6.8, Table 6.9 and Table 6.10.

![Figure 6.8: Histogram of the retained earnings for the large bank](image1)

![Figure 6.9: Histogram of the retained earnings for the retail bank](image2)
Figure 6.10: Histogram of the retained earnings for the investment bank

It follows from the three histograms that the distribution of the retained earnings for the investment bank has far more downside tail risk than the other two types of banks. These downside losses are compensated by the upside returns, such that the expected retained earnings are the highest among the three types of banks. Thus, in periods where the prevailing market trend is upward moving (i.e., bull markets), it is optimal for banks to be involved in investment banking activities. However, in periods where the prevailing trend is downward moving (i.e., bear markets), large losses may be suffered. This is in accordance with what happened during the credit crisis in 2007-2008, when investment banks suffered far more from the crisis than the other two types of banks. These results indicate that the model is capable of optimizing a stylized balance sheet of any type of bank.
CHAPTER 7

Conclusion

This thesis started with the goal to set up a model capable of optimizing the balance sheet of any type of bank. A balance sheet of a bank is a complex statement that has to satisfy many requirements set forth by the Basel accords. The most recent Basel accord, Basel III, imposed a set of new requirements for banks. Therefore, banks are seeking for tools and models to be in compliance with the Basel III regulation, while still maximizing their profits. The model proposed in this thesis can be used by every type of bank as a top-down strategic balance sheet management tool to obtain an optimal balance sheet allocation.

7.1 Bank balance sheet optimization model

The difficulty in finding a model to be used for strategic balance sheet management under Basel III lies in the complexity of the problem. Banks are involved in different banking business activities that are subjected to different types of risks. Many different types of approaches were attempted throughout the research of this thesis. In the end, the portfolio modeling approach proved to be the approach capable of tackling the problem. Eventually, a CVaR portfolio optimization approach is used in the final model. The big advantage of this approach is that besides meeting all the Basel III constraints, the portfolio risk can be minimized, while a minimum level of retained earnings is met. This section will briefly discuss the most important findings during the research of this thesis.

Scenarios set

As the scenario set is based on the historical data over the last 10 years, every possible prevailing market trend is taken into consideration. This mean that the model is optimized over all possible events. Consequently, the model is able to generate a robust and optimal portfolio allocation.

Optimization objective

Two different types of optimization objectives are proposed in this thesis. Firstly, it is possible maximize the expected retained earnings over all scenarios. This portfolio allocation can serve as an upper bound to obtain the highest expected retained earnings, while disregarding the risk of that portfolio allocation. Secondly, the conditional expected loss at a certain confidence level can be minimized, while meeting a minimum amount of required retained earnings.
Consequently, when both optimization objectives are combined, it is possible to generate an efficient frontier. This gives the bank the opportunity to choose a portfolio allocation that fits its risk appetite.

**Modeling**

The difficulty in modeling the balance sheet is the non-linearity of a number of Basel III constraints. Using the NLOPT package in the software program R for statistical computing, proved to be the best way to find the optimal portfolio solution.

Another important feature of the model is that the objective values of the constraints by Basel III can be easily adjusted, such that a more conservative bank can choose to use higher objective values. Thereby reducing the risk of the portfolio allocation.

A balance sheet of a bank consists of many different types of instruments. The big advantage of the model is that additional portfolio instruments can be added very easily. Therefore it is possible to fully adjust the model to any specific bank. Even with 28 portfolio instruments considered on the stylized balance sheet, the model takes approximately 20 minutes to calculate the optimal solution.

**Model performance**

The optimization results presented in the previous chapter confirms that the model is capable of optimizing the balance sheet for different types of banks. The model was tested on the balance sheet of a large bank, retail bank and investment bank. The initial portfolio weights were based upon the annual reports of three of the largest Dutch banks. The optimization results showed that the model was able to generate far better portfolio allocations compared to the initial allocations for each different type of bank. The retained earnings nearly doubled for all three banks, while the portfolio risk was reduced and the Basel ratios also improved slightly.

Another interesting fact from the results in the previous chapter is that a bank primarily involved in investments banking activities will have difficulties with meeting the NSFR, that will be introduced in 2018. As the proposed model was unable to meet the NSFR constraint for the investment bank in the previous chapter.

**7.2 Further research**

The number of other research projects on models for strategic balance sheet management under Basel III is scarce. Therefore it is hard to compare the model to other existing models. But
the fact that the presented model is this thesis is capable of optimizing the balance sheet of any type of bank is a very big result itself.

Further research could be directed to further expanding the model proposed in this thesis. For instance, the expansion of the model to a multi-period optimization problem instead of a single-period optimization problem. A lot of successful research has been done on multi-period CVaR portfolio optimization. It would be optimal to already take the requirements for 2019 into account, as Basel III is transitionally implemented from 2012 to 2019; gradually re-composing the balance sheet will be less costly and will have smaller impact on the overall performance of the bank.

Secondly, the standardized approach is used to calculate the RWA for credit risk. Alternatively, the internal ratings based (IRB) approach could be considered in order get a more accurate calculation of the RWA. Consequently, the basic indicator approach is considered to calculate the RWA for market risk in the proposed model. It could make the model more accurate when a more advanced approach is used.

In the last place, transaction costs could be taken into account to make the solution more realistic. Large shifts in portfolio weights now have no influence on the retained earnings in the model. However, when offsetting large positions of certain portfolio instruments, a haircut should be taken into account to make the model more realistic.
BIBLIOGRAPHY


BCBS (1996), ‘Amendment to the capital accord to incorporate market risks’, BIS publication.


APPENDIX A
Starting values

A.1 Test case 1: large bank

The initial portfolio weights, the lower bounds and the upper bounds are given by

\[
\begin{pmatrix}
  x_1^0 \\
x_2^0 \\
\vdots \\
x_n^0
\end{pmatrix}
= 
\begin{pmatrix}
  0.0200 \\
0.0500 \\
\vdots \\
0.0200
\end{pmatrix}
\quad \begin{pmatrix}
  lb_1 \\
  lb_2 \\
\vdots \\
  lb_n
\end{pmatrix}
= 
\begin{pmatrix}
  0.0067 \\
0.0500 \\
\vdots \\
0.0067
\end{pmatrix}
\quad \begin{pmatrix}
  ub_1 \\
  ub_2 \\
\vdots \\
  ub_n
\end{pmatrix}
= 
\begin{pmatrix}
  0.0600 \\
0.4500 \\
\vdots \\
0.0600
\end{pmatrix}
\]

\[
\begin{pmatrix}
  x_1^0 \\
x_2^0 \\
\vdots \\
x_n^0
\end{pmatrix}
= 
\begin{pmatrix}
  0.1500 \\
0.0200 \\
\vdots \\
0.0050
\end{pmatrix}
\quad \begin{pmatrix}
  lb_1 \\
  lb_2 \\
\vdots \\
  lb_n
\end{pmatrix}
= 
\begin{pmatrix}
  0.0067 \\
0.0500 \\
\vdots \\
0.0067
\end{pmatrix}
\quad \begin{pmatrix}
  ub_1 \\
  ub_2 \\
\vdots \\
  ub_n
\end{pmatrix}
= 
\begin{pmatrix}
  0.0600 \\
0.4500 \\
\vdots \\
0.0600
\end{pmatrix}
\]

A.2 Test case 2: retail bank

The initial portfolio weights, the lower bounds and the upper bounds are given by
The initial portfolio weights, the lower bounds and the upper bounds are given by

\[
\begin{pmatrix}
    x^0_1 \\
    x^0_2 \\
    x^0_{3,1} \\
    x^0_{3,2} \\
    x^0_{3,3} \\
    x^0_{4,1} \\
    x^0_{4,2} \\
    x^0_{4,3} \\
    x^0_{5,1} \\
    x^0_{5,2} \\
    x^0_{5,3} \\
    x^0_{6,1} \\
    x^0_{6,2} \\
    x^0_{6,3} \\
    x^0_{6,4} \\
    x^0_{7,1} \\
    x^0_{7,2} \\
    x^0_{7,3} \\
    x^0_{7,4} \\
    x^0_{7,5} \\
    x^0_{8,1} \\
    x^0_{8,2} \\
    x^0_{9,1} \\
    x^0_{9,2} \\
    x^0_{10} \\
    x^0_{11,1} \\
    x^0_{11,2} \\
    x^0_{11,3}
\end{pmatrix} =
\begin{pmatrix}
    0.0200 \\
    0.1500 \\
    0.0250 \\
    0.0500 \\
    0.0200 \\
    0.0400 \\
    0.0500 \\
    0.1150 \\
    0.0120 \\
    0.0300 \\
    0.3880 \\
    0.0250 \\
    0.0250 \\
    0.0250 \\
    0.0000 \\
    0.0000 \\
    0.0000 \\
    0.0000 \\
    0.0000 \\
    0.0000 \\
    0.0000 \\
    0.0000 \\
    0.1900 \\
    0.1600 \\
    0.2900 \\
    0.0500 \\
    0.1150 \\
    0.0400 \\
    0.0400 \\
    0.0300
\end{pmatrix}
\]

\[
\begin{pmatrix}
    lb_1 \\
    lb_2 \\
    lb_{3,1} \\
    lb_{3,2} \\
    lb_{3,3} \\
    lb_{4,1} \\
    lb_{4,2} \\
    lb_{4,3} \\
    lb_{5,1} \\
    lb_{5,2} \\
    lb_{5,3} \\
    lb_{6,1} \\
    lb_{6,2} \\
    lb_{6,3} \\
    lb_{6,4} \\
    lb_{7,1} \\
    lb_{7,2} \\
    lb_{7,3} \\
    lb_{7,4} \\
    lb_{7,5} \\
    lb_{8,1} \\
    lb_{8,2} \\
    lb_{9,1} \\
    lb_{9,2} \\
    lb_{10} \\
    lb_{11,1} \\
    lb_{11,2} \\
    lb_{11,3}
\end{pmatrix} =
\begin{pmatrix}
    0.0067 \\
    0.0500 \\
    0.0083 \\
    0.0167 \\
    0.0067 \\
    0.0133 \\
    0.0167 \\
    0.0383 \\
    0.0040 \\
    0.0100 \\
    0.1293 \\
    0.0083 \\
    0.0083 \\
    0.0083 \\
    0.0000 \\
    0.0000 \\
    0.0000 \\
    0.0000 \\
    0.0000 \\
    0.0633 \\
    0.0533 \\
    0.0967 \\
    0.0167 \\
    0.0383 \\
    0.0133 \\
    0.0133 \\
    0.0100
\end{pmatrix}
\]

\[
\begin{pmatrix}
    ub_1 \\
    ub_2 \\
    ub_{3,1} \\
    ub_{3,2} \\
    ub_{3,3} \\
    ub_{4,1} \\
    ub_{4,2} \\
    ub_{4,3} \\
    ub_{5,1} \\
    ub_{5,2} \\
    ub_{5,3} \\
    ub_{6,1} \\
    ub_{6,2} \\
    ub_{6,3} \\
    ub_{6,4} \\
    ub_{7,1} \\
    ub_{7,2} \\
    ub_{7,3} \\
    ub_{7,4} \\
    ub_{7,5} \\
    ub_{8,1} \\
    ub_{8,2} \\
    ub_{9,1} \\
    ub_{9,2} \\
    ub_{10} \\
    ub_{11,1} \\
    ub_{11,2} \\
    ub_{11,3}
\end{pmatrix} =
\begin{pmatrix}
    0.0600 \\
    0.4500 \\
    0.0750 \\
    0.1500 \\
    0.0600 \\
    0.1200 \\
    0.1500 \\
    0.3450 \\
    0.0360 \\
    0.0900 \\
    0.5000 \\
    0.1000 \\
    0.1000 \\
    0.1000 \\
    0.1000 \\
    0.0500 \\
    0.0500 \\
    0.0500 \\
    0.0500 \\
    0.0500 \\
    0.5000 \\
    0.4800 \\
    0.5000 \\
    0.1500 \\
    0.3450 \\
    0.1200 \\
    0.1200 \\
    0.0900
\end{pmatrix}
\]

A.3 Test case 3: investment bank

The initial portfolio weights, the lower bounds and the upper bounds are given by
\[
\begin{align*}
\mathbf{x}^0 &= \\
\begin{pmatrix}
x_1^0 \\ x_2^0 \\ x_3^0,1 \\ x_3^0,2 \\ x_3^0,3 \\ x_4^0,1 \\ x_4^0,2 \\ x_4^0,3 \\ x_5^0,1 \\ x_5^0,2 \\ x_5^0,3 \\ x_6^0,1 \\ x_6^0,2 \\ x_6^0,3 \\ x_6^0,4 \\ x_7^0,1 \\ x_7^0,2 \\ x_7^0,3 \\ x_7^0,4 \\ x_7^0,5 \\ x_8^0,1 \\ x_8^0,2 \\ x_8^0,3 \\ x_8^0,4 \\ x_8^0,5 \\ x_9^0,1 \\ x_9^0,2 \\ x_9^0,3 \\ x_9^0,4 \\ x_{10}^0 \\ x_{11,1}^0 \\ x_{11,2}^0 \\ x_{11,3}^0
\end{pmatrix} &= \\
\begin{pmatrix}
0.0200 \\ 0.1200 \\ 0.0250 \\ 0.0050 \\ 0.0100 \\ 0.0250 \\ 0.0250 \\ 0.0650 \\ 0.0100 \\ 0.0250 \\ 0.1500 \\ 0.0300 \\ 0.0300 \\ 0.0300 \\ 0.0800 \\ 0.0800 \\ 0.0800 \\ 0.0800 \\ 0.0800 \\ 0.0850 \\ 0.1000 \\ 0.2000 \\ 0.0300 \\ 0.2050 \\ 0.0750 \\ 0.1600 \\ 0.0600
\end{pmatrix} \\
\mathbf{lb} &= \\
\begin{pmatrix}
lb_1 \\ lb_2 \\ lb_3,1 \\ lb_3,2 \\ lb_3,3 \\ lb_4,1 \\ lb_4,2 \\ lb_4,3 \\ lb_5,1 \\ lb_5,2 \\ lb_5,3 \\ lb_6,1 \\ lb_6,2 \\ lb_6,3 \\ lb_6,4 \\ lb_7,1 \\ lb_7,2 \\ lb_7,3 \\ lb_7,4 \\ lb_7,5 \\ lb_8,1 \\ lb_8,2 \\ lb_8,3 \\ lb_8,4 \\ lb_8,5 \\ lb_9,1 \\ lb_9,2 \\ lb_9,3 \\ lb_{10} \\ lb_{11,1} \\ lb_{11,2} \\ lb_{11,3}
\end{pmatrix} &= \\
\begin{pmatrix}
0.0067 \\ 0.0400 \\ 0.0083 \\ 0.0017 \\ 0.0033 \\ 0.0083 \\ 0.0083 \\ 0.0217 \\ 0.0033 \\ 0.0083 \\ 0.0500 \\ 0.0100 \\ 0.0100 \\ 0.0100 \\ 0.0267 \\ 0.0267 \\ 0.0267 \\ 0.0267 \\ 0.0267 \\ 0.0283 \\ 0.0333 \\ 0.0667 \\ 0.0100 \\ 0.0683 \\ 0.0250 \\ 0.0533 \\ 0.0200
\end{pmatrix} \\
\mathbf{ub} &= \\
\begin{pmatrix}
ub_1 \\ ub_2 \\ ub_3,1 \\ ub_3,2 \\ ub_3,3 \\ ub_4,1 \\ ub_4,2 \\ ub_4,3 \\ ub_5,1 \\ ub_5,2 \\ ub_5,3 \\ ub_6,1 \\ ub_6,2 \\ ub_6,3 \\ ub_6,4 \\ ub_7,1 \\ ub_7,2 \\ ub_7,3 \\ ub_7,4 \\ ub_7,5 \\ ub_8,1 \\ ub_8,2 \\ ub_8,3 \\ ub_8,4 \\ ub_8,5 \\ ub_9,1 \\ ub_9,2 \\ ub_{10} \\ ub_{11,1} \\ ub_{11,2} \\ ub_{11,3}
\end{pmatrix} &= \\
\begin{pmatrix}
0.0800 \\ 0.4800 \\ 0.1000 \\ 0.0200 \\ 0.0400 \\ 0.1000 \\ 0.1000 \\ 0.1000 \\ 0.2600 \\ 0.0400 \\ 0.1000 \\ 0.5000 \\ 0.1200 \\ 0.1200 \\ 0.1200 \\ 0.1200 \\ 0.1200 \\ 0.1200 \\ 0.3200 \\ 0.3200 \\ 0.3200 \\ 0.3200 \\ 0.3200 \\ 0.3200 \\ 0.3400 \\ 0.4000 \\ 0.5000 \\ 0.1200 \\ 0.5000 \\ 0.5000 \\ 0.5000 \\ 0.2400
\end{pmatrix}
\end{align*}
\]
APPENDIX B
Optimization results

B.1 Optimal portfolio weights

The optimal portfolio weights for the two optimizations of the large bank, retail bank and investment bank are respectively given in Table B.1, Table B.2, Table B.3, Table B.4, Table B.5 and Table B.6.
Table B.1: Optimal portfolio weights for the large bank including LCR and NSFR

### Asset instruments part 1

<table>
<thead>
<tr>
<th></th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_{3,1}$</th>
<th>$x_{3,2}$</th>
<th>$x_{3,3}$</th>
<th>$x_{4,1}$</th>
<th>$x_{4,2}$</th>
<th>$x_{4,3}$</th>
<th>$x_{5,1}$</th>
<th>$x_{5,2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>p=1</td>
<td>0.0067</td>
<td>0.0500</td>
<td>0.0750</td>
<td>0.0150</td>
<td>0.0600</td>
<td>0.0750</td>
<td>0.0117</td>
<td>0.0383</td>
<td>0.0450</td>
<td>0.0083</td>
</tr>
<tr>
<td>p=2</td>
<td>0.0067</td>
<td>0.0500</td>
<td>0.0750</td>
<td>0.0150</td>
<td>0.0600</td>
<td>0.0250</td>
<td>0.0117</td>
<td>0.0383</td>
<td>0.0101</td>
<td>0.0083</td>
</tr>
<tr>
<td>p=3</td>
<td>0.0067</td>
<td>0.0500</td>
<td>0.0750</td>
<td>0.0150</td>
<td>0.0600</td>
<td>0.0250</td>
<td>0.0117</td>
<td>0.0383</td>
<td>0.0050</td>
<td>0.0083</td>
</tr>
<tr>
<td>p=4</td>
<td>0.0067</td>
<td>0.0500</td>
<td>0.0750</td>
<td>0.0150</td>
<td>0.0600</td>
<td>0.0250</td>
<td>0.0117</td>
<td>0.0383</td>
<td>0.0050</td>
<td>0.0083</td>
</tr>
<tr>
<td>p=5</td>
<td>0.0067</td>
<td>0.0500</td>
<td>0.0750</td>
<td>0.0150</td>
<td>0.0600</td>
<td>0.0250</td>
<td>0.0117</td>
<td>0.0383</td>
<td>0.0050</td>
<td>0.0083</td>
</tr>
<tr>
<td>p=6</td>
<td>0.0067</td>
<td>0.0500</td>
<td>0.0750</td>
<td>0.0150</td>
<td>0.0600</td>
<td>0.0250</td>
<td>0.0117</td>
<td>0.0383</td>
<td>0.0050</td>
<td>0.0083</td>
</tr>
</tbody>
</table>

### Asset instruments part 2

<table>
<thead>
<tr>
<th></th>
<th>$x_{5,3}$</th>
<th>$x_{6,1}$</th>
<th>$x_{6,2}$</th>
<th>$x_{6,3}$</th>
<th>$x_{6,4}$</th>
<th>$x_{7,1}$</th>
<th>$x_{7,2}$</th>
<th>$x_{7,3}$</th>
<th>$x_{7,4}$</th>
<th>$x_{7,5}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>p=1</td>
<td>0.2742</td>
<td>0.1058</td>
<td>0.1200</td>
<td>0.0450</td>
<td>0.0450</td>
<td>0.0050</td>
<td>0.0050</td>
<td>0.0050</td>
<td>0.0050</td>
<td>0.0050</td>
</tr>
<tr>
<td>p=2</td>
<td>0.3476</td>
<td>0.1089</td>
<td>0.1200</td>
<td>0.0450</td>
<td>0.0450</td>
<td>0.0050</td>
<td>0.0050</td>
<td>0.0050</td>
<td>0.0050</td>
<td>0.0133</td>
</tr>
<tr>
<td>p=3</td>
<td>0.3328</td>
<td>0.1087</td>
<td>0.1200</td>
<td>0.0450</td>
<td>0.0450</td>
<td>0.0050</td>
<td>0.0050</td>
<td>0.0050</td>
<td>0.0050</td>
<td>0.0335</td>
</tr>
<tr>
<td>p=4</td>
<td>0.3080</td>
<td>0.1083</td>
<td>0.1200</td>
<td>0.0450</td>
<td>0.0450</td>
<td>0.0050</td>
<td>0.0050</td>
<td>0.0050</td>
<td>0.0187</td>
<td>0.0450</td>
</tr>
<tr>
<td>p=5</td>
<td>0.2726</td>
<td>0.1079</td>
<td>0.1200</td>
<td>0.0450</td>
<td>0.0450</td>
<td>0.0050</td>
<td>0.0050</td>
<td>0.0145</td>
<td>0.0450</td>
<td>0.0450</td>
</tr>
<tr>
<td>p=6</td>
<td>0.2023</td>
<td>0.1077</td>
<td>0.1200</td>
<td>0.0450</td>
<td>0.0450</td>
<td>0.0050</td>
<td>0.0450</td>
<td>0.0450</td>
<td>0.0450</td>
<td>0.0450</td>
</tr>
</tbody>
</table>

### Liability instruments

<table>
<thead>
<tr>
<th></th>
<th>$x_{8,1}$</th>
<th>$x_{8,2}$</th>
<th>$x_{9,1}$</th>
<th>$x_{9,2}$</th>
<th>$x_{10}$</th>
<th>$x_{11,1}$</th>
<th>$x_{11,2}$</th>
<th>$x_{11,3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>p=1</td>
<td>0.0583</td>
<td>0.0442</td>
<td>0.5000</td>
<td>0.1350</td>
<td>0.0500</td>
<td>0.1116</td>
<td>0.0267</td>
<td>0.0150</td>
</tr>
<tr>
<td>p=2</td>
<td>0.0583</td>
<td>0.0442</td>
<td>0.5000</td>
<td>0.1350</td>
<td>0.0500</td>
<td>0.1110</td>
<td>0.0267</td>
<td>0.0150</td>
</tr>
<tr>
<td>p=3</td>
<td>0.0583</td>
<td>0.0442</td>
<td>0.5000</td>
<td>0.1350</td>
<td>0.0500</td>
<td>0.1105</td>
<td>0.0267</td>
<td>0.0150</td>
</tr>
<tr>
<td>p=4</td>
<td>0.0583</td>
<td>0.0442</td>
<td>0.5000</td>
<td>0.1350</td>
<td>0.0500</td>
<td>0.1099</td>
<td>0.0267</td>
<td>0.0150</td>
</tr>
<tr>
<td>p=5</td>
<td>0.0583</td>
<td>0.0442</td>
<td>0.5000</td>
<td>0.1350</td>
<td>0.0500</td>
<td>0.1093</td>
<td>0.0267</td>
<td>0.0150</td>
</tr>
<tr>
<td>p=6</td>
<td>0.0583</td>
<td>0.0442</td>
<td>0.5000</td>
<td>0.1350</td>
<td>0.0500</td>
<td>0.1091</td>
<td>0.0267</td>
<td>0.0150</td>
</tr>
</tbody>
</table>
Table B.2: Optimal portfolio weights for the large bank excluding LCR and NSFR

### Asset instruments part 1

<table>
<thead>
<tr>
<th></th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_{3,1}$</th>
<th>$x_{3,2}$</th>
<th>$x_{3,3}$</th>
<th>$x_{4,1}$</th>
<th>$x_{4,2}$</th>
<th>$x_{4,3}$</th>
<th>$x_{5,1}$</th>
<th>$x_{5,2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>p=1</td>
<td>0.0067</td>
<td>0.0500</td>
<td>0.0750</td>
<td>0.0150</td>
<td>0.0600</td>
<td>0.1395</td>
<td>0.0117</td>
<td>0.0383</td>
<td>0.0450</td>
<td>0.0083</td>
</tr>
<tr>
<td>p=2</td>
<td>0.0067</td>
<td>0.0500</td>
<td>0.0750</td>
<td>0.0150</td>
<td>0.0600</td>
<td>0.0359</td>
<td>0.0117</td>
<td>0.0383</td>
<td>0.0450</td>
<td>0.0083</td>
</tr>
<tr>
<td>p=3</td>
<td>0.0067</td>
<td>0.0500</td>
<td>0.0750</td>
<td>0.0150</td>
<td>0.0600</td>
<td>0.0250</td>
<td>0.0117</td>
<td>0.0383</td>
<td>0.0050</td>
<td>0.0083</td>
</tr>
<tr>
<td>p=4</td>
<td>0.0067</td>
<td>0.0500</td>
<td>0.0750</td>
<td>0.0150</td>
<td>0.0600</td>
<td>0.0250</td>
<td>0.0117</td>
<td>0.0383</td>
<td>0.0050</td>
<td>0.0083</td>
</tr>
<tr>
<td>p=5</td>
<td>0.0067</td>
<td>0.0500</td>
<td>0.0750</td>
<td>0.0150</td>
<td>0.0600</td>
<td>0.0250</td>
<td>0.0117</td>
<td>0.0383</td>
<td>0.0050</td>
<td>0.0083</td>
</tr>
<tr>
<td>p=6</td>
<td>0.0067</td>
<td>0.0500</td>
<td>0.0750</td>
<td>0.0150</td>
<td>0.0600</td>
<td>0.0250</td>
<td>0.0117</td>
<td>0.0383</td>
<td>0.0050</td>
<td>0.0083</td>
</tr>
</tbody>
</table>

### Asset instruments part 2

<table>
<thead>
<tr>
<th></th>
<th>$x_{5,3}$</th>
<th>$x_{6,1}$</th>
<th>$x_{6,2}$</th>
<th>$x_{6,3}$</th>
<th>$x_{6,4}$</th>
<th>$x_{7,1}$</th>
<th>$x_{7,2}$</th>
<th>$x_{7,3}$</th>
<th>$x_{7,4}$</th>
<th>$x_{7,5}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>p=1</td>
<td>0.3022</td>
<td>0.0133</td>
<td>0.1200</td>
<td>0.0450</td>
<td>0.0450</td>
<td>0.0050</td>
<td>0.0050</td>
<td>0.0050</td>
<td>0.0050</td>
<td>0.0050</td>
</tr>
<tr>
<td>p=2</td>
<td>0.4023</td>
<td>0.0133</td>
<td>0.1200</td>
<td>0.0450</td>
<td>0.0450</td>
<td>0.0050</td>
<td>0.0050</td>
<td>0.0050</td>
<td>0.0050</td>
<td>0.0084</td>
</tr>
<tr>
<td>p=3</td>
<td>0.4356</td>
<td>0.0133</td>
<td>0.1200</td>
<td>0.0450</td>
<td>0.0450</td>
<td>0.0050</td>
<td>0.0050</td>
<td>0.0050</td>
<td>0.0050</td>
<td>0.0084</td>
</tr>
<tr>
<td>p=4</td>
<td>0.4097</td>
<td>0.0133</td>
<td>0.1200</td>
<td>0.0450</td>
<td>0.0450</td>
<td>0.0050</td>
<td>0.0050</td>
<td>0.0050</td>
<td>0.0120</td>
<td>0.0450</td>
</tr>
<tr>
<td>p=5</td>
<td>0.3716</td>
<td>0.0133</td>
<td>0.1200</td>
<td>0.0450</td>
<td>0.0450</td>
<td>0.0050</td>
<td>0.0050</td>
<td>0.0050</td>
<td>0.0450</td>
<td>0.0450</td>
</tr>
<tr>
<td>p=6</td>
<td>0.2967</td>
<td>0.0133</td>
<td>0.1200</td>
<td>0.0450</td>
<td>0.0450</td>
<td>0.0050</td>
<td>0.0050</td>
<td>0.0050</td>
<td>0.0450</td>
<td>0.0450</td>
</tr>
</tbody>
</table>

### Liability instruments

<table>
<thead>
<tr>
<th></th>
<th>$x_{8,1}$</th>
<th>$x_{8,2}$</th>
<th>$x_{9,1}$</th>
<th>$x_{9,2}$</th>
<th>$x_{10}$</th>
<th>$x_{11,1}$</th>
<th>$x_{11,2}$</th>
<th>$x_{11,3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>p=1</td>
<td>0.0583</td>
<td>0.0442</td>
<td>0.5000</td>
<td>0.1350</td>
<td>0.0500</td>
<td>0.1114</td>
<td>0.0267</td>
<td>0.0150</td>
</tr>
<tr>
<td>p=2</td>
<td>0.0583</td>
<td>0.0442</td>
<td>0.5000</td>
<td>0.1350</td>
<td>0.0500</td>
<td>0.1107</td>
<td>0.0267</td>
<td>0.0150</td>
</tr>
<tr>
<td>p=3</td>
<td>0.0583</td>
<td>0.0442</td>
<td>0.5000</td>
<td>0.1350</td>
<td>0.0500</td>
<td>0.1101</td>
<td>0.0267</td>
<td>0.0150</td>
</tr>
<tr>
<td>p=4</td>
<td>0.0583</td>
<td>0.0442</td>
<td>0.5000</td>
<td>0.1350</td>
<td>0.0500</td>
<td>0.1095</td>
<td>0.0267</td>
<td>0.0150</td>
</tr>
<tr>
<td>p=5</td>
<td>0.0583</td>
<td>0.0442</td>
<td>0.5000</td>
<td>0.1350</td>
<td>0.0500</td>
<td>0.1088</td>
<td>0.0267</td>
<td>0.0150</td>
</tr>
<tr>
<td>p=6</td>
<td>0.0583</td>
<td>0.0442</td>
<td>0.5000</td>
<td>0.1350</td>
<td>0.0500</td>
<td>0.1085</td>
<td>0.0267</td>
<td>0.0150</td>
</tr>
</tbody>
</table>
Table B.3: Optimal portfolio weights for the retail bank including LCR and NSFR

<table>
<thead>
<tr>
<th></th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_{3,1}$</th>
<th>$x_{3,2}$</th>
<th>$x_{3,3}$</th>
<th>$x_{4,1}$</th>
<th>$x_{4,2}$</th>
<th>$x_{4,3}$</th>
<th>$x_{5,1}$</th>
<th>$x_{5,2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>p=1</strong></td>
<td>0.0067</td>
<td>0.0500</td>
<td>0.0750</td>
<td>0.1500</td>
<td>0.0600</td>
<td>0.0133</td>
<td>0.0167</td>
<td>0.0383</td>
<td>0.0360</td>
<td>0.0100</td>
</tr>
<tr>
<td><strong>p=2</strong></td>
<td>0.0067</td>
<td>0.0500</td>
<td>0.0750</td>
<td>0.1500</td>
<td>0.0600</td>
<td>0.0133</td>
<td>0.0167</td>
<td>0.0383</td>
<td>0.0040</td>
<td>0.0100</td>
</tr>
<tr>
<td><strong>p=3</strong></td>
<td>0.0067</td>
<td>0.0500</td>
<td>0.0750</td>
<td>0.1500</td>
<td>0.0600</td>
<td>0.0133</td>
<td>0.0167</td>
<td>0.0383</td>
<td>0.0040</td>
<td>0.0100</td>
</tr>
<tr>
<td><strong>p=4</strong></td>
<td>0.0067</td>
<td>0.0500</td>
<td>0.0750</td>
<td>0.1500</td>
<td>0.0349</td>
<td>0.0133</td>
<td>0.0167</td>
<td>0.0383</td>
<td>0.0040</td>
<td>0.0100</td>
</tr>
<tr>
<td><strong>p=5</strong></td>
<td>0.0067</td>
<td>0.0500</td>
<td>0.0750</td>
<td>0.1500</td>
<td>0.0071</td>
<td>0.0133</td>
<td>0.0167</td>
<td>0.0383</td>
<td>0.0040</td>
<td>0.0100</td>
</tr>
<tr>
<td><strong>p=6</strong></td>
<td>0.0067</td>
<td>0.0500</td>
<td>0.0750</td>
<td>0.1500</td>
<td>0.0067</td>
<td>0.0133</td>
<td>0.0167</td>
<td>0.0383</td>
<td>0.0040</td>
<td>0.0100</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$x_{5,3}$</th>
<th>$x_{6,1}$</th>
<th>$x_{6,2}$</th>
<th>$x_{6,3}$</th>
<th>$x_{6,4}$</th>
<th>$x_{7,1}$</th>
<th>$x_{7,2}$</th>
<th>$x_{7,3}$</th>
<th>$x_{7,4}$</th>
<th>$x_{7,5}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>p=1</strong></td>
<td>0.1295</td>
<td>0.1000</td>
<td>0.1000</td>
<td>0.1000</td>
<td>0.1000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td><strong>p=2</strong></td>
<td>0.1470</td>
<td>0.1000</td>
<td>0.1000</td>
<td>0.1000</td>
<td>0.1000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td><strong>p=3</strong></td>
<td>0.1299</td>
<td>0.1000</td>
<td>0.1000</td>
<td>0.1000</td>
<td>0.1000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td><strong>p=4</strong></td>
<td>0.1293</td>
<td>0.1000</td>
<td>0.1000</td>
<td>0.1000</td>
<td>0.1000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0217</td>
</tr>
<tr>
<td><strong>p=5</strong></td>
<td>0.1293</td>
<td>0.1000</td>
<td>0.1000</td>
<td>0.1000</td>
<td>0.1000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0495</td>
</tr>
<tr>
<td><strong>p=6</strong></td>
<td>0.1293</td>
<td>0.0500</td>
<td>0.1000</td>
<td>0.1000</td>
<td>0.1000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0500</td>
<td>0.0500</td>
<td>0.0500</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$x_{8,1}$</th>
<th>$x_{8,2}$</th>
<th>$x_{9,1}$</th>
<th>$x_{9,2}$</th>
<th>$x_{10}$</th>
<th>$x_{11,1}$</th>
<th>$x_{11,2}$</th>
<th>$x_{11,3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>p=1</strong></td>
<td>0.0638</td>
<td>0.0533</td>
<td>0.5000</td>
<td>0.1500</td>
<td>0.0383</td>
<td>0.1082</td>
<td>0.0133</td>
<td>0.0100</td>
</tr>
<tr>
<td><strong>p=2</strong></td>
<td>0.0652</td>
<td>0.0533</td>
<td>0.5000</td>
<td>0.1500</td>
<td>0.0383</td>
<td>0.1063</td>
<td>0.0133</td>
<td>0.0100</td>
</tr>
<tr>
<td><strong>p=3</strong></td>
<td>0.0633</td>
<td>0.0533</td>
<td>0.5000</td>
<td>0.1500</td>
<td>0.0383</td>
<td>0.1065</td>
<td>0.0133</td>
<td>0.0113</td>
</tr>
<tr>
<td><strong>p=4</strong></td>
<td>0.0633</td>
<td>0.0533</td>
<td>0.5000</td>
<td>0.1500</td>
<td>0.0383</td>
<td>0.1065</td>
<td>0.0133</td>
<td>0.0108</td>
</tr>
<tr>
<td><strong>p=5</strong></td>
<td>0.0633</td>
<td>0.0533</td>
<td>0.5000</td>
<td>0.1500</td>
<td>0.0383</td>
<td>0.1065</td>
<td>0.0133</td>
<td>0.0104</td>
</tr>
<tr>
<td><strong>p=6</strong></td>
<td>0.1341</td>
<td>0.0533</td>
<td>0.5000</td>
<td>0.1500</td>
<td>0.0383</td>
<td>0.0351</td>
<td>0.0133</td>
<td>0.0107</td>
</tr>
</tbody>
</table>
Table B.4: Optimal portfolio weights for the retail bank excluding LCR and NSFR

### Asset instruments part 1

<table>
<thead>
<tr>
<th></th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_{3,1}$</th>
<th>$x_{3,2}$</th>
<th>$x_{3,3}$</th>
<th>$x_{4,1}$</th>
<th>$x_{4,2}$</th>
<th>$x_{4,3}$</th>
<th>$x_{5,1}$</th>
<th>$x_{5,2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>p=1</td>
<td>0.0067</td>
<td>0.0500</td>
<td>0.0750</td>
<td>0.1500</td>
<td>0.0600</td>
<td>0.0388</td>
<td>0.0167</td>
<td>0.0383</td>
<td>0.0360</td>
<td>0.0279</td>
</tr>
<tr>
<td>p=2</td>
<td>0.0067</td>
<td>0.0500</td>
<td>0.0750</td>
<td>0.1500</td>
<td>0.0600</td>
<td>0.0133</td>
<td>0.0167</td>
<td>0.0383</td>
<td>0.0278</td>
<td>0.0100</td>
</tr>
<tr>
<td>p=3</td>
<td>0.0067</td>
<td>0.0500</td>
<td>0.0750</td>
<td>0.1500</td>
<td>0.0600</td>
<td>0.0133</td>
<td>0.0167</td>
<td>0.0383</td>
<td>0.0040</td>
<td>0.0100</td>
</tr>
<tr>
<td>p=4</td>
<td>0.0067</td>
<td>0.0500</td>
<td>0.0750</td>
<td>0.1500</td>
<td>0.0600</td>
<td>0.0133</td>
<td>0.0167</td>
<td>0.0383</td>
<td>0.0040</td>
<td>0.0100</td>
</tr>
<tr>
<td>p=5</td>
<td>0.0067</td>
<td>0.0500</td>
<td>0.0750</td>
<td>0.1500</td>
<td>0.0600</td>
<td>0.0133</td>
<td>0.0167</td>
<td>0.0383</td>
<td>0.0040</td>
<td>0.0100</td>
</tr>
<tr>
<td>p=6</td>
<td>0.0067</td>
<td>0.0500</td>
<td>0.0750</td>
<td>0.1500</td>
<td>0.0600</td>
<td>0.0133</td>
<td>0.0167</td>
<td>0.0383</td>
<td>0.0040</td>
<td>0.0100</td>
</tr>
</tbody>
</table>

### Asset instruments part 2

<table>
<thead>
<tr>
<th></th>
<th>$x_{5,3}$</th>
<th>$x_{6,1}$</th>
<th>$x_{6,2}$</th>
<th>$x_{6,3}$</th>
<th>$x_{6,4}$</th>
<th>$x_{7,1}$</th>
<th>$x_{7,2}$</th>
<th>$x_{7,3}$</th>
<th>$x_{7,4}$</th>
<th>$x_{7,5}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>p=1</td>
<td>0.1856</td>
<td>0.0083</td>
<td>0.1000</td>
<td>0.1000</td>
<td>0.1000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0067</td>
</tr>
<tr>
<td>p=2</td>
<td>0.2214</td>
<td>0.0083</td>
<td>0.1000</td>
<td>0.1000</td>
<td>0.1000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0224</td>
</tr>
<tr>
<td>p=3</td>
<td>0.2249</td>
<td>0.0083</td>
<td>0.1000</td>
<td>0.1000</td>
<td>0.1000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0042</td>
</tr>
<tr>
<td>p=4</td>
<td>0.1945</td>
<td>0.0083</td>
<td>0.1000</td>
<td>0.1000</td>
<td>0.1000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0232</td>
</tr>
<tr>
<td>p=5</td>
<td>0.1530</td>
<td>0.0083</td>
<td>0.1000</td>
<td>0.1000</td>
<td>0.1000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0146</td>
<td>0.0500</td>
<td>0.0500</td>
</tr>
<tr>
<td>p=6</td>
<td>0.1293</td>
<td>0.0083</td>
<td>0.0883</td>
<td>0.1000</td>
<td>0.1000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0500</td>
<td>0.0500</td>
<td>0.0500</td>
</tr>
</tbody>
</table>

### Liability instruments

<table>
<thead>
<tr>
<th></th>
<th>$x_{8,1}$</th>
<th>$x_{8,2}$</th>
<th>$x_{9,1}$</th>
<th>$x_{9,2}$</th>
<th>$x_{10}$</th>
<th>$x_{11,1}$</th>
<th>$x_{11,2}$</th>
<th>$x_{11,3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>p=1</td>
<td>0.0633</td>
<td>0.0533</td>
<td>0.5000</td>
<td>0.1500</td>
<td>0.0383</td>
<td>0.1084</td>
<td>0.0133</td>
<td>0.0100</td>
</tr>
<tr>
<td>p=2</td>
<td>0.0633</td>
<td>0.0533</td>
<td>0.5000</td>
<td>0.1500</td>
<td>0.0383</td>
<td>0.1079</td>
<td>0.0133</td>
<td>0.0100</td>
</tr>
<tr>
<td>p=3</td>
<td>0.0633</td>
<td>0.0533</td>
<td>0.5000</td>
<td>0.1500</td>
<td>0.0383</td>
<td>0.1073</td>
<td>0.0133</td>
<td>0.0100</td>
</tr>
<tr>
<td>p=4</td>
<td>0.0633</td>
<td>0.0533</td>
<td>0.5000</td>
<td>0.1500</td>
<td>0.0383</td>
<td>0.1067</td>
<td>0.0133</td>
<td>0.0100</td>
</tr>
<tr>
<td>p=5</td>
<td>0.0633</td>
<td>0.0533</td>
<td>0.5000</td>
<td>0.1500</td>
<td>0.0383</td>
<td>0.1061</td>
<td>0.0133</td>
<td>0.0100</td>
</tr>
<tr>
<td>p=6</td>
<td>0.0633</td>
<td>0.0533</td>
<td>0.5000</td>
<td>0.1500</td>
<td>0.0383</td>
<td>0.1058</td>
<td>0.0133</td>
<td>0.0100</td>
</tr>
</tbody>
</table>
Table B.5: Optimal portfolio weights for the investment bank including LCR and NSFR

<table>
<thead>
<tr>
<th>Asset instruments part 1</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_{3,1}$</th>
<th>$x_{3,2}$</th>
<th>$x_{3,3}$</th>
<th>$x_{4,1}$</th>
<th>$x_{4,2}$</th>
<th>$x_{4,3}$</th>
<th>$x_{5,1}$</th>
<th>$x_{5,2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p=1$</td>
<td>0,0067</td>
<td>0,0400</td>
<td>0,0617</td>
<td>0,0183</td>
<td>0,0033</td>
<td>0,0083</td>
<td>0,0083</td>
<td>0,0217</td>
<td>0,0033</td>
<td>0,0083</td>
</tr>
<tr>
<td>$p=2$</td>
<td>0,0072</td>
<td>0,0400</td>
<td>0,0544</td>
<td>0,0200</td>
<td>0,0033</td>
<td>0,0083</td>
<td>0,0083</td>
<td>0,0217</td>
<td>0,0033</td>
<td>0,0083</td>
</tr>
<tr>
<td>$p=3$</td>
<td>0,0084</td>
<td>0,0400</td>
<td>0,0471</td>
<td>0,0200</td>
<td>0,0033</td>
<td>0,0083</td>
<td>0,0083</td>
<td>0,0217</td>
<td>0,0033</td>
<td>0,0083</td>
</tr>
<tr>
<td>$p=4$</td>
<td>0,0095</td>
<td>0,0400</td>
<td>0,0397</td>
<td>0,0200</td>
<td>0,0033</td>
<td>0,0083</td>
<td>0,0083</td>
<td>0,0217</td>
<td>0,0033</td>
<td>0,0083</td>
</tr>
<tr>
<td>$p=5$</td>
<td>0,0107</td>
<td>0,0400</td>
<td>0,0324</td>
<td>0,0200</td>
<td>0,0033</td>
<td>0,0083</td>
<td>0,0083</td>
<td>0,0217</td>
<td>0,0033</td>
<td>0,0083</td>
</tr>
<tr>
<td>$p=6$</td>
<td>0,0112</td>
<td>0,0400</td>
<td>0,0288</td>
<td>0,0200</td>
<td>0,0033</td>
<td>0,0083</td>
<td>0,0083</td>
<td>0,0217</td>
<td>0,0033</td>
<td>0,0083</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Asset instruments part 2</th>
<th>$x_{5,3}$</th>
<th>$x_{6,1}$</th>
<th>$x_{6,2}$</th>
<th>$x_{6,3}$</th>
<th>$x_{6,4}$</th>
<th>$x_{7,1}$</th>
<th>$x_{7,2}$</th>
<th>$x_{7,3}$</th>
<th>$x_{7,4}$</th>
<th>$x_{7,5}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p=1$</td>
<td>0,0500</td>
<td>0,1200</td>
<td>0,1200</td>
<td>0,0100</td>
<td>0,1200</td>
<td>0,0267</td>
<td>0,0267</td>
<td>0,0267</td>
<td>0,0267</td>
<td>0,2933</td>
</tr>
<tr>
<td>$p=2$</td>
<td>0,0500</td>
<td>0,1200</td>
<td>0,1200</td>
<td>0,0100</td>
<td>0,1200</td>
<td>0,0267</td>
<td>0,0267</td>
<td>0,0267</td>
<td>0,0267</td>
<td>0,2984</td>
</tr>
<tr>
<td>$p=3$</td>
<td>0,0500</td>
<td>0,1200</td>
<td>0,1200</td>
<td>0,0100</td>
<td>0,1200</td>
<td>0,0267</td>
<td>0,0267</td>
<td>0,0267</td>
<td>0,0267</td>
<td>0,3046</td>
</tr>
<tr>
<td>$p=4$</td>
<td>0,0500</td>
<td>0,1200</td>
<td>0,1200</td>
<td>0,0100</td>
<td>0,1200</td>
<td>0,0267</td>
<td>0,0267</td>
<td>0,0267</td>
<td>0,0267</td>
<td>0,3108</td>
</tr>
<tr>
<td>$p=5$</td>
<td>0,0500</td>
<td>0,1200</td>
<td>0,1200</td>
<td>0,0100</td>
<td>0,1200</td>
<td>0,0267</td>
<td>0,0267</td>
<td>0,0267</td>
<td>0,0267</td>
<td>0,3169</td>
</tr>
<tr>
<td>$p=6$</td>
<td>0,0500</td>
<td>0,1200</td>
<td>0,1200</td>
<td>0,0100</td>
<td>0,1200</td>
<td>0,0267</td>
<td>0,0267</td>
<td>0,0267</td>
<td>0,0267</td>
<td>0,3200</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Liability instruments</th>
<th>$x_{8,1}$</th>
<th>$x_{8,2}$</th>
<th>$x_{9,1}$</th>
<th>$x_{9,2}$</th>
<th>$x_{10}$</th>
<th>$x_{11,1}$</th>
<th>$x_{11,2}$</th>
<th>$x_{11,3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p=1$</td>
<td>0,0283</td>
<td>0,1165</td>
<td>0,5000</td>
<td>0,1200</td>
<td>0,0683</td>
<td>0,0250</td>
<td>0,0533</td>
<td>0,0200</td>
</tr>
<tr>
<td>$p=2$</td>
<td>0,0283</td>
<td>0,1164</td>
<td>0,5000</td>
<td>0,1200</td>
<td>0,0683</td>
<td>0,0250</td>
<td>0,0533</td>
<td>0,0200</td>
</tr>
<tr>
<td>$p=3$</td>
<td>0,0283</td>
<td>0,1164</td>
<td>0,5000</td>
<td>0,1200</td>
<td>0,0683</td>
<td>0,0250</td>
<td>0,0533</td>
<td>0,0200</td>
</tr>
<tr>
<td>$p=4$</td>
<td>0,0283</td>
<td>0,1164</td>
<td>0,5000</td>
<td>0,1200</td>
<td>0,0683</td>
<td>0,0250</td>
<td>0,0533</td>
<td>0,0200</td>
</tr>
<tr>
<td>$p=5$</td>
<td>0,0283</td>
<td>0,1163</td>
<td>0,5000</td>
<td>0,1200</td>
<td>0,0683</td>
<td>0,0250</td>
<td>0,0533</td>
<td>0,0200</td>
</tr>
<tr>
<td>$p=6$</td>
<td>0,0283</td>
<td>0,1163</td>
<td>0,5000</td>
<td>0,1200</td>
<td>0,0683</td>
<td>0,0250</td>
<td>0,0533</td>
<td>0,0200</td>
</tr>
</tbody>
</table>
Table B.6: Optimal portfolio weights for the investment bank excluding LCR and NSFR

### Asset instruments part 1

<table>
<thead>
<tr>
<th></th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_{3,1}$</th>
<th>$x_{3,2}$</th>
<th>$x_{3,3}$</th>
<th>$x_{4,1}$</th>
<th>$x_{4,2}$</th>
<th>$x_{4,3}$</th>
<th>$x_{5,1}$</th>
<th>$x_{5,2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>p=1</td>
<td>0.0067</td>
<td>0.0400</td>
<td>0.1000</td>
<td>0.0200</td>
<td>0.0400</td>
<td>0.0083</td>
<td>0.0083</td>
<td>0.0217</td>
<td>0.0033</td>
<td>0.0083</td>
</tr>
<tr>
<td>p=2</td>
<td>0.0067</td>
<td>0.0400</td>
<td>0.1000</td>
<td>0.0200</td>
<td>0.0400</td>
<td>0.0083</td>
<td>0.0083</td>
<td>0.0217</td>
<td>0.0033</td>
<td>0.0083</td>
</tr>
<tr>
<td>p=3</td>
<td>0.0067</td>
<td>0.0400</td>
<td>0.1000</td>
<td>0.0200</td>
<td>0.0266</td>
<td>0.0083</td>
<td>0.0083</td>
<td>0.0217</td>
<td>0.0033</td>
<td>0.0083</td>
</tr>
<tr>
<td>p=4</td>
<td>0.0067</td>
<td>0.0400</td>
<td>0.1000</td>
<td>0.0064</td>
<td>0.0083</td>
<td>0.0083</td>
<td>0.0217</td>
<td>0.0033</td>
<td>0.0083</td>
<td></td>
</tr>
<tr>
<td>p=5</td>
<td>0.0067</td>
<td>0.0400</td>
<td>0.0083</td>
<td>0.0017</td>
<td>0.0083</td>
<td>0.0083</td>
<td>0.0217</td>
<td>0.0033</td>
<td>0.0083</td>
<td></td>
</tr>
<tr>
<td>p=6</td>
<td>0.0067</td>
<td>0.0400</td>
<td>0.0083</td>
<td>0.0017</td>
<td>0.0083</td>
<td>0.0083</td>
<td>0.0217</td>
<td>0.0033</td>
<td>0.0083</td>
<td></td>
</tr>
</tbody>
</table>

### Asset instruments part 2

<table>
<thead>
<tr>
<th></th>
<th>$x_{5,3}$</th>
<th>$x_{6,1}$</th>
<th>$x_{6,2}$</th>
<th>$x_{6,3}$</th>
<th>$x_{6,4}$</th>
<th>$x_{7,1}$</th>
<th>$x_{7,2}$</th>
<th>$x_{7,3}$</th>
<th>$x_{7,4}$</th>
<th>$x_{7,5}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>p=1</td>
<td>0.0500</td>
<td>0.0100</td>
<td>0.0433</td>
<td>0.1200</td>
<td>0.1200</td>
<td>0.0267</td>
<td>0.0267</td>
<td>0.0267</td>
<td>0.0267</td>
<td>0.2933</td>
</tr>
<tr>
<td>p=2</td>
<td>0.0500</td>
<td>0.0100</td>
<td>0.0236</td>
<td>0.1200</td>
<td>0.1200</td>
<td>0.0267</td>
<td>0.0267</td>
<td>0.0267</td>
<td>0.0267</td>
<td>0.3131</td>
</tr>
<tr>
<td>p=3</td>
<td>0.0500</td>
<td>0.0100</td>
<td>0.0100</td>
<td>0.1200</td>
<td>0.1200</td>
<td>0.0267</td>
<td>0.0267</td>
<td>0.0267</td>
<td>0.0267</td>
<td>0.3200</td>
</tr>
<tr>
<td>p=4</td>
<td>0.0500</td>
<td>0.0100</td>
<td>0.0100</td>
<td>0.1200</td>
<td>0.1200</td>
<td>0.0267</td>
<td>0.0267</td>
<td>0.0267</td>
<td>0.0267</td>
<td>0.3200</td>
</tr>
<tr>
<td>p=5</td>
<td>0.0500</td>
<td>0.0100</td>
<td>0.0100</td>
<td>0.1029</td>
<td>0.1200</td>
<td>0.0267</td>
<td>0.0267</td>
<td>0.0267</td>
<td>0.0267</td>
<td>0.3200</td>
</tr>
<tr>
<td>p=6</td>
<td>0.0500</td>
<td>0.0100</td>
<td>0.0100</td>
<td>0.0100</td>
<td>0.1200</td>
<td>0.0267</td>
<td>0.0267</td>
<td>0.0267</td>
<td>0.0267</td>
<td>0.3200</td>
</tr>
</tbody>
</table>

### Liability instruments

<table>
<thead>
<tr>
<th></th>
<th>$x_{8,1}$</th>
<th>$x_{8,2}$</th>
<th>$x_{9,1}$</th>
<th>$x_{9,2}$</th>
<th>$x_{10}$</th>
<th>$x_{11,1}$</th>
<th>$x_{11,2}$</th>
<th>$x_{11,3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>p=1</td>
<td>0.0283</td>
<td>0.0333</td>
<td>0.5000</td>
<td>0.1200</td>
<td>0.0683</td>
<td>0.1042</td>
<td>0.0533</td>
<td>0.0200</td>
</tr>
<tr>
<td>p=2</td>
<td>0.0283</td>
<td>0.0333</td>
<td>0.5000</td>
<td>0.1200</td>
<td>0.0683</td>
<td>0.1037</td>
<td>0.0533</td>
<td>0.0200</td>
</tr>
<tr>
<td>p=3</td>
<td>0.0283</td>
<td>0.0333</td>
<td>0.5000</td>
<td>0.1200</td>
<td>0.0683</td>
<td>0.1032</td>
<td>0.0533</td>
<td>0.0200</td>
</tr>
<tr>
<td>p=4</td>
<td>0.0283</td>
<td>0.0333</td>
<td>0.5000</td>
<td>0.1200</td>
<td>0.0683</td>
<td>0.1027</td>
<td>0.0533</td>
<td>0.0200</td>
</tr>
<tr>
<td>p=5</td>
<td>0.0283</td>
<td>0.0333</td>
<td>0.5000</td>
<td>0.1200</td>
<td>0.0683</td>
<td>0.1021</td>
<td>0.0533</td>
<td>0.0200</td>
</tr>
<tr>
<td>p=6</td>
<td>0.0283</td>
<td>0.0333</td>
<td>0.5000</td>
<td>0.1200</td>
<td>0.0683</td>
<td>0.1019</td>
<td>0.0533</td>
<td>0.0200</td>
</tr>
</tbody>
</table>