I got lost after (13.23) early 2023 while I was having fun with

$$\Gamma(x)\Gamma(1-x) = \frac{\pi}{\sin \pi x} \tag{0.1}$$

and all that. Turns out I suffered a stroke on February 10 and I had to avoid computer screens for a while. I only taught one session of the complex analysis course I was scheduled to do this spring. I had been writing on an appetizer for complex analysis in Section 13.6, also because of an upcoming bachelor project on the Riemann ζ-function. Will see when I continue making illustrations like the one just before Section 18.2. Corrected typos and started from (13.23. Edited (13.22) and further. Last editing in Section 31.6.

Earlier I had been working on Chapter 18, also on complex analysis. Part of that chapter was conceived in Dutch first. Sectie 18.7 is still in Dutch and under construction. So is Section 18.8. Section 18.9 is more recent. Not sure yet that Section 18.9.7 is of real help in the theory of analytic halfgroups generated by sectorial operators. Section 18.9.5 suggests to think of hyperbolic regions rather than sectors.

Chapter 34 is about Lebesgue spaces of functions defined on open subsets of Euclidean space. I tried to keep measure theory from playing a crucial role. Section 34.3 for instance explains how any kosher extension of the Riemann integral will suffice for sophisticated purposes such as Theorem 34.12, which is about what I call the good set of a locally integrable function. Theorem 34.19 is new (to me) in this context. Section 34.6 offers a shortcut sufficient for a proper treatment of Sobolev spaces in Chapter 35 and further via Section 34.9 and Sections 20.3 and 20.5 in Chapter 20. Chapter 38 improves on the regularity theory in Evans' PDE book. Michael wants me do $p \neq 2$-estimates.

Chapter 31 on Fourier theory also avoids Lebesgue theory. The results in Section 31.2.2 are also new (to me) and prepare for a rather nice treatment for the multi-variable case, merging Folland's approach in his Real Analysis book with that of Olver's in his PDE text, based on Riemann integrals only. Section 31.9 indicates how to extend the theory to the distributional context.

Chapter 15 has a new introduction explaining why in the end it is cleaner to first do the inverse function, which in itself is a special case of the implicit function theorem. The latter however is easily obtained by adapting the proof the inverse function theorem, rather than applying the inverse function theorem to some specially chosen function to artificially derive the implicit function theorem from the inverse function theorem. Thus the proof of the pudding is in the method.

On a different note: Chapter 16 did away with the implicit function theorem in the proof of the Morse Lemma, returning in Section 16.6 to a method suggested by YBC7289. Section 16.5 remains instructive though, and the implicit function theorem is of course fundamental for the Lagrange multiplier method, which has many applications in Chapter 29, in particular in machine learning. Thanks to Anh I learnt the approach in the lost section on Kuhn-Tucker theory.

There's more to discuss. For now I thank Michael Winkler (he's crazy too).

I thank Finn, Lotte, Anh, Feddrick, and more students.

I also thank Thomas, Bob and Bob, Daniele, Michiel, and many more friends.

Large parts were written while corona shook the world and some went nuts.

But that's another story.

About the videos

Each playlist has a description of the playlist, and a list of the videos in it. The videos have informative names, and descriptions with corrections and links: to the notes, to every previous video and every next video. The first 6 playlists are for a first Analysis course. Do sneak around in the others too. Whereas the course notes are organised by topic, the videos are organised in part by technique.

The short first playlist corresponds to the introductory Chapter 1 and sets the scene with YBC7289, Archimedes, and the (set of) real numbers. The long second playlist then covers what can be done with $\varepsilon - N$ arguments in the context of convergence of sequences of real numbers, the concept of continuity for functions, and sequences of continuous functions. The number sequences include sequences obtained from the iteration of contractive maps, such as in particular Heron's sequence for $\sqrt{2}$.

Section 4.5 summarises the results on sequences of numbers, continuity of functions, sequences of continuous functions, and spaces of continuous functions. It is followed by the first (and only) completely abstract chapter: Chapter 5 is about abstract metric spaces, includes an *outlook on topology*, and concludes the first part (period 4) of the course.

The next four playlists 3-6 are about integration and differentiation, Chapters 6-11 in the notes. These constitute part two (period 5) of the course, which starts from square one again: Chapter 6, on the integration of monotone functions, requires little more than Chapter 1.

Chapter 7 then builds on all of the first part of the course. It covers the general theory for Riemann integration and uses the Banach Fixed Point Theorem from Chapter 5 to solve integral equations in spaces of integrable functions.

Only thereafter, Chapter 8 and playlist 6 introduce and exploit $\varepsilon - \delta$ arguments, to establish in particular the integrability of continuous functions and addresses the issue of the existence of convergent subsequences in Section 4.5. For bounded sequences of real numbers this was settled in Chapter 3.

Chapter 9 in turn, on the algebraic approach to differentiation for power series, is largely independent of everything done before, although some of the language from Chapter 4 is used to formulate the results. It introduces the fundamental approach to differentiation with linear approximations and *no limits but* error estimates.

The general theory of differentiation in Chapters 10 and 11 uses the $\varepsilon - \delta$ machinery again, to establish the relation between integration and differentiation, and most of the facts for and from calculus. *The pointwise continuity statement in Theorem 8.1 is used in Section 10.2.*