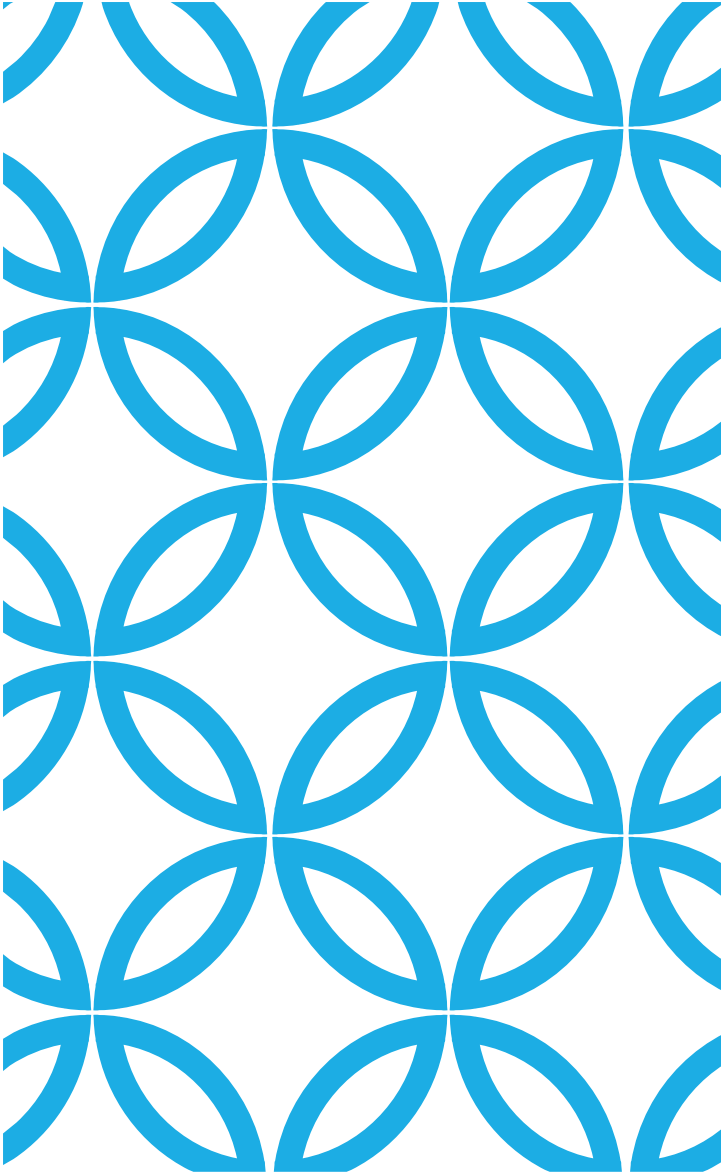




ERDÖS PROBLEMS AND HIGH SCHOOL MATH

Lecture in honor of André Ran

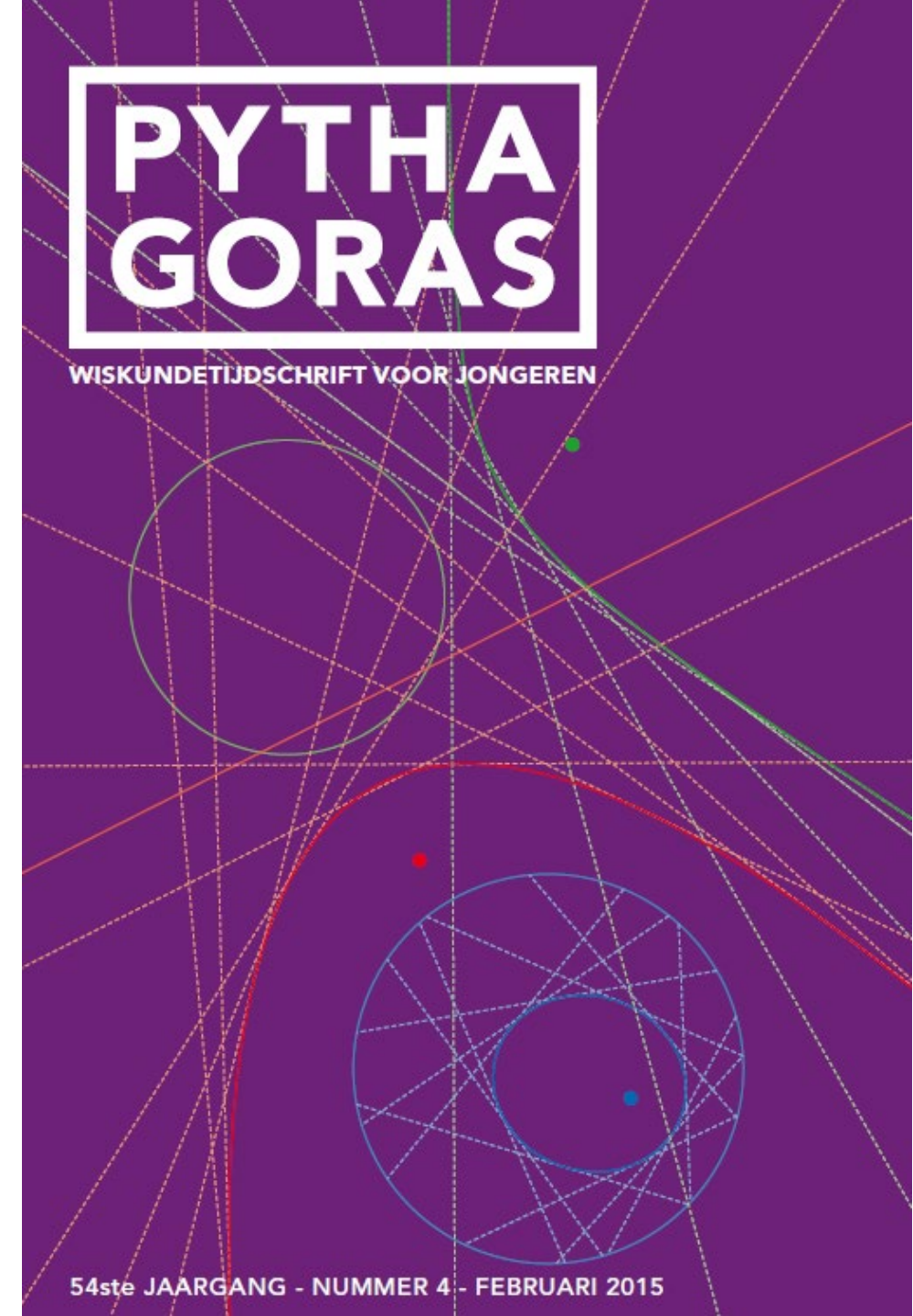
Derk Pik, drpik@uva.nl



UNIVERSITEIT VAN AMSTERDAM

IN HONOR OF ANDRÉ RAN

- Lecturer (Systems theory)
- PhD (Operator Group Kaashoek/Gohberg)
 - Rooms at the same corridor
 - Many joint conferences
- Head of the KWG-board
 - Strong support of Pythagoras (Youth Math Magazine)



PROGRAM

An Erdős result

Points in the plane with integer distance
(Pythagoras Erdős Series 14)

Computational thinking

Problem solving in School mathematics

Conclusion



Erdős-Anning Theorem:

An infinite number of points in the plane can have mutual integer distances only if all the points lie on a straight line.



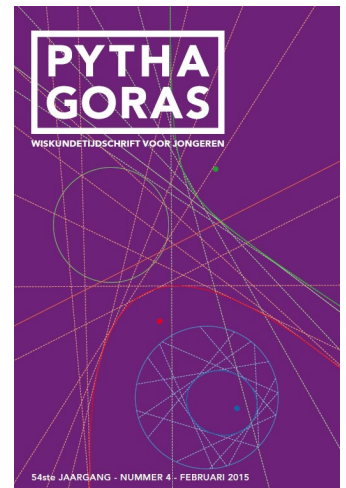
Erdős-Anning Theorem:

An infinite number of points in the plane can have mutual integer distances only if all the points lie on a straight line.



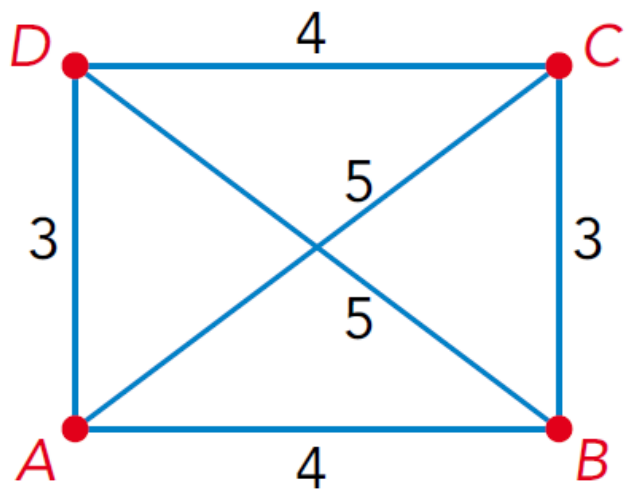
Pythagoras 54 – 4:

What about finitely many points?

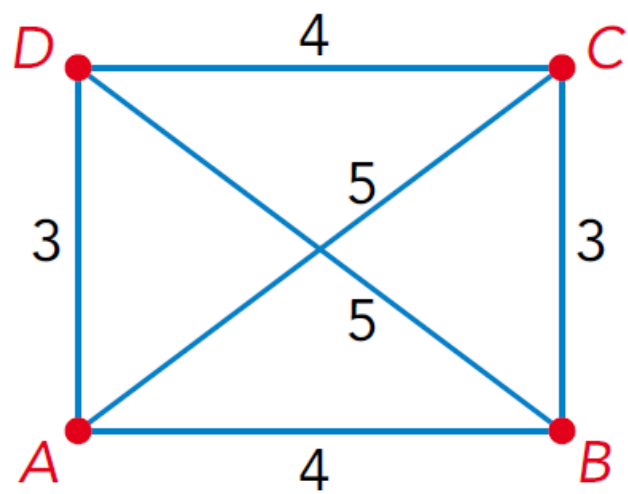


Four points

Four points

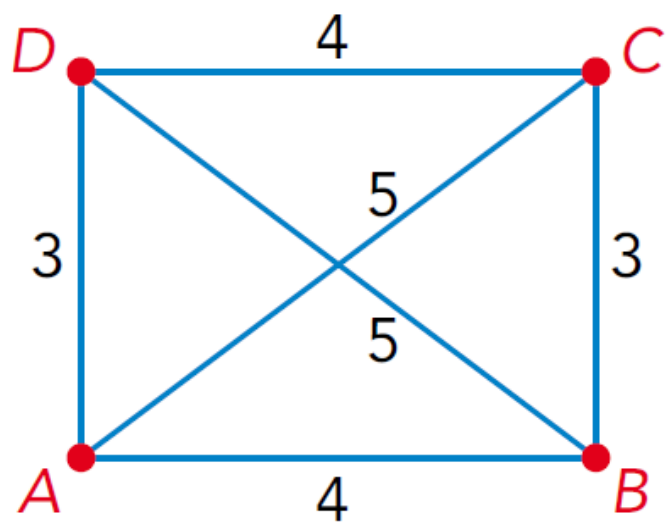


Four points

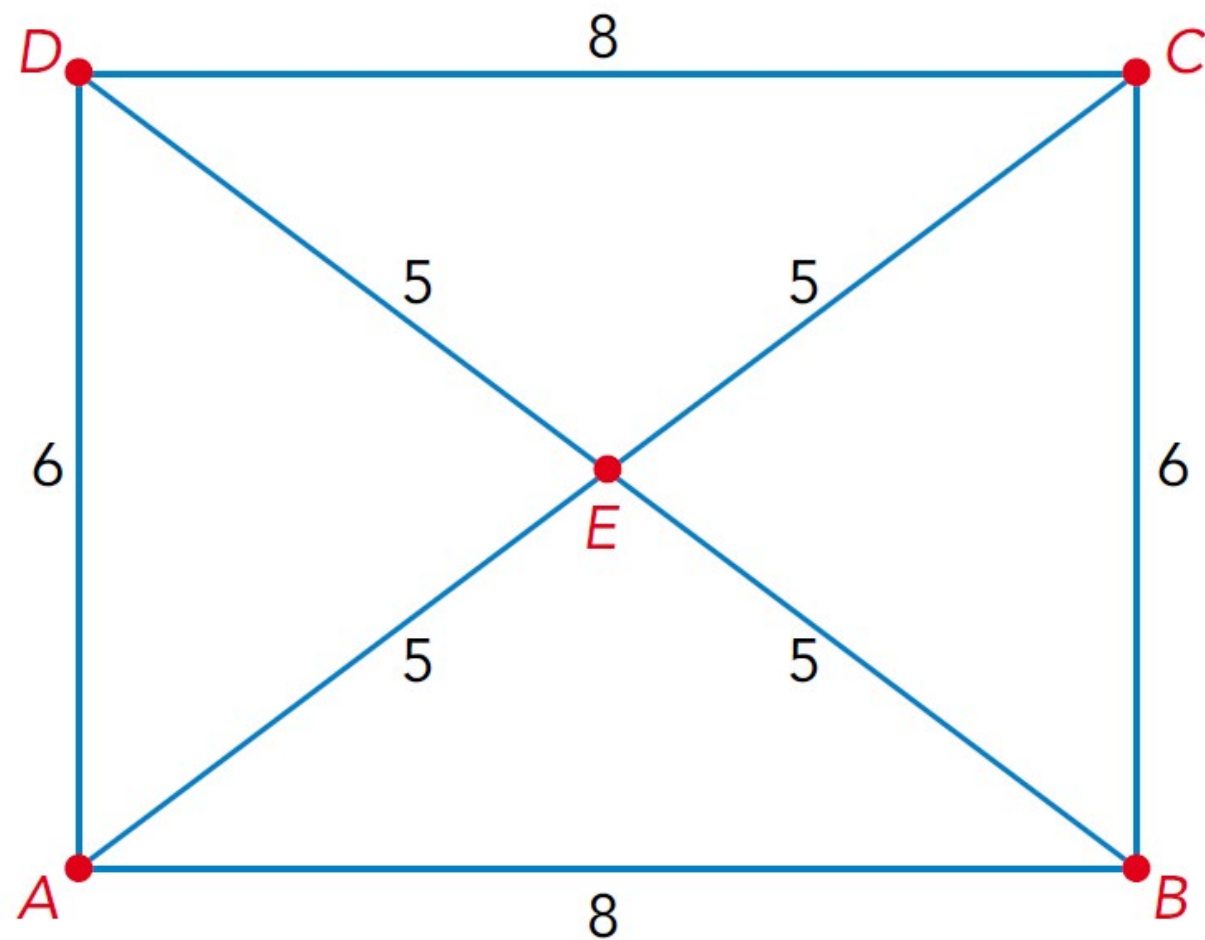


Five points

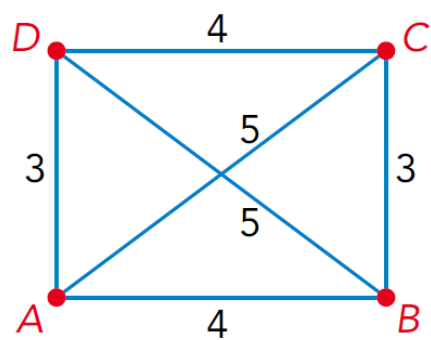
Four points



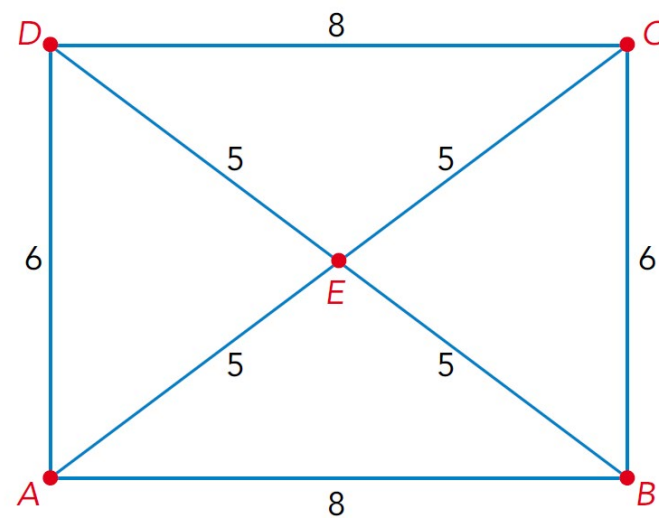
Five points



Four points



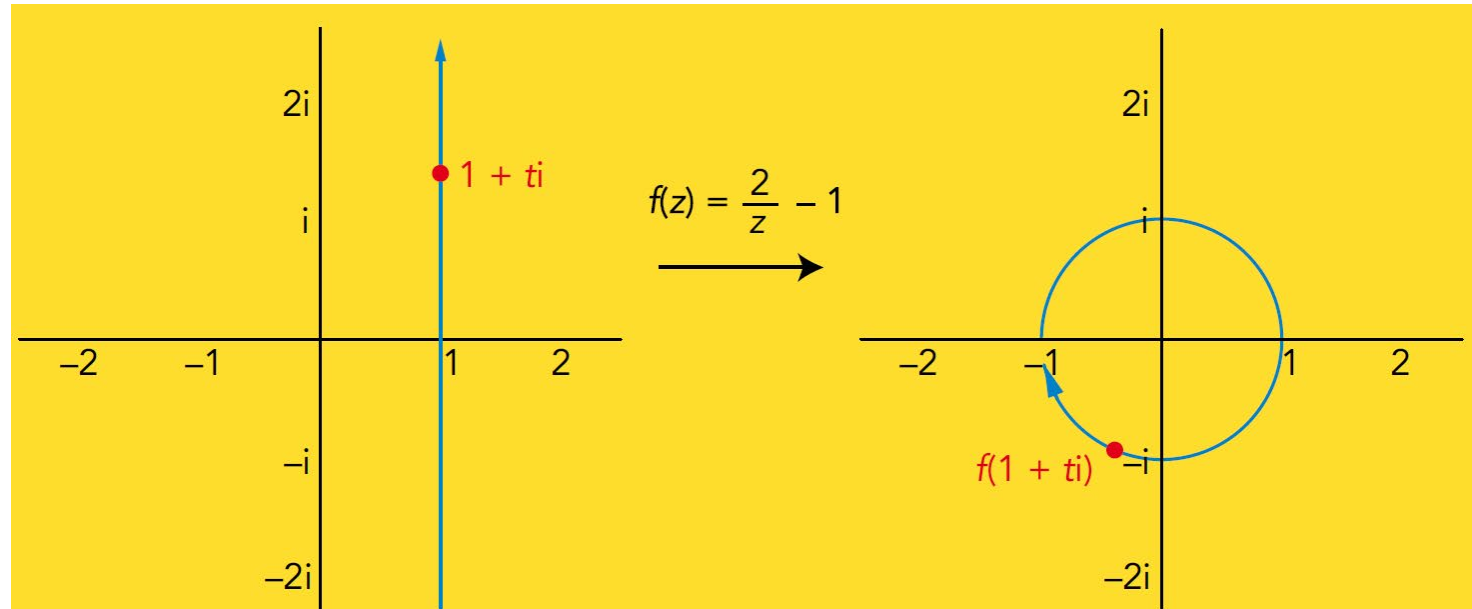
Five points



Six Points?

Finitely many points: construction

Complex plane:



If for w, z in $\mathbb{C}$

$|w|, |z|, |w - z|$ are rational,

then also

$$|f(w) - f(z)| = \left| \left(\frac{2}{w} - 1 \right) - \left(\frac{2}{z} - 1 \right) \right| = \left| \frac{2}{w} - \frac{2}{z} \right| = \frac{2|w - z|}{|w| \cdot |z|}$$

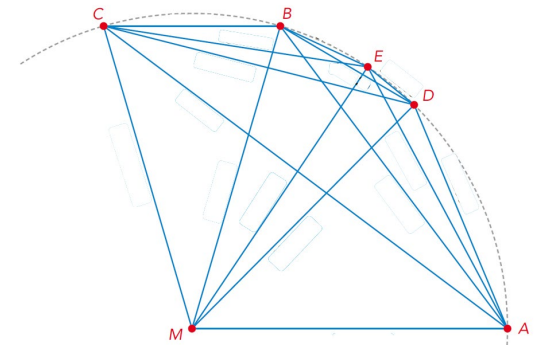
Finitely many points: construction

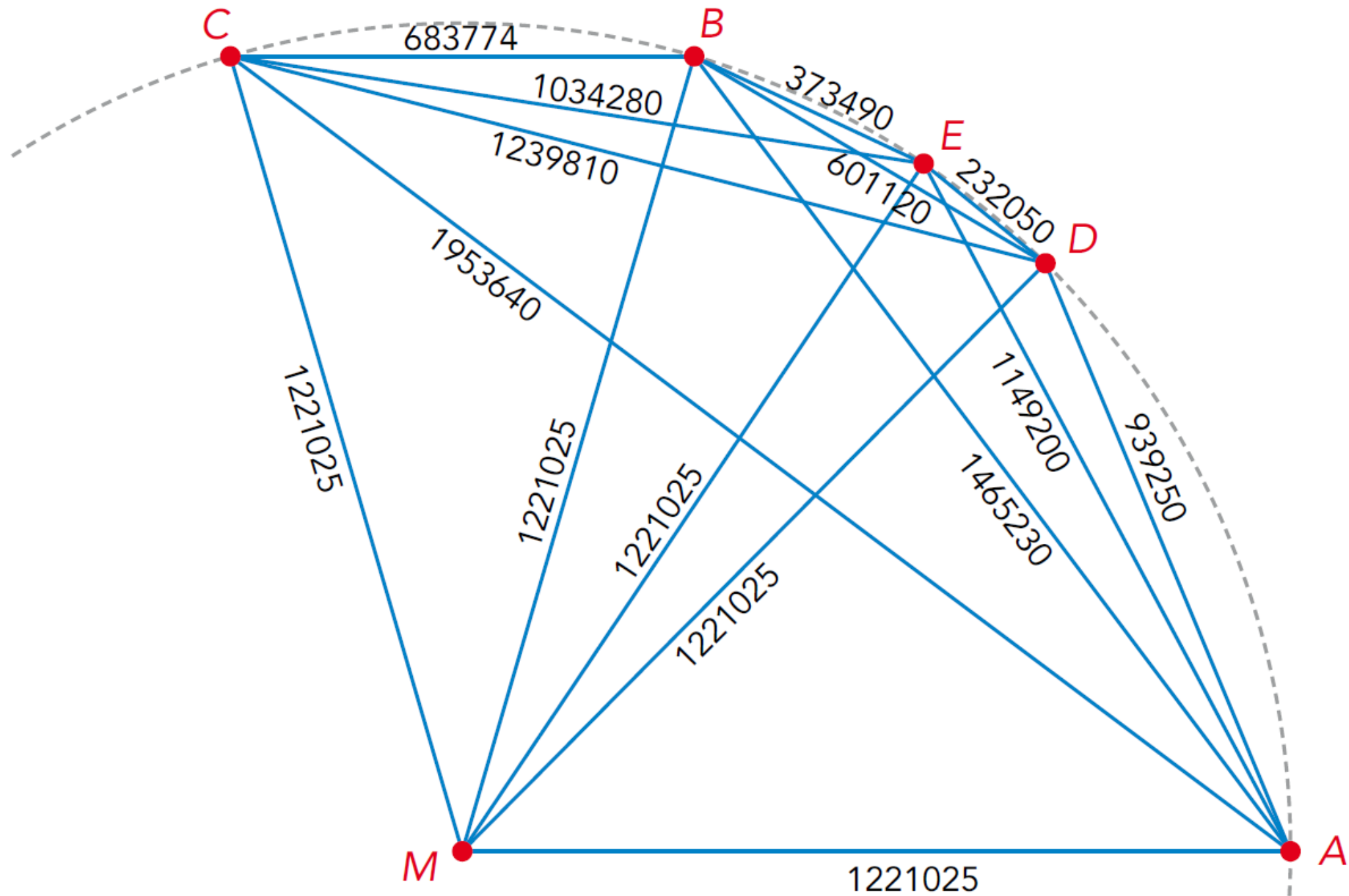
A fraction $t = a/b$ is called Pythagorean, if $a^2 + b^2 = c^2$ has an integer solution c .

If t and s are Pythagorean fractions, and $z = 1 + ti$ and $w = 1 + si$,

then $|w|$, $|z|$, $|w-z|$ are rational.

Thus $|f(w) - f(z)|$ is rational.





New requirements

Find points in the plane with:

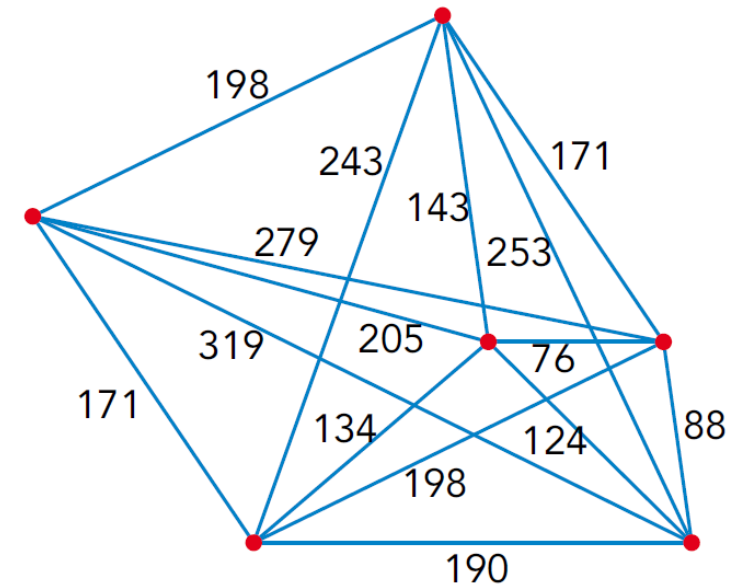
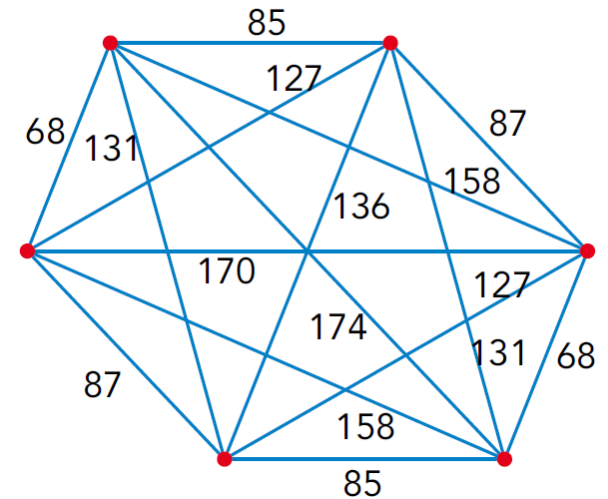
- each pair of points: integer distance
- no more than two points on a straight line
- no more than three points on a circle

New requirements

Find points in the plane with:

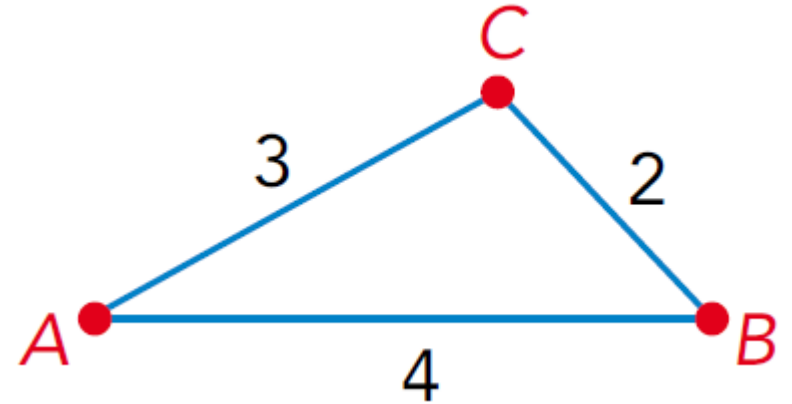
- each pair of points: integer distance
- no more than two points on a straight line
- no more than three points on a circle

John Leech (1981), Arnfried Kemnitz (1990)





Can we construct a point D with integer distance to A, B, C?



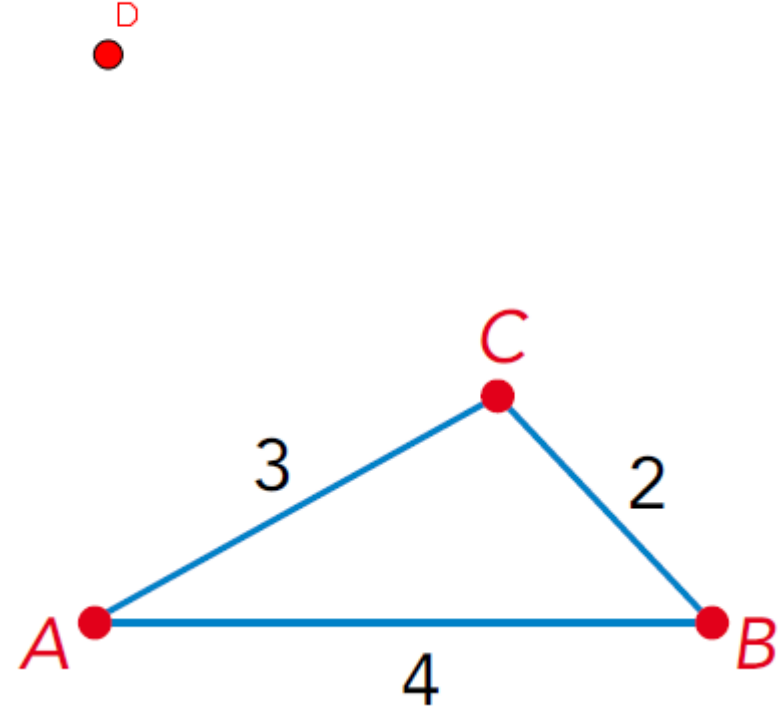
Triangle inequalities

$$4 = AB \geq |DB - DA|, \quad 3 = AC \geq |DC - DA|, \quad 2 = CB \geq |DB - DC|$$

$$|AD - BD| = 0, 1, 2, 3, 4,$$

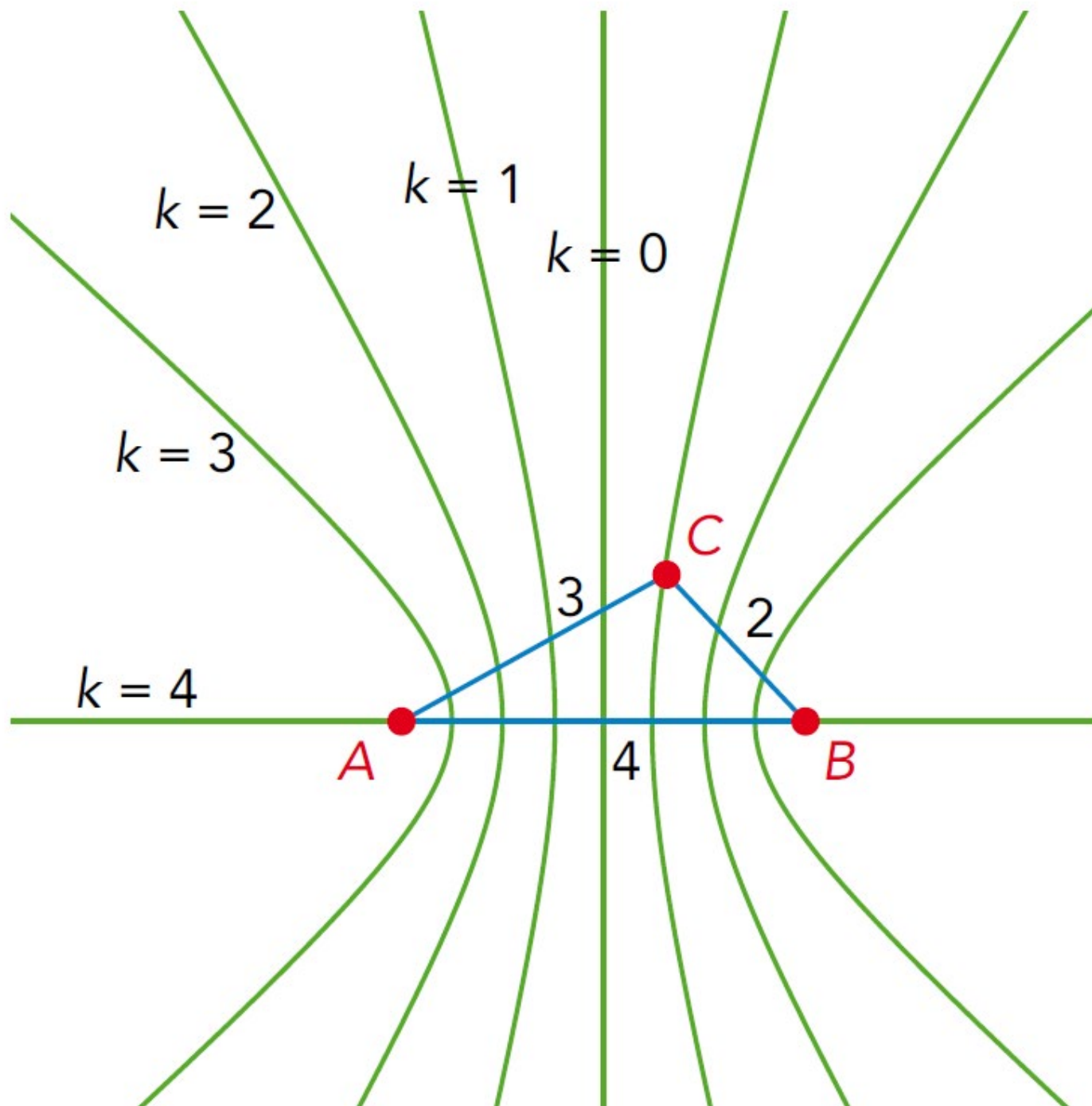
$$|AD - CD| = 0, 1, 2, 3,$$

$$|CD - BD| = 0, 1, 2.$$



Triangle inequality

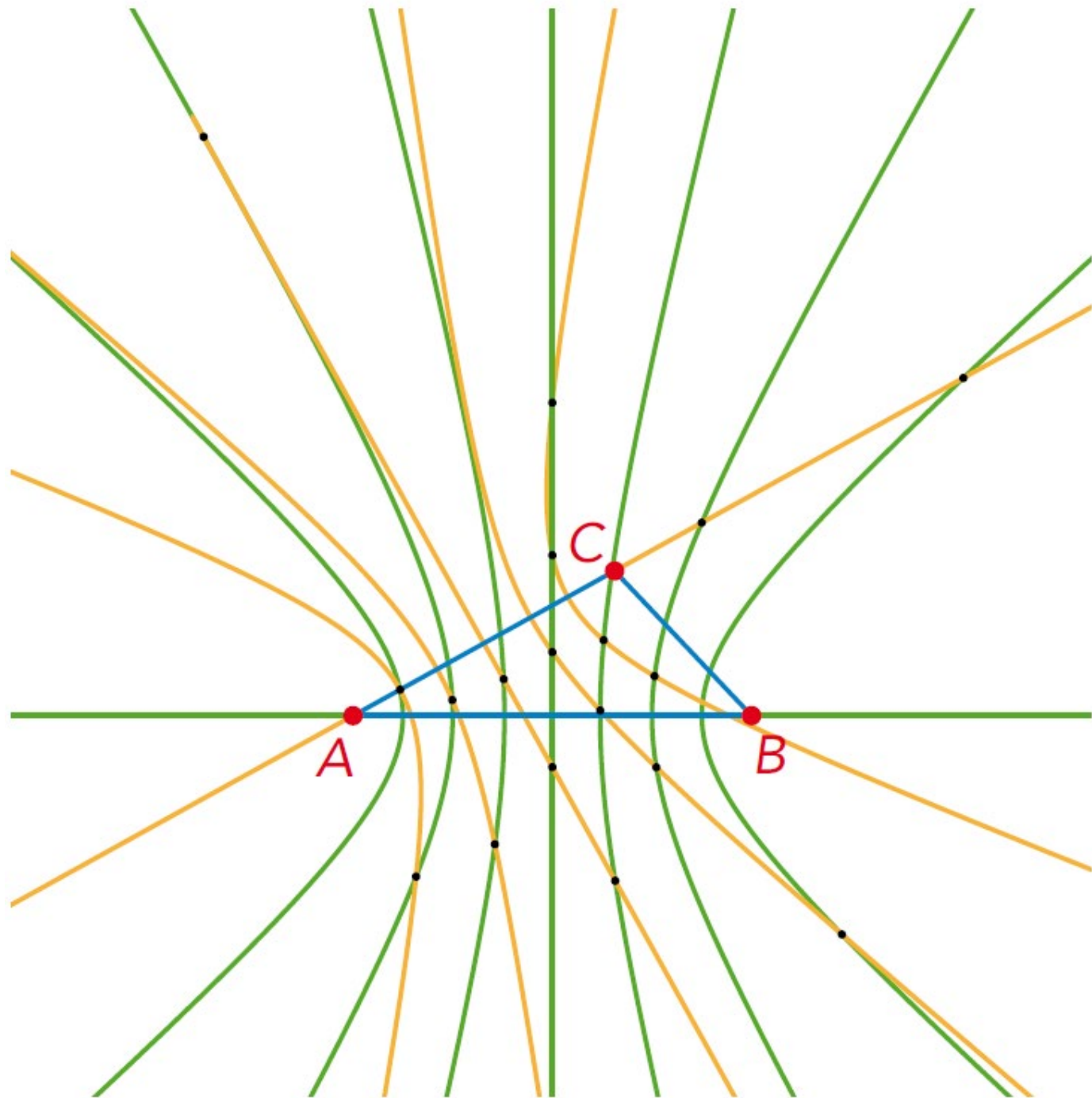
$$|AD - BD| = k$$

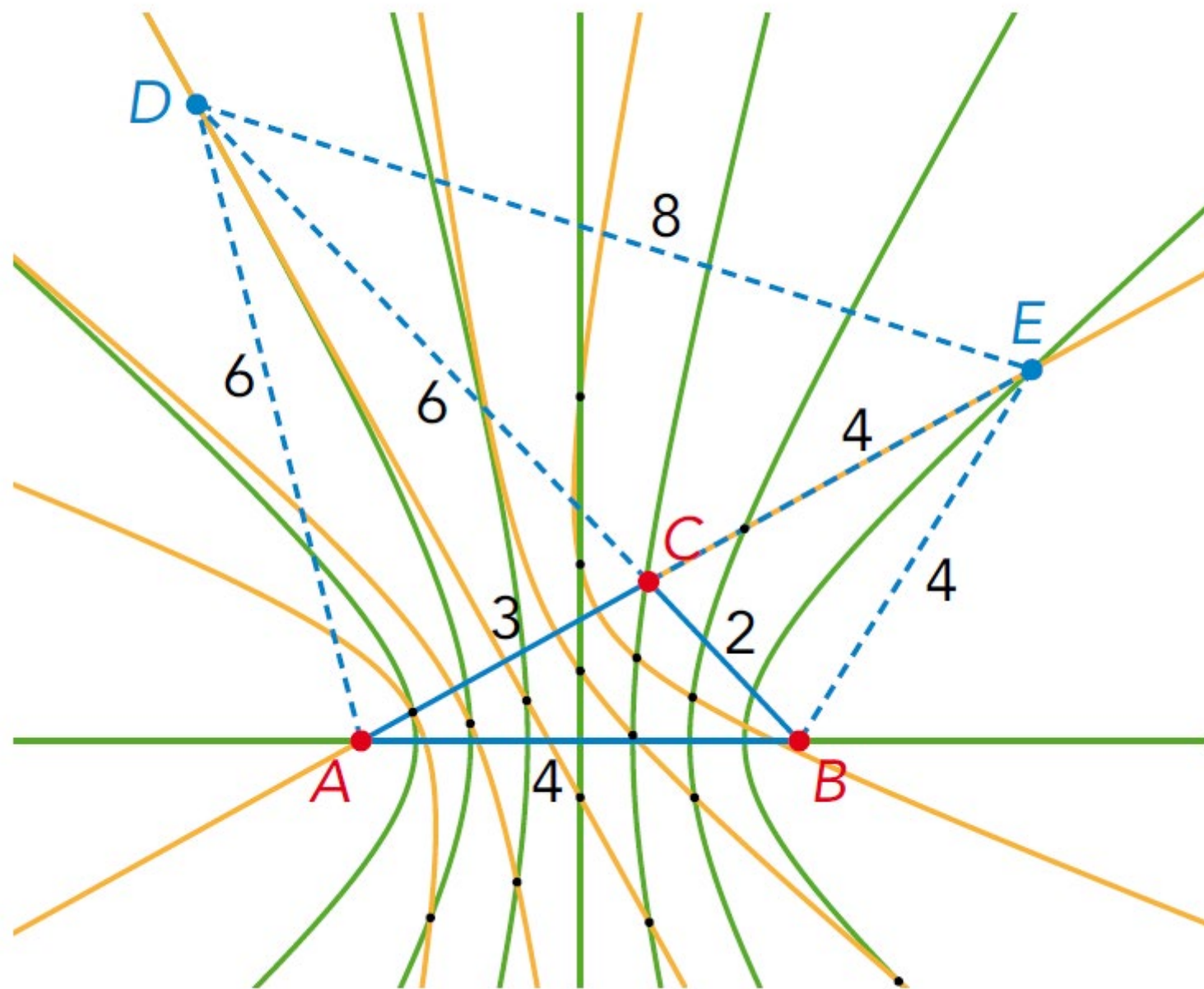


Triangle inequality

$$|AD - BD| = k$$

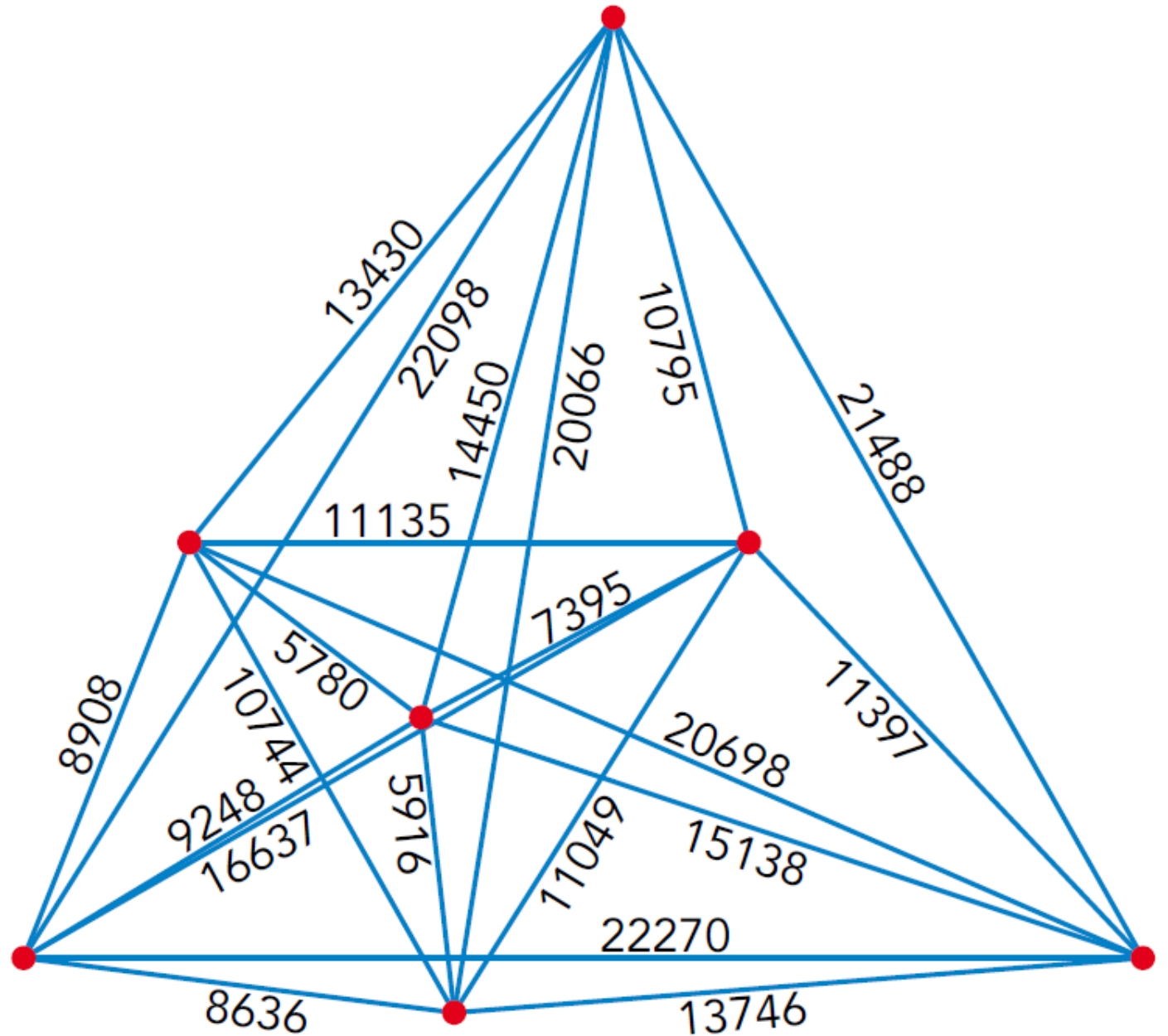
$$|AD - CD| = n$$





Seven points construction

Tobias Kreisl, Sacha Kurz: 7 points



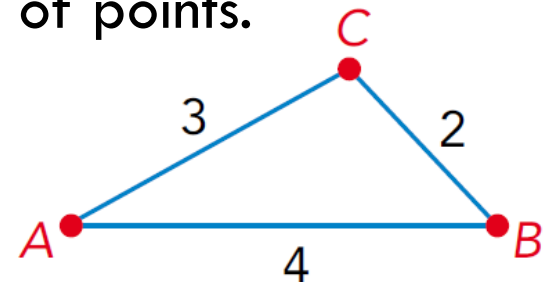
COMPUTATIONAL THINKING

Geometric problem -> Make it fit for the computer
Check finitely many points.

Pythagoras high school readers: R, Python programs

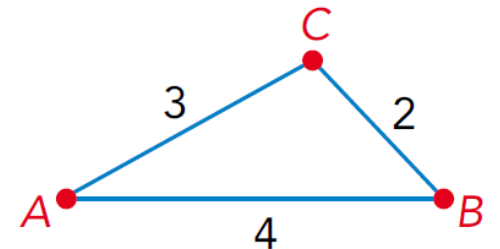
How do you check this with a computer?

- knowledge needed about linear algebra, conics and perp bisectors, geometry and proof
- programming
- put more details into your checking to bring down the number of points.



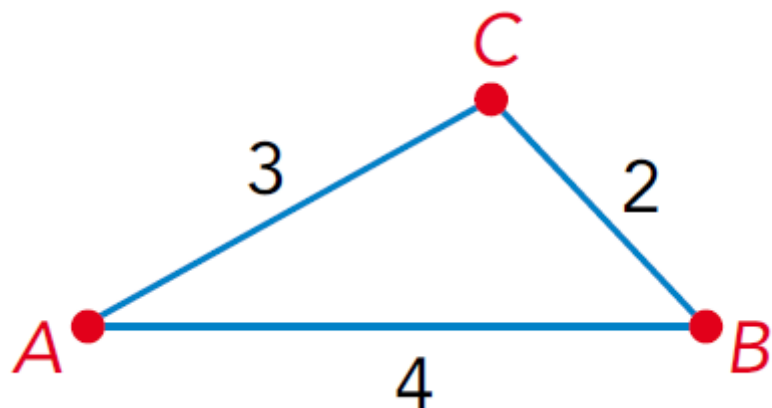
EDUCATIONAL VIEWPOINT

- School math is completely organized around closed problems, problems with a definite answer. Open problems give an outlook on math as science.
- Open problems are good from a mind-set point of view (Dweck, Boaler, Kaplinsky). Different students, different approaches.
- Curriculum: what skills do we want to teach to raise mathematically creative students? Which germs are the most fruitful?



Thank you for your attention

Questions?



PYTHA GORAS

WISKUNDETIJDSCHRIFT VOOR JONGEREN



52ste JAARGANG - NUMMER 3 - JANUARI 2013