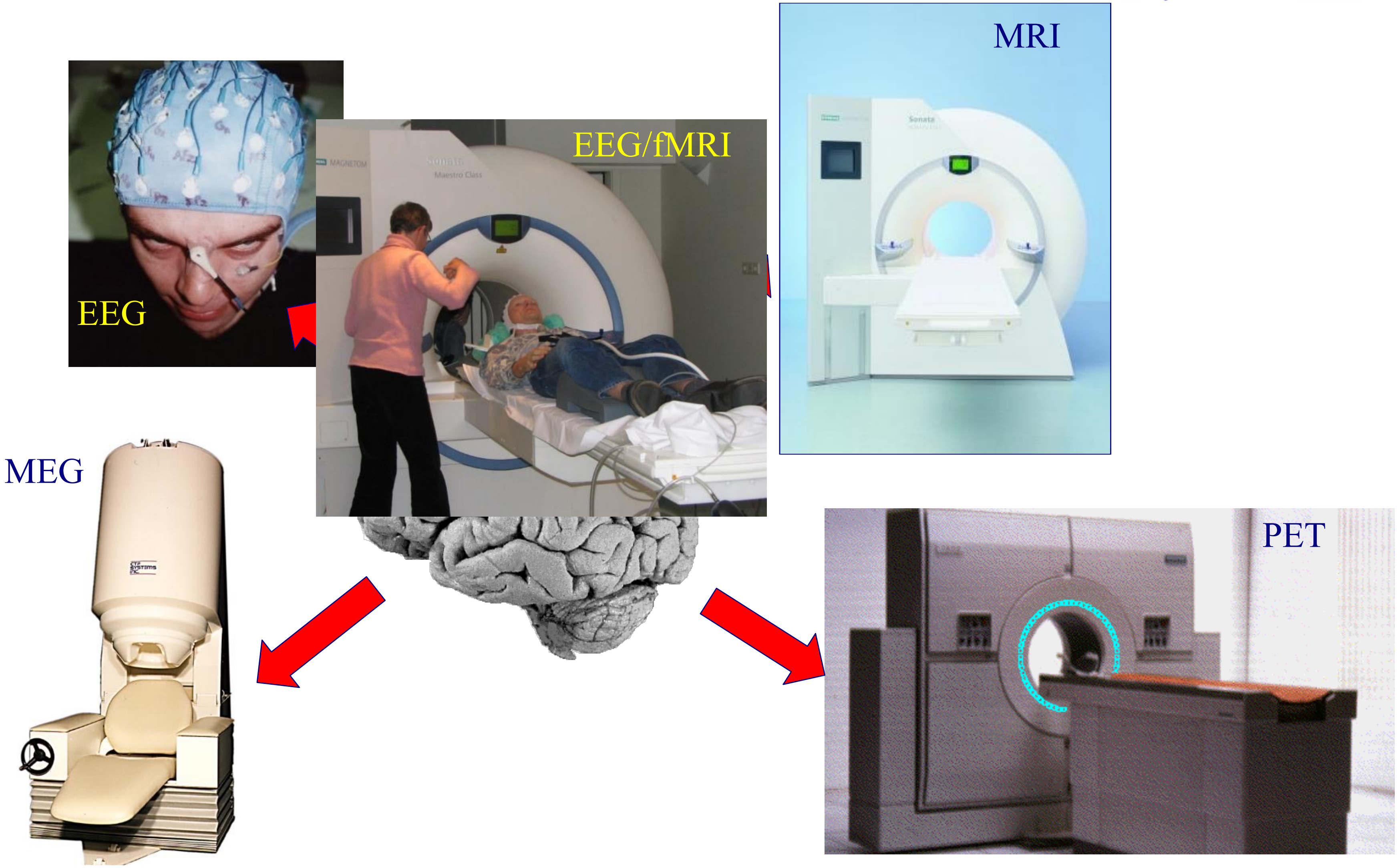
Mathematical modelling in medicine and industry

JC de Munck
PostNL
Den Haag

Brain Imaging

VUmc VU University Medical Center Amsterdam



Brain Imaging

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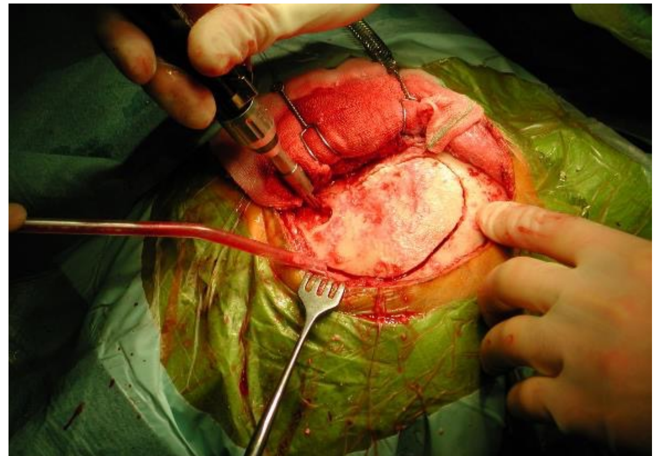

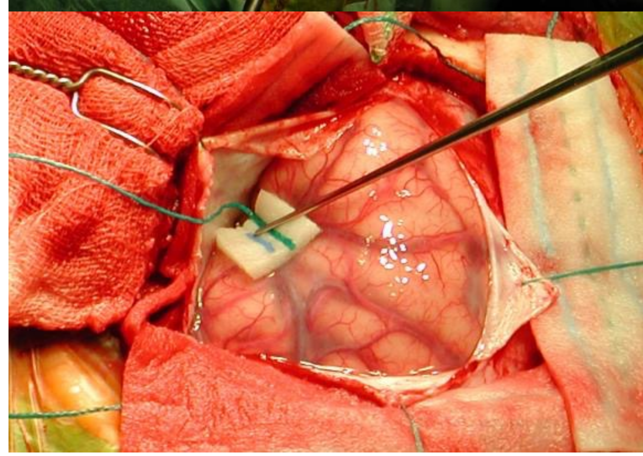

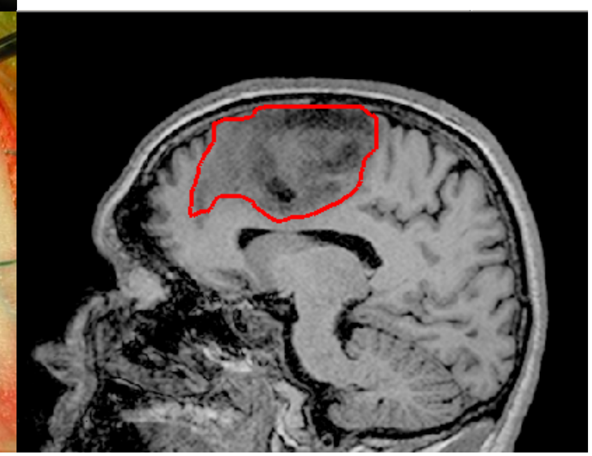

Brain imaging has many practical applications in medicine: neurosurgery, neurology, radiology and radiotherapy.

Brain Imaging

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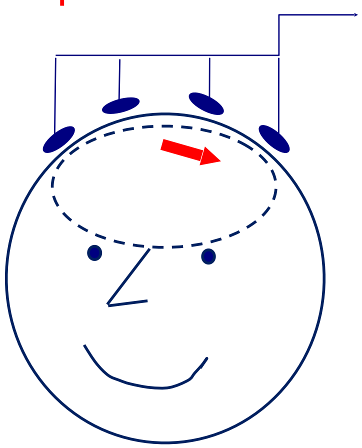
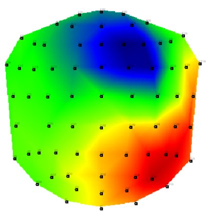
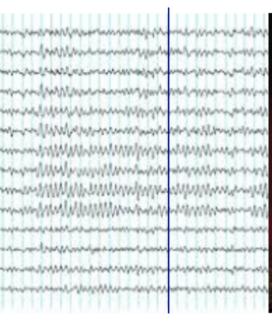
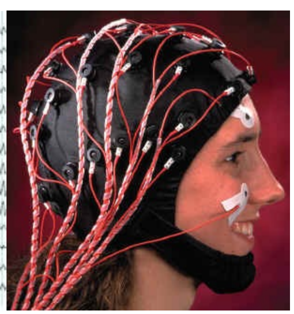
Dipole model

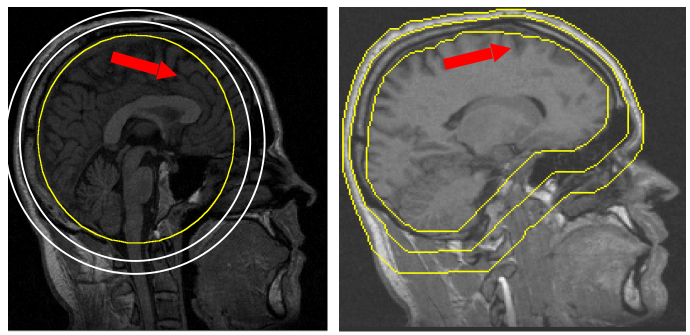






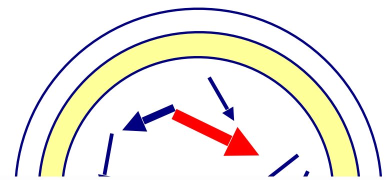


Conductor model



- Poisson equation
- Numerical methods
- Image processing
- Shape analysis
- Algorithm design and implementation

Noise model



Where is the source?

You need inspiration…

… and excellent teachers

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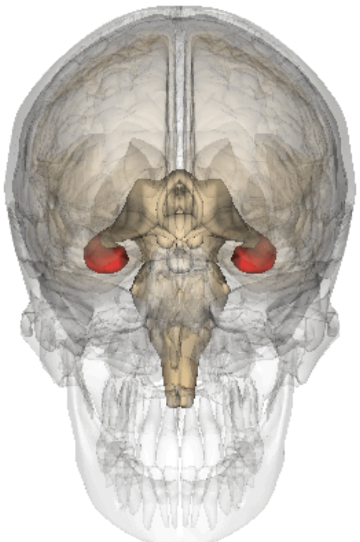
The hippocampus



“hippo”:	horse
“kampos”:	sea

Radiology: Assess disease progression.
Radiotherapy: Make hippocampal sparing treatment plans in whole brain radiation therapy.

The hippocampus

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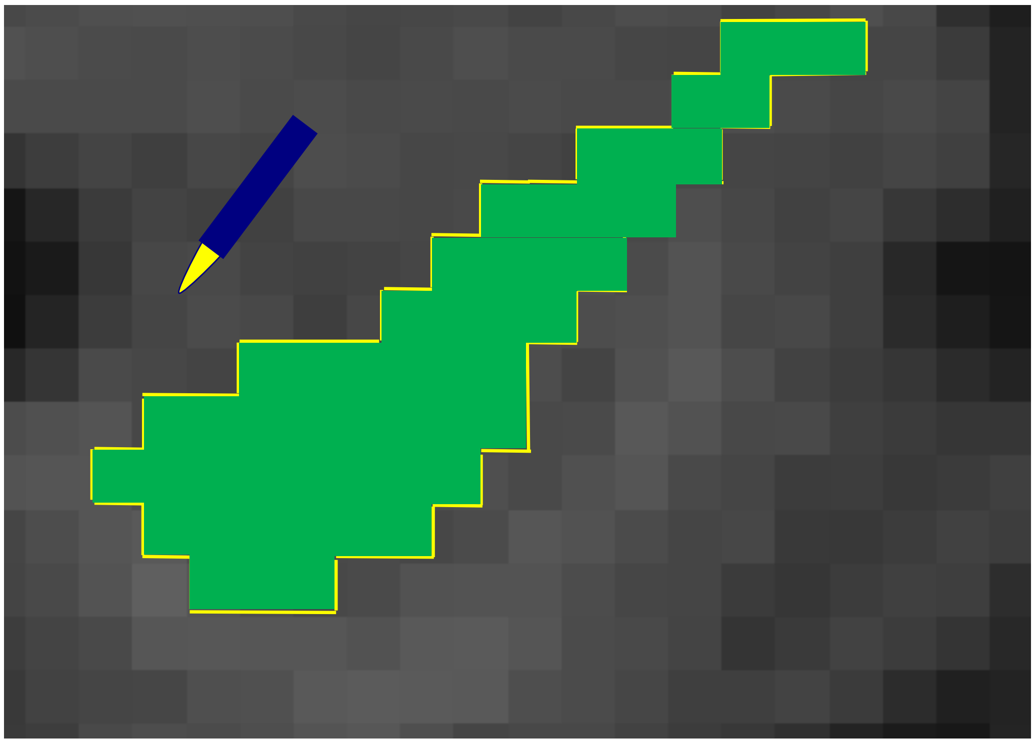

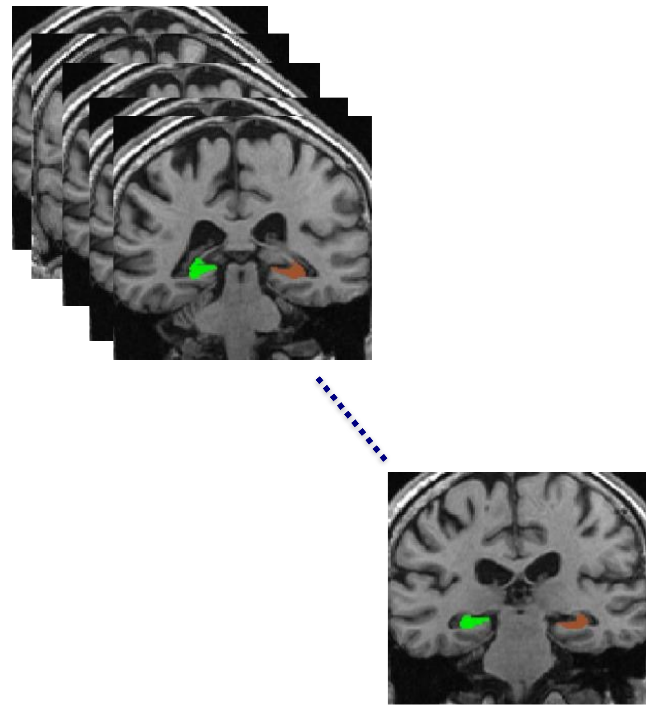

Hippocampus delineation

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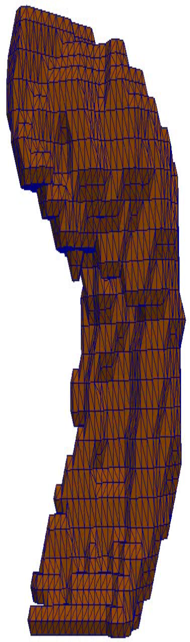
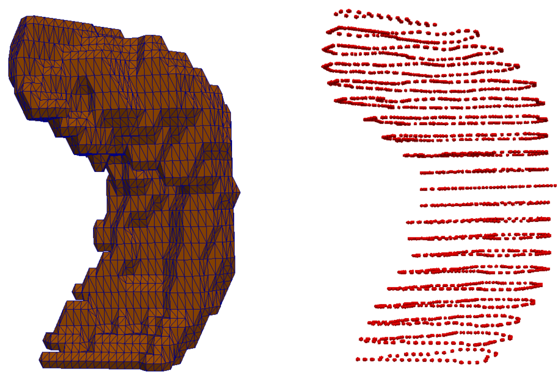


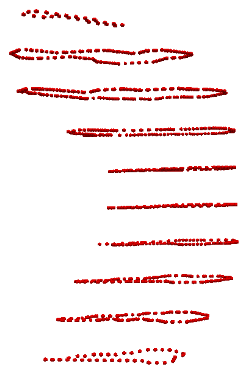
Manual hippocampus segmentation takes 2 h for a trained expert.

Surface reconstruction

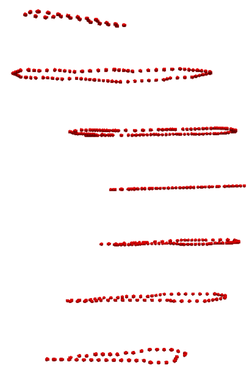
All 21 contours



10 Contours



7 Contours

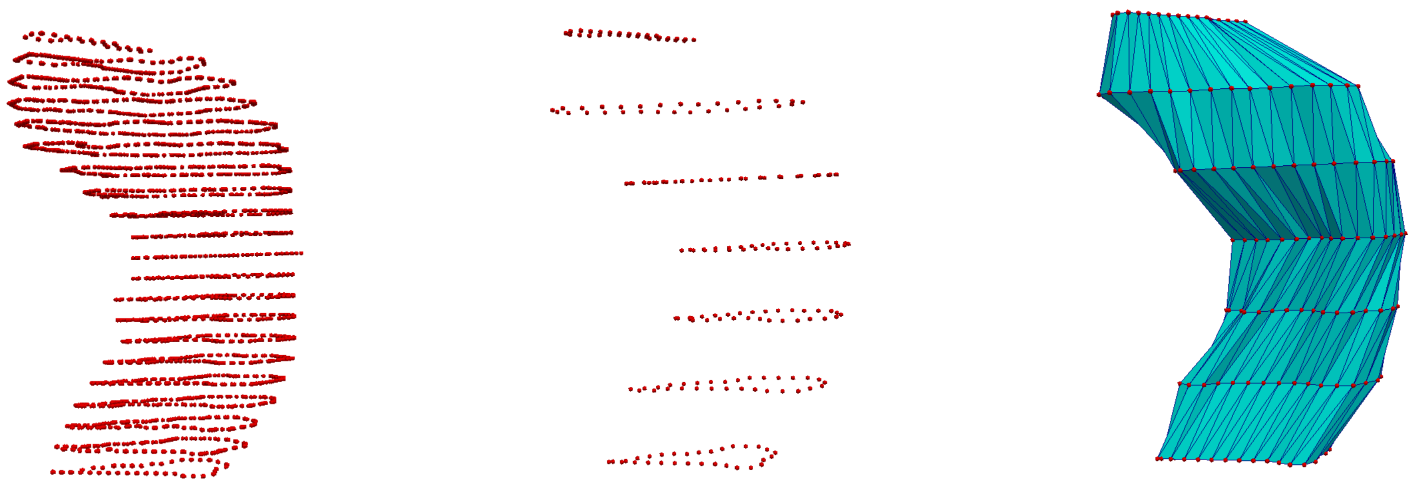


4 Contours

Potential time reduction: ~50% ~65% ~80%

Surface reconstruction

Straight line connection



Reconstruction by straight lines leads to inaccurate results and self intersections.

Surface reconstruction

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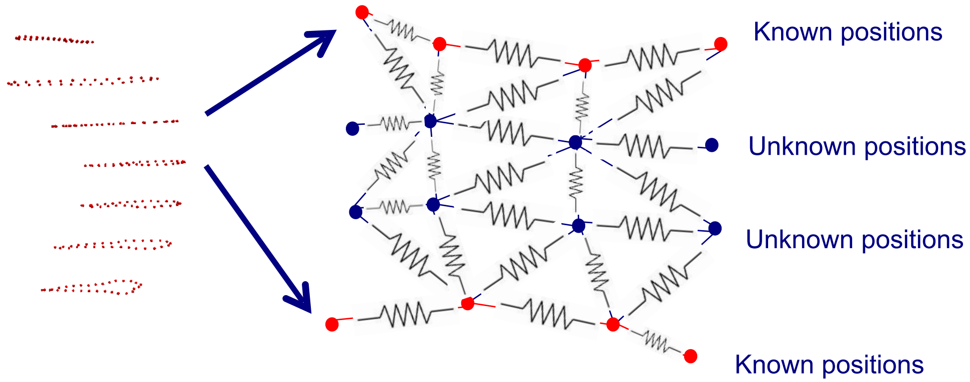
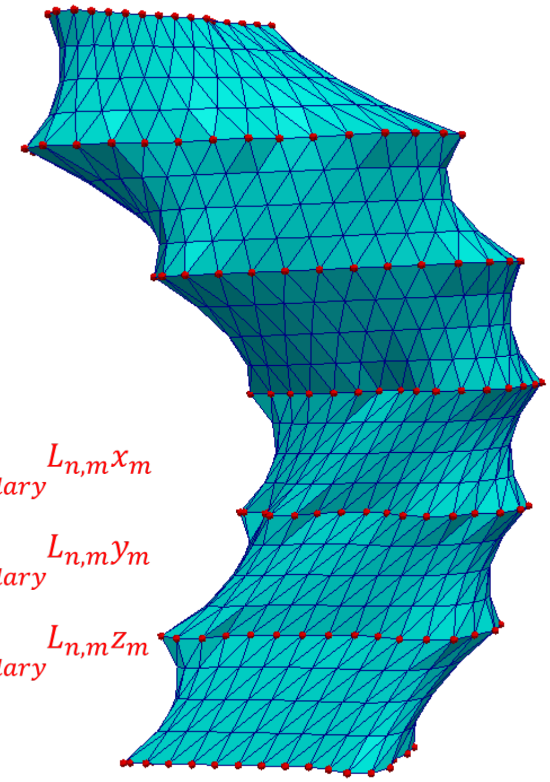


Reconstruct su
imposing force



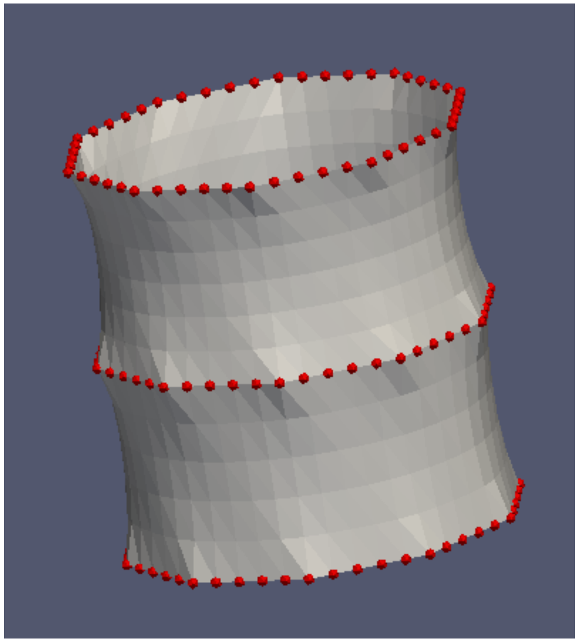
$$\sum_m L_{n,m} x_m = 0$$
$$\sum_m L_{n,m} y_m = 0$$
$$\sum_m L_{n,m} z_m = 0$$

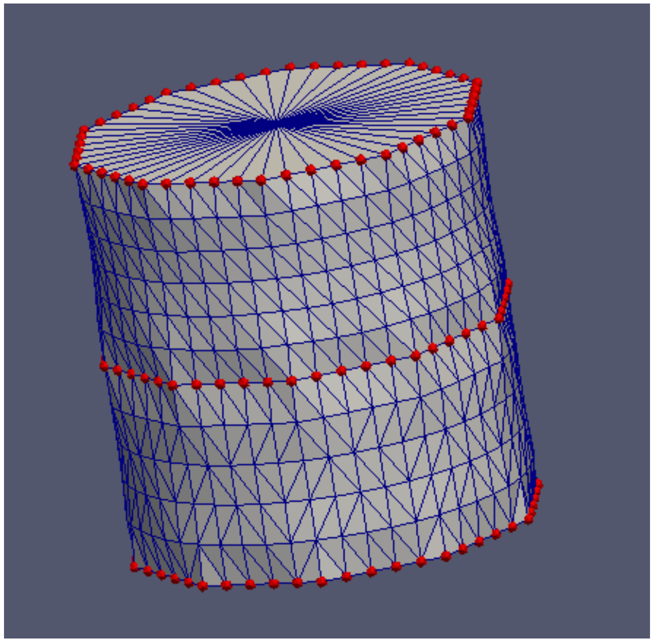
$$\sum_{m \in Interior} L_{n,m} x_m = -\sum_{m \in Boundary} L_{n,m} x_m$$
$$\sum_{m \in Interior} L_{n,m} y_m = -\sum_{m \in Boundary} L_{n,m} y_m$$
$$\sum_{m \in Interior} L_{n,m} z_m = -\sum_{m \in Boundary} L_{n,m} z_m$$

Laplacian matrix

$$L_{n,m} = \begin{cases} 1 & \text{if } n=m \\ \dfrac{-1}{N_{\mathrm{Neighbors}}(v_n)} & v_n \text{ is adjacent to } v_m \\ 0 & \text{otherwise} \end{cases}$$

Surface reconstruction



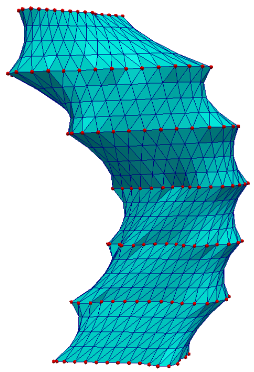


Minimizing squared mean curvature seems to work, but computationally it is very slow

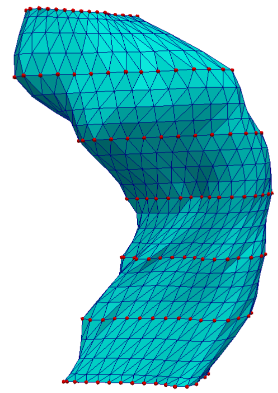
Time for help!

Surface reconstruction

$$\sum_m L_{n,m} x_m = 0$$
$$\sum_m L_{n,m} y_m = 0$$
$$\sum_m L_{n,m} z_m = 0$$



$$\sum_m L_{n,m}{}^2 x_m = 0$$
$$\sum_m L_{n,m}{}^2 y_m = 0$$
$$\sum_m L_{n,m}{}^2 z_m = 0$$

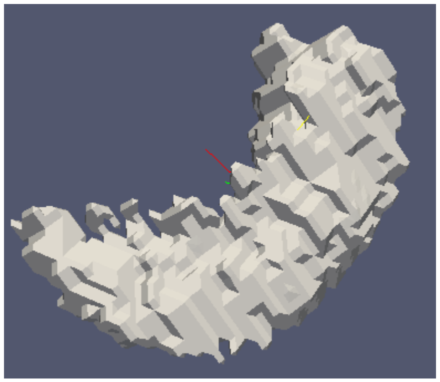


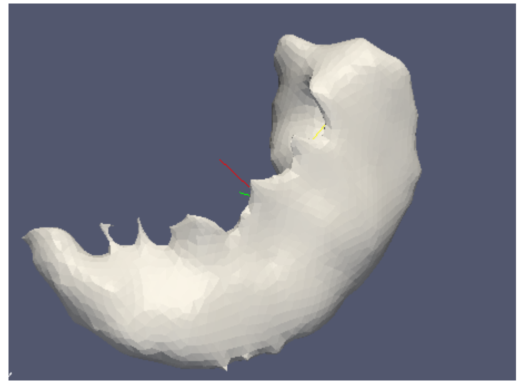
Minimum area reconstruction

Minimum *curvature* reconstruction

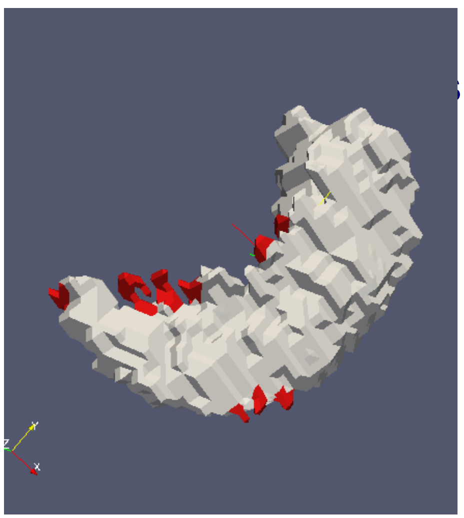
Systematic comparison shows that 7 contours are sufficient for accurate surface reconstruction of hippocampus.

Surface smoothing?

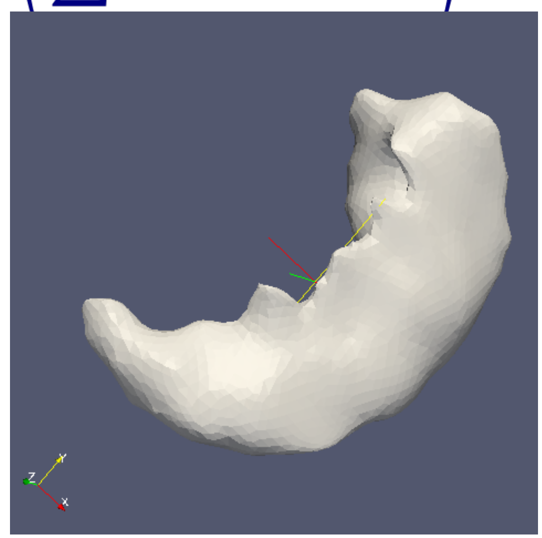




$$L_{12}(\boldsymbol{\xi}, \boldsymbol{\eta}, \boldsymbol{\zeta}) = \boldsymbol{\xi}^T M \boldsymbol{\xi} + \boldsymbol{\eta}^T M \boldsymbol{\eta} + \boldsymbol{\zeta}^T M \boldsymbol{\zeta} + \mu \left(\sum D_{nn}^{inv} \|\boldsymbol{v}_n - \boldsymbol{x}_n\|_2^1 \right)$$

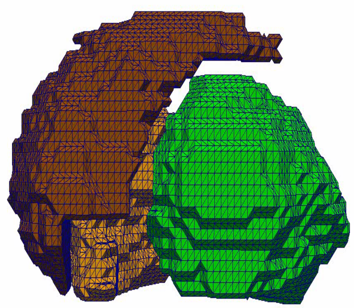


smoothing

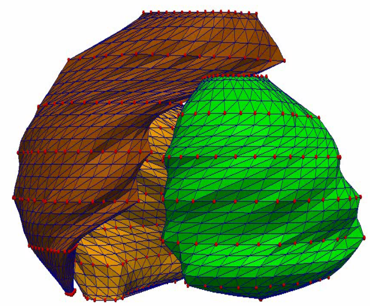


Further applications

Manual



Reconstructions



Brown: Caudate
Green: Thalamus
Orange: Putamen

Surface reconstruction from sparse delineations also proved very useful for fast delineation of other brain structures.

All's well that ends well…

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VRIJE UNIVERSITEIT

Surfing the Hippocampus Wave

ACADEMISCH PROEFSCHRIFT

ter verkrijging van de graad Doctor of Philosophy
aan de Vrije Universiteit Amsterdam,
op gezag van de rector magnific
prof.dr. V. Subramania
in het openbaar te v
ten overstaan van d
van de Facu
op donderd

door

Fabian Bartel

geboren te Haselünne, Germany

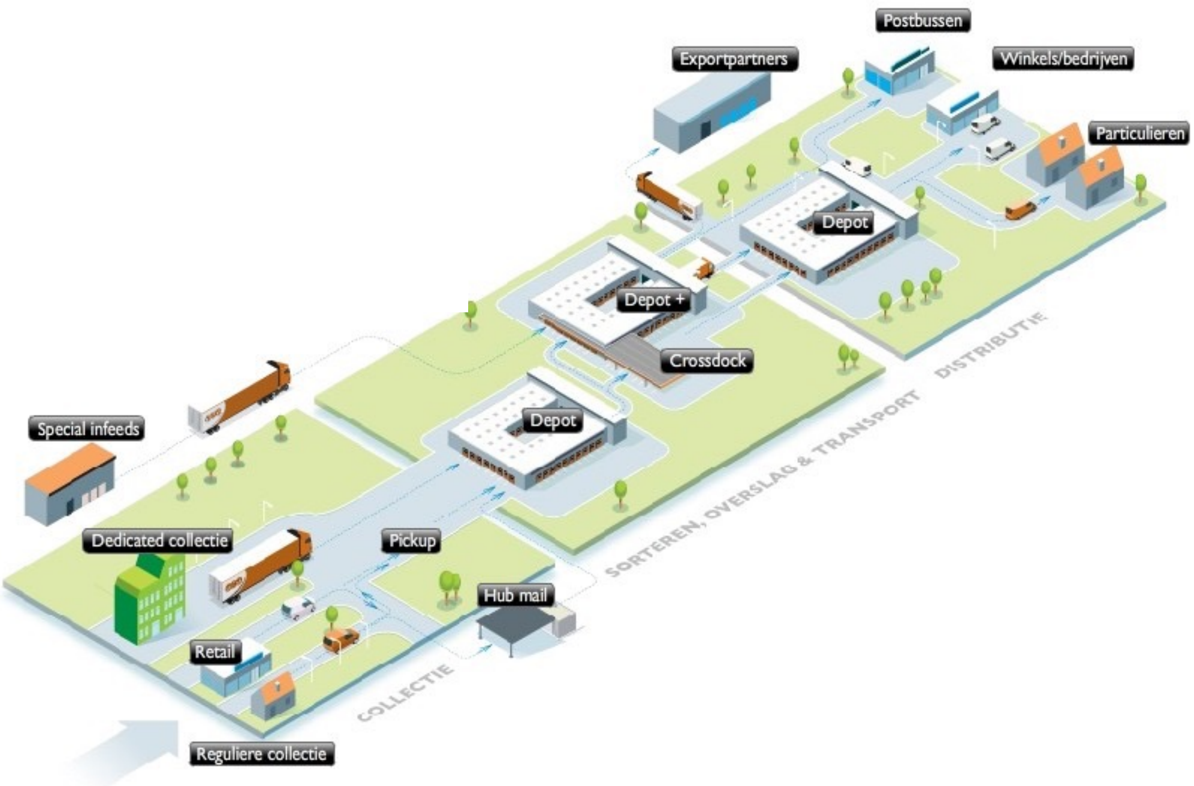
VU € 90.000

… but a new start in industry.

- 10^6 parcels per weekday
- 7 x 10^6 letters per weekday in Benelux
- 3,8 x 10^4 employees
- Largest transporter of The Netherlands
- Exists for > 220 years

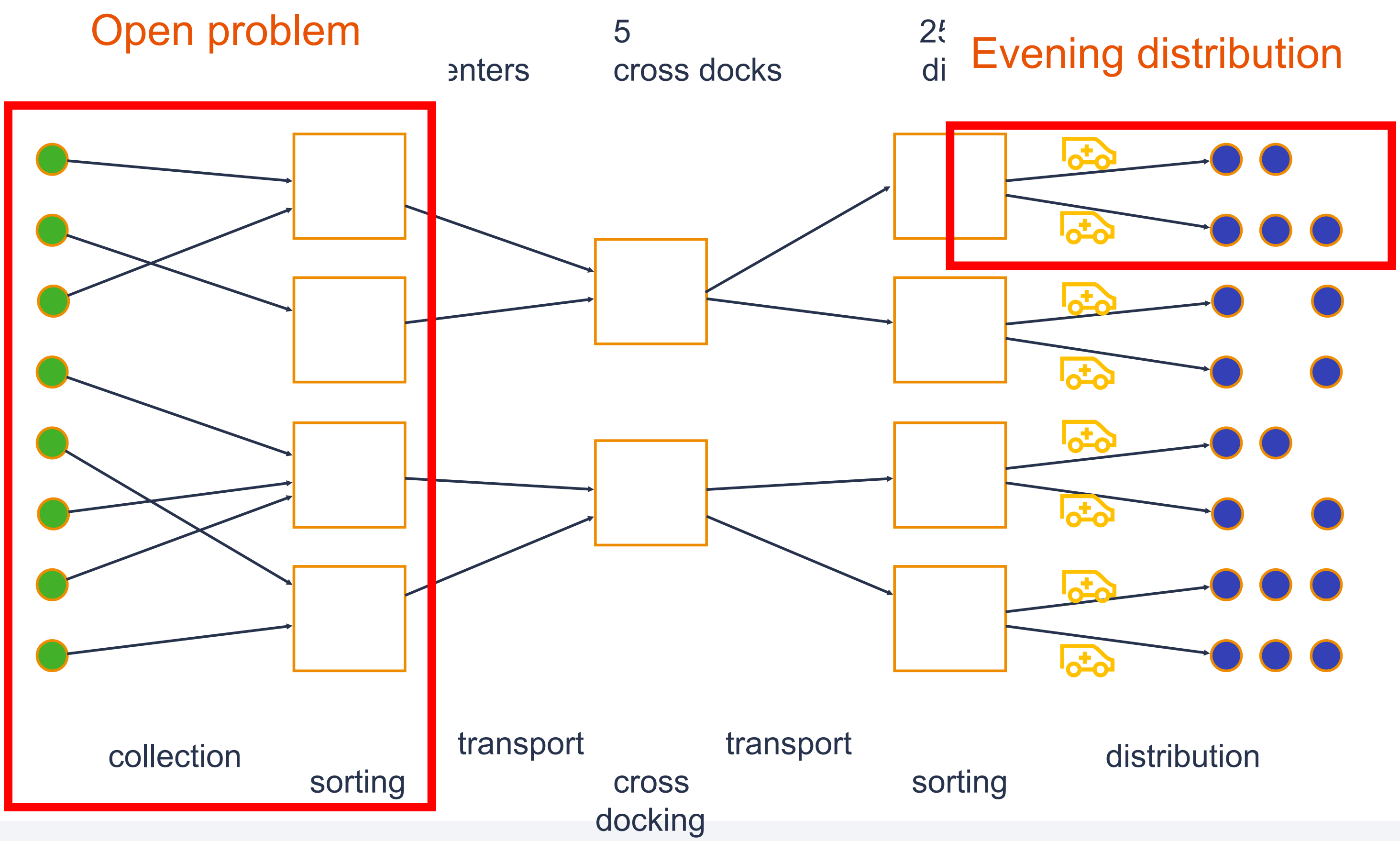
To handle these amounts of volumes in an increasing market of parcel and decreasing numbers of letters, PostNL has decided to make itself a data driven organization, with data awareness and data science in all aspects of its business.

Complex logistics

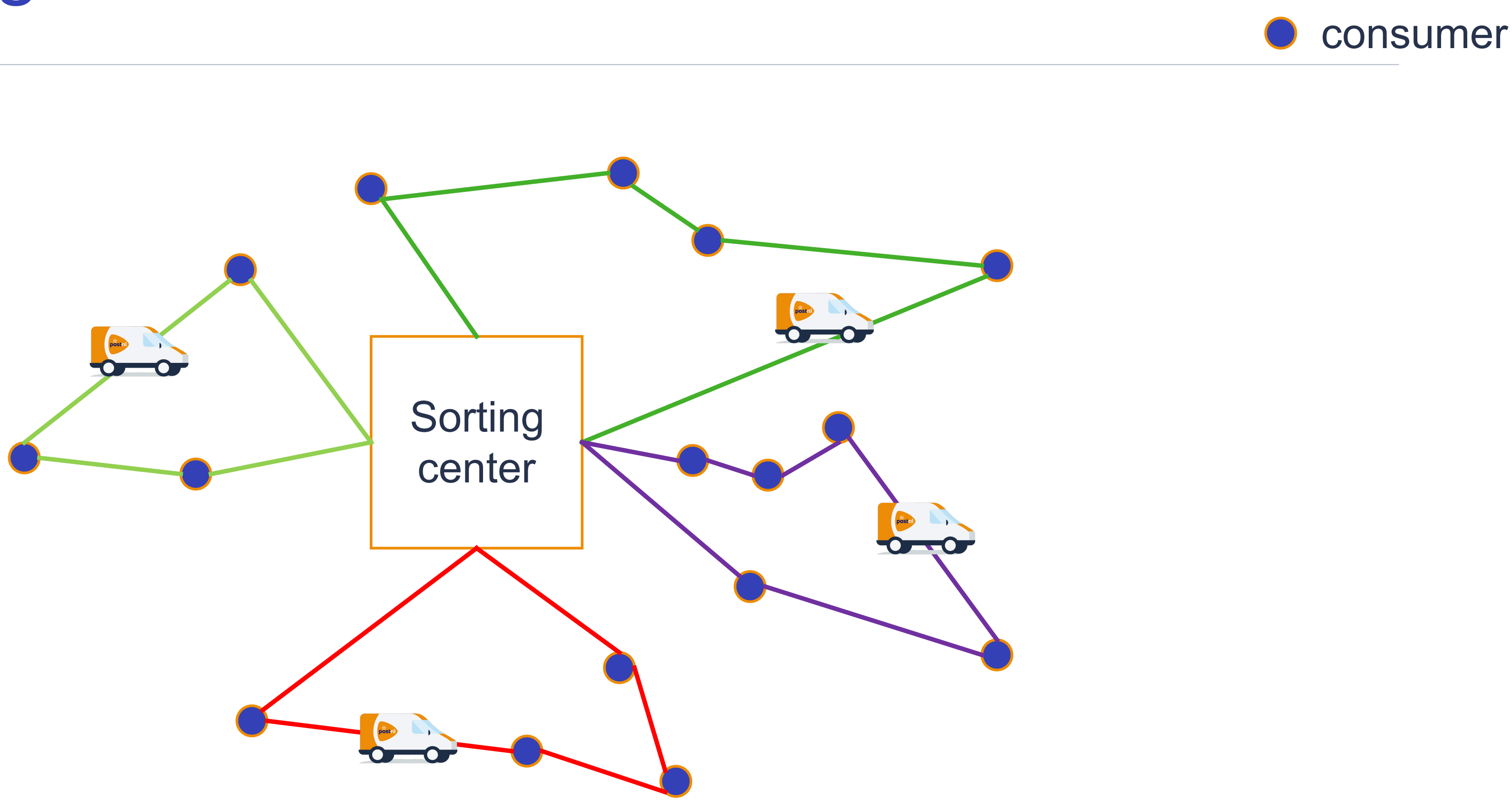


Parcel logistics consists of a chain of processes that can be analyzed and optimized more or less independently.

Complex logistics



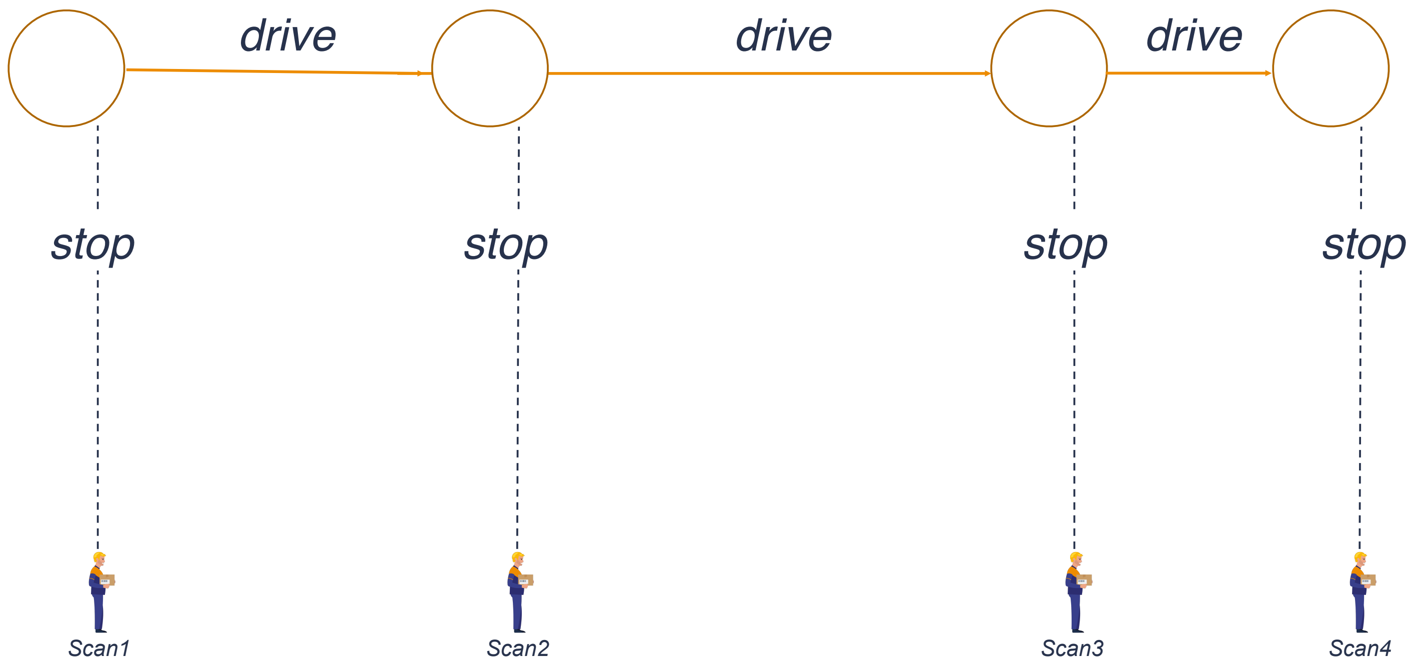
Evening distribution



The MVRP: Given all time distances and all stop times and time constraints. How many cars are needed? Which routes are optimal?

Evening distribution

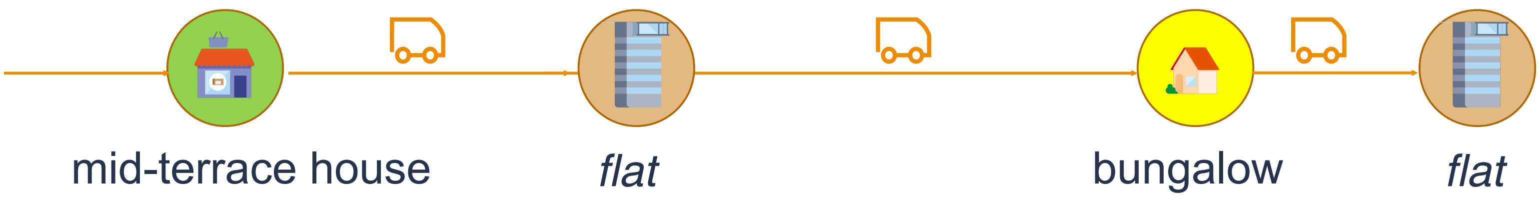
Business question: can you extract reliable stopping and driving times from scan data?

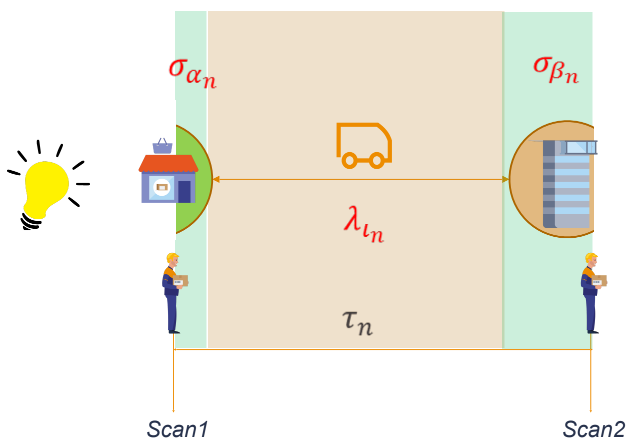


Assumption 1: stop time depends on building type.
Assumption 2: drive time is a correction factor on OSRM per postal code.

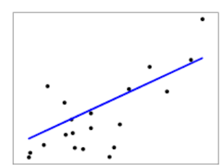
Evening distribution

Business question: can you extract reliable stopping and driving times from scan data?





$$X\boldsymbol{\theta} = \boldsymbol{y} + \boldsymbol{\varepsilon}$$



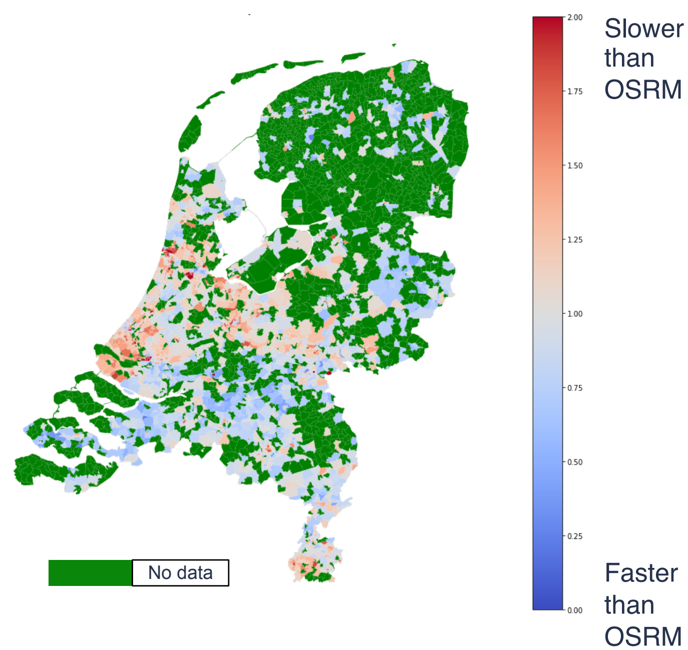
one of the milion segments

Results evening distribution

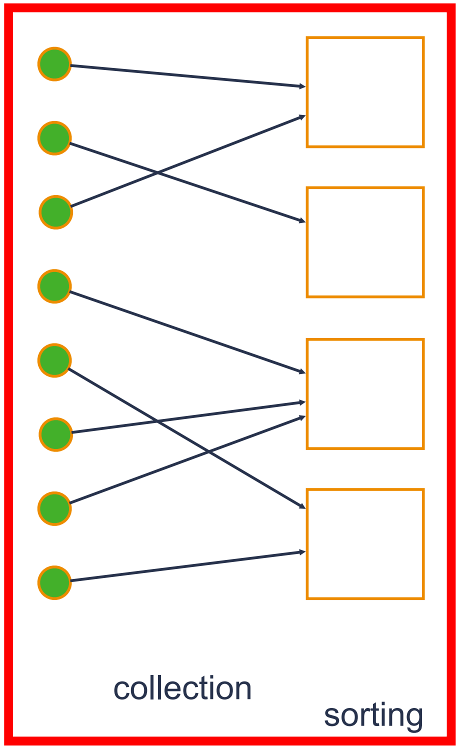
Stop times per building type

Building type	Stop time	sigma
Eengezinswoning zonder tuin / Hoekwoning	02:38 min	4 sec
Woning met tuin / Etage woning	02:58 min	2 sec
Boerderij / Agrarisch met woongedeelte	02:59 min	3 sec
Eengezinswoning zonder tuin / Etage woning	03:10 min	5 sec
Hoogbouw / Flat-appartement	03:48 min	1 sec
Bejaardetehuis / Doelgroepwoning	4:17 min	4 sec
etc	etc	etc

Drive times compared to Open Street Map



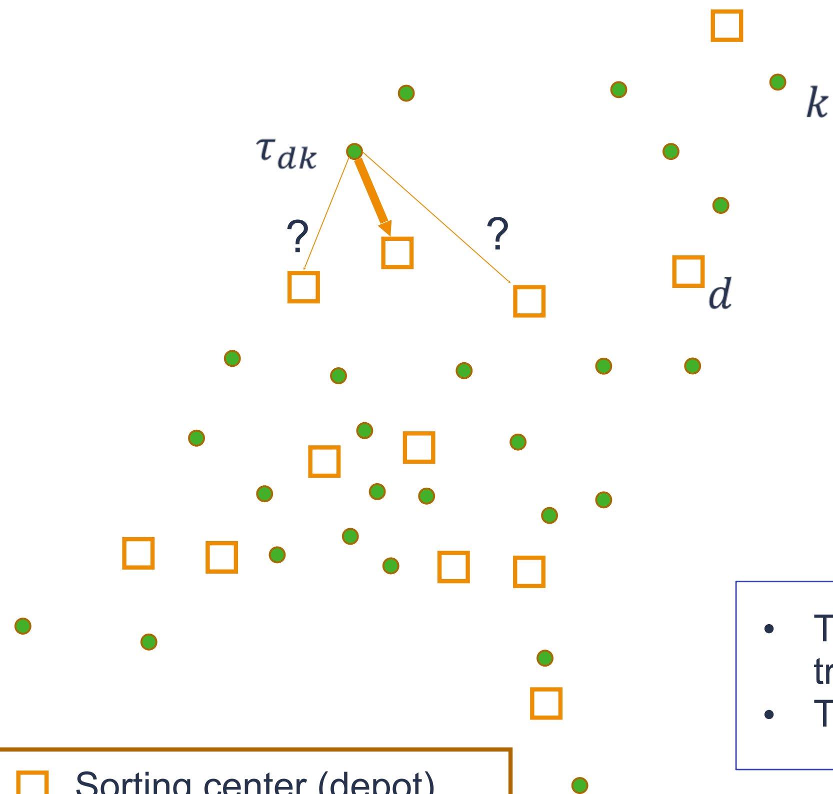
Open problem



Joost, where are you?

To which sorting center should each customer ideally be connected?

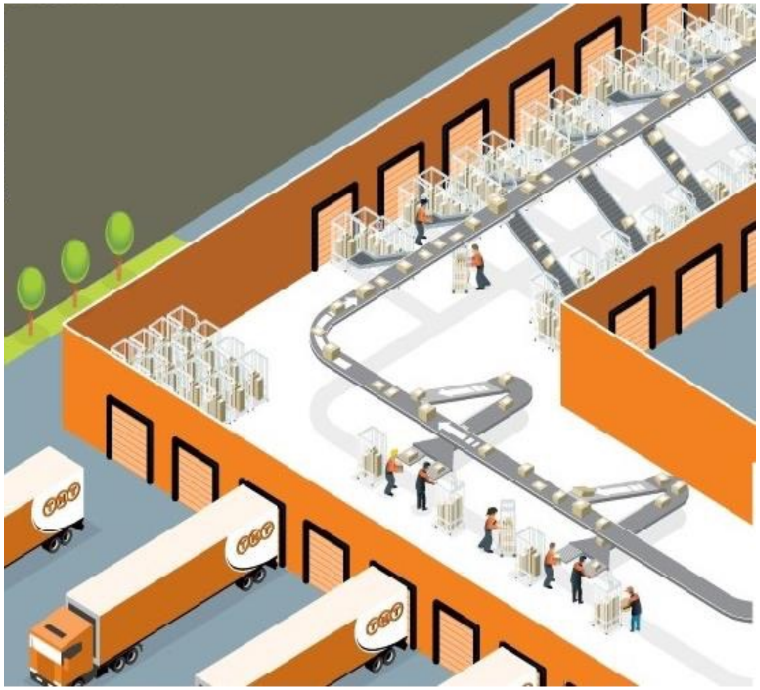
The depot allocation problem



- Transportation costs are proportional to travel time and volumes per customer.
- There are more customers in the south.

Sorting center (depot)
Customer (e.g. webshop)

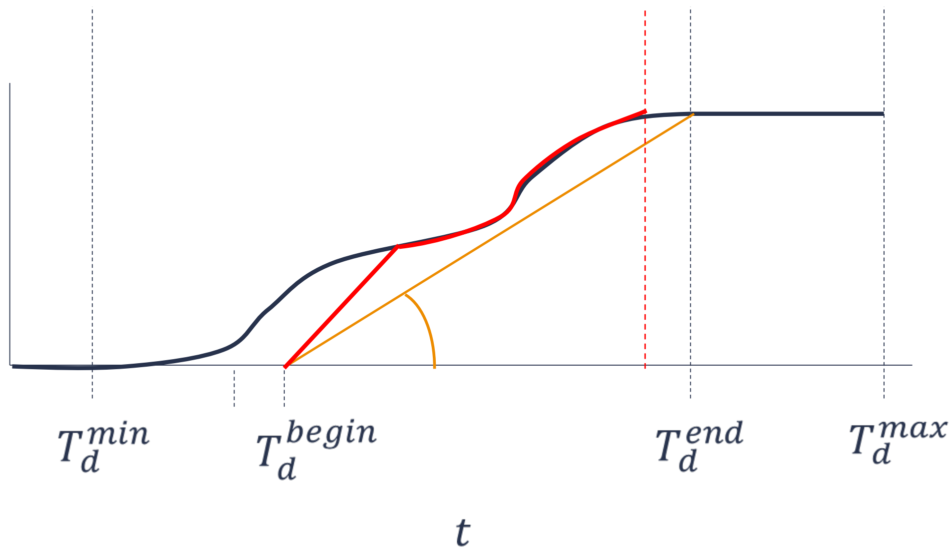
The depot allocation problem



- There is a maximum number of 840 parcels per hour per belt that can be added to the sorter.
- Sorting costs are proportional to number of conveyer belts and sorting time.

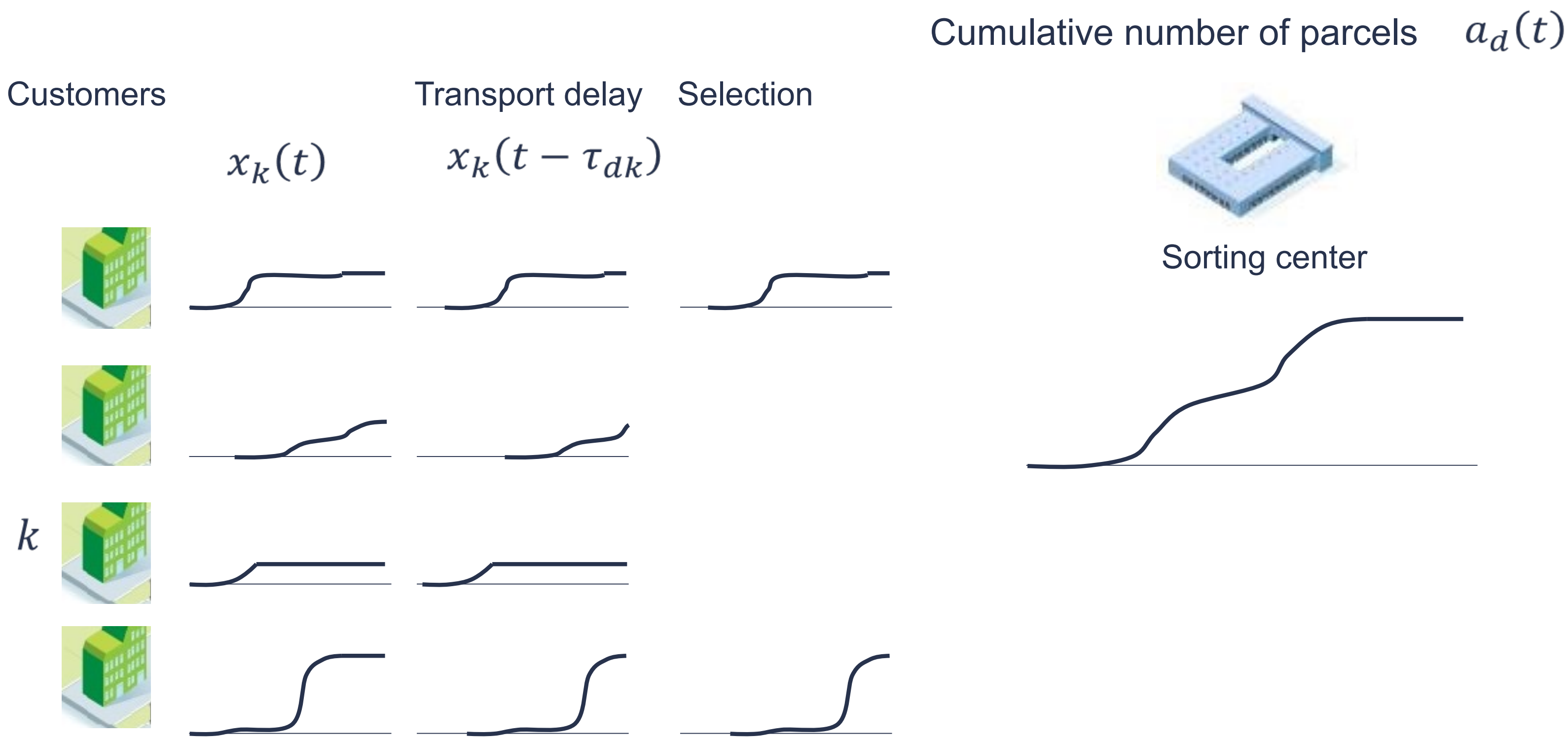
The depot allocation problem

Cumulative number of parcels $a_d(t)$



For a given number of conveyer belts (and labor) the minimum sorting costs can be graphically derived from the supply curve $a_d(t)$.

The depot allocation problem



The supply curves $a_d(t)$ depend on transport delays and allocation of customer to sorting center.

The depot allocation problem

The depot allocation problem is to assign customers to depots (sorting centers) such that the total cost $(J_T + J_S)$ is minimum.

$$a_d(t) = \sum_k f_{dk} x_k(t - \tau_{dk})$$

Supply line at depot d

$$f_{dk} \in \{0,1\}$$

Unknown allocation

$$J_T(f_{dk}) = \lambda_T \sum_k f_{dk} x_k(\infty) \tau_{dk}$$

Transport cost

$$J_S(f_{dk}, n_d) = \lambda_S \sum_d n_d (T_d^{end} - T_d^{begin})$$

Sorting cost

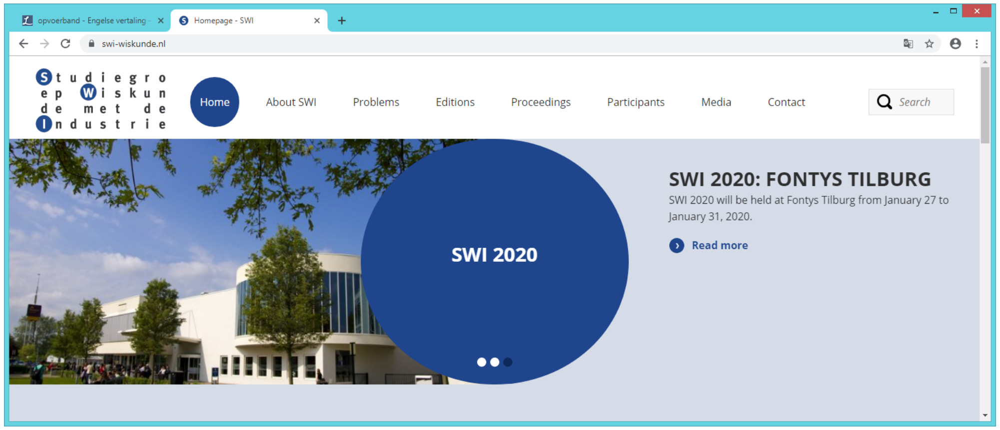
$$n_d \in \{0,1,\ldots,12\}$$

Unknown number of conveyer belts

The depot allocation problem

Furter improvements:

- Add constraints on number of truck docking positions
- Allow connection of customer to multiple depots
- Save costs when not using certain sorting centers
- Include transport process more realistically in the model
- Etc, etc, etc.



There are many refinements possible and necessary. Hopefully this leads to a versatile approach to address the collection problem at PostNL.

Conclusion

The weak law of mathematical modelling:

Mathematical modelling is ubiquitous.

$$\lim_{n\to\infty} P(\ |Y_n - Y| < \epsilon) = 1$$

The strong law of mathematical modelling:

If it has no mathematical model, it is not science.

$$P(\ \lim_{n\to\infty}\ ||Y_n - Y|| < \epsilon) = 1$$