After the first 11 chapters, the student...

1. ... knows basic definitions concerning limits and continuity (convergence, Cauchy sequence, limit, completeness, continuity, uniform continuity) and is able to determine whether a sequence, series or function satisfies these definitions;

2. ... knows the definition of differentiability (i.e., that a function can be approximated by a linear one), can determine whether a function is differentiable, and is familiar with the more algebraic approach for power series);

3. ... knows the definition of Riemann integrability and can prove that certain functions (in particular, polynomials, monotone and uniformly continuous functions) are Riemann integrable, and knows the limit theorems about limits of integrals of uniformly convergent sequences of functions on [a,b], and of pointwise convergent monotone functions[1];

4. ... knows the definition of basic concepts from metric topology (metric, convergence, completeness, Banach space) and can prove that certain sets of functions satisfy these definitions, such as $C([a,b])$, the space of continuous functions $f : [a,b] \to \mathbb{R}$ with the uniform metric, and knows that convergence in this space corresponds to uniform convergence[2];

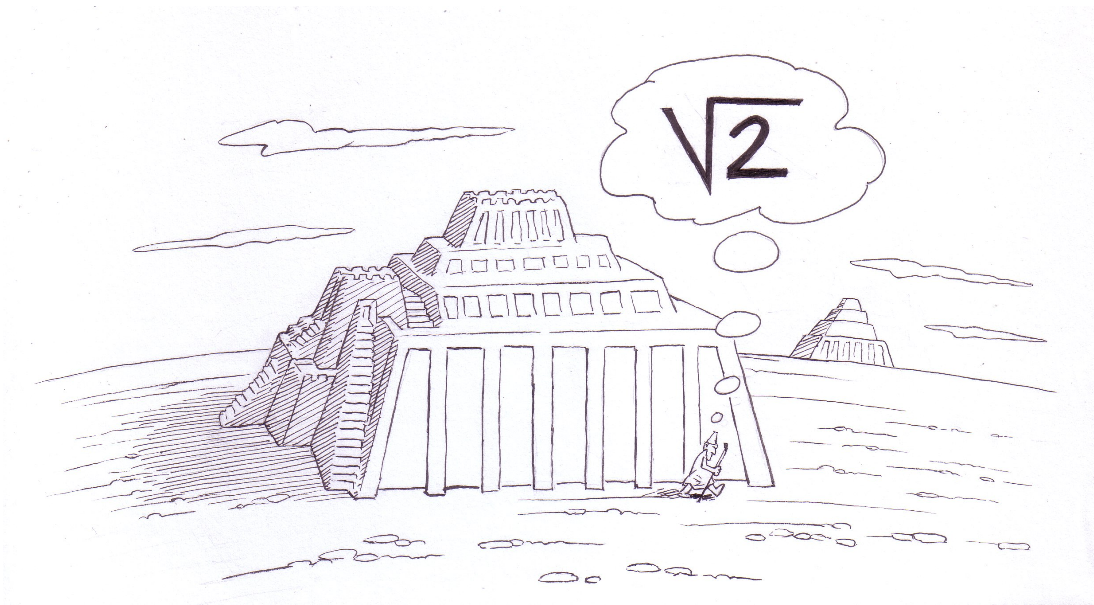
5. ... knows the statement of the Banach Fixed Point Theorem, and can apply this theorem to solve fixed point equations, in particular integral equations in $C([a,b])$ for solutions of differential equations.

[1] This will be a not too hard part at the start of the second block.
[2] This will be a hard part at the end of the first block.

Course content

This course treats the rigorous mathematical theory behind Calculus: limits, continuity, linear approximation, differentiability, integrability, and the mutual relation between these concepts. The mathematical tools that are necessary for formulating and proving the essential results of Calculus are first presented in the context of real valued sequences and real valued functions of a real variable, in such a way that everything can later be generalised (to Y-valued functions of variables in X, with X and Y Banach spaces). The space $C([a,b])$ of real valued continuous functions on an interval $[a,b]$ will appear as the first example of such a Banach space.

Starting point of the course are an ancient iterative scheme for solving equations, and the fundamental properties of (the set of) real numbers, in relation to two if you like geometric numbers: $\sqrt{2}$ and $\frac{1}{3}$.


$\sqrt{2}$