Modeling the Dynamics of Mood and Depression

Fiemke Both and **Mark Hoogendoorn** and **Michel Klein** and **Jan Treur**[1]

Abstract. Both for developing human-like virtual agents and for developing intelligent systems that make use of knowledge about the emotional state of the user, it is important to model the mood of a person. In this paper, a model for simulating the dynamics of mood is presented, based on psychological theories about a uni-polar clinical depression. The model was analyzed mathematically and by means of simulations, and it was shown that the model exhibits the most important characteristics of the theories. It shows how stress factors under some conditions can lead to a depression, while it will not lead to a depression under other conditions.

1 INTRODUCTION

Traditionally, emotions were often left out of consideration in the areas of cognitive, agent, and user modeling. Only few computational models of mood and depression have been developed [12]. However, emotions undoubtedly influence the behavior of humans. In recent years, there is a growing awareness of this role of emotions within human behavior.

Modeling of emotions is important for the development of agents that should exhibit human-like behavior. For example, agents that are used in Intelligent Tutoring Systems to train humans should behave human-like, and show emotions as well. Similarly, virtual agents in games that should interact in a realistic manner have to incorporate the effect of emotions on behavior.

Secondly, also systems that reason over the state of humans should take the emotions and mood of humans into account. For example, one may think of ambient intelligence applications that react on the mood and emotional state of humans. Another category of applications that use knowledge about the mood of humans are systems that support therapy, such as systems the help to quit smoking addiction, or internet-based therapy for depression (online counseling systems). In this paper, a formal model is presented that can be used to simulate the dynamics in the mood of humans, and more specifically, whether they develop longer periods a undesired moods, as in depressions. To come to this model, commonly used psychological theories about uni-polar (i.e., uncomplicated) depressions are discussed in Section 2. Then, in Section 3 the main concepts and relations are extracted from the theories. This results in a formal representation of the aspects of mood and depression, used in Section 4 to simulate several situations. In addition, in Section 5 by way of a mathematical analysis it is shown that indeed two equilibria can be found in the model: one in which a depression occurs, and one in which a good mood is maintained as desired. In Section 6, the adherence of the model to the theories is validated by automatic verification of a number of properties that should hold for the model according to the theories. Finally, Section 7 concludes the paper.

2 THEORIES ABOUT DEPRESSION

A clinical depression is one of the most prominent disturbances in mood. It is a common psychiatric disorder, affecting about 7–18% of the people at least once in their lives. In the USA, the prevalence is approximately 14 million adults per year [1]. Symptoms of a depression are a deep feeling of sadness, and a noticeable loss of interest or pleasure in favorite activities. There is not one specific cause of a depression, most experts believe that both biological and psychological factors play a role.

In the last decennia, several theories have been developed about the course and treatment of a depression. A classic behavioral model of uni-polar depression by **Lewinsohn** [2] states that depression results from a stressful event that disrupts normal behavior patterns. According to Lewinsohn, a low rate of behavior (often caused by inadequate social skills) is the essence of the depression and the cause of all other symptoms. Part of his theory is the hypothesis that there is a causal relationship between lack of positive reinforcement from the environment and the depression.

According to many psychologists, the mood of a person is influenced by stressful events and the abilities a person has to cope with these events. In the stress, appraisal and coping theory by **Lazarus and Folkman** [5] they emphasize that stress is not a direct response to a stressor, but a response to a situation that has been appraissed as taxing or exceeding ones resources. When a situation has been identified as stressful, coping skills are applied. Coping is defined as “constantly changing cognitive and behavioral efforts to manage specific external and/or internal demands that are appraised as taxing or exceeding the resources of the person”. In the theory of Lazarus and Folkman, vulnerability is conceptualized in terms of coping resources. A vulnerable person is said to have “deficient coping resources”. Commitments are also claimed to be important by Lazarus and Folkman. It is stated that “commitments are an expression of what is important to persons, and they underlie the choices persons make”. Hence, commitments play a role in selecting the situation.

Aaron Beck developed a cognitive theory of depression [4]. He believes that depression is due to negative views towards the self, world, and future in particular. Depressed persons have thoughts like “nobody cares about me” or “I can’t do this task”. In his theory depressed persons also use faulty information processing, like selective attention, to maintain their negative views, even if the situation is actually more positive.

Gross [6, 7] developed a theory about emotion regulation. The core idea is that you can regulate emotions by choosing situations, subsituations, aspects and meanings with emotion levels that are near the preferred emotion level. By choosing and changing the emotional value of situations, the mood level can

[1] Vrije Universiteit Amsterdam, Department of Artificial Intelligence, De Boelelaan 1081a, 1081 HV Amsterdam, The Netherlands, email: {fboth, mhoogen, mcaklein, treur}@cs.vu.nl

be regulated so that it is close to the prospected mood level. How well a person is able to do this depends on the sensitivity of a person.

The diathesis-stress model was first introduced by **Zubin and Spring** [8] for schizophrenia. Now, there are many different stress-vulnerability models [9], most of which involve predisposed factors (vulnerability or diathesis) and external influences (stress) that together determine whether a person develops a mental disorder or not. There is no typical definition, but most theories assume that vulnerability is a trait, is stable but can change, is endogenous to individuals and is usually latent [10].

3 MODEL OF MOOD DYNAMICS

3.1 Concepts from theory

In the model basic concepts form the theories summarized above will be used. From the behavioral theory of Lewinsohn the idea is adopted that a lack of reinforcement from the environment (i.e., *situation*) influences the choice of new situation via the mood. According to Beck the perception of the situation will be negatively influenced by the thoughts, so also a *subjective emotional value of the situation* is used. This is also in line with the ideas of stress by Lazarus and Folkman: a part of the model represents the actual environmental situation experienced, whereas there is also a part that represents the interpretation of the situation specific for the person the model concerns. In addition, their idea of *vulnerability* (also called *diathesis)* is used, as having deficient coping resources and Zubin and Spring's idea of vulnerability as having a predisposition for developing a disorder. *Coping* is used in the model presented in this paper by means of continuously trying to adapt the situation in such a way that an improvement is achieved. This is done through a regulation system in the person, which is inspired by Gross's theory about emotion regulation by striving for a specific *prospected mood level*.

3.2 Conceptual model of mood and depression

In the model, it is assumed that every situation has an emotional value, which represents the extent to which a situation is experienced as something positive. The *objective emotional value of situation* (OEVS) represents how an average person would perceive the situation. A situation can be an event or series of events one has no control over, or that are chosen or influenced by the person. The *subjective emotional value of situation* (SEVS) can differ from OEVS when the *thoughts* of the person are more positive or more negative than average. Negative thoughts will cause the SEVS to be lower than OEVS, which is often the case with a depression. How one perceives the situation (SEVS) influences the *mood* one is in and the *thoughts* one has. When the person is in a positive situation, mood level and thoughts will increase. For example, attending a birthday party, which is usually a positive experience, causes a better mood and more positive thoughts. In contrast, an argument with a close friend has a low emotional value and causes a bad mood and negative thoughts. By changing or choosing a situation, one can influence their own mood level (e.g. choosing to go to the birthday party when one feels down increases the mood level). The complex notion of mood is represented by the simplified concept *mood level*, ranging from low corresponding to a bad mood to high corresponding to a good mood. The mood level influences and is influenced by *thoughts*. Positive thinking has a positive effect on the mood and vice versa. The mood level someone strives for, whether conscious or unconscious, is represented by *prospected mood level*. This notion is split into a *long term prospected mood level*, an evolutionary drive to be in a good mood, and a *short term prospected mood level*, representing a temporary prospect when mood level is far from the prospected mood level. The node *sensitivity* represents the ability to change or choose situations in order to bring mood level closer to prospected mood level. A high sensitivity means that someone's behavior is very much affected by thoughts and mood, while a low sensitivity means that someone is very unresponsive. The level of sensitivity itself is influenced by mood level and thoughts. A low mood level and negative thoughts can decrease the sensitivity and a high mood level and positive thoughts can increase the sensitivity. Mood level, prospected mood level and sensitivity together influence *OEVS* by choosing or changing a situation.

The new value of a node is determined by preceding nodes and the previous value of that node. Decay factors determine how fast the previous value of the node decays. For the entire model there are two decay factors: *diatheses* for downward regulation and *coping* for upward regulation. The term *diatheses* represents the vulnerability one has for developing a depression. The term *coping* represents the skills one has to deal with negative moods and situations. A person with very low diatheses will probably never get a depression, because mood, thoughts and sensitivity will go down very slowly with a negative event. That person is therefore always capable of choosing situations that have a positive influence on his/her mood level and emotions. High diatheses and low coping skills will cause a person to get a depression very easily when a negative event occurs, because mood, thoughts and sensitivity will decrease fast. It will be very difficult to climb out of a depression: the upward regulation of mood, thoughts and sensitivity will go very slow.

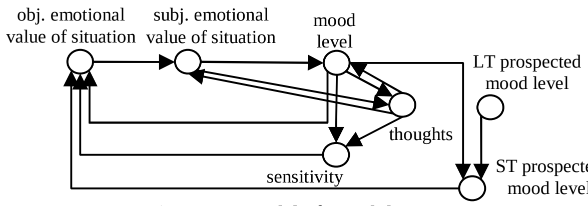

Figure 1. Model of mood dynamics.

3.3 Formalization

The model described above is formed into a quantitative model in this section. The basic principle used to make this translation is to find equations that incorporate the qualitative trends observed in the literature described before. All nodes in the model have values between 0 and 1. The first formula (equation 1) describes the calculation of the new value for *OEVS* using *sensitivity*, *mood*, *LT prospected mood* and *beta*. If the mood level is below the *ST prospected mood* level (beta times LT prospected mood level), the new situation will be higher than the previous one. The amount of change is mediated by sensitivity (the higher the sensitivity, the faster the optimal situation is reached) and by the previous situation (the closer to the optimal situation, the smaller the steps).

$$oevs(t+\Delta t) = oevs(t) - sensitivity(t) \cdot \varphi \cdot \Delta t$$
$$\varphi = \begin{cases} oevs(t) \cdot \delta_{mood} & \delta_{mood} >= 0 \\ (1 - oevs(t)) \cdot \delta_{mood} & \delta_{mood} < 0 \end{cases} \quad (1)$$
$$\delta_{mood} = mood(t) - \beta \cdot lt\_prosp\_mood$$

The *SEVS* (equation 2) depends on the *OEVS*, *thoughts* and *coping* and *diatheses*. The value for thoughts has a negative influence on SEVS by projecting thoughts (within range [0,1]) onto the part of the scale below the old SEVS value (range [0, SEVS]). Thoughts have a positive influence by projection of thoughts onto the upper part of SEVS (range [SEVS, 1]). The factors coping and diatheses determine the degree of positive and negative influence respectively.

Persons with a high vulnerability for depression will use thoughts mostly to downregulate their SEVS, whereas balanced persons will consider their SEVS as similar to the OEVS and their thoughts.

$$\begin{aligned} &sevs(t+\Delta t) = sevs(t) + (\gamma - sevs(t)) \cdot \Delta t \\ &\gamma = diatheses \cdot oevs(t) \cdot thoughts(t) + \\ &\quad coping \cdot (1 - (1 - oevs(t)) \cdot (1 - thoughts(t))) \end{aligned} \tag{2}$$

The new *mood level* (equation 3) is influenced by the *SEVS* (with weight w_{sevs_mood}) and *thoughts* (with weight $w_{thoughts\_mood}$). If the new mood level is to be increased, the decay factor of the previous mood is *coping*. If the new mood level is to be decreased, *diatheses* is used to determine the speed of the fall.

Good coping skills result in a faster increase of mood when the situation and thoughts are increasing. Bad coping skills result in a faster decrease of mood when SEVS and thoughts are low.

$$mood(t+\Delta t) = \begin{cases} mood(t) + coping \cdot (\phi - mood(t)) \cdot \Delta t & \phi >= mood(t) \\ mood(t) + diatheses \cdot (\phi - mood(t)) \cdot \Delta t & \phi < mood(t) \end{cases} \tag{3}$$

$$\phi = sevs(t) \cdot w_{sevs\_mood} + thoughts(t) \cdot w_{thoughts\_mood}$$

The formula for the new *thoughts* value (equation 4) is similar to the formula for mood: thoughts is influenced by *SEVS* (with weight $w_{sevs\_thoughts}$) and *mood* (with weight $w_{mood\_thoughts}$) The decay factors are *coping* and *diatheses* for upwards and downwards regulation respectively.

$$thoughts(t+\Delta t) = \begin{cases} thoughts(t) + coping \cdot (\phi - thoughts(t)) \cdot \Delta t & \phi >= thoughts(t) \\ thoughts(t) + diatheses \cdot (\phi - thoughts(t)) \cdot \Delta t & \phi < thoughts(t) \end{cases} \tag{4}$$

$$\phi = sevs(t) \cdot w_{sevs\_thoughts} + mood(t) \cdot w_{mood\_thoughts}$$

Again, equation 5 is similar: *sensitivity* is calculated using the values for *mood* and *thoughts* with the corresponding weights and is mediated by coping for upwards regulation and diatheses for downwards regulation.

$$sens(t+\Delta t) = \begin{cases} sens(t) + coping \cdot (\phi - sens(t)) \cdot \Delta t & \phi >= sens(t) \\ sens(t) + diatheses \cdot (\phi - sens(t)) \cdot \Delta t & \phi < sens(t) \end{cases} \tag{5}$$

$$\phi = mood(t) \cdot w_{mood\_sens} + thoughts(t) \cdot w_{thoughts\_sens}$$

The *ST prospected mood level* (equation 6) is a percentage (β) of the *LT prospected mood level*. The percentage is adjusted towards *mood level* with a factor (*diatheses*) and towards LT prospected mood level with factor *coping*.

Persons with bad coping skills will let their ST prospected mood level be heavily influenced by mood and not by LT prospected mood. Healthy persons use a balanced influence to determine their new ST prospected mood level.

$$\begin{aligned} &st\_prosp\_mood(t+\Delta t) = \beta(t+\Delta t) \cdot lt\_prosp\_mood \\ &\beta(t+\Delta t) = \beta(t) + \Delta t \cdot ((diatheses \cdot (mood(t) - \beta(t) \cdot lt\_prosp\_mood) + \\ &\quad coping \cdot (lt\_prosp\_mood - \beta(t) \cdot lt\_prosp\_mood) + \\ &\quad \beta(t) \cdot lt\_prosp\_mood) / lt\_prosp\_mood - \beta(t)) \\ &= \beta(t) + (diatheses \cdot (mood(t) / lt\_prosp\_mood - \beta(t)) + coping \cdot (1 - \beta(t))) \cdot \Delta t \end{aligned} \tag{6}$$

4 SIMULATIONS

The model for depression was used to simulate three types of people in different situations. The different types are accomplished by setting the parameters coping, diatheses and LT prospected mood level. The six weights between mood, thoughts and SEVS can also be varied to simulate different personal characteristics. However, in these simulations they have been set at the following values: w_{sevs_mood} 0.7, $w_{thoughts\_mood}$ 0.3 (mood is influenced by SEVS for 70% and by thoughts for 30%), $w_{sevs\_thoughts}$ 0.6, $w_{mood\_thoughts}$ 0.4 (thoughts is influenced by SEVS for 60% and by mood for 40%), w_{mood_sevs} 0.5, $w_{thoughts\_sevs}$ 0.5 (SEVS is influenced by mood and thoughts for both 50%). The first type, an emotionally stable person, is defined by having good coping skills that balance out any diatheses, and by having the desire to have a good mood: coping value is 0.5, diatheses 0.5 and LT prospected mood level 0.8. An emotionally slightly unstable person is defined by having some diatheses and bad coping skills and the desire to have a medium mood: settings 0.1, 0.9 and 0.6 respectively. The third type, an emotionally very unstable person, is characterized by settings 0.01, 0.99 and 0.6. The start value for OEVS needs to be calculated for each type so that when no events occur, the person stays balanced with al variables equal to LT prospected mood level. For type the OEVS is 0.8, for type 2 it is 0.94 and for type 3 the stable OEVS is 0.999.

Each type of person has been simulated in different scenarios during 1000 time steps (representing 1000 hours, 1.5 months), resulting in a total of five simulations. In the first scenario (traces 1 and 5) one or two minor negative events with an emotional value 0.5 occur. In the second scenario (traces 2 and 4) a negative event with value 0.2 or 0.3 occurs and in the third scenario (trace 3) six major negative events occur. Table 1 describes the simulation settings (type of person and scenario) and results (min and max mood levels, occurrence and duration of depression and recovery within 1.5 months). A depression is defined as a mood level below 0.5 during at least 336 time steps (two weeks, [11]).

Table 1. Settings and results of the simulations. *: there is no recovery within the simulation time, but mood level is increasing and the person will recover after approximately 22 weeks.

#	Person	Scenario	Mood levels	Depression	Recovery
1	1	2 events 0.5	[0.63; 0.81]	no (0)	-
2	1	1 event 0.2	[0.52; 0.81]	no (0)	-
3	1	6 events 0.1	[0.36; 0.81]	yes (572)	yes
4	2	1 event 0.3	[0.09; 0.61]	yes (888)	no*
5	3	1 event 0.5	[0.01; 0.60]	yes (887)	no

The results show that an emotionally stable person is unlikely to develop a depression. Figure 2 shows the values of OEVS, SEVS, mood and the events for simulationtrace 1. Note that the x-axis indicates time and the y-axis the respective values. Persons with a very high vulnerability develop a depression already after one minor event (simulationtrace 5, Figure 3).

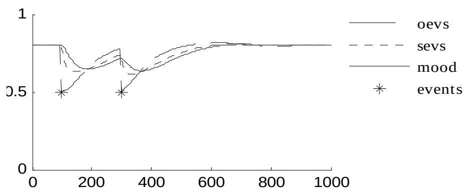

Figure 2. Simulation 1: emotionally stable person, 2 negative events.

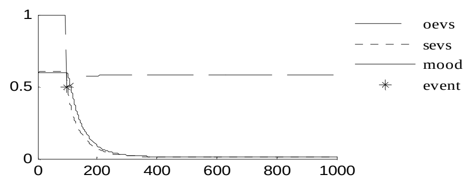

Figure 3. Simulation 5: emotionally unstable person, 1 negative event.

5 MATHEMATICAL ANALYSIS

It is also possible to show mathematically that two different situations can be distinguished in the model: one in which stressful events lead to a depression, and one in which this won't lead to a depression. To do so, the model is rewritten in a continuous form of the system of (nonlinear) differential equations shown below. Here *O* denotes *oevs*, *M mood*, *T thoughts*, *S sevs*, σ *sensitivity*, and α *coping*. Moreover, *Pos(x) = x* when $x \geq 0$, and *0* otherwise; alternatively $Pos(x) = (x + |x|)/2$.

$$\frac{dO(t)}{dt} = \sigma(t)\,(-\,O(t)\,Pos(M(t) - \beta(t)\lambda) + (1 - O(t))Pos(\beta(t)\lambda - M(t)))$$

$$\frac{dS(t)}{dt} = \alpha\,(1 - (1 - O(t))(1 - T(t))) + (1 - \alpha)\,O(t)\,T(t) - S(t)$$

$$\frac{dM(t)}{dt} = \alpha\,Pos(w_{sm}S(t) + (1 - w_{sm})T(t) - M(t))\ -$$

$$(1 - \alpha)\, Pos(M(t) - w_{sm}S(t) - (1-w_{sm})T(t))$$

$$\frac{dT(t)}{dt} = \alpha\, Pos(w_{st}S(t) + (1-w_{st})M(t) - T(t)) - (1 - \alpha)\, Pos(T(t) - w_{st}S(t) - (1-w_{st})M(t))$$

$$\frac{d\sigma(t)}{dt} = \alpha\, Pos(w_{m\sigma}M(t) + (1-w_{m\sigma})T(t) - \sigma(t)) - (1 - \alpha)\, Pos(\sigma(t) - w_{m\sigma}M(t) - (1-w_{m\sigma})T(t))$$

$$\frac{d\beta(t)}{dt} = \alpha\,(1 - \beta(t)) + (1 - \alpha)\,(M(t)/\lambda - \beta(t))$$

Equilibria satisfy the following equations in $O, S, M, T, \sigma, \beta$:

(i) $\sigma(- O\, Pos(M - \beta\lambda) + (1 - O)Pos(\beta\lambda - M) = 0$

(ii) $\alpha\,(1 - (1 - O)(1 - T)) + (1 - \alpha)\, O\, T - S = 0$

(iii) $\alpha\, Pos(w_{sm}S + (1-w_{sm})T - M) - (1 - \alpha)\, Pos(M - w_{sm}S - (1-w_{sm})T) = 0$

(iv) $\alpha\, Pos(w_{st}S + (1-w_{st})M - T) - (1 - \alpha)\, Pos(T - w_{st}S - (1-w_{st})M) = 0$

(v) $\alpha\, Pos(w_{m\sigma}M + (1-w_{m\sigma})T - \sigma) - (1 - \alpha)\, Pos(\sigma - w_{m\sigma}M - (1-w_{m\sigma})T) = 0$

(vi) $\alpha\,(1 - \beta) + (1 - \alpha)\,(M/\lambda - \beta) = 0$

Note that $Pos(x) \neq 0 \Rightarrow Pos(-x) = 0$. From (vi) it follows that either $M = \lambda\beta$ (if $\alpha = 0$) or $M = \lambda\,(\beta - \alpha)/(1 - \alpha)$ (if $0<\alpha<1$) or $\beta = 1$ (if $\alpha = 1$). Using this, (i) provides for $0<\alpha<1$:

$$\sigma\, \lambda\, (\alpha/(1-\alpha))\, (- O\, Pos(\beta - 1) + (1 - O)Pos(1 - \beta)) = 0$$

This implies $\sigma = 0$ or $(- O\, Pos(\beta - 1) + (1 - O)Pos(1 - \beta) = 0$, which implies either $\beta = 1$ or $\beta > 1$ and $O = 0$, or $\beta < 1$ and $O = 1$. Two specific cases are as follows.

The case with $M = \lambda$ In this case $\beta = 1$ (by (vi)), and by (iii) and (iv) it follows:

$$\lambda = w_{sm}S + (1-w_{sm})T \qquad T = w_{st}S + (1-w_{st})\lambda$$

Hence

$$\lambda = w_{sm}S + (1-w_{sm})(w_{st}S + (1-w_{st})\lambda) = (w_{sm} + (1-w_{sm})\, w_{st})S + (1-w_{sm})(1-w_{st})\lambda$$

or

$$(1-(1-w_{sm})(1-w_{st}))\lambda = (w_{sm} + (1-w_{sm})\, w_{st})S = (1 - (1 - w_{sm}) + (1-w_{sm})\, w_{st})S = (1 - (1 - w_{sm})\, (1-w_{st})\, S$$

So, assuming the weights < 1, it follows $S = \lambda$, and from (iv) also $T = \lambda$. From (v) it follows $\sigma = \lambda$, and by (ii) O can be determined. This is an equilibrium which would be considered a good situation.

The case with $M = 0$ Another special case of an equilibrium is when the mood M is 0. From (iii) and (iv) it follows that also $S = 0$, and from (iii) that $T = 0$ (assuming the weights <1). By (ii) $O = 0$, and by (v) $\sigma = 0$. Finally, from (vi) it follows $\beta = \alpha$. This is an equilibrium that would be classified as a depression.

6 VALIDATION: ADHERENCE TO THEORIES

In order to verify whether the model indeed produces results that follow psychological observations, a number of properties have been identified in the psychological literature, which have been verified against representative traces (the ones described in Section 4 and additional traces). In order to conduct such a verification, the properties have been specified in a language called TTL (for Temporal Trace Language, cf. [13]) that features an automated checker. This predicate logical temporal language supports formal specification and analysis of dynamic properties, covering both qualitative and quantitative aspects. TTL is built on atoms referring to *states* of the world, *time points* and *traces*, i.e. trajectories of states over time. In addition, *dynamic properties* are temporal statements that can be formulated with respect to traces based on the state ontology Ont in the following manner. Given a trace γ over state ontology Ont, the state in γ at time point t is denoted by state(γ, t). These states can be related to state properties via the formally defined satisfaction relation denoted by the infix predicate |=, comparable to the Holds-predicate in the Situation Calculus: state(γ, t) |= p denotes that state property p holds in trace γ at time t. Based on these statements, dynamic properties can be formulated in a formal manner in a sorted first-order predicate logic, using quantifiers over time and traces and the usual first-order logical connectives such as ¬, ∧, ∨, ⇒, ∀, ∃. Below, the properties and the results of the verification upon the representative traces are shown.

The first property (P1) expresses that a person with bad coping and diatheses values will get depressed after having encountered at least one negative situation. This property is supported by the theory of Lazarus and Folkman [5].

P1: Bad coping person gets depressed after negative situation

```
∀γ:TRACE, t:TIME, R1, R2, R3:REAL
[ [∀t':TIME [state(γ, t') |= has_value(coping_factor, R1) &
   R1 < AVERAGE_COPING &
   state(γ, t') |= has_value(diatheses_factor, R2) &
   R2 > AVERAGE_DIATHESES ] &
   state(γ, t) |= has_value(objective_situation, R3) &
   R3 < AVERAGE_SITUATION ]
 ⇒ ∃t2:TIME > t [depression(γ , t, MIN_DUR, MAX_LEVEL) ] ]
```

A depression is defined as having a mood value below a certain maximum for a certain time period.

```
depression(γ:TRACE, t:TIME, MIN_DUR:INTEGER, MAX_LEVEL:REAL) ≡
∀t2:TIME > t & t2 < t + MIN_DUR
[ ∃R:REAL state(γ, t2) |= has_value(mood, R) & R < MAX_LEVEL ]
```

The property has been verified against the set of traces generated by the model. Hereby MIN_DUR has been set to 336 and MAX_LEVEL is set to 0.5 (see Section 4). Furthermore, the AVERAGE_COPING has been set to 0.4, AVERAGE_DIATHESES to 0.6 and AVERAGE_SITUATION to 0.5. Given these settings, this property indeed holds for the set of traces.

Property P2 expresses a similar property for persons with a healthy coping and diatheses factor, stating that an emotionally stable person will not get depressed from one negative experience. This property is supported by the theories of Zubin and Spring [8] and Lazarus and Folkman [5].

P2: Good coping person does not get depressed after one negative situation

```
∀γ:TRACE, t:TIME, R1, R2, R3, R4:REAL
[[∀t':TIME  [state(γ, t') |= has_value(coping_factor, R1) &
  R1 ≥ AVERAGE_COPING &
  state(γ, t') |= has_value(diatheses_factor, R2) &
  R2 ≥ AVERAGE_DIATHESES ] &
   state(γ, t) |= has_value(objective_situation, R3) &
   R3 < AVERAGE_SITUATION &
   ¬∃t'':TIME > t + MAX_DUR, R4:REAL
    [state(γ, t'') |= has_value(objective_situation, R4) &
     R4 < AVERAGE_SITUATION ] ]
 ⇒ ¬∃t2:TIME > t [depression(γ , t, MIN_DUR, MAX_LEVEL) ] ]
```

Using the same parameters as stated before, this property was shown to be satisfied for all traces. Hereby, the MAX_DUR has been set to 75 hours.

Besides the influence of a situation upon the internal states of the person, the internal levels also influence the choice of situation. In case the thoughts are more negative for the same objective situation, then the judgment of the situation (i.e. the subjective situation) will be lower. This phenomenon is expressed in property P3. The theory of Beck [4] supports this property.

P3: Negative thoughts result in lower subjective situations

```
∀γ:TRACE, t1, t2:TIME, R1, R2, R3:REAL
[ [ state(γ, t1) |= has_value(objective_situation, R1) &
   state(γ, t2) |= has_value(objective_situation, R1) &
   state(γ, t1) |= has_value(thoughts, R2) &
   state(γ, t2) |= has_value(thoughts, R3) & R2 < R3 ]
 ⇒ ∃R4, R5 :REAL, D:integer < MAX_DELAY
  [ state(γ, t1+D) |= has_value(subjective_situation, R4) &
    state(γ, t2+D) |= has_value(subjective_situation, R5) & R4 < R5 ] ]
```

Again, this property is satisfied for the given traces with parameter MAX_DELAY set to 5.

A key element in avoiding a depression is to choose increasingly better situations. Property P4 draws inspiration from this observation. In case a person can constantly choose better situations during a certain period, this avoids a depression. The behavioural theory of Lewinsohn et al [2] supports property P4.

P4: Increasingly more positive situations avoid depression

∀γ:TRACE, t:TIME, R:REAL
[[state(γ, t) |= has_value(objective_situation, R) &
R < AVERAGE_SITUATION &
increasingly_better_situations(γ, t, EPSILON, MIN_DURATION)
⇒ ¬∃t2:TIME > t & t2 < t + MIN_DURATION
[depression(γ , t, MIN_DUR, MAX_LEVEL)

Hereby, the increasingly better situations are defined by means of an epsilon that determines the minimum increase.

increasingly_better_situations(γ:TRACE, t:TIME, EPSILON:REAL, MIN_DUR:INTEGER) ≡
∀t2:TIME > t & t2 < t + MIN_DUR
[∀R1:REAL state(γ, t2) |= has_value(subjective_situation, R1) ⇒
∃R2:REAL state(γ, t2 + 1) |= has_value(subjective_situation, R2) &
R2 > R1 * EPSILON]

This property is satisfied for all traces with an epsilon value of 1.005 and a MIN_DUR of 336. These are chosen such that a person would be above a mood value considered too low from a mood level of 0.1 within 336 hours (i.e. before officially being in state of depression).

Property P5 specifies that in case thoughts are negative for a certain period, the person will become depressed. This property, together with property P3, is supported by the ideas of Beck [4].

P5: Negative thoughts result in depression

∀γ:TRACE, t:TIME, R:REAL
[state(γ, t) |= has_value(thoughts, R) & R ≥ AVG_THOUGHTS &
negative_thoughts(γ, t + 1, MIN_DURATION, AVERAGE_THOUGHTS)
⇒ ∃t2:TIME > t [depression(γ , t2, MIN_DUR, MAX_LEVEL)]]

Hereby, negative thoughts means that the thoughts are below a certain threshold during the specified period.

negative_thoughts(γ:TRACE, t:TIME, MIN_DUR:INTEGER, AVG_THOUGHTS:REAL) ≡
∀t2:TIME > t & t2 < t + MIN_DUR, R :REAL
[state(γ, t2) |= has_value(thoughts, R) ⇒ R < AVG_THOUGHTS]

It was shown that this property is satisfied using the values as identified before, and in addition, a value of 0.5 for AVERAGE_THOUGHTS, and a MIN_DUR of 336, i.e. following the parameters used for the depression.

The final property to be satisfied is the monotonic increase of the mood level of an emotionally stable person in case no negative external situations are encountered. Gross' theory [6, 7] about emotion regulation supports this property.

P6: Monotonic increase of mood level for healthy person

∀γ:TRACE, t:TIME, R1, R2:REAL
[[[∀t':TIME [state(γ, t') |= has_value(coping_factor, R1) &
R1 ≥ AVERAGE_COPING &
state(γ, t') |= has_value(diatheses_factor, R2) &
R2 ≥ AVERAGE_DIATHESES]
state(γ, t-1) |= has_value(mood, R3) & R3 ≥ AVG_MOOD &
state(γ, t) |= has_value(mood, R4) & R4 < AVG_MOOD] &
¬∃t'':TIME > t + MAX_DUR, R5:REAL
[state(γ, t'') |= has_value(objective_situation, R5) &
R5 < AVERAGE_SITUATION]]
⇒ ∃t2:TIME < t + MAX_DIP [monotonic_increase_mood(γ , t2, MIN_DUR)]]

Hereby, the monotonic increase is defined as follows.

monotonic_increase_mood(γ:TRACE, t:TIME, MIN_DUR:INTEGER) ≡
∀t2:TIME > t & t2 < t + MIN_DUR, R1 :REAL
[state(γ, t2) |= has_value(mood, R1) ⇒ ∃R2:REAL state(γ, t2) |= has_value(mood, R2) & R1 ≥ R2]

This final property is also satisfied for the given traces with the parameters used throughout this section, and AVG_MOOD set to 0.5, MAX_DIP to 100 and MAX_DUR to 30.

7 CONCLUSION AND FUTURE WORK

A model of mood dynamics has been shown that incorporates concepts from general theories about depression, stress and coping. This model has been used to simulate different scenarios in which personal characteristics determine the effect of stressful effects on the (long-term) mood of a person. A mathematical analysis illustrated the existence of different equilibriums in the model for persons with different characteristics. By formally checking properties of the simulation traces, the adherence of the model to the most important ideas in the theories was validated. The resulting model can be useful for developing human-like virtual agents. In addition, models like these can be used in systems that use knowledge about the mood of humans, such as systems for internet-based therapy.

Currently, only the major general theories about depression have been involved. For specific applications, it might be required to use more detailed theories that focus on certain aspects. In future work will be investigated whether a model in line with the presented model can help to improve an existing online counselling system.

REFERENCES

[1] Kessler RC, McGonagle KA, Zhao S, et al. Lifetime and 12-month prevalence of DSM-III-R psychiatric disorders in the United States: results from the National Comorbidity Survey. Arch Gen Psychiatry 1994;51:8-19.

[2] Lewinsohn, P.M., Youngren, M.A., & Grosscup, S.J. (1979). Reinforcement and depression. In R. A. Dupue (Ed.), *The psychobiology of depressive disorders: Implications for the effects of stress* (pp. 291-316). New York: Academic Press.

[3] Andersson, G., J. Bergstrom, F. Hollandare, P. Carlbring, V. Kaldo and L. Ekselius. Internet-based self-help for depression: randomised controlled trial. The British J. of Psy. 187, Nov. 2005.

[4] Beck, A.T., Depression: Causes and Treatment. University of Pennsylvania Press, 1972. - ISBN 978-0812276527

[5] Lazarus, R. S., & Folkman, S. (1984). *Stress, Appraisal, and Coping*. New York: Springer.

[6] Gross, J.J. The Emerging Field of Emotion Regulation: An Integrative Review. *Rev. of General Psy.*, 1998, vol. 2,pp. 271-299.

[7] Gross, J.J. "Emotion Regulation in Adulthood: Timing is Everything." *Current directions in psychological science*, 2001, Vol. 10, No. 6, pp. 214-219.

[8] Zubin, J. & Spring, B. (1977). Vulnerability: A new view of schizophrenia. *Journal of Abnormal Psychology, 86*, 103-126.

[9] Ingram, R. E. & Luxton, D. D. (2005). *Vulnerability-Stress Models.* In J. Abela (Ed.). Development of Psychopathology: Stress-Vulnerability Perspectives. New York: Sage.

[10] Ingram, R. E., Miranda, J., & Segal, Z. V. (1998). *Cognitive vulnerability to depression.* New York: Guilford Press.

[11] American Psychiatric Association (1994) *Diagnostic and Statistical Manual of Mental Disorders, 4th ed. (DSM-IV).* Washington (DC), American Psychiatric Association.

[12] Davidson, R.J., D.A. Lewis, L.B. Alloy, D.G. Amaral, G. Bush, J.D. Cohen, W.C. Drevets, M.J. Farah, J. Kagan, J.L. McClelland, S. Nolen-Hoeksema & B.S. Peterson (2002) Neural and behavioral substrates of mood and mood regulation. *Biological Psychiatry, Volume 52, Issue 6*, pp. 478-502.

[13] Bosse, T., C.M. Jonker, L. van der Meij, A. Sharpanskykh and J. Treur, Specification and Verification of Dynamics in Cognitive Agent Models, In: *Proc. of IAT'06*, IEEE C.Soc. Press, pp.247-254, 2006.