An Agent Model Integrating an Adaptive Model for Environmental Dynamics

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Abstract. The environments in which agents are used often may be described by dynamical models, for example in the form of a set of differential equations. In this paper an agent model is proposed that can perform model-based reasoning about the environment, based on a numerical (dynamical system) model of the environment. Moreover, it does so in an adaptive manner by adjusting the parameter values in the environment model that represent believed environmental characteristics, thus adapting these beliefs to the real characteristics of the environment.

Keywords: Agent model, Adaptive Model, Environmental dynamics

1 Introduction

Agents are often used in environments that have a highly dynamic nature. In many applications of agent systems, varying from robot contexts to information brokering and virtual world contexts, some form of world model plays an important role; e.g., [7, 8, 9, 12]. Usually in such applications a world model represents a state of the world that is built up by using different types of inputs and is updated with some frequency. Examples of challenges addressed are, for example, (i) organize data collection and signal processing from different types of sensor, (ii) produce local and global world models using the multi-sensor information about the environment, and (iii) integrate the information from the different sensors into a continuously updated model (cf. [8, 9]).

For dynamic environments another challenge for such an agent, which goes beyond gathering and integrating information from different sources, is to be able to reason about the environmental dynamics in order to predict future states of the environment and to (avoid or) achieve the occurrence of certain (un)desired states in the future. One way to address this challenge is to equip the agent with a model for the environmental dynamics. Examples of environmental dynamics that can be described by such models can be found not only in the natural physical and biological reality surrounding an agent, but also in many relevant (local and global) processes in human-related autonomic environments in the real and/or virtual world such as epidemics, gossiping, crowd movements, social network dynamics, traffic flows.

When the environmental dynamics has a continuous character, such a model often has the form of a numerical dynamical system. A dynamical system model usually involves two different types of concepts:

- *state variables* representing *state aspects* of the environment
- *parameters* representing *characteristics* of the environment

When values for the parameters are given, and initial values for the state variables, the model is used to determine values for the state variables at future points in time. In this way, from beliefs about environmental characteristics represented as parameter values, and beliefs about the initial state of the environment represented as values for the state variables, the agent derives predictions on future states of the environment.

A particular problem here is that values for parameters often are not completely known initially: the beliefs the agent maintains about the environmental characteristics may (initially) not correspond well to the real characteristics. In fact the agent needs to be able to perform parameter estimation or tuning on the fly. The agent model proposed here maintains in an adaptive manner a dynamic model of the environment using results from mathematical analyses from the literature such as [13]. On the one hand, based on this model predictions about future states of the environment can be made, and actions can be generated that fulfill desires of the agent concerning such future states. On the other hand, the agent can adapt its beliefs about the environmental characteristics (represented by the model parameters) to the real characteristics. This may take place whenever observations on environmental state variables can be obtained and compared to predicted values for them. The model is illustrated for an environmental model involving the environment's ground water level and its effect (via the moisture of the soil) on two interacting populations of species both for interactions of type competition and of type parasitism.

The paper is organized as follows. In Section 2 the example model for the environment is briefly introduced. The overall model-based agent model is described in Section 3. Section 4 presents the method by which the agent adapts to the environmental characteristics. In Section 5 some simulation results are discussed. Finally, Section 6 is a discussion.

2 An Example Environmental Model

The example environment for the agent considered involves physical (abiotic) and biological (biotic) elements. Within this environment certain factors can be manipulated, for example, the (ground) water level. As desired states may also involve different species in the environment, the dynamics of interactions between species and abiotic factors and between different species are to be taken into account by the agent. The example model for the environmental dynamics, used for the purpose of illustration deals with two species s_1 and s_2; both depend on the abiotic factor moisture of the soil; see **Fig. 1** for a causal diagram. Quantitative variants of the model for both a competitive and a parasitic environment are given as below.

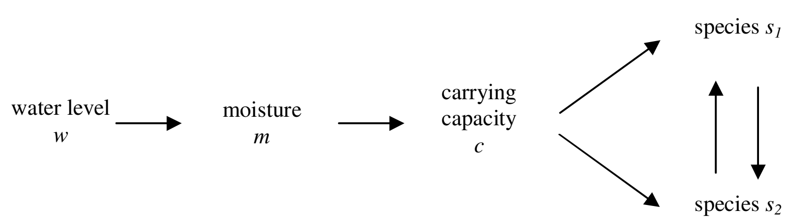

Fig 1. Causal relations for the example environmental model

Competition
A differential equation form consisting of 4 first-order differential equations for the example environment model is as follows:

$$\frac{ds_1(t)}{dt} = \beta\, s_1(t)\big(c(t) - a_1 s_1(t) - a_2 s_2(t)\big)$$

$$\frac{ds_2(t)}{dt} = \gamma\, s_2(t)\big(c(t) - b_1 s_1(t) - b_2 s_2(t)\big)$$

$$\frac{dc(t)}{dt} = \omega\,(\eta\, m(t) - c(t))$$

$$\frac{dm(t)}{dt} = \theta\,(\lambda\, w - m(t))$$

Here $s_1(t)$ and $s_2(t)$ are the densities of species s_1 and s_2 at time point t; moreover, $c(t)$ denotes the carrying capacity for s_1 and s_2 at t, which depends on the moisture $m(t)$. The moisture depends on the water level, indicated by w. This w is considered a parameter that can be controlled by the agent, and is kept constant over longer time periods. Moreover the parameters β, γ are growth rates for species s_1, s_2. For carrying capacity and moisture respectively, η and λ are proportion parameters, and Θ and ω are speed factors. The parameters a_1, a_2 and b_1, b_2 are the proportional contributions in the environment for species s_1 and s_2 respectively.

Parasitism

Similarly as given above, the following set of differential equations represents a model for the case of a parasitic environment.

$$\frac{ds_1(t)}{dt} = \beta\, s_1(t)\big(c(t) - a_1 s_1(t) - b_1 s_2(t)\big)$$

$$\frac{ds_2(t)}{dt} = -\gamma\, s_2(t)\big(c(t) - b_2 s_1(t)\big)$$

$$\frac{dc(t)}{dt} = \omega\,(\eta\, m(t) - c(t))$$

$$\frac{dm(t)}{dt} = \theta\,(\lambda\, w - m(t))$$

3 Using a Model for Environmental Dynamics in an Agent Model

As a point of departure the agent model described in [2] was taken that focuses on the class of Ambient Intelligence applications where the ambient software has context awareness about environmental aspects including human behaviours and states and (re) acts on these accordingly, and is formally specified within the dynamical modelling language LEADSTO; cf. [3]. In this language, direct temporal dependencies between two state properties in successive states are modeled by *executable dynamic properties.* The LEADSTO format is defined as follows. Let α and β be state properties of the form 'conjunction of ground atoms or negations of ground atoms'. In the LEADSTO language the notation $\alpha \twoheadrightarrow_{e, f, g, h} \beta$, means:

> *If state property α holds for a certain time interval with duration g,*
> *then after some delay (between e and f) state property β will hold*
> *for a certain time interval of length h.*

Here, atomic state properties can have a qualitative, logical format, such as an expression desire(d), expressing that desire d occurs, or a quantitative, numerical format such as an expression has_value(x, v) which expresses that variable x has value v. Within this agent model specification a dynamical domain model is represented in the format

belief(leads_to_after(I:INFO_EL, J:INFO_EL, D:REAL))

which represents that the agent believes that state property I leads to state property J with a certain time delay specified by D. Some of the temporal relations for the functionality within the agent model are as follows.

observed_from(I, W) & belief(is_reliable_for(W, I)) $\twoheadrightarrow$ belief(I)
communicated_from_ to(I,Y,X) & belief(is_reliable_for(X, I)) $\twoheadrightarrow$ belief(I)
belief(at(I, T)) & belief(leads_to_after(I, J, D)) $\twoheadrightarrow$ belief(at(J, T+D))
belief(I_1) & … & belief(I_n) $\twoheadrightarrow$ belief(and(I_1, .., I_n))

A differential equation such as

$$\frac{ds_1(t)}{dt} = \beta\, s_1(t)\big(c(t) - a_1 s_1(t) - a_2 s_2(t)\big)$$

can be represented in this leads-to-after format as follows:

```
belief(leads_to_after(
        and(at(has_value(s1, V1), t),   at(has_value(s2, V2), t), at(has_value(c, V3), t)),
        at(has_value(s1, V1+ W*V1*[V3 – W1*V1 – W2*V2]*D), t),
        D))
```

Here W, W1, W2 are the believed parameter values for respectively parameters *β*, a1, a2 representing certain environmental characteristics. As pointed out above, the agent has capabilities for two different aims: (1) to predict future states of the environment and generate actions to achieve desired future states, and (2) to adapt its environment model to the environmental characteristics. The latter will be discussed in more detail in the next section. For the former, a central role is played by the sensitivities of the values of the variables at future time points for certain factors in the environment that can be manipulated. More specifically, for the example, this concerns the sensitivities of s_1 and s_2 for the water level *w,* denoted by $\partial s_1/\partial w$ and $\partial s_2/\partial w$. These sensitivities cannot be determined directly, but differential equations for them can be found by differentiating the original differential equations with respect to *w.* This process is illustrated for both competition and parasitism as follows.

Competition

The following equations describe over time how the values of species s_1, s_2, moisture *m* and carrying capacity *c* at time point t are sensitive to the change in the value of the water level parameter *w.* The idea is that the orginal differential equations are differentiated with respect to the parameter *w*, and using commutation rules such as

$$\frac{\partial \frac{\partial s_1}{\partial t}}{\partial w} = \frac{\partial \frac{\partial s_1}{\partial w}}{\partial t}$$

the following differential equations for the sensitivities $\partial s_1/\partial w$, $\partial s_2/\partial w$, $\partial c/\partial w$ and $\partial m/\partial w$ are obtained:

$$\frac{\partial \frac{\partial s_1(t)}{\partial w}}{\partial t} = \beta\, \frac{\partial s_1(t)}{\partial w}\,(c(t) - a_1 s_1(t) - a_2 s_2(t)) + \beta\, s_1(t)\left(\frac{\partial c(t)}{\partial w} - a_1 \frac{\partial s_1(t)}{\partial w} - a_2 \frac{\partial s_2(t)}{\partial w}\right)$$

$$\frac{\partial \frac{\partial s_2(t)}{\partial w}}{\partial t} = \gamma\, \frac{\partial s_2(t)}{\partial w}\,(c(t) - b_1 s_1(t) - b_2 s_2(t)) + \gamma\, s_2(t)\left(\frac{\partial c(t)}{\partial w} - b_1 \frac{\partial s_1(t)}{\partial w} - b_2 \frac{\partial s_2(t)}{\partial w}\right)$$

$$\frac{\partial \frac{\partial c(t)}{\partial w}}{\partial t} = \left(\eta\, \frac{\partial m(t)}{\partial w} - \frac{\partial c(t)}{\partial w}\right)\omega$$

$$\frac{\partial \frac{\partial m(t)}{\partial w}}{\partial t} = \left(\lambda - \frac{\partial m(t)}{\partial w}\right)\Theta$$

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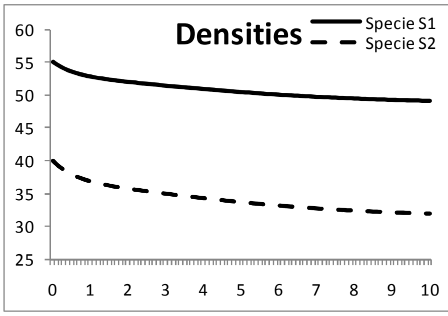

Fig. 2: Predicted densities after 10 years (w=200, m(0)=110, c(0)=88, β=0.01, ω=0.4)

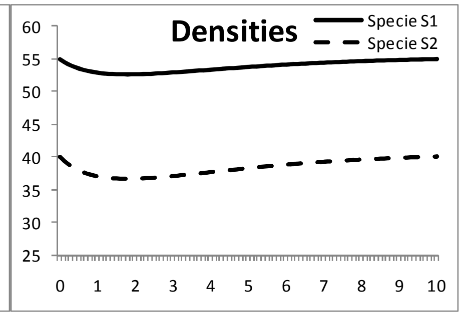

Fig. 3: Densities, over 10 years, after incorporating (w=240, m(0)=110, c(0)=88, ω=0.4), Δw = 40

Fig 2, where the vertical axis represents the densities of species s_1 and s_2 and the horizontal axis represents number of years, shows the trend in change of densities of species over 10 years given the initial values of the water level w. Using the following formula, the agent can determine the change (Δw) in circumstance w to achieve the goal at some specific time point t in future:

$$\Delta w = \left(s_1(w + \Delta w) - s_1(w)\right) / \left(\frac{\partial s_1}{\partial w}\right)$$

where $s_1(w+\Delta w)$ is the desired density at time t, $s_1(w)$ the predicted density s_1 at time t for water level w, and $(\partial s_1/\partial w)$ the change in density of s_1 at time t against the change in w. Within the agent model this is specified in a generic manner as (where X can be taken s1 and P can be taken w):

```
desire(at(has_value(X, VD), T)) &  belief(at(has_value(X, V), T))  &
belief(has_sensitivity_for_at(X, P, S, T)) &  belief(has_value(P, W))
→› intention(has_value(P, W+(VD-V)/S))
```

Fig. 2 depicts a situation where the densities of species s_1 and s_2 are predicted to decrease, given $w = 200$. The initial values for species s_1 and s_2 could be taken random but for this example scenario species s_1 has density 55 and s_2 has density 40. Under these settings the density of species s_1 will be *49*. If the agent wants to aim it to become *55* after 10 years, then according to the model described above it has to change w to *240* (see **Fig. 3**).

Parasitism

Similarly for parasitism the following set of equations have been obtained to describe how the values of species s_1, s_2, moisture m and carrying capacity c at time point t are sensitive to the change in the value of the water level parameter w.

$$\frac{\partial \frac{\partial s_1(t)}{\partial w}}{\partial t} = \beta \frac{\partial s_1(t)}{\partial w} \left(c(t) - a_1 s_1(t) - b_1 s_2(t)\right) + \beta\, s_1(t) \left(\frac{\partial c(t)}{\partial w} - a_1 \frac{\partial s_1(t)}{\partial w} - b_1 \frac{\partial s_2(t)}{\partial w}\right)$$

$$\frac{\partial \frac{\partial s_2(t)}{\partial w}}{\partial t} = -\gamma \frac{\partial s_2(t)}{\partial w} \left(c(t) - b_2 s_1(t)\right) - \gamma\, s_2(t) \left(\frac{\partial c(t)}{\partial w} - b_2 \frac{\partial s_1(t)}{\partial w}\right)$$

$$\frac{\partial \frac{\partial c(t)}{\partial w}}{\partial t} = \left(\eta \frac{\partial m(t)}{\partial w} - \frac{\partial c(t)}{\partial w}\right) \omega$$

$$\frac{\partial \frac{\partial m(t)}{\partial w}}{\partial t} = \left(\lambda - \frac{\partial m(t)}{\partial w}\right) \Theta$$

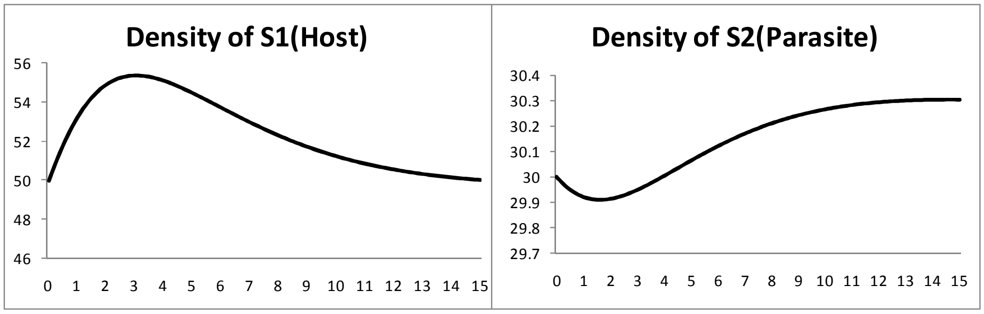

Fig. 4: Predicted densities after 15 years

(w=200, m(0)=110, c(0)=88, β=0.01, γ= 0.0005, θ = 0.4, Δt = 0.1, ω=0.4)

Fig 4, where the vertical axis represents the densities of species s_1 and s_2 and the horizontal axis represents number of years, shows the trend in change of densities of species over 15 years given the initial values of the water level w = 200 Using the same formula for "Δw" given above, the agent can determine the change (Δw) in circumstance w to achieve the goal at some specific time point t in future:

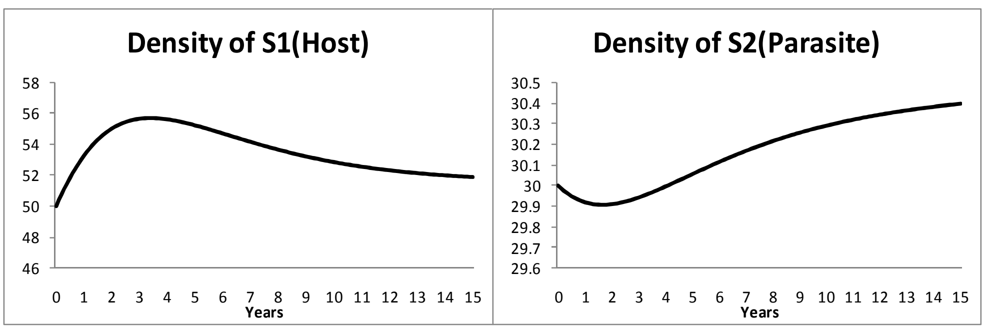

Fig. 5: Densities, over 15 years, after incorporating

(w=205, m(0)=110, c(0)=88, β=0.01, γ= 0.0005, θ = 0.4, Δt = 0.1, ω=0.4), Δw = 5

Fig. 4 depicts a situation where the densities of species s_1 and s_2 are predicted, given $w = 200$. The initial values for species s_1 and s_2 could be taken random but for this example scenario species s_1 has density 50 and s_2 has density 30. Under these settings the density of species s_1 will be *50.* If the agent wants to aim it to become *52* after 15 years, then according to the model described above it has to change w to *205* (see **Fig. 5**).

4 Adaptation within the Agent Model

This section describes the method by which the agent adapts its beliefs concerning parameters representing environmental characteristics to the real characteristics. The agent initially receives rough estimations of the values for these parameters, and maintains them as initial beliefs. With these initial beliefs the agent predicts the environmental state for after say X years. When at a certain time point the real value of some state variable is observed, as a next step the predicted value and observed value of that state variable at time X are passed to the adaptation process of the agent. The agent then tries to minimize the difference between predicted and desired value and adjust the beliefs on the environmental characteristics (i.e., the parameter values which were initially assumed). This process of adaptation is kept on going until the difference is negligible, i.e., until the agent has an accurate set of beliefs about the environmental characteristics (see **Fig 6**).

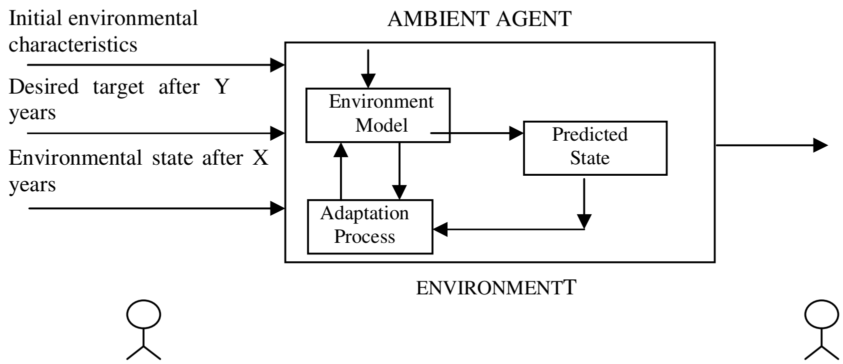

Fig. 6: The Ambient Agent's Adaptation Process of the Environment Model

Within this adaptation process sensitivities of state variables for changes in parameter values for environmental characteristics play an important role. For example, differential equations for the sensitivities of values of the variables w.r.t. the parameter a_1 are obtained by differentiating the original differential equations for a_1:

$$\left(\frac{\partial s_1}{\partial a_1}\right)(t+\Delta t) = \left(\frac{\partial s_1}{\partial a_1}\right)(t) + \beta\left(\left(\frac{\partial s_1}{\partial a_1}\right)(t)\left(c(t) - a_1 s_1(t) - a_2 s_2(t)\right) + s_1(t)\left(\left(\frac{\partial c}{\partial a_1}\right)(t) - s_1(t) - a_1\left(\frac{\partial s_1}{\partial a_1}\right)(t) - a_2\left(\frac{\partial s_2}{\partial a_1}\right)(t)\right)\right)\Delta t$$

$$\left(\frac{\partial s_2}{\partial a_1}\right)(t+\Delta t) = \left(\frac{\partial s_2}{\partial a_1}\right)(t) + \gamma\left(\left(\frac{\partial s_2}{\partial a_1}\right)(t)\left(c(t) - b_1 s_1(t) - b_2 s_2(t)\right) + s_2(t)\left(\left(\frac{\partial c}{\partial a_1}\right)(t) - b_1\left(\frac{\partial s_1}{\partial a_1}\right)(t) - b_2\left(\frac{\partial s_2}{\partial a_1}\right)(t)\right)\right)\Delta t$$

$$\left(\frac{\partial c}{\partial a_1}\right)(t+\Delta t) = \left(\frac{\partial c}{\partial a_1}\right)(t) + \left(\eta\left(\frac{\partial m}{\partial a_1}\right)(t) - \left(\frac{\partial c}{\partial a_1}\right)(t)\right)\omega\,\Delta t$$

$$\left(\frac{\partial m}{\partial a_1}\right)(t+\Delta t) = \left(\frac{\partial m}{\partial a_1}\right)(t)(1 - \theta\ \Delta t)$$

These equations describe how the values of species s_1, s_2, moisture m and carrying capacity c at time point t are sensitive to the change in the value of the proportional contributing parameter a_1 in a competitiveenvironment. The adaptation formulas for other parameters of the focus set are shown in Appendix A (competition) and Appendix B (parasitism).

In a manner as described in Section 3, by the differential equations given above, the agent can derive its beliefs on sensitivities S represented as

belief(has_sensitivity_for_at(X, P, S, T))

for each variable X with respect to each parameter P.

Once these beliefs on sensitivities are available, the agent can use them to adapt certain parameter values. Here a control choice can be made. A first element of this choice is that a *focus set* of parameters can be chosen which are considered for adaptation (the values for parameters not in this focus set are considered fixed). A second control element is whether the value of only *one parameter* at a time is changed, or the values of *all parameters* in the focus set. In Section 5 example simulations are discussed in which only one of the parameters are adapted at a time, and also when more than one are adapted. A third choice element is on the *adaptation speed*: for example, will the agent attempt to get the right value in one step, or will it adjust only halfway; the latter choice may obtain more stable results. In Section 5 example simulations are discussed in which the agent attempt to get the right value in four steps. Specification of the adaptation model is based on relationships such as:

predicted_for(at(has_value(X, V1), P, W, T)) & observed(at(has_value(X, V2), T) &
belief(has_sensitivity_for_at(X, P, S, T)) & belief(has_value(P, W)) &
belief(adaptation_speed(AS))
→→ belief(adjustment_option(has_value(P, W+AS*(V2-V1)/S))

5 Simulation Results

In this section the approach will be illustrated by some example simulations for the two types of environments used as a case study.

Competition

To test the behaviour of the model to adapt the agent's beliefs on the environmental characteristics (represented by the parameters) to the real characteristics, it has been used to perform a number of simulation runs, using standard numerical simulation software, resulting in a variety of interesting patterns. The *focus set* of parameters for adaptation includes a_1, a_2, b_1, b_2, β, and γ. For the real environment characteristics these parameters were given the values $a_1=1$, $a_2=1$, $b_1=1$, $b_2=1$, $\beta=0.01$, $\gamma=0.02$.

Fig 7(a, b) shows the patterns in change of densities of species during the adaptation process of *a1* given the initial values of other parameters. Using the following formula, the agent can determine the change (Δa_1) in *a1* to minimize the difference both in predicted and observed densities:

$$\Delta a_1 = \left(s_1(a_1 + \Delta a_1) - s_1(a_1)\right) / \left(\frac{\partial s_1}{\partial a_1}\right)$$

where, $s_1(a_1+\Delta a_1)$ is the observed density after X years, $s_1(a_1)$ is the predicted density of s_1 after X years for a_1, and $(\partial s_1/\partial a_1)$ is the change in density of s_1 after X years against the change in a_1.

In **Fig 7(a)**, vertical axis represents the predicted density of s_1 and in **Fig 7 (b)** vertical axis represents adapted value of *a1* by the agent. Where as in both **Fig 7(a)** and **(b)** horizontal axis represents number of iterations performed. For this simulation the initial value assumed by agent for a_1 is 0.5 and the given observed density of s_1 is 52.78. First iteration in the adaptation process reflects that the predicted density of $s_1(a_1)$ deviates a lot from its observed density $s_1(a_1+\Delta a_1)$ (see **Fig 7(a)**), which means that the adapted value of a_1 by agent is not correct (see **Fig 5 (b)**) and it may required to modify a_1 (Δa_1) to a certain degree to attain the observed density. In next iterations this deviation becomes smaller and it needs small modification in the value of a_1. Finally the agent fully adapts the value of *a1* in fourth iteration to attain the observed density of s_1. The agent can utilize this value to achieve the target, as depicted in Section 3. The details are described in [6].

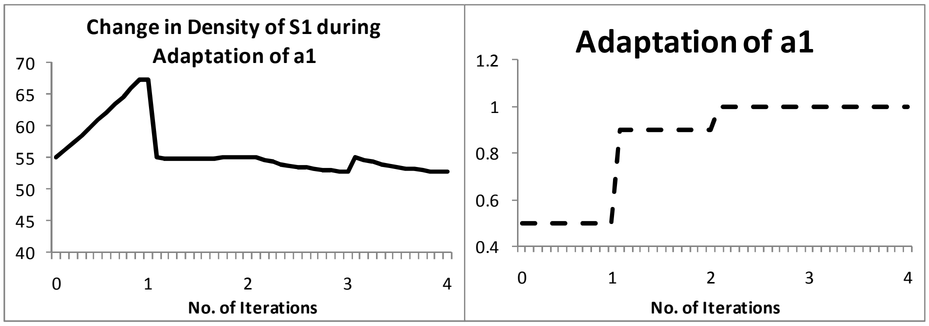

Fig 7 (a): Change of densities of species s_1 during the adaptation process of a_1, **(b):** Adaptation process of a_1

As shown in **Fig 7** the agent attains the observed density of s_1 for the initially assumed value 0.5 for a_1, but the proposed model in this paper is so generic that it can achieve the observed density for any initially assumed values for a_1. For this purpose **Fig 8 (a)**, and **(b)** show the adaptation process of a_1 with different initial values assumed by the agent.

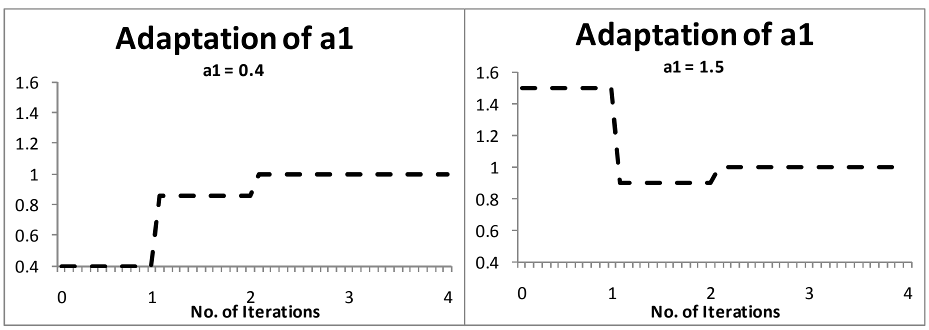

Fig 8. (a): Adaption of a_1 with initial value 0.4, **(b):** Adaption of a_1 with initial value 1.5

Similarly the simulations for the adaptation process of other parameters of *focus set* i.e. a_2, b_1, b_2, β, and γ are shown in **Fig 9, 10, 11, 12**, and **13** respectively. The agent determines the change (Δa_2) in a_2 and Δb_1 in b_1 as follows (see Figures**9**,**10**).

$$\Delta a_2 = \left(s_1(a_2 + \Delta a_2) - s_1(a_2)\right) / \left(\frac{\partial s_1}{\partial a_2}\right) \qquad \Delta b_1 = \left(s_1(b_1 + \Delta \mathrm{b}_1) - s_1(b_1)\right) / \left(\frac{\partial s_1}{\partial b_1}\right)$$

The agent determines the change (Δb_2) in b_2 as follows (see **Fig11**).

$$\Delta b_2 = \left(s_1(b_2 + \Delta b_2) - s_1(b_2)\right) / \left(\frac{\partial s_1}{\partial b_2}\right)$$

The agent determines the change ($\Delta\beta$, $\Delta\gamma$) in β resp. γ as follows (see **Fig 12** resp. **13**).

$$\Delta\beta = \left(s_1(\beta + \Delta\mathrm{\beta}) - s_1(\beta)\right) / \left(\frac{\partial s_1}{\partial \beta}\right) \qquad \Delta\gamma = \left(s_1(\gamma + \Delta\gamma) - s_1(\gamma)\right) / \left(\frac{\partial s_1}{\partial \gamma}\right)$$

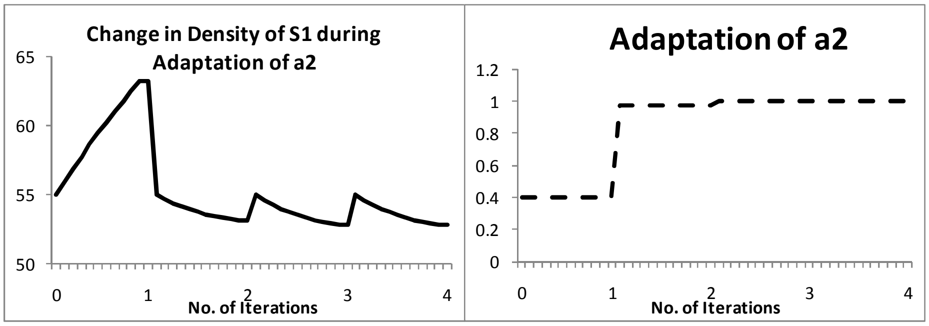

Fig. 9: Adaptation of a_2

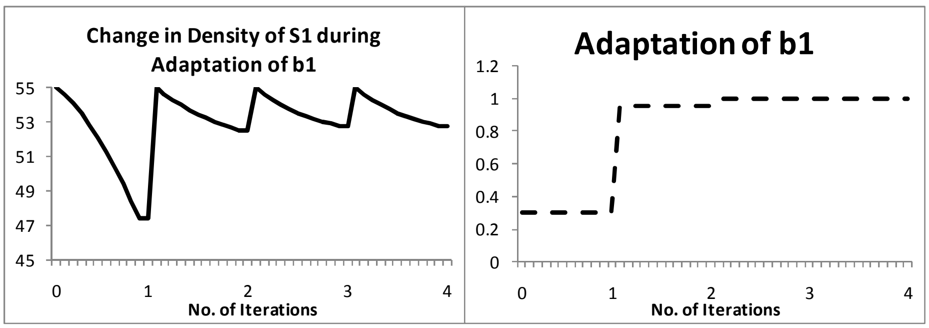

Fig. 10: Adaptation of b_1

Fig 14 shows the adaptation process of *all parameters* in the *focus set.* For this simulation the initial values assumed by agent for the parameters a_1, a_2, b_1, b_2, β,and γ are 0.5, 0.4, 0.3, 0.2, 0.0005, and 0.07 respectively. The given observed density of s_1 is 52.78 .

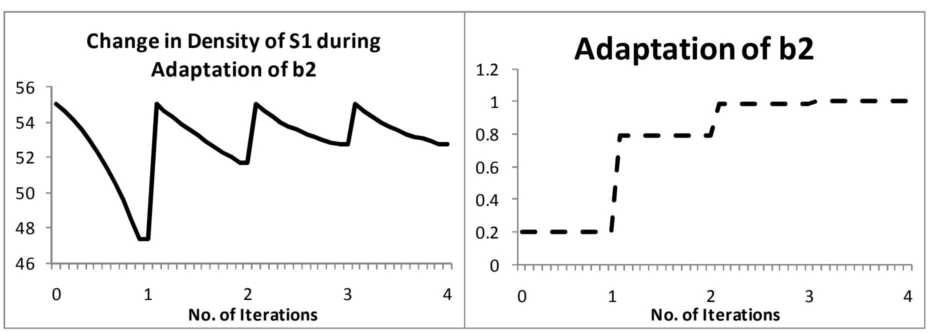

Fig. 11: Adaptation of b_2

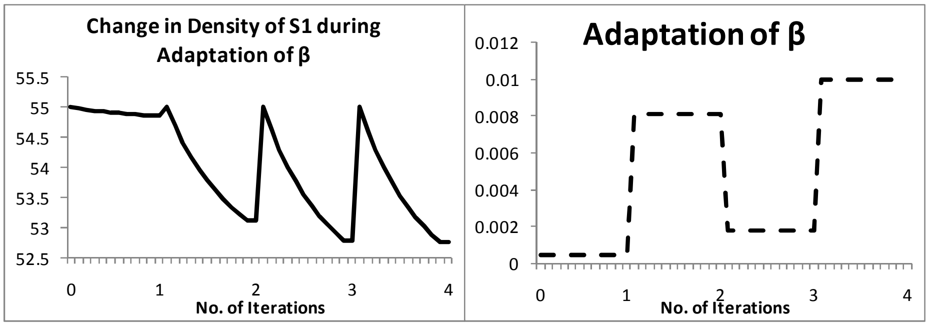

Fig.12: Adaptation of β

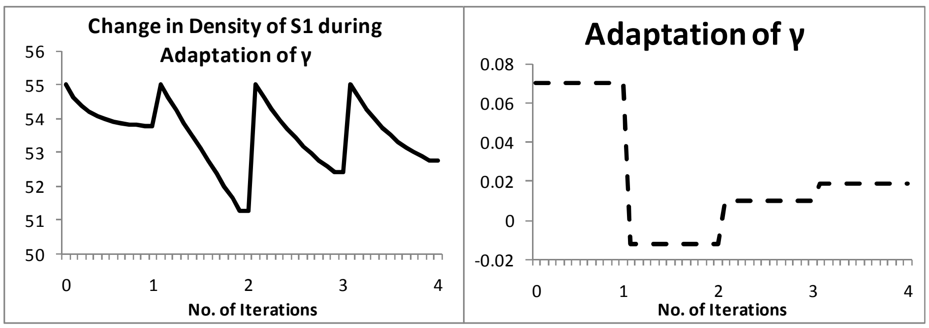

Fig. 13: Adaptation of γ

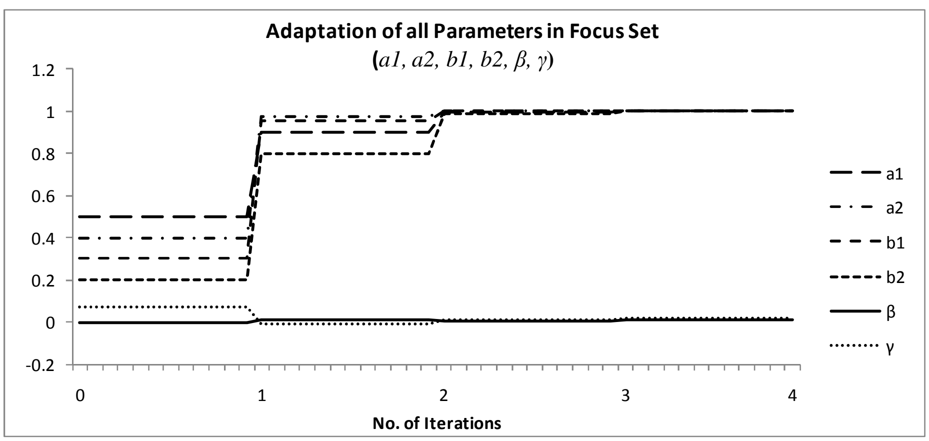

Fig. 14: Adaptation of all parameters in the focus set

In a nutshell the change in values of the parameters during the adaptation process is given in **Table 1**.

Table 1. Values during the adaptation process

		After Iteration(s)				
	Initial	1st	2nd	3rd	4th	Target
a_1	0.5	0.90163	0.996124	0.999994	1	1
b_1	0.3	0.956724	0.999756	1	1	1
a_2	0.4	0.9774253	0.9999606	1	1	1
b_2	0.2	0.79576312	0.98698014	0.99994745	1	1
β	0.0005	0.00810025	0.00991723	0.00999984	0.01	0.01
γ	0.07	-0.0120269	0.01025877	0.01902967	0.01999006	0.02

Parasitism

In the same manner as above, to test the behaviour of the model to adapt the agent's beliefs on the environmental characteristics (represented by the parameters) to the real characteristics for a parasitism case, it has been used to perform a number of simulation runs, using standard numerical simulation software, resulting in a variety of interesting patterns. The *focus set* of parameters for adaptation includes a_1, b_1, b_2, β,and γ. For the real environment characteristics these parameters were given the values $a_1=1$, $b_1=1$, $b_2=1.6$, $\beta=0.01$, $\gamma=0.0005$.

Fig 15(a, b) shows the patterns in change of densities of species during the adaptation process of a_1 given the initial values of other parameters. Using the same formula used in previous section on competition, the agent can determine the change (Δa_1) in a_1 to minimize the difference both in predicted and observed densities:

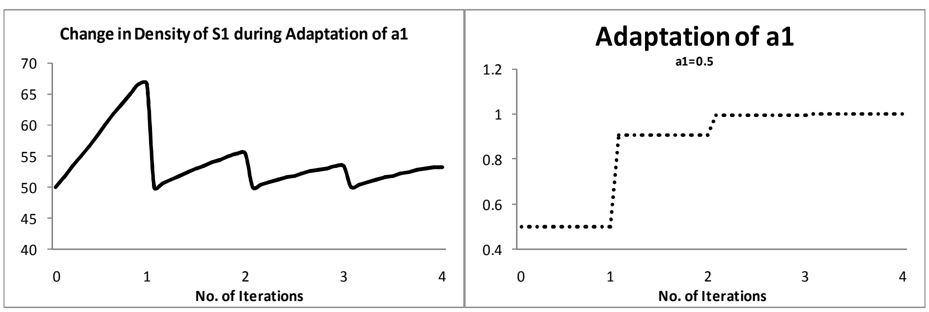

Fig 15 (a): Change of densities of species s_1 during the adaptation process of a_1, **(b):** Adaptation process of a_1

In **Fig 15(a)**, the vertical axis represents the predicted density of s_1 and in **Fig 15 (b)** the vertical axis represents the adapted value of a_1 by the agent. In both **Fig 15(a)** and **(b)** the horizontal axis represents the number of iterations performed. For this simulation the initial value assumed by agent for a_1 is 0.5 and the given observed density of s_1 is 53.25. The first iteration in the adaptation process reflects that the predicted density of $s_1(a_1)$ deviates a lot from its observed density $s_1(a_1+\Delta a_1)$ (see **Fig 15(a)**), which means that the adapted value of a_1 by agent is far from correct (see **Fig 15 (b)**) and it will be required to modify a_1 (Δa_1) to a certain degree to attain the observed density. In next iterations this deviation becomes smaller and it needs smaller modifications in the value of a_1. Finally the agent fully adapts the value of a_1 in the fourth iteration to attain the observed density of s_1. The agent can utilize this value to achieve the target, as depicted in Section 3. The details are described in [6].

As shown in **Fig 15** the agent attains the observed density of s_1 for the initially assumed value 0.5 for a_1, but the proposed model can achieve the observed density for any initially assumed values for a_1. For this purpose **Fig 16 (a)**, and **(b)** show the adaptation process of a_1 with different initial values assumed by the agent.

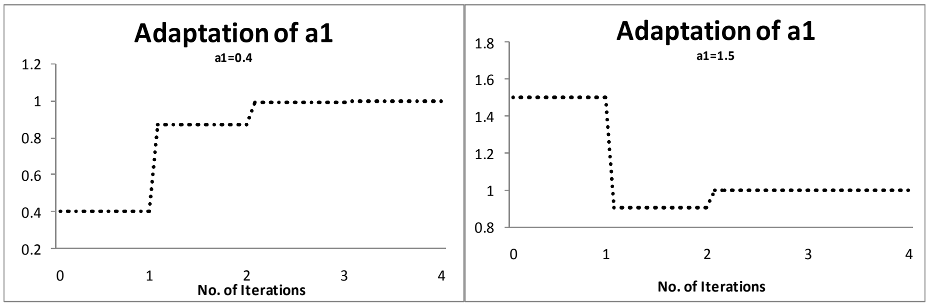

Fig 16. (a): Adaption of a_1 with initial value 0.4, **(b):** Adaption of a_1 with initial value 1.5

One more interesting scenario was the situation in which the agent assumed the right value in the beginning: it is observed that if the agent assumed the value correctly then during adaptation process there will be no change in this assumed value, as $\Delta a_1 = 0$. The results for such a situation are shown in **Fig 17 (a)** and **(b).**

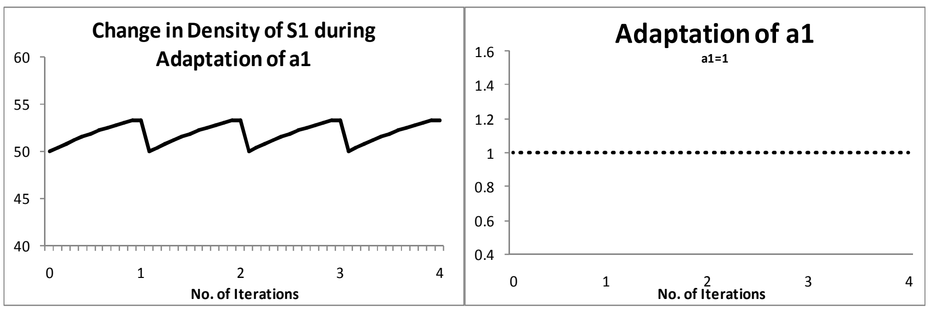

Fig 17 (a): Change of densities of species s_1 during the adaptation process of a_1, **(b):** Adaptation process of a_1

Similarly the simulations for the adaptation process of the other parameters of the focus set i.e. b_1, b_2, β, and γ are shown in **Fig 18, 19, 20, 21**, and **22** respectively. The agent determines the change (Δb_1) in b_1 and (Δb_2) in b_2 in the same fashion as described earlier for the competition case (see **Fig 18** and **Fig 19**).

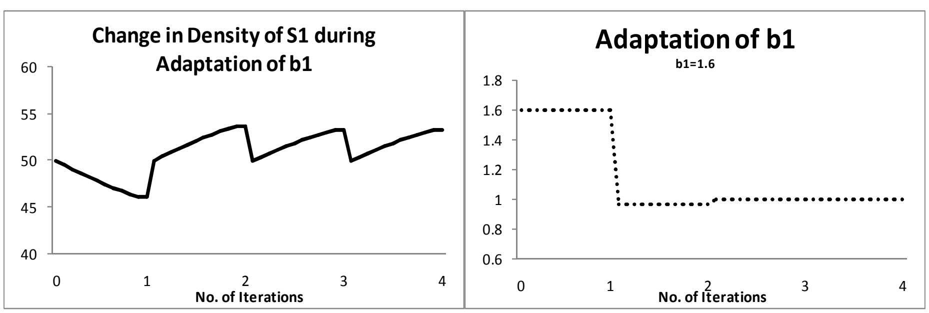

Fig. 18: Adaptation of b_1

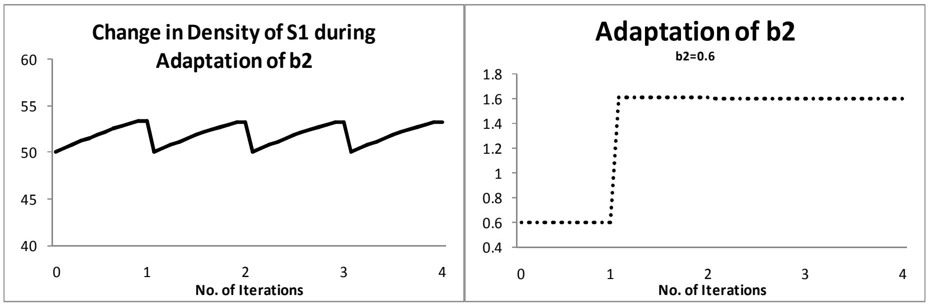

Fig. 19: Adaptation of b_2

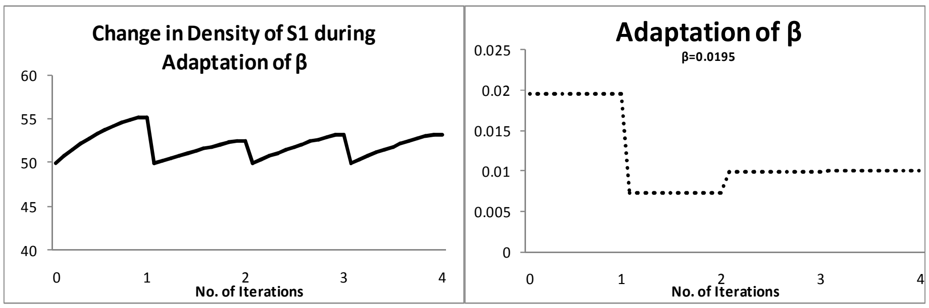

Fig. 20: Adaptation of β

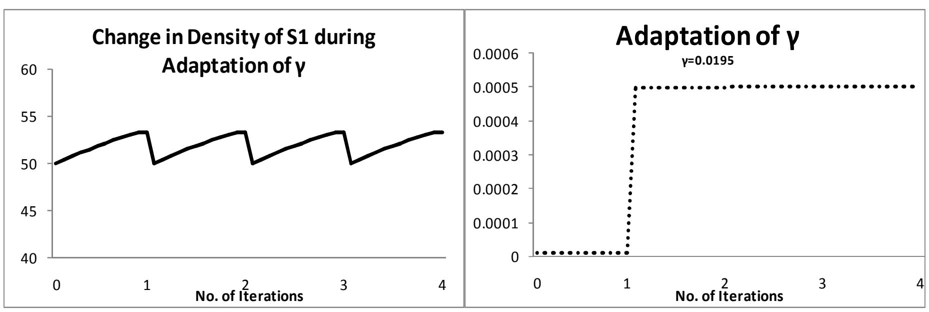

Fig. 21: Adaptation of γ

Fig 22 shows the adaptation process of *all parameters* in the *focus set*. For this simulation the initial values assumed by agent for the parameters a_1, b_1, b_2, β, and γ are 0.5, 1.6, 0.8, 0.0195, and 0.00001 respectively. The given observed density of s_1 is 53.25.

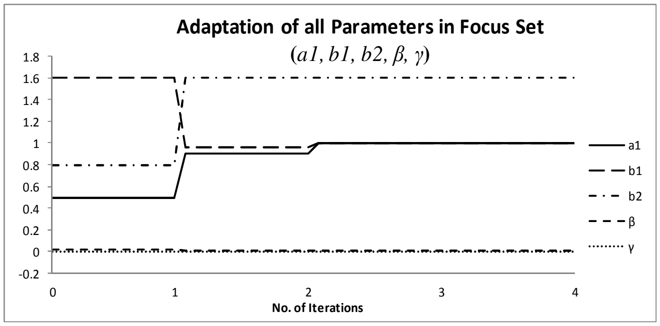

Fig. 22: Adaptation of all parameters in the focus set

In a nutshell the change in values of the parameters during the adaptation process is given in **Table 2**.

Table 2. Values during the adaptation process

		After Iteration(s)				
	Initial	1st	2nd	3rd	4th	Target
a_1	0.5	0.907427	0.996778	0.999996	1	1
b_1	1.6	0.964458	0.999884	1	1	1
b_2	0.8	1.603607	1.6	1.6	1.6	1.6
β	0.0195	0.007257	0.009837	0.009999	0.01	0.01
γ	0.00001	0.000499	0.0005	0.0005	0.0005	0.0005

6 Discussion

In this paper an agent model is proposed that maintains a model of the environmental dynamics, based on a numerical (dynamical system). Moreover, it does so in an adaptive manner by adjusting the parameter values in the environment model that represent believed environmental characteristics, thus adapting these beliefs to the real characteristics of the environment. The described agent model can be used for any agent with an environment (local and/or global) that can be described in a continuous manner by a dynamical system (based on a set of first-order differential equations).

The results shown in the previous section prove that the target which was set initially for the adaptation of the parameters of the focus set is achieved both in cases of competition and of parasitism. The graphs presented in the section completely show the adaptation process of different parameters. Moreover the results show that the method used is significantly precise and accurate.

In [6] as a special case a decision support system model was presented that takes into account an ecological model of the temporal dynamics of environmental species and inter species interaction; see also [4, 5, 10, 11]. Approaches to environmental dynamics such as the one described in [6] are rigid in the sense that the parameters of environmental characteristics were assumed known initially and fixed which in reality is not possible because the internal dynamics of the species is a characteristic which would not be known by everyone. The current paper extends and generalizes this idea by making the parameters representing environmental characteristics adaptive (see **Fig 7** to **Fig 22**). For future research, one of the plans is to validate the model using empirical data within an example domain. Moreover, other approaches for sensitivity analysis will be used to compare the convergence and speed of the adaptation process.

Domains for which the presented agent model may be relevant do not only concern natural physical and biological domains but also to human-related autonomic environments for example in logistic, economic, social and medical domains. Such applications of the approach may involve both local information and global information of the environment. An example of the former is monitoring a human's gaze over time and using a dynamical model to estimate the person's attention distribution over time, as described in [1]. Examples of the latter may concern monitoring and analysis of (statistical) population information about (real or virtual) epidemics, gossiping, or traffic flows. This allows to combine in an agent both local (e.g., using information on individual agents) and global (e.g., using information on groups of agents) perspectives on the environment.

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APPENDIX – A: Adaptation Formulas for parameters in focus set (competition)

Differential equations for the sensitivities of values of the variables w.r.t. the parameter a_2 are obtained by differentiating the original differential equations for a_2:

$$\left(\frac{\partial s1}{\partial a_2}\right)(t+\Delta t) = \left(\frac{\partial s1}{\partial a_2}\right)(t) + \beta\left(\left(\frac{\partial s_1}{\partial a_2}\right)(t)\left(c(t) - a_1 s_1(t) - a_2 s_2(t)\right) + s_1(t)\left(\left(\frac{\partial c}{\partial a_2}\right)(t) - a_1\left(\frac{\partial s_1}{\partial a_2}\right)(t) - a_2\left(\frac{\partial s_2}{\partial a_2}\right)(t) - s_2(t)\right)\right)\Delta t$$

$$\left(\frac{\partial s2}{\partial a_2}\right)(t+\Delta t) = \left(\frac{\partial s2}{\partial a_2}\right)(t) + \gamma\left(\left(\frac{\partial s_2}{\partial a_2}\right)(t)\left(c(t) - b_1 s_1(t) - b_2 s_2(t)\right) + s_2(t)\left(\left(\frac{\partial c}{\partial a_2}\right)(t) - b_1\left(\frac{\partial s_1}{\partial a_2}\right)(t) - b_2\left(\frac{\partial s_2}{\partial a_2}\right)(t)\right)\right)\Delta t$$

$$\left(\frac{\partial c}{\partial a_2}\right)(t+\Delta t) = \left(\frac{\partial c}{\partial a_2}\right)(t) + \left(\eta\left(\frac{\partial m}{\partial a_2}\right)(t) - \left(\frac{\partial c}{\partial a_2}\right)(t)\right)\omega\,\Delta t$$

$$\left(\frac{\partial m}{\partial a_2}\right)(t+\Delta t) = \left(\frac{\partial m}{\partial a_2}\right)(t)(1-\Theta\,\Delta t)$$

Differential equations for the sensitivities of values of the variables w.r.t. the parameter b_1 are obtained by differentiating the original differential equations for b_1:

$$\left(\frac{\partial s1}{\partial b_1}\right)(t+\Delta t) = \left(\frac{\partial s1}{\partial b_1}\right)(t) + \beta\left(\left(\frac{\partial s_1}{\partial b_1}\right)(t)\left(c(t) - a_1 s_1(t) - a_2 s_2(t)\right) + s_1(t)\left(\left(\frac{\partial c}{\partial b_1}\right)(t) - a_1\left(\frac{\partial s_1}{\partial b_1}\right)(t) - a_2\left(\frac{\partial s_2}{\partial b_1}\right)(t)\right)\right)\Delta t$$

$$\left(\frac{\partial s2}{\partial b_1}\right)(t+\Delta t) = \left(\frac{\partial s2}{\partial b_1}\right)(t) + \gamma\left(\left(\frac{\partial s_2}{\partial b_1}\right)(t)\left(c(t) - b_1 s_1(t) - b_2 s_2(t)\right) + s_2(t)\left(\left(\frac{\partial c}{\partial b_1}\right)(t) - b_1\left(\frac{\partial s_1}{\partial b_1}\right)(t) - s_1(t) - b_2\left(\frac{\partial s_2}{\partial b_1}\right)(t)\right)\right)\Delta t$$

$$\left(\frac{\partial c}{\partial b_1}\right)(t+\Delta t) = \left(\frac{\partial c}{\partial b_1}\right)(t) + \left(\eta\left(\frac{\partial m}{\partial b_1}\right)(t) - \left(\frac{\partial c}{\partial b_1}\right)(t)\right)\omega\,\Delta t$$

$$\left(\frac{\partial m}{\partial b_1}\right)(t+\Delta t) = \left(\frac{\partial m}{\partial b_1}\right)(t)(1-\Theta\,\Delta t)$$

Differential equations for the sensitivities of values of the variables w.r.t. the parameter b_2 are obtained by differentiating the original differential equations for b_2:

$$\left(\frac{\partial s1}{\partial b_2}\right)(t+\Delta t) = \left(\frac{\partial s1}{\partial b_2}\right)(t) + \beta\left(\left(\frac{\partial s_1}{\partial b_2}\right)(t)\left(c(t) - a_1 s_1(t) - a_2 s_2(t)\right) + s_1(t)\left(\left(\frac{\partial c}{\partial b_2}\right)(t) - a_1\left(\frac{\partial s_1}{\partial b_2}\right)(t) - a2\left(\frac{\partial s_2}{\partial b_2}\right)(t)\right)\right)\Delta t$$

$$\left(\frac{\partial s_2}{\partial b_2}\right)(t+\Delta t) = \left(\frac{\partial s_2}{\partial b_2}\right)(t) + \gamma\left(\left(\frac{\partial s_2}{\partial b_2}\right)(t)\left(c(t) - b_1\, s_1(t) - b_2 s_2(t)\right) + s_2(t)\left(\left(\frac{\partial c}{\partial b_2}\right)(t) - b_1\left(\frac{\partial s_1}{\partial b_2}\right)(t) - b_2\left(\frac{\partial s_2}{\partial b_2}\right)(t) - s_2(t)\right)\right)\ \Delta t$$

$$\left(\frac{\partial c}{\partial b_2}\right)(t+\Delta t) = \left(\frac{\partial c}{\partial b_2}\right)(t) + \left(\eta\left(\frac{\partial m}{\partial b_2}\right)(t) - \left(\frac{\partial c}{\partial b_2}\right)(t)\right)\omega\ \Delta t$$

$$\left(\frac{\partial m}{\partial b_2}\right)(t+\Delta t) = \left(\frac{\partial m}{\partial b_2}\right)(t)(1-\Theta\ \Delta t)$$

Differential equations for the sensitivities of values of the variables w.r.t. the parameter β are obtained by differentiating the original differential equations for β:

$$\left(\frac{\partial s_1}{\partial \beta}\right)(t+\Delta t) = \left(\frac{\partial s_1}{\partial \beta}\right)(t) + \left(\left(s_1(t) + (\partial s_1/\partial \beta)(t)\ \beta\right)\left(c(t) - a_1 s_1(t) - a_2 s_2(t)\right) + s_1(t)\,\beta\left(\left(\frac{\partial c}{\partial \beta}\right)(t) - a_1\left(\frac{\partial s_1}{\partial \beta}\right)(t) - a_2\left(\frac{\partial s_2}{\partial \beta}\right)(t)\right)\right)\ \Delta t$$

$$\left(\frac{\partial s_2}{\partial \beta}\right)(t+\Delta t) = \left(\frac{\partial s_2}{\partial \beta}\right)(t) + \gamma\left((\partial s_2/\partial \beta)(t)\left(c(t) - b_1 s_1(t) - b_2 s_2(t)\right) + s_2(t)\left(\left(\frac{\partial c}{\partial \beta}\right)(t) - b_1\left(\frac{\partial s_1}{\partial \beta}\right)(t) - b_2\left(\frac{\partial s_2}{\partial \beta}\right)(t)\right)\right)\ \Delta t$$

$$\left(\frac{\partial c}{\partial \beta}\right)(t+\Delta t) = \left(\frac{\partial c}{\partial \beta}\right)(t) + \left(\eta\left(\frac{\partial m}{\partial \beta}\right)(t) - \left(\frac{\partial c}{\partial \beta}\right)(t)\right)\omega\ \Delta t$$

$$\left(\frac{\partial m}{\partial \beta}\right)(t+\Delta t) = \left(\frac{\partial m}{\partial \beta}\right)(t)(1-\Theta\ \Delta t)$$

Differential equations for the sensitivities of values of the variables w.r.t. the parameter γ are obtained by differentiating the original differential equations for γ:

$$\left(\frac{\partial s_1}{\partial \gamma}\right)(t+\Delta t) = \left(\frac{\partial s_1}{\partial \gamma}\right)(t)\left(1 + \left(c(t) - a_1 s_1(t) - a_2 s_2(t)\right)\beta\,\Delta t\right) + s_1(t)\,\beta\left(\left(\frac{\partial c}{\partial \gamma}\right)(t) - a_1\left(\frac{\partial s_1}{\partial \gamma}\right)(t) - a2\left(\frac{\partial s_2}{\partial \gamma}\right)(t)\right)\ \Delta t$$

$$\left(\frac{\partial s_2}{\partial \gamma}\right)(t+\Delta t) = \left(\frac{\partial s_2}{\partial \gamma}\right)(t)\left(1 + \left(c(t) - b_1 s_1(t) - b_2 s_2(t)\right)\gamma\,\Delta t\right) + s_2(t)\left(c(t) - b_1 s_1(t) - b_2 s_2(t) + \gamma\left(\left(\frac{\partial c}{\partial \gamma}\right)(t) - b_1\left(\frac{\partial s_1}{\partial \gamma}\right)(t) - b_2\left(\frac{\partial s_2}{\partial \gamma}\right)(t)\right)\right)\ \Delta t$$

$$\left(\frac{\partial c}{\partial \gamma}\right)(t+\Delta t) = \left(\frac{\partial c}{\partial \gamma}\right)(t) + \left(\eta\left(\frac{\partial m}{\partial \gamma}\right)(t) - \left(\frac{\partial c}{\partial \gamma}\right)(t)\right)\omega\ \Delta t$$

$$\left(\frac{\partial m}{\partial \gamma}\right)(t+\Delta t) = \left(\frac{\partial m}{\partial \gamma}\right)(t)(1-\Theta\ \Delta t)$$

APPENDIX – B: Adaptation Formulas for parameters in focus set (parasitism)

Basic Model

$$\frac{ds_1(t)}{dt} = \beta\, s_1(t)\big(c(t) - a_1 s_1(t) - b_1 s_2(t)\big)$$
$$\frac{ds_2(t)}{dt} = -\gamma\, s_2(t)\big(c(t) - b_2 s_1(t)\big)$$
$$\frac{dc(t)}{dt} = \omega\,(\eta\, m(t) - c(t))$$
$$\frac{dm(t)}{dt} = \theta\,(\lambda\, w - m(t))$$

Sensitivities calculations for water level

$$\frac{\partial \frac{\partial s_1}{\partial w}}{\partial t} = \beta\, \frac{\partial s_1(t)}{\partial w}\,(c(t) - a_1 s_1(t) - b_1 s_2(t)) + \beta\, s_1(t)\left(\frac{\partial c(t)}{\partial w} - a_1 \frac{\partial s_1(t)}{\partial w} - b1 \frac{\partial s_2(t)}{\partial w}\right)$$

$$\frac{\partial \frac{\partial s_2}{\partial w}}{\partial t} = -\gamma\, \frac{\partial s_2(t)}{\partial w}\,\big(c(t) - b_2 s_1(t)\big) - \gamma\, s_2(t)\left(\frac{\partial c(t)}{\partial w} - b_2 \frac{\partial s_1(t)}{\partial w}\right)$$

$$\frac{\partial \frac{\partial c}{\partial w}}{\partial t} = \omega\left(\eta\, \frac{\partial m(t)}{\partial w} - \frac{\partial c(t)}{\partial w}\right)$$

$$\frac{\partial \frac{\partial m}{\partial w}}{\partial t} = \theta\left(\lambda - \frac{\partial m(t)}{\partial w}\right)$$

Change required in water level to achieve target for s$_1$

$$\Delta w = (s_1(w + \Delta w) - s_1(w))/\left(\frac{\partial s_1}{\partial w}\right)$$

Change required in different parameters

$$\Delta a_1 = \big(s_1(a_1 + \Delta a_1) - s_1(a_1)\big)/\left(\frac{\partial s_1}{\partial a_1}\right)$$

$$\Delta b_1 = \big(s_1(b_1 + \Delta b_1) - s_1(b_1)\big)/\left(\frac{\partial s_1}{\partial b_1}\right)$$

$$\Delta b_2 = \big(s_1(b_2 + \Delta b_2) - s_1(b_2)\big)/\left(\frac{\partial s_1}{\partial b_2}\right)$$

$$\Delta \beta = \big(s_1(\beta + \Delta\beta) - s_1(\beta)\big)/\left(\frac{\partial s_1}{\partial \beta}\right)$$

$$\Delta \gamma = \big(s_1(\gamma + \Delta\gamma) - s_1(\gamma)\big)/\left(\frac{\partial s_1}{\partial \gamma}\right)$$

Sensitivities Calculations

For *a₁*

$$\left(\frac{\partial s_1}{\partial a_1}\right)(t+\Delta t) = \left(\frac{\partial s_1}{\partial a_1}\right)(t) + \beta\left(\left(\frac{\partial s_1}{\partial a_1}\right)(t)\left(c(t) - a_1 s_1(t) - b_1 s_2(t)\right) + s_1(t)\left(\left(\frac{\partial c}{\partial a_1}\right)(t) - s_1(t) - a_1\left(\frac{\partial s_1}{\partial a_1}\right)(t) - b_1\left(\frac{\partial s_2}{\partial a_1}\right)(t)\right)\right)\Delta t$$

$$\left(\frac{\partial s_2}{\partial a_1}\right)(t+\Delta t) = \left(\frac{\partial s_2}{\partial a_1}\right)(t) - \gamma\left(\left(\frac{\partial s_2}{\partial a_1}\right)(t)\left(c(t) - b_2 s_1(t)\right) + s_2(t)\left(\left(\frac{\partial c}{\partial a_1}\right)(t) - b2\left(\frac{\partial s_1}{\partial a_1}\right)(t)\right)\right)\Delta t$$

$$\left(\frac{\partial c}{\partial a_1}\right)(t+\Delta t) = \left(\frac{\partial c}{\partial a_1}\right)(t) + \left(\eta\left(\frac{\partial m}{\partial a_1}\right)(t) - \left(\frac{\partial c}{\partial a_1}\right)(t)\right)\omega\ \Delta t$$

$$\left(\frac{\partial m}{\partial a_1}\right)(t+\Delta t) = \left(\frac{\partial m}{\partial a_1}\right)(t)(1 - \Theta\ \Delta t)$$

For *b₁*

$$\left(\frac{\partial s_1}{\partial b_1}\right)(t+\Delta t) = \left(\frac{\partial s_1}{\partial b_1}\right)(t) + \beta\left(\left(\frac{\partial s_1}{\partial b_1}\right)(t)\left(c(t) - a_1 s_1(t) - b_1 s_2(t)\right) + s_1(t)\left(\left(\frac{\partial c}{\partial b_1}\right)(t) - a_1\left(\frac{\partial s_1}{\partial b_1}\right)(t) - b_1\left(\frac{\partial s_2}{\partial b_1}\right)(t) - s_2(t)\right)\right)\Delta t$$

$$\left(\frac{\partial s_2}{\partial b_1}\right)(t+\Delta t) = \left(\frac{\partial s_2}{\partial b_1}\right)(t) - \gamma\left(\left(\frac{\partial s_2}{\partial b_1}\right)(t)\left(c(t) - b_2 s_1(t)\right) + s_2(t)\left(\left(\frac{\partial c}{\partial b_1}\right)(t) - b_2\left(\frac{\partial s_1}{\partial b_1}\right)(t)\right)\right)\Delta t$$

$$\left(\frac{\partial c}{\partial b_1}\right)(t+\Delta t) = \left(\frac{\partial c}{\partial b_1}\right)(t) + \left(\eta\left(\frac{\partial m}{\partial b_1}\right)(t) - \left(\frac{\partial c}{\partial b_1}\right)(t)\right)\omega\ \Delta t$$

$$\left(\frac{\partial m}{\partial b_1}\right)(t+\Delta t) = \left(\frac{\partial m}{\partial b_1}\right)(t)(1 - \Theta\ \Delta t)$$

For *b₂*

$$\left(\frac{\partial s_1}{\partial b_2}\right)(t+\Delta t) = \left(\frac{\partial s_1}{\partial b_2}\right)(t) + \beta\left(\left(\frac{\partial s_1}{\partial b_2}\right)(t)\left(c(t) - a_1 s_1(t) - b_1 s_2(t)\right) + s_1(t)\left(\left(\frac{\partial c}{\partial b_2}\right)(t) - a_1\left(\frac{\partial s_1}{\partial b_2}\right)(t) - b_1\left(\frac{\partial s_2}{\partial b_2}\right)(t)\right)\right)\Delta t$$

$$\left(\frac{\partial s_2}{\partial b_2}\right)(t+\Delta t) = \left(\frac{\partial s_2}{\partial b_2}\right)(t) - \gamma\left(\left(\frac{\partial s_2}{\partial b_2}\right)(t)\left(c(t) - b_2 s_1(t)\right) + s_2(t)\left(\left(\frac{\partial c}{\partial b_2}\right)(t) - s_1(t) - b_2\left(\frac{\partial s_1}{\partial b_2}\right)(t)\right)\right)\Delta t$$

$$\left(\frac{\partial c}{\partial b_2}\right)(t+\Delta t) = \left(\frac{\partial c}{\partial b_2}\right)(t) + \left(\eta\left(\frac{\partial m}{\partial b_2}\right)(t) - \left(\frac{\partial c}{\partial b_2}\right)(t)\right)\omega\ \Delta t$$

$$\left(\frac{\partial m}{\partial b_2}\right)(t+\Delta t) = \left(\frac{\partial m}{\partial b_2}\right)(t)(1 - \Theta\ \Delta t)$$

For β

$$\left(\frac{\partial s_1}{\partial \beta}\right)(t+\Delta t) = \left(\frac{\partial s_1}{\partial \beta}\right)(t) + \Bigg(\left(s_1(t) + \beta\, (\partial s_1/\partial\beta)\,(t) \right) \left(c(t) - a_1 s_1(t) - b_1 s_2(t)\right) + \beta\, s_1(t) \left(\left(\frac{\partial c}{\partial \beta}\right)(t) - a_1 \left(\frac{\partial s_1}{\partial \beta}\right)(t) - b_1 \left(\frac{\partial s_2}{\partial \beta}\right)(t) \right) \Bigg)\ \Delta t$$

$$\left(\frac{\partial s_2}{\partial \beta}\right)(t+\Delta t) = \left(\frac{\partial s_2}{\partial \beta}\right)(t) - \gamma \left((\partial s_2/\partial\beta)\,(t) \left(c(t) - b_2 s_1(t)\right) + s_2(t) \left(\left(\frac{\partial c}{\partial \beta}\right)(t) - b_2 \left(\frac{\partial s_1}{\partial \beta}\right)(t) \right) \right)\ \Delta t$$

$$\left(\frac{\partial c}{\partial \beta}\right)(t+\Delta t) = \left(\frac{\partial c}{\partial \beta}\right)(t) + \left(\eta \left(\frac{\partial m}{\partial \beta}\right)(t) - \left(\frac{\partial c}{\partial \beta}\right)(t) \right) \omega\ \Delta t$$

$$\left(\frac{\partial m}{\partial \beta}\right)(t+\Delta t) = \left(\frac{\partial m}{\partial \beta}\right)(t)(1 - \Theta\ \Delta t)$$

For γ

$$\left(\frac{\partial s_1}{\partial \gamma}\right)(t+\Delta t) = \left(\frac{\partial s_1}{\partial \gamma}\right)(t) \left(1 + \left(c(t) - a_1 s_1(t) - b_1 s_2(t)\right) \beta\, \Delta t\right) + s_1(t)\, \beta \left(\left(\frac{\partial c}{\partial \gamma}\right)(t) - a_1 \left(\frac{\partial s_1}{\partial \gamma}\right)(t) - b_1 \left(\frac{\partial s_2}{\partial \gamma}\right)(t) \right)\ \Delta t$$

$$\left(\frac{\partial s_2}{\partial \gamma}\right)(t+\Delta t) = \left(\frac{\partial s_2}{\partial \gamma}\right)(t) \left(1 - \left(c(t) - b_2 s_1(t)\right) \gamma\, \Delta t\right) - s_2(t) \left(c(t) - b_2 s_1(t) + \gamma \left(\left(\frac{\partial c}{\partial \gamma}\right)(t) - b_2 \left(\frac{\partial s_1}{\partial \gamma}\right)(t) \right) \right)\ \Delta t$$

$$\left(\frac{\partial c}{\partial \gamma}\right)(t+\Delta t) = \left(\frac{\partial c}{\partial \gamma}\right)(t) + \left(\eta \left(\frac{\partial m}{\partial \gamma}\right)(t) - \left(\frac{\partial c}{\partial \gamma}\right)(t) \right) \omega\ \Delta t$$

$$\left(\frac{\partial m}{\partial \gamma}\right)(t+\Delta t) = \left(\frac{\partial m}{\partial \gamma}\right)(t)(1 - \Theta\ \Delta t)$$