

**Example 1. (genus 2, rational 5-torsion point)**

For non-zero  $b \in \mathbb{Z}$

$$y^2 + ((4b - 3)x^2 - (5b - 1)x + b)y + x^5 = 0.$$

defines a nonsingular curve of genus 2 with a rational 5-torsion point  $(0, 0)$ .

The 2-torsion polynomial has three rational factors

$$\begin{aligned} m_1(x) &= x - 1, \\ m_2(x) &= 4x - 1, \\ m_3(x) &= x^3 - (4b^2 - 6b + 1)x^2 + (5b^2 - b)x - b^2. \end{aligned}$$

$$\Lambda' = \begin{cases} \langle M_1, \dots, M_k \rangle, & f(0) \neq \pm 1, \\ \langle M_1, \dots, M_k, \mathbb{M} \rangle, & f(0) = \pm 1, d = 2g + 1, \\ \langle M_1, \dots, M_k, 2\mathbb{M} \rangle, & f(0) = \pm 1, d = 2g + 2, \end{cases}$$

$$\Lambda = \Lambda' \cap K_2(C; \mathbb{Z})$$

$\Lambda = \langle M_1, M_2, M_3, \mathbb{M} \rangle$  for  $b = \pm 1$  and  $\Lambda = \langle M_1, M_2 \rangle$  otherwise.

TABLE 1. Genus 2 curves  $y^2 + ((4b - 3)x^2 - (5b - 1)x + b)y + x^5 = 0$

| $b$ | Conductor                                             | $\Lambda$                                                                  | $L^*(0)$          | $L^*(0)/R(\Lambda)$             |
|-----|-------------------------------------------------------|----------------------------------------------------------------------------|-------------------|---------------------------------|
| -10 | $2^2 \cdot 3 \cdot 5^2 \cdot 7 \cdot 11^2 \cdot 1031$ | $\langle M_1, M_2 \rangle$                                                 | 161973.28326750   | $2^2 \cdot 3 \cdot 5^2 \cdot 7$ |
| -9  | $3^3 \cdot 23 \cdot 8461$                             | $\langle M_1, M_2 \rangle$                                                 | 2647.35530780277  | $2^2 \cdot 3^2$                 |
| -8  | $2^2 \cdot 3 \cdot 61 \cdot 6113$                     | $\langle M_1, M_2 \rangle$                                                 | 2402.40716016213  | $3 \cdot 23/2$                  |
| -7  | $3 \cdot 7^2 \cdot 53 \cdot 4243$                     | $\langle M_1, M_2 \rangle$                                                 | 16142.83542365697 | $13 \cdot 19$                   |
| -6  | $2^2 \cdot 3^3 \cdot 5 \cdot 2797$                    | $\langle M_1, M_2 \rangle$                                                 | 1090.93388314409  | $2 \cdot 3^2$                   |
| -5  | $3 \cdot 5^2 \cdot 37 \cdot 1721$                     | $\langle M_1, M_2 \rangle$                                                 | 1602.46040140665  | 29                              |
| -4  | $2^2 \cdot 3 \cdot 29 \cdot 31^2$                     | $\langle M_1, M_2 \rangle$                                                 | 196.40102935124   | $2^2$                           |
| -3  | $3^3 \cdot 7 \cdot 463$                               | $\langle M_1, M_2 \rangle$                                                 | 41.79313813774    | 1                               |
| -2  | $2^2 \cdot 3 \cdot 13 \cdot 173$                      | $\langle M_1, M_2 \rangle$                                                 | 16.33803025572    | $1/2$                           |
| -1  | $3 \cdot 5 \cdot 37$                                  | $\langle M_1, M_2, M_3, \mathbb{M} \rangle$<br>$41M_1 + 56M_2 - 44M_3 = 0$ | 0.22831231664     | 1                               |
| 1   | $3 \cdot 11^2$                                        | $\langle M_1, M_2, M_3, \mathbb{M} \rangle$<br>$4M_1 - 26M_2 + 29M_3 = 0$  | 0.12984963471     | $1/2$                           |
| 2   | $2^2 \cdot 3 \cdot 13 \cdot 19$                       | $\langle M_1, M_2 \rangle$                                                 | 1.90319317512     | $1/2^2 \cdot 5$                 |
| 3   | $3^3 \cdot 47$                                        | $\langle M_1, M_2 \rangle$                                                 | 0.62178975663     | $1/2 \cdot 3^3$                 |
| 4   | $2^2 \cdot 3 \cdot 5 \cdot 7 \cdot 223$               | $\langle M_1, M_2 \rangle$                                                 | 42.90813731245    | 1                               |
| 5   | $3 \cdot 5^2 \cdot 43 \cdot 569$                      | $\langle M_1, M_2 \rangle$                                                 | 802.87262799199   | $2^4$                           |
| 6   | $2^2 \cdot 3^3 \cdot 17^2 \cdot 67$                   | $\langle M_1, M_2 \rangle$                                                 | 1575.43548439888  | $2^2 \cdot 7$                   |
| 7   | $3 \cdot 7^2 \cdot 59 \cdot 1987$                     | $\langle M_1, M_2 \rangle$                                                 | 6154.43484637855  | $2^2 \cdot 5^2$                 |
| 8   | $2^2 \cdot 3 \cdot 67 \cdot 3167$                     | $\langle M_1, M_2 \rangle$                                                 | 1788.15229247201  | $3^3$                           |
| 9   | $3^3 \cdot 5 \cdot 4733$                              | $\langle M_1, M_2 \rangle$                                                 | 281.80083335457   | $2^2$                           |
| 10  | $2^2 \cdot 3 \cdot 5^2 \cdot 23 \cdot 83 \cdot 293$   | $\langle M_1, M_2 \rangle$                                                 | 85596.822531781   | $2^7 \cdot 3^2$                 |

**Example 2. (genus 2, rational 6-torsion point)**

The curves given by

$$y^2 + (2x^3 - 4bx^2 - x + b)y + x^6 = 0, \quad b \in \mathbb{N}$$

are of genus 2 and have a rational 6-torsion point  $(0, 0)$ . The 2-torsion polynomial has four factors

$$\begin{aligned} m_1(x) &= x - b, \\ m_2(x) &= 2x - 1, \\ m_3(x) &= 2x + 1, \\ m_4(x) &= 4bx^2 + x - b, \end{aligned}$$

$M_1, M_2, M_3, M_4$  in  $K_2^T(C)/\text{torsion}$  satisfy  $M_1 + M_2 + M_3 - M_4 = 0$ .

TABLE 2. Genus 2 curves  $y^2 + (2x^3 - 4bx^2 - x + b)y + x^6 = 0$

| $b$ | Conductor                                           | $\Lambda$                                                                | $L^*(0)$      | $L^*(0)/R(\Lambda)$     |
|-----|-----------------------------------------------------|--------------------------------------------------------------------------|---------------|-------------------------|
| 1   | $2^4 \cdot 3 \cdot 17$                              | $\langle M_1, M_2, M_3, 2\mathbb{M} \rangle$<br>$10M_1 - 2M_2 + M_3 = 0$ | 0.35283625317 | $1/2^3$                 |
| 2   | $2^5 \cdot 3 \cdot 5^2 \cdot 13$                    | $\langle M_2, M_3 \rangle$                                               | 22.9767849706 | $1/2$                   |
| 3   | $2^4 \cdot 3^2 \cdot 5^2 \cdot 7 \cdot 29$          | $\langle M_2, M_3 \rangle$                                               | 347.644931187 | $2 \cdot 3$             |
| 4   | $2^5 \cdot 3 \cdot 7 \cdot 257$                     | $\langle M_2, M_3 \rangle$                                               | 134.428839854 | 2                       |
| 5   | $2^4 \cdot 3 \cdot 5^2 \cdot 11 \cdot 401$          | $\langle M_2, M_3 \rangle$                                               | 2694.74551646 | $2^2 \cdot 3^2$         |
| 6   | $2^5 \cdot 3^2 \cdot 11 \cdot 13 \cdot 577$         | $\langle M_2, M_3 \rangle$                                               | 20021.3775652 | $2 \cdot 3 \cdot 41$    |
| 8   | $2^5 \cdot 3 \cdot 5^2 \cdot 17 \cdot 41$           | $\langle M_2, M_3 \rangle$                                               | 1106.79592308 | $2^2 \cdot 3$           |
| 10  | $2^5 \cdot 3 \cdot 5^2 \cdot 7 \cdot 19 \cdot 1601$ | $\langle M_2, M_3 \rangle$                                               | 416121.1462   | $2^2 \cdot 3 \cdot 7^3$ |
| 12  | $2^5 \cdot 3^2 \cdot 5^2 \cdot 23 \cdot 461$        | $\langle M_2, M_3 \rangle$                                               | 58663.7341258 | $2^2 \cdot 3^3 \cdot 5$ |
| 13  | $2^4 \cdot 3 \cdot 5^2 \cdot 13^2 \cdot 541$        | $\langle M_2, M_3 \rangle$                                               | 46380.28556   | $2 \cdot 3^2 \cdot 23$  |
| 14  | $2^5 \cdot 3 \cdot 7^2 \cdot 29 \cdot 3137$         | $\langle M_2, M_3 \rangle$                                               | 322837.4973   | $2 \cdot 3 \cdot 467$   |

### Example 3. (genus 3, rational 7-torsion point)

A special case is the two-parameter family

$$y^2 + ((4b-3)x^3 - (4a+5b-1)x^2 + (5a+b)x - a)y + x^7 = 0$$

of genus 3 curves with rational 7-torsion point  $(0, 0)$  and two rational 2-torsion points with  $x$ -coordinates 1 and  $1/4$ .

$$t(x) = -x^7 + ((4b-3)x^3 - (4a+5b-1)x^2 + (5a+b)x + a)^2/4 = -\frac{1}{4}m_1(x)m_2(x)m_3(x)$$

$$m_1(x) = x - 1,$$

$$m_2(x) = 4x - 1,$$

$$m_3(x) = x^5 - (4b^2 - 6b + 1)x^4 + (5b^2 + 8ab - 2b - 6a)x^3 - (b^2 + 10ab + 4a^2 - 2a)x^2 + (5a^2 + 2ab)x - a^2.$$

For  $a = \pm 1$  we get  $\Lambda = \langle M_1, M_2, M_3, \mathbb{M} \rangle \subseteq K_2(C; \mathbb{Z})$ .

If  $a=1, b=3$  we have

$$m_3(x) = m_{3,1}(x)m_{3,2}(x) = (x^2 - 3x + 1)(x^3 - 16x^2 + 8x - 1)$$

so  $\Lambda = \langle M_1, M_2, M_{3,1}, M_{3,2}, \mathbb{M} \rangle \subseteq K_2(C; \mathbb{Z})$  and we expect a linear dependency.

TABLE 3. Genus 3 curves  $y^2 + ((4b-3)x^3 - (4a+5b-1)x^2 + (5a+b)x - a)y + x^7 = 0$

| $a, b$ | Conductor                                | $\Lambda$                                                                                                | $L^*(0)$        | $L^*(0)/R(\Lambda)$   |
|--------|------------------------------------------|----------------------------------------------------------------------------------------------------------|-----------------|-----------------------|
| -1, -6 | $3 \cdot 5 \cdot 13 \cdot 62773$         | $\langle M_1, M_2, M_3, \mathbb{M} \rangle$                                                              | 157.32845991763 | $2 \cdot 3$           |
| -1, -5 | $2 \cdot 3 \cdot 5 \cdot 34543$          | $\langle M_1, M_2, M_3, \mathbb{M} \rangle$                                                              | 13.71462235524  | 1                     |
| -1, -4 | $3^3 \cdot 7^2 \cdot 73$                 | $\langle M_1, M_2, M_3, \mathbb{M} \rangle$                                                              | 0.99948043865   | $1/2 \cdot 3$         |
| -1, -2 | $3 \cdot 19 \cdot 1051$                  | $\langle M_1, M_2, M_3, \mathbb{M} \rangle$                                                              | 0.68841617093   | $1/2 \cdot 7$         |
| -1, 0  | $3 \cdot 5 \cdot 7 \cdot 997$            | $\langle M_1, M_2, M_3, \mathbb{M} \rangle$                                                              | 1.30028297707   | $1/2 \cdot 3$         |
| -1, 1  | $2 \cdot 3 \cdot 43 \cdot 599$           | $\langle M_1, M_2, M_3, \mathbb{M} \rangle$                                                              | 1.85769147358   | $1/2$                 |
| -1, 2  | $3^3 \cdot 17 \cdot 199$                 | $\langle M_1, M_2, M_3, \mathbb{M} \rangle$                                                              | 0.95009772023   | $1/2 \cdot 3 \cdot 5$ |
| -1, 3  | $2 \cdot 3 \cdot 7^2 \cdot 59 \cdot 599$ | $\langle M_1, M_2, M_3, \mathbb{M} \rangle$                                                              | 117.81139857649 | $2^2$                 |
| 1, 0   | $3 \cdot 29^2 \cdot 71$                  | $\langle M_1, M_2, M_3, \mathbb{M} \rangle$                                                              | 1.91931648711   | $1/3$                 |
| 1, 2   | $3 \cdot 13 \cdot 971$                   | $\langle M_1, M_2, M_3, \mathbb{M} \rangle$                                                              | 0.41650834991   | $1/19$                |
| 1, 3   | $2 \cdot 3 \cdot 5^2 \cdot 229$          | $\langle M_1, M_2, M_{3,1}, M_{3,2}, \mathbb{M} \rangle$<br>$36M_1 - 111M_2 - 41M_{3,1} + 71M_{3,2} = 0$ | 0.33653518886   | 1                     |
| 1, 4   | $3^2 \cdot 7877$                         | $\langle M_1, M_2, M_3, \mathbb{M} \rangle$                                                              | 0.84449758753   | $2/3 \cdot 5$         |
| 1, 5   | $2 \cdot 3 \cdot 11 \cdot 44071$         | $\langle M_1, M_2, M_3, \mathbb{M} \rangle$                                                              | 39.13475965835  | 2                     |
| 1, 6   | $3 \cdot 5^2 \cdot 19 \cdot 46273$       | $\langle M_1, M_2, M_3, \mathbb{M} \rangle$                                                              | 631.40978672220 | $2^2 \cdot 5$         |
| 1, 7   | $2 \cdot 3^2 \cdot 479 \cdot 2011$       | $\langle M_1, M_2, M_3, \mathbb{M} \rangle$                                                              | 170.28405530696 | $2^2$                 |

#### Example 4. (genus 4 and 5)

For  $g = 4$  and  $g = 5$  a computer search gives some examples of curves of the form

$$y^2 + f(x)y + x^{10} = 0$$

with  $f(x)$  of degree 5 and

$$y^2 + f(x)y + x^{12} = 0$$

with  $f(x)$  of degree 6 respectively, where  $4t(x) = -4x^{10} + f(x)^2$  (resp.  $-4x^{12} + f(x)^2$ ) has  $g$  factors in  $\mathbb{Z}[x]$  with constant term  $\pm 1$ .  $(0, 0)$  is a point of order 10 (resp. 12). In all cases,  $\Lambda \subseteq K_2(C; \mathbb{Z})$  has rank  $g$ .

TABLE 4. Genus 4 curves  $y^2 + f(x)y + x^{10} = 0$

| $f(x)$                               | Conductor                                    | $L^*(0)$    | $L^*(0)/R(\Lambda)$ |
|--------------------------------------|----------------------------------------------|-------------|---------------------|
| $2x^5 + 2x^4 + x^3 + x^2 - 3x - 1$   | $2^{11} \cdot 5^3 \cdot 19 \cdot 29$         | 35.85879769 | $1/2$               |
| $2x^5 + 2x^4 + 2x^3 - 3x^2 - 2x + 1$ | $2^{11} \cdot 3^2 \cdot 17 \cdot 59$         | 5.336928011 | $1/2$               |
| $2x^5 + 3x^4 - 3x^3 - 2x^2 - x - 1$  | $2^3 \cdot 3^3 \cdot 5 \cdot 19 \cdot 331$   | 1.865694255 | $1/2^2 \cdot 3$     |
| $2x^5 + 3x^4 - x^3 - 4x^2 - 3x + 1$  | $2^3 \cdot 3^4 \cdot 5 \cdot 7 \cdot 20759$  | 126.4283012 | 1                   |
| $2x^5 + 3x^4 + x^3 - 3x - 1$         | $2^4 \cdot 3^3 \cdot 7 \cdot 11 \cdot 4793$  | 41.29358643 | $1/2$               |
| $2x^5 + 4x^4 - 3x^3 - 2x + 1$        | $2^4 \cdot 5^3 \cdot 7 \cdot 11 \cdot 103$   | 3.546483598 | $1/2^3$             |
| $2x^5 + 4x^4 - x^3 - 3x^2 - 3x - 1$  | $2^{10} \cdot 3 \cdot 7 \cdot 1051$          | 6.484247251 | $1/2^3$             |
| $2x^5 + 4x^4 - x^3 - 3x^2 - x + 1$   | $2^{12} \cdot 3^6 \cdot 13$                  | 11.50901911 | $1/2^2$             |
| $2x^5 + 4x^4 + 3x^3 - 5x^2 - 5x - 1$ | $2^{12} \cdot 3 \cdot 5 \cdot 19 \cdot 79$   | 27.69939565 | $1/2$               |
| $2x^5 + 4x^4 + 3x^3 + 3x^2 - x - 1$  | $2^{12} \cdot 3 \cdot 5^2 \cdot 23 \cdot 43$ | 89.28895569 | 1                   |
| $2x^5 + 4x^4 + 5x^3 + 2x^2 + 2x + 1$ | $2^4 \cdot 3^3 \cdot 7^2 \cdot 379$          | 2.157167657 | $1/2^4$             |

TABLE 5. Genus 5 curves  $y^2 + f(x)y + x^{12} = 0$

| $f(x)$                                      | Conductor                                         | $L^*(0)$         | $L^*(0)/R(\Lambda)$ |
|---------------------------------------------|---------------------------------------------------|------------------|---------------------|
| $2x^6 + 2x^5 - 4x^4 - 3x^3 - 2x^2 + 4x - 1$ | $2^8 \cdot 3^4 \cdot 5 \cdot 7 \cdot 11 \cdot 19$ | 0.97906446422637 | $1/2^2 \cdot 3^3$   |
| $2x^6 + 4x^5 - 5x^4 - 3x^3 + 2x^2 - x - 1$  | $2^7 \cdot 3^2 \cdot 5^2 \cdot 107 \cdot 139$     | 2.86707608488323 | $1/2^2 \cdot 3$     |
| $2x^6 + 6x^5 - 5x^3 - 3x^2 + x + 1$         | $2^{16} \cdot 3^2 \cdot 5 \cdot 13^2$             | 3.21518014215484 | $1/2^2 \cdot 3$     |
| $2x^6 + 2x^5 + 2x^4 - x^3 - 3x^2 - 3x - 1$  | $2^{16} \cdot 3 \cdot 5^3 \cdot 31$               | 4.04748393920751 | $1/2^3 \cdot 3$     |
| $2x^6 + 2x^5 + x^3 - 3x^2 - x + 1$          | $2^{16} \cdot 3^4 \cdot 5 \cdot 181$              | 28.41118880946   | $1/3$               |

**Example 5. (arbitrary genus)** Consider a curve of the form

$$y^2 + \left(2x^{g+1} + \prod_{j=1}^g (v_j x + 1)\right) y + x^{2g+2} = 0 ,$$

where  $0 < v_1 < \dots < v_g$  are distinct non-zero integers, and we are assuming that the 2-torsion polynomial  $t(x)$  has no multiple roots. Then  $t(x)$  has at least  $g + 1$  factors in  $\mathbb{Z}[x]$ ,

$$v_1 x + 1, v_2 x + 1, \dots, v_g x + 1, 4x^{g+1} + \prod_{j=1}^g (v_j x + 1) .$$

TABLE 6. Curves  $y^2 + (2x^3 + (v_1 x + 1)(v_2 x + 1))y + x^6 = 0$  (genus 2)

| $v$   | Conductor                                           | $\Lambda$                               | $L^*(0)$           | $L^*(0)/R(\Lambda)$     |
|-------|-----------------------------------------------------|-----------------------------------------|--------------------|-------------------------|
| 9, 10 | $2^3 \cdot 3 \cdot 5 \cdot 5261$                    | $\langle M_1, M_2, 2\mathbb{M} \rangle$ | 275.183140339336   | $2^2 \cdot 3$           |
| 8, 10 | $2^6 \cdot 5 \cdot 2221$                            | $\langle M_1, M_2, 2\mathbb{M} \rangle$ | 417.499914032374   | 13                      |
| 2, 11 | $2^3 \cdot 3 \cdot 11 \cdot 6053$                   | $\langle M_1, M_2, 2\mathbb{M} \rangle$ | 689.050402379536   | $3 \cdot 7$             |
| 5, 12 | $2^3 \cdot 3 \cdot 5^2 \cdot 7^2 \cdot 61$          | $\langle M_1, M_2, 2\mathbb{M} \rangle$ | 860.242245920864   | $2 \cdot 3^2$           |
| 2, 12 | $2^6 \cdot 3 \cdot 5 \cdot 2341$                    | $\langle M_1, M_2, 2\mathbb{M} \rangle$ | 1267.775571888346  | $2^2 \cdot 3^2$         |
| 7, 15 | $2 \cdot 3 \cdot 5^2 \cdot 7^2 \cdot 313$           | $\langle M_1, M_2, 2\mathbb{M} \rangle$ | 1075.913430726619  | $2 \cdot 3^2$           |
| 8, 17 | $2 \cdot 3 \cdot 17 \cdot 23321$                    | $\langle M_1, M_2, 2\mathbb{M} \rangle$ | 1201.189781491657  | $2 \cdot 3^2$           |
| 5, 10 | $2^3 \cdot 5^2 \cdot 59 \cdot 263$                  | $\langle M_1, M_2, 2\mathbb{M} \rangle$ | 1440.017545464245  | $2^2 \cdot 3^2$         |
| 4, 10 | $2^6 \cdot 3 \cdot 5 \cdot 17 \cdot 197$            | $\langle M_1, M_2, 2\mathbb{M} \rangle$ | 1562.214360456082  | $2^3 \cdot 5$           |
| 6, 10 | $2^6 \cdot 3 \cdot 5 \cdot 3797$                    | $\langle M_1, M_2, 2\mathbb{M} \rangle$ | 1888.148590577887  | $2^4 \cdot 3$           |
| 4, 13 | $2^3 \cdot 3 \cdot 11 \cdot 13^2 \cdot 89$          | $\langle M_1, M_2, 2\mathbb{M} \rangle$ | 2048.440414253531  | $2 \cdot 3 \cdot 7$     |
| 8, 16 | $2^6 \cdot 109 \cdot 601$                           | $\langle M_1, M_2, 2\mathbb{M} \rangle$ | 2029.415672068518  | $2^5$                   |
| 4, 14 | $2^6 \cdot 5 \cdot 7^2 \cdot 373$                   | $\langle M_1, M_2, 2\mathbb{M} \rangle$ | 3298.078842289480  | $2^6$                   |
| 6, 12 | $2^6 \cdot 3^5 \cdot 431$                           | $\langle M_1, M_2, 2\mathbb{M} \rangle$ | 3779.698402282094  | $2 \cdot 3 \cdot 13$    |
| 4, 11 | $2^4 \cdot 7 \cdot 11 \cdot 23 \cdot 239$           | $\langle M_1, M_2, 2\mathbb{M} \rangle$ | 3065.942241509888  | $2^3 \cdot 3^2$         |
| 3, 10 | $2^3 \cdot 3 \cdot 5 \cdot 7^2 \cdot 1307$          | $\langle M_1, M_2, 2\mathbb{M} \rangle$ | 3909.737148728376  | $2^2 \cdot 3^3$         |
| 7, 10 | $2^3 \cdot 3 \cdot 5^2 \cdot 7 \cdot 43 \cdot 59$   | $\langle M_1, M_2, 2\mathbb{M} \rangle$ | 4653.068964642799  | $2 \cdot 3^2 \cdot 7$   |
| 5, 13 | $2^5 \cdot 5 \cdot 13 \cdot 31 \cdot 263$           | $\langle M_1, M_2, 2\mathbb{M} \rangle$ | 8592.680376054557  | $2^3 \cdot 3 \cdot 7$   |
| 7, 16 | $2^5 \cdot 3 \cdot 7^2 \cdot 4493$                  | $\langle M_1, M_2, 2\mathbb{M} \rangle$ | 11344.672982180578 | $2^2 \cdot 3^2 \cdot 5$ |
| 5, 11 | $2^3 \cdot 3 \cdot 5 \cdot 11 \cdot 26573$          | $\langle M_1, M_2, 2\mathbb{M} \rangle$ | 16672.060582310110 | $2 \cdot 3^3 \cdot 7$   |
| 6, 14 | $2^6 \cdot 3 \cdot 7 \cdot 41 \cdot 677$            | $\langle M_1, M_2, 2\mathbb{M} \rangle$ | 20507.523505991035 | $2^4 \cdot 23$          |
| 7, 14 | $2^3 \cdot 7^2 \cdot 117541$                        | $\langle M_1, M_2, 2\mathbb{M} \rangle$ | 20242.092778779142 | $2^3 \cdot 3^2 \cdot 5$ |
| 3, 13 | $2^3 \cdot 3 \cdot 5^2 \cdot 13 \cdot 6553$         | $\langle M_1, M_2, 2\mathbb{M} \rangle$ | 20094.811949554845 | $2 \cdot 3^2 \cdot 5^2$ |
| 6, 16 | $2^6 \cdot 3 \cdot 5 \cdot 73 \cdot 773$            | $\langle M_1, M_2, 2\mathbb{M} \rangle$ | 26721.552091088677 | $2^4 \cdot 3^3$         |
| 6, 15 | $2^3 \cdot 3^5 \cdot 5 \cdot 59 \cdot 101$          | $\langle M_1, M_2, 2\mathbb{M} \rangle$ | 28628.778269858470 | $2 \cdot 3^5$           |
| 5, 14 | $2^3 \cdot 3 \cdot 5 \cdot 7 \cdot 95621$           | $\langle M_1, M_2, 2\mathbb{M} \rangle$ | 32866.799798423930 | $2 \cdot 3 \cdot 101$   |
| 5, 15 | $2^3 \cdot 3 \cdot 5^2 \cdot 29 \cdot 4673$         | $\langle M_1, M_2, 2\mathbb{M} \rangle$ | 41114.005206032957 | $2^4 \cdot 3^2 \cdot 5$ |
| 8, 18 | $2^6 \cdot 3 \cdot 5^2 \cdot 73 \cdot 353$          | $\langle M_1, M_2, 2\mathbb{M} \rangle$ | 59468.612113790229 | $2^2 \cdot 3 \cdot 71$  |
| 6, 13 | $2^3 \cdot 3 \cdot 7^2 \cdot 13 \cdot 17 \cdot 619$ | $\langle M_1, M_2, 2\mathbb{M} \rangle$ | 81849.542063836685 | $2 \cdot 3^3 \cdot 29$  |

TABLE 7. Curves  $y^2 + (2x^4 + (v_1x+1)(v_2x+1)(v_3x+1))y + x^8 = 0$  (genus 3)

| $v$      | Conductor                                   | $\Lambda$                                                                    | $L^*(0)$          | $L^*(0)/R(\Lambda)$ |
|----------|---------------------------------------------|------------------------------------------------------------------------------|-------------------|---------------------|
| 2, 4, 6  | $2^8 \cdot 3^3 \cdot 13$                    | $\langle M_1, M_2, M_3, 2\mathbb{M} \rangle$                                 | 1.237888702787    | $1/2^7$             |
| 1, 3, 4  | $2^8 \cdot 3^2 \cdot 7^2$                   | $\langle M_1, M_2, M_3, 2\mathbb{M} \rangle$                                 | 1.319632692018    | $1/2^6$             |
| 1, 2, 4  | $2^6 \cdot 3^3 \cdot 67$                    | $\langle M_1, M_2, M_3, 2\mathbb{M} \rangle$                                 | 1.326260789773    | $1/2^5$             |
| 1, 2, 3  | $2^6 \cdot 3^3 \cdot 73$                    | $\langle M_1, M_2, M_3, 2\mathbb{M} \rangle$                                 | 1.360553792128    | $1/2^4$             |
| 1, 5, 6  | $2^8 \cdot 3^3 \cdot 5^3$                   | $\langle M_1, M_2, M_3, M_4, 2\mathbb{M} \rangle$<br>$M_1+M_2+11M_3-12M_4=0$ | 9.556528296211    | $1/2$               |
| 1, 3, 5  | $2^6 \cdot 3^3 \cdot 5 \cdot 179$           | $\langle M_1, M_2, M_3, 2\mathbb{M} \rangle$                                 | 15.763316182605   | $1/2^3$             |
| 3, 4, 7  | $2^8 \cdot 3^2 \cdot 7 \cdot 113$           | $\langle M_1, M_2, M_3, 2\mathbb{M} \rangle$                                 | 21.282782316338   | $1/2^3$             |
| 1, 2, 5  | $2^8 \cdot 3^3 \cdot 5 \cdot 53$            | $\langle M_1, M_2, M_3, 2\mathbb{M} \rangle$                                 | 18.574599809025   | $1/2^2$             |
| 2, 6, 8  | $2^{14} \cdot 3^2 \cdot 17$                 | $\langle M_1, M_2, M_3, 2\mathbb{M} \rangle$                                 | 41.533490724724   | $1/2^3$             |
| 2, 3, 5  | $2^6 \cdot 3^2 \cdot 5^3 \cdot 89$          | $\langle M_1, M_2, M_3, 2\mathbb{M} \rangle$                                 | 72.152711581916   | 1                   |
| 1, 2, 6  | $2^6 \cdot 3^3 \cdot 5^3 \cdot 43$          | $\langle M_1, M_2, M_3, 2\mathbb{M} \rangle$                                 | 105.836805455409  | 1                   |
| 1, 3, 6  | $2^6 \cdot 3^2 \cdot 5 \cdot 47 \cdot 73$   | $\langle M_1, M_2, M_3, 2\mathbb{M} \rangle$                                 | 98.432671578138   | $1/2$               |
| 2, 3, 6  | $2^6 \cdot 3^2 \cdot 7^2 \cdot 373$         | $\langle M_1, M_2, M_3, 2\mathbb{M} \rangle$                                 | 113.826557165808  | 1                   |
| 1, 3, 7  | $2^6 \cdot 3^2 \cdot 7 \cdot 2837$          | $\langle M_1, M_2, M_3, 2\mathbb{M} \rangle$                                 | 137.788942178887  | 1                   |
| 1, 4, 6  | $2^6 \cdot 3^2 \cdot 5 \cdot 5669$          | $\langle M_1, M_2, M_3, 2\mathbb{M} \rangle$                                 | 198.333924314685  | $3/2$               |
| 1, 2, 9  | $2^7 \cdot 3^3 \cdot 7 \cdot 883$           | $\langle M_1, M_2, M_3, 2\mathbb{M} \rangle$                                 | 206.791570180633  | 1                   |
| 3, 4, 8  | $2^5 \cdot 3^3 \cdot 5 \cdot 7^2 \cdot 101$ | $\langle M_1, M_2, M_3, 2\mathbb{M} \rangle$                                 | 219.415523333470  | 1                   |
| 4, 5, 9  | $2^8 \cdot 3^3 \cdot 5^3 \cdot 29$          | $\langle M_1, M_2, M_3, 2\mathbb{M} \rangle$                                 | 278.222780993318  | 1                   |
| 2, 4, 10 | $2^{11} \cdot 3^3 \cdot 5 \cdot 101$        | $\langle M_1, M_2, M_3, 2\mathbb{M} \rangle$                                 | 375.379161057633  | 1                   |
| 1, 2, 8  | $2^6 \cdot 3^3 \cdot 7 \cdot 29 \cdot 103$  | $\langle M_1, M_2, M_3, 2\mathbb{M} \rangle$                                 | 429.330911878658  | $5/2$               |
| 1, 4, 9  | $2^7 \cdot 3^2 \cdot 5 \cdot 9281$          | $\langle M_1, M_2, M_3, 2\mathbb{M} \rangle$                                 | 554.543543287698  | 2                   |
| 1, 3, 9  | $2^6 \cdot 3^2 \cdot 137437$                | $\langle M_1, M_2, M_3, 2\mathbb{M} \rangle$                                 | 831.548080326194  | $2^2$               |
| 1, 5, 8  | $2^7 \cdot 3^3 \cdot 5 \cdot 7 \cdot 659$   | $\langle M_1, M_2, M_3, 2\mathbb{M} \rangle$                                 | 798.968120685204  | 3                   |
| 1, 4, 7  | $2^8 \cdot 3^2 \cdot 7 \cdot 5879$          | $\langle M_1, M_2, M_3, 2\mathbb{M} \rangle$                                 | 930.405544167007  | 5                   |
| 1, 2, 7  | $2^6 \cdot 3^3 \cdot 5 \cdot 7 \cdot 1999$  | $\langle M_1, M_2, M_3, 2\mathbb{M} \rangle$                                 | 1105.646745724194 | $2^3$               |
| 1, 3, 8  | $2^7 \cdot 3^3 \cdot 5^3 \cdot 7 \cdot 41$  | $\langle M_1, M_2, M_3, 2\mathbb{M} \rangle$                                 | 1395.300068143365 | $2^3$               |
| 1, 4, 8  | $2^5 \cdot 3^3 \cdot 7 \cdot 21773$         | $\langle M_1, M_2, M_3, 2\mathbb{M} \rangle$                                 | 1402.036187242234 | $2 \cdot 3$         |
| 2, 4, 8  | $2^{11} \cdot 3^3 \cdot 4093$               | $\langle M_1, M_2, M_3, 2\mathbb{M} \rangle$                                 | 3341.033608754311 | $2^2 \cdot 3$       |

 TABLE 8. Curves  $y^2 + (2x^5 + (v_1x+1) \cdots (v_4x+1))y + x^{10} = 0$  (genus 4)

| $v$        | Conductor                           | $\Lambda$                                         | $L^*(0)$        | $L^*(0)/R(\Lambda)$ |
|------------|-------------------------------------|---------------------------------------------------|-----------------|---------------------|
| 1, 2, 3, 4 | $2^9 \cdot 3^2 \cdot 17 \cdot 113$  | $\langle M_1, M_2, M_3, M_4, 2\mathbb{M} \rangle$ | 2.497987723694  | $1/2^5 \cdot 5$     |
| 2, 3, 4, 6 | $2^8 \cdot 3^2 \cdot 80251$         | $\langle M_1, M_2, M_3, M_4, 2\mathbb{M} \rangle$ | 45.006091920102 | $1/2^5$             |
| 1, 2, 3, 5 | $2^9 \cdot 3^2 \cdot 5 \cdot 10007$ | $\langle M_1, M_2, M_3, M_4, 2\mathbb{M} \rangle$ | 68.192003860287 | $1/2^3$             |