Invited talks at the workshop "An algebraic view of dynamics" 16-17 april 2014

Badly approximable numbers

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One of the meeting points between number theory and dynamical systems is the subject of badly approximable numbers which plays an important role in the normalization of perturbed quasi-periodic systems. It is here that the mysterious properties of the real numbers express themselves through dynamics. In this lecture we shall highlight some remarkable properties of badly approximable numbers and indicate some problems that are still unsolved.

Automorphic Lie Algebras and Classical Invariant Theory

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The concept of *Automorphic Lie Algebras* arises in the context of reduction groups introduced in the early 1980s [1] in the field of integrable systems. Automorphic Lie Algebras are obtained by imposing a discrete group symmetry on a Lie algebra over a field of rational functions.

Since their official introduction in 2005 [2] the goal has been to classify Automorphic Lie Algebras. Past work shows remarkable uniformity between algebras associated to different reduction groups. For example, if the base Lie algebra is $\mathfrak{sl}_2(\mathbb{C})$ then the Automorphic Lie Algebra is independent of the reduction group [3, 4].

Several restrictions were in place however, on the poles of the rational functions and the group representations. In the case where the reduction group is a dihedral group all these restrictions have now been lifted, and for all reduction groups we present a classification result for $\mathfrak{sl}_n(\mathbb{C})$-based Automorphic Lie Algebras.

Finally, there are some intriguing, though not yet understood, observations suggesting ways to describe the Automorphic Lie Algebras by surprisingly simple classical methods. An understanding of this would enable us to circumvent a tremendous amount of computations and make predictions for Automorphic Lie Algebras based on any semisimple Lie algebra.

References

[1] A. V. Mikhailov, *The reduction problem and the inverse scattering method*, Physica D, 3(1 and 2) (1981), pp. 73117.

[2] S. Lombardo and A. V. Mikhailov, *Reduction Groups and Automorphic Lie Algebras*, Communications in Mathematical Physics, 258 (2005), pp. 179–202.

[3] Rhys Thomas Bury, *Automorphic Lie Algebras, Corresponding Integrable Systems and their Soliton Solutions*, PhD thesis, (2010), The University of Leeds, UK

[4] Sara Lombardo, Jan A. Sanders, *On the classification of Automorphic Lie Algebras*, Communications in Mathematical Physics, 299 (2010), pp. 793–824.

Bogdanov-Takens Bifurcations: An Interplay Between Symbolic and Numerical Analysis

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The talk will address various aspects of symbolic and numerical bifurcation analysis of codim 2 and 3 Bogdanov-Takens (BT) bifurcations of equilibria in $\mathbb{R}^n$ [1, 2, 3]. Some global properties of the classical unfoldings of codim 2 and 3 BT singularities reduced to the 2D center manifolds will be discussed. For the codim 2 BT bifurcation, explicit higher-order approximations for the saddle homoclinic solutions will be given. For the "elliptic case" of a codim 3 bifurcation, global bifurcations will be studied, revealing its similarity to the "focus case". It will also be shown how coefficents of the 2D unfoldings can be effectiently computed via a combined reduction/normalization method based on Fredholm's decomposition.

References

[1] Kuznetsov, Yu.A.: Practical computation of normal forms on center manifolds at degenerate Bogdanov-Takens bifurcations. *Int. J. Bifurcation & Chaos* **15** (2005) 3535–3546

[2] Baer, S.M., Kooi, B.W., Kuznetsov, Yu.A., and Thieme, H.R.: Multiparametric bifurcation analysis of a basic two-stage population model. *SIAM J. Appl. Math.* **66** (2006) 1339-1365

[3] Kuznetsov, Yu.A., Meijer, H.G.E., Al Hdaibat, B., Govaerts, W.: Improved homoclinic predictor for Bogdanov-Takens bifurcation. *Int. J. Bifurcation & Chaos* **24** (2014) [in press]

Title to be announced

Alexander V Mikhailov

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Box products of Stanley decompositions for normal forms of nilpotent dynamical systems

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Jan Sanders and Richard Cushman, realizing that a nilpotent system $\dot{\mathbf{x}} = N\mathbf{x} + \mathbf{v}$ of differential equations is in normal form when the higher order terms $\mathbf{v}$ are equivariant with respect to the flow of a nilpotent matrix M different from N, introduced the use of classical invariant theory and modern commutative algebra (Stanley decompositions) into the study of such normal forms. More recently Jan and I, [1], simplified this work by introducing the *box product* method, which allows the form of the normal form for systems with given N to be determined if the form of the normal form is known for the simpler systems having the individual Jordan blocks of N for their linear parts. I will present a new way of computing box products, which is conceptually clearer than the previous method and produces simpler results, although the computation itself is more difficult. The new method also extends to invariants that are of interest in quantum computing, although the previous method does not.

References

[1] Murdock, James and Sanders, Jan A.: A new transvectant algorithm for nilpotent normal forms. *J. Diff. Eq.* **238** (2007) 234-256.

Integrable systems in 3D: deformations of dispersionless limits.

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Classification of integrable systems remains as a topic of active research from the beginning of soliton theory. Numerous classification results are obtained in 1 + 1 dimensions by means of the symmetry approach. Although the symmetry approach is also applicable to 2 + 1-dimensional systems, one encounters additional difficulties due to the appearance of nonlocal variables. There are several techniques to tackle the problem (e.g. the perturbative symmetry approach). In the perturbative symmetry approach one starts with a linear equation having a degenerate dispersion law and reconstructs the allowed nonlinearity.

In this talk we present a novel perturbative approach to the classification problem. Based on the method hydrodynamic reductions, we first classify integrable quasilinear systems which may potentially occur as dispersionless limits of integrable 2 + 1-dimensional soliton equations. Subsequently we construct dispersive deformations preserving integrability deforming the hyrdrodynamic reductions by dispersive deformations and requiring that all hydrodynamic reductions of the dispersionless limit will be inherited by the corresponding dispersive counterpart. The method also allows to effectively reconstruct Lax representations of the deformed systems. We present various classification results obtained in the frame of the new approach, e.g. the classification of scalar 2+1-dimensional equations generalizing KP, BKP/CKP, the classification of Davey-Stewartson type systems as well as various classifications of 2 + 1-dimensional differential-difference equations.

The talk is based on joint work with E. Ferapontov, A. Moro, B. Huard and I. Roustemoglou.

Walking with Jan: Lyapunov-Schmidt meets Normal Form

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The classical Lyapunov-Schmidt method for the study of the bifurcation of small periodic orbits from an equilibrium (generalised Hopf bifurcation) uses a purely functional analytic approach, concentrating solely on the existence of periodic orbits and discarding almost all dynamical aspects of the problem. Inspired by some work of Jan Sanders we found an alternative approach to the method which incorporates normal forms and which leads to a much more natural formulation of the reduction result. Although in our talk we will only discuss the case of plain continuous systems it is possible to adapt the approach to systems with additional structures, such as equivariant systems, reversible systems or Hamiltonian systems, and also to the discrete counterparts of such systems; some of these cases are discussed in more detail in the references.

References

[1] Vanderbauwhede, A. and J.-C. van der Meer: A general reduction method for periodic solutions near equilibria in Hamiltonian systems. *Normal Forms and Homoclinic Chaos*, W.F. Langford and W. Nagata (Eds.), A.M.S. Providence (1995), 273-294.

[2] Takens, F. and A. Vanderbauwhede: Local invariant manifolds and normal forms. *Handbook of Dynamical Systems*, Vol. 3, H.W. Broer, B. Hasselblatt and F. Takens (Eds.), Elsevier (2010), 89-124.

[3] Vanderbauwhede, A.: Lyapunov-Schmidt – A discrete revision. *Discrete Dynamics and Difference Equations*, S. Elaydi, H. Oliveira, J.M. Ferreira and J.F. Alves (Eds.), World Scientific (2010), 120-143.

Are integrable systems really interesting?

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Non-generic behaviour in Hamiltonian systems can be a sign of integrability, but it is not a conclusive indication. We will briefly review the integrability of Hamiltonian normal forms in two and three degrees of freedom. In addtion two integrable normal form Hamiltonian chains, FPU and $1 : 2 : \ldots : 2$, are discussed and three non-integrable normal form chains.

References

[1] Jan A. Sanders: Are higher-order resonances really interesting? *Celestial Mech.* **16** (1977) 421–440.

Symbolic representation and its applications

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Symbolic representation can be viewed as a simplified notation for a Fourier transform. It transforms the problems in differential algebra into the ones in algebra of symmetric polynomials. This enables us to use powerful results from Diophantine equations, algebraic geometry and commutative algebra. In this talk, I'll give a brief account on its applications in three aspects:

1. The global classification of integrable scalar evolutionary equations
2. The construction of an infinite family of differential operators producing Lie subalgebras
3. The structure of $(2 + 1)$–dimensional commutative and noncommutative integrable equations

The first two aspects are joint works with Jan Sanders.
