Homework 2, Percolation 2014

Please give crisp and clear computations/argumentations. The solutions should be given to René Conijn (as hard-copy, or as pdf file by e-mail) before 14:00, Thursday, November 13. His e-mail address is r.p.conijn@vu.nl

1. Consider bond percolation on the square lattice with parameter $p > 1/2$. Let $f_n = f_n(p)$ denote the probability that there is an open path in the box $[0, n] \times [0, n]$ from $(0, 0)$ to (n, n).
Show that $\lim_{n\to\infty} f_n$ exists and is strictly larger than 0.

2. Again, consider bond percolation on the square lattice. Show that there are $C > 0$ and $\alpha > 0$ such that, for all $p > 1/2$,

$$\theta(p) \leq C(p - \frac{1}{2})^{\alpha}.$$

Hint: Use that, for each n, $\theta(p) \leq P_p(0 \leftrightarrow \partial B(n))$. Then use Theorem 2.10 in my lecture notes (and a very general and simple result which gives, for an event A which is defined in terms of finitely many edges, a lower bound for $P_{p_1}(A)$ in terms of $P_{p_2}(A)$, for $p_1 < p_2$). Finally, choose n properly (depending on p).

3. Grimmett, page 119, a few lines above (5.58) says: "For example, the contribution from z' if its face is at the bottom (but not at the corner) of Γ^n is

$$\frac{1}{n}[(\tau + \tau^2)H_1^n(z') - (1 + \tau)H_\tau^n(z')] \text{ ''}.$$

Check this. Don't give a long story; just a few lines pointing out from which (combination of) terms in (5.56) the above contribution comes; a picture may be helpful.