Homework 3, Percolation 2014

Please give crisp and clear computations/argumentations.
The solutions should be given to René Conijn (as hard-copy, or as pdf file by e-mail) before 14:00, Monday, December 1. His e-mail address is r.p.conijn@vu.nl

1. Do part (1) of Exercise 5.1 in the lecture notes by Camia. From this, obtain *directly* (just by explicit differentiation) the Loewner equation for this curve.

2. Make the exercise (both parts) on page 5 of my lecture notes "Introduction to Schramm-Loewner Evolutions".

3. *(Corrected version).* Suppose the plane consists of 'polders' $(x - 1/2, x + 1/2) \times (y - 1/2, y + 1/2)$, $x, y \in \mathbb{Z}$. Each pair of neighbouring polders is separated by a dike. (So each polder is surrounded by four dikes). We assume that the four dikes surrounding the polder containing the point $(0, 0)$ have height 0.51, and that the heights of all the other dikes are independent random variables, uniformly distributed in the interval $(0, 1)$.
Now let n be a positive integer. Suppose that the polder containing the point $(n, 0)$ has a water source, from which water keeps flowing and spreads from there in the 'natural' way, namely as follows: First the water level in that polder will rise until it reaches the height (which we call a) of the lowest of the four dikes of that polder. Then the water starts flowing over that dike into the neighbouring polder on the other side of that dike. Let b denote the smallest height of the three other dikes in that neighbouring polder. If $b > a$, then the water in the polder containing $(n, 0)$ will stay at level a until the water in the mentioned neighbouring polder has also reached that level, after which the level in both polders will rise simultaneously until it starts flowing over the lowest of the six dikes bounding the union of the two polders. If $b < a$, the water in the neighbouring polder rises to level b and starts flowing over the corresponding dike, while the water in the polder of $(n, 0)$ stays (for the time being) at level a. Etcetera.
Let f_n denote the probability that the ('special') polder of $(0, 0)$ becomes wet. Show that there are constants $C_1, C_2 > 0$ such that, for all $n \geq 1$,

$$f_n \leq C_1 \exp(-C_2 n).$$