Content The majority of physical phenomena can be described by partial differential equations. This module discusses these equations and methods for their solution. In particular, use is made of the remarkable result of Fourier that almost any periodic function (i.e. one whose graph endlessly repeats the same pattern) can be represented as a sum of sines and cosines, called its Fourier series. An analogous representation for non-periodic functions is provided by the Fourier transform and the closely related Laplace transform.

Goals To provide an understanding of, and methods of solution for, the most important types of partial differential equations that arise in mathematical physics. In particular to discuss Fourier series and Fourier and Laplace transform and their application to the solution of classical Partial Differential Equations of mathematical physics. On completion of this module, students should be able to:

a) Obtain the whole or half range Fourier series of a simple function;

b) Apply the method of separation of variables to the solution of boundary and initial value problems for the classical PDEs of mathematical physics.

c) Obtain Fourier and Laplace transforms of simple functions and apply them to the solution of initial value problems for linear differential equations with constant coefficients.

d) Use Green's functions and fundamental solutions to solve PDE's.

Toetsvorm Two written exams.

Literatuur Drábek and Holubová, Elements of Partial Differential Equations, De Gruyter, ISBN 978-3-11-019124-0.

Prerequisites Differentieren en Integreren 1,2,3 en Lineaire Algebra.

Doelgroep 2N, 2W, 3W, mMath

About this course

The majority of physical phenomena can be described by partial differential equations. In this course we explain how these equations are derived from physical principles, notably conservation laws and constitutive laws, and introduce some of the standard methods for their solution, as well as methods to understand the qualitative behaviour of solutions, such as the difference between diffusive and wavelike propagation.

Partial differential equations are hard, basically because solutions are less smooth than solutions of ordinary differential equations. Computing a solution is often easier than formulating in which sense the solution formula actually defines a solution.This is already an issue for linear equations. For practical purposes such questions are only mentioned but not addressed, as this is a first introduction to the field.

For linear equations with constant coefficients ample use is made of the remarkable result of Fourier that almost any periodic function (i.e. one whose graph endlessly repeats the same pattern) can be represented as a sum of sines and cosines, called its Fourier series.

All this is connected to the method of separation of variables and leads to generalised Fourier methods in which eigenfunctions (the natural oscillation patterns of the underlying domain) of the Laplacian play an important role. This Laplacian appears all over the place. Functions with a vanishing Laplacian are called harmonic function and in two dimensions they appear as real and imaginary parts of complex analytic functions. Maximum principles for and integral representations (potential theory) of solutions as those encountered in complex analysis play an important role.

Complex methods are important in the study of PDE's: the representation of non-periodic functions provided by the Fourier transform, and the closely related Laplace transform (this last topic to be treated only if time allows), the spectral properties of partial differential operators, the construction of fundamental solutions, etc.

1 Outline of the course

1.1 Week 1,2

Chapter 1. Notation, mathematical models, constitutive laws, conservation laws. The Gauss divergence theorem is essential.
Chapter 2. Linearity and its consequences. Homogeneous and inhomogeneous linear equations. Examples of equations and boundary conditions. Wellposedness. Classification of second order e quations for $u = u(x, y)$.
Chapter 3. First order equations. The method of characteristics.

Exercise class:
1.6: 1,3,4,5.
2.3: 1,2,4,5,6,8,9
3.4: 1,2,4, 7, 10,11,17,27

1.2 Week 3,4

Chapter 4,5. The linear heat and wave equations on the real line. Homogeneous and inhomogeneous.

Exercise class:
4.4: 1,3,4,5,8,9,11,13,14,15,21
5.4: 1,4,5,6,7,8,9,10,11,12,13,14,16

1.3 Week 5,6

Chapter 7. Initial and boundary value problems, separation of variables and Fourier series, see also Sections 2 and 3 in http://www.few.vu.nl/~jhulshof/CF2011/DEEL2/CFTdeel2.pdf

Exercise class:
7.5: 2,3,4,5,6,8,9,10,11,12,15,16,17,19,20,22,25,28,29,32,33,35.

1.4 Week 7

Equations with the Laplacian in two dimensions. See also Churchill&Brown, Complex Variables and Applications, Chapter 9 and 10.

Exercise class:
6.4: 2,5,8,9,10,11,12,13,14

1.5 Week 8

First exam

1.6 Week 9

Equations with the Laplacian in rectangles and discs, separation of variables, Poisson formula, see §123 in C&B.

Exercise class:
8.4: 4,5,6,7,8,9,10,11,12,13,15,16,18

1.7 Week 10,11

Laplace transforms and Fourier transforms.

Exercise class:
9.3: 1,2,4,5,6,7,9,10,11,12,14,
7 is impossible for $\beta \neq \gamma$. 12 answers wrong?

1.8 Week 12

Properties of solutions.

Exercise class:
10.8 2,3,6,8,9,10,13,16,19

1.9 Week 13

Equations with the Laplacian in more dimensions. Potentials and Green's functions.

Exercise class:
11.7 5,6,7,8,9,11,12,15,16.

1.10 Week 14

Heat equation in more dimensions.

Exercise class:
12.4 1,2,4,5,7,9

1.11 Week 15

Wave equation in more dimensions.

Exercise class:
13.7

1.12 Week 16

Second exam