Research on One-dimensional Modified Quadratic Shepard Interpolants at a Glance

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Abstract
This paper is concerned with comparative study of 1D modified quadratic Shepard interpolants developed so far. It consolidates useful information on the topic. The aim of this paper is to provide, users of the Shepard interpolants and / or researchers in this area, with a quick reference document. The process evaluates different versions of the Shepard interpolants in term of timing, storage, smoothness, shape preservation, visual pleasantness, symmetry, and slope interpolation. We give representative perspective plots of several curves to indicate the flavor of the results regarding visual pleasantness. We also construct a table, for this purpose, to summarize Shepard interpolants with respect to some important attributes of evaluation.

1. Introduction

Historically speaking techniques fairly similar to those known as Shepard interpolants had been in the use by Cressman 1959 and Crain and Bhattacharyya 1967 in one form or the other. Since D. Shepard was first to formally introduce them in 1968, they have been named after him as Shepard family of interpolants. This family of interpolants is a famous approach (see Shepard [1]). They are being used all over the world. Shepard method is mainly used for 2D and 3D scattered data, but is equally applicable to the data on regular mesh in these dimensions or as a curve interpolation technique. Moreover, study of whatever aspect of a method begins with 1D and Shepard methods are no exception.

The problem being addressed here (by various versions of Shepard method in 1D) is the evaluation of function F(x) which is at the least C^1 continuous, takes certain prescribed values $F(x_i) = f_i$, for i = 1,2,. .. , N, and is subject to making various provisions such as shape preservation, slope interpolation and least-square fit to the data.

An emphasis on shape preservation classifies this paper as the one on the topic of a review on constraind interpolation. The data points $(x_1, f_1), (x_2, f_2),\ldots,(x_N, f_N)$ do not satisfy any particular condition as to spacing or density which in higher dimensions means scattered data. However, as a matter of convenience we assume that x-coordinates of these data points are subject to, $x_1 < x_2 < \ldots < x_N$, order. Basically the Shepard interpolants are a distance-weighted average of basis functions defined for each data point. Each basis function interpolates its respective data point and is the best weighted least-square fit to others. The interpolation by individual basis functions results in interpolation by their distance weighted average, F(x), and their least-square fit property indirectly makes F(x) dependent on all data points and hence global in nature.

It is not feasible to use a global method for a large N and may not be manageable when N is very large. This inspired Gregory M. Nielson to localize the Shepard method. As a consequence of modification on his part we have modified quadratic Shepard interpolant now. This interpolant with basis functions that are quadratic, is the most popular version in this family (see Nielson [2]). The reason for popularity lies in localization of Shepard method which reduced its computational cost by limiting the least-squares fitting process to a local subset of the data.

The Shepard methods have C^1 continuity (see Lancaster and Salkauskas [3]) and are applicable to a space of any dimension for data of any distribution. However, to their disadvantage they did not preserve any of the local shapes implied by the data (see Nielson [2]), until recently. Asim and Brodlie [4] was first to introduce a 1D positive modified quadratic Shepard curve. The idea of positivity by 1D modified quadratic Shepard curve was then extended to the constrained least-square fit to data (see Asim and Brodlie [5]). Asim and Brodlie [6] incorporates the provision of slope interpolation to the constrained 1D modified quadratic Shepard method presented in Asim and Brodlie [4]. The data transformation to log space used in conjunction with modified quadratic Shepard method is an other version for positive curve. Asim and Brodlie [7] extend the interpolant in Asim and Brodlie [4] to 2D. A variety of other extensions of this research work are desirable.

Out of this situation arises a desire to look at the achievements in this research area at a glance. So that we may answer a number of questions, basically all related: Which of these methods are worth extending/using in terms of complexity, cost, visual pleasantness and some other attributes. This motivates the need for a review paper on one-dimensional modified quadratic Shepard interpolants. We may mention that we have, used small data sets for experimentation, and treated all the versions of modified quadratic Shepard method, global. Still we believe that our findings and results equally represent those due to localized counterparts of these global versions of modified quadratic Shepard method. We believe it so because ultimately, even localized versions of modified quadratic Shepard method are to be applied to small sub sets of the large data sets at a time.

2. Analysis Criterion

This Section elaborates characteristics on which various versions of modified quadratic Shepard method are to be assessed, and the way they are to be weighted in this research paper, are somewhat subjective. Neither the list of characteristics considered is exhaustive nor do we expect everybody to fully agree to it. Each reader may like to weigh various aspects of the findings, such as entries in the summary table and perspective graphs, to suit his own requirements.

Tested Attributes

Computational cost versus Visual appearance: Since visual appearance is subjective, different persons may differ in their ratings. At the same time we do not expect their views to be altogether contradictory. Because resolution and viewpoint of a perspective curve could obscure its ugliness, but it is doubtful that any really bad looking curve has escaped detection. Visual appearance is quite important, in graphics, and can not be ignored.

Computational cost is secondary and is normally ignored except when it is too high. So when high quality visual appearance or smoothness is required we are constrained to employ more costly interpolants. Simultaneously, an easy to compute interpolant is preferable over a costly one. In fact there is a trade off between cost and visual appearance or cost and smoothness.

Smoothness: The importance of smoothness lies in its key role to make the curve nice-looking. It intuitively follows that a smooth curve should be free of discontinuities and corners. Mathematically speaking, a curve is smooth if, firstly, it is continuous and secondly it is continuously differentiable. We denote the class of all continuous curves by C^0. A subset C^1 of C^0 consists of continuously differentiable curves. Further, a subset C^2 of C^1 consists of the curves whose second order derivatives are also continuous.

Shape preservation: Sometimes it is desirable for an interpolant to preserve the inherent shape of the data. An interpolant fitted to the data sampled as concentrations of a chemical reactant during the course of a chemical reaction is one such example. We mean the interpolant must be positive everywhere, for it does not make any sense otherwise.

Slope interpolation: In some cases, like the numerical solution of differential equations where data is sampled computationally, we need to interpolate the slope data as well.

Global versus local: A global interpolant is the one which is constructed with a simultaneous participation of all the data points involved. Whereas, a local interpolant can be defined with the help of small set of the neighbouring (local) data points. Both techniques have their own advantages and disadvantages. The global ones give higher degree of smoothness but are costly or may be unmanageable for large amounts of data. Also a small perturbation of one data point changes the whole global interpolant. Despite these disadvantages sometimes it is preferable to use global methods. An example in 3D is that modern neuroscientists are interested in understanding the global activity of the brain by simultaneously analyzing the results of several electrodes placed over the scalp (see Neilson [8]).On the other hand, when the curve drawing and the data collection are to be carried out side by side, as in case of cardiogram drawn by an electrocardiograph using a local technique.

Symmetry: By symmetry of an algorithm we mean the same curve is generated by processing data from start-point to end-point, or from end-point to start-point. When a curve is to be drawn while collecting data, and the running data is not sure to be sampled in ascending or descending order of the independent variable, we are constrained to use a symmetric interpolation scheme.

The Testing Process:

We started with experimentation on the original modified quadratic Shepard method to examine the trends followed by its basis functions. The study progressed through evolution of ideas. Consequently some aspects of the process have been unexpectedly complicated for some of the test data.

For a consistent evaluation of all the versions, we developed a standard program for each version to obtain timing statistics and generate interpolation data for each curve. A separate program was written to draw and label perspective plots of curves. The programs were written to minimize the computational effort involved.

We used a total of ten data sets to compare the methods. The representative graphs consist of those drawn through four data sets. Two of these are taken from literature and the other two are self prepared. Their choice is based on special reasons. For instance data in Table 2 is chosen to show the failure of Method-4 for a data involving zero data values. Similarly data in Table 3 is crucial in that it involves both convex and concave basis functions simultaneously. We may mention that only those versions have been thoroughly tested and reported which performed well in initial testing.

3 Description of Tested Versions of the Modified Quadratic Shepard Method

In this section we describe different versions of the modified quadratic Shepard method which have been developed tested and tried so far. Since we apply them to various data sets as a whole and in a way treat them as global methods, we use simple weights instead of those proposed by Nielson [2].

The resultant deviation from the actual state of the art has been explained to be negligible in section 2. We may mention that all the methods discussed below are, C^1 continuous except the data transformation technique which is C^{∞}. Also they are conceptually same in that all of them are defined by (1). There is however, a significant difference that their basis functions are constructed differently so as to make them suitable for varying needs of a user. As such construction of basis function for each method has been elaborated instead of complete description of the method concerned.

3.1 The Origina11D Modified Quadratic Shepard Method

The modified quadratic Shepard curve *F(x)* is defined such that

$$F(x) = \left[\sum_{i=1}^{N} w_i(x)\, Q_i(x)\right] / \left[\sum_{i=1}^{N} w_i(x)\right] \qquad \textbf{(1)}$$

where the weight function $Wi(x)=1/(x-x_i)^2$ and the basis function

$$Q_i(x) = f_i + b_i(x - x_i) + a_i(x - x_i)^2$$

is related to data point (x_i, f_i) By definition, $Q_i(x_i) = f_i$ and the coefficients a_i and b_i are chosen so that Q_i is a best distance-weighted least-squares approximation to the other data points.

x	0	2	4	10	28	30	32
y	20.8	8.8	4.2	.5	3.9	6.2	9.6

Table 1: Oxygen levels in flue gas.

The modified quadratic Shepard curve is C^1 continuous. It does not guarantee to preserve any of the shapes inherent from data including positivity throughout the domain

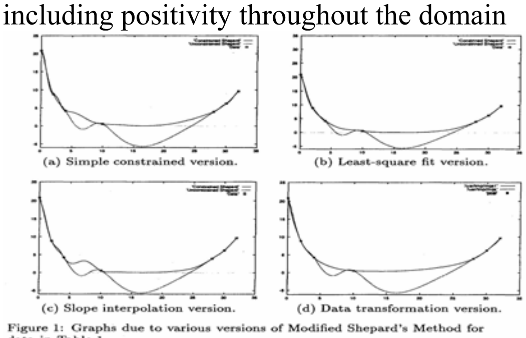

Figure 1: Graphs due to various versions of Modified Shepard's Method for data in Table 1

of definition. The dotted curves in figures of this paper practically illustrate it to be negative.

3.2 The Simple Constrained ID Modified Quadratic Shepard Method

Since *F(x)* is a weighted average of the basis quadratics $Q_i(x)$, positivity of *F(x)* can be assured by taking each $Q_i(X)$ positive. For this we rewrite the original Q_i in the form

$$Q_i(x) = y_s + a_i(x - x_s)^2 \qquad \textit{(2)}$$

where (x_s, y_s) is the stationary point of Q_i, with $x_s = x_i - b_i / 2a_i$ and $y_s = f_i - b_i^{\,2} / 4\, a_i$

By stationary point we mean a point on the curve where derivative of the function vanishes. The interpolation condition $Q_i(x_i) = f_i$, gives us

$$a_i = [f_i - y_s] / [x_i - x_s]^2 \qquad \textit{(3)}$$

The condition imposed on y_s (or a_i) to ensure positivity is that they are respectively replaced by $\hat{y}_s$ (or $\hat{a}_i$) as follows, (see Asim and Brodlie [4]) for details.

- a convex quadratic: $\hat{y}_s = 0$,
- a concave quadratic:

with $x_s \neq x_i$,

$$\hat{y}_s = [f_i\,(x_N - x_s)^2\,] / [\,x_N - x_s)^2 - (x_i - x_s)^2] \qquad \textit{if } x_s - x_1 \leq x_N - x_s$$

and

$$\hat{y}_s = [f_i\,(x_1 - x_s)^2\,] / [\,x_1 - x_s)^2 - (x_i - x_s)^2] \qquad \textit{if } x_s - x_1 > x_N - x_s \qquad \textit{(4)}$$

with $x_s = x_i$, $\hat{a}_i = -y_s / (x_1 - x_s)^2$ or $-y_s / (x_N - x_s)^2$

3.3 Constrained Least-square Fit ID Modified Quadratic Shepard Method

We give only a brief view of the method in this subsection and refer readers to Asim and Brodlie [5] for details. Two cases depending upon convexity and concavity of basis functions are involved. We deal with them one by one.

Convex Case

The essential idea used is that we replace a convex quadratic basis function by a convex quadratic spline. We insert a knot to join two pieces of spline and provide freedom necessary for the least-square fit in this case. Suppose first that $x_i < x_s$, so the interpolation point lies to the left of the minimum of the original basis function. Mathematically, the constrained basis function is a quadratic spline,

$$Q_i(x) = S_1(x) \text{ if } x_1 \le x < x_{knot}$$

$$Q_i(x) = S_2(x) \text{ if } x_{knot} \le x \le x_N \qquad (5)$$

where (omitting subscript i for simplicity of notation)

$$S_1(x)=f_{knot}+ b_{knot}(x-x_{knot}) + a_{knot}(x-x_{knot})^2 \qquad (6)$$

and

$$S_2(x) = a_s(x-x_s)^2 \qquad (7)$$

On expressing f_{knot} and b_{knot} in terms of a_s and for

$$a_{knot} = (f_i / h^2) - 3a_s \qquad (8)$$

with $h = x_{knot} - x_i$, the problem of best weighted least-squares fit to the other data points reduces to the choice of optimum value of a_s. To achieve this we minimize:

$$\sum_{x_k \neq x_i} w_i(x_k)\,[Q_i(x_k) - f_k]^2 \qquad (9)$$

subject to the conditions $a_s > 0$ and $a_{knot} > 0$ where the weight function $w_i(x) = 1/(x-x_i)^2$ The value of a_s thus obtained is then restricted to the interval $[0, f_i/3h^2]$ such that:

- If $a_s < 0$, then we set $a_s = 0$ - in which case $a_{knot} > 0$ from (8).
- If $a_s > 0$, then again from (8), $a_{knot} = (1/h^2)f_i-3a_s$ which is positive provided $a_s \geq f_i/3h^2$ so we constrain $a_s = f_i/3h^2$ as a maximum value.

Similarly for $x_i > x_s$, that is when minimum is on the left side of the data point, we proceed in the same way as we did in the case $x_i < x_s$ except that:

- quadratics S_1 and S_2 swap their domains of definition.
- $x_{knot} \in [x_s, x_i]$

X	1	2	4	6	7
Y	0	0.006	3.5	0.06	0

Table 2: Data that provides concave basis functions.

Concave Case

In the case where $Q_i(x)$ is concave, we raise values at one or both end-points to zero

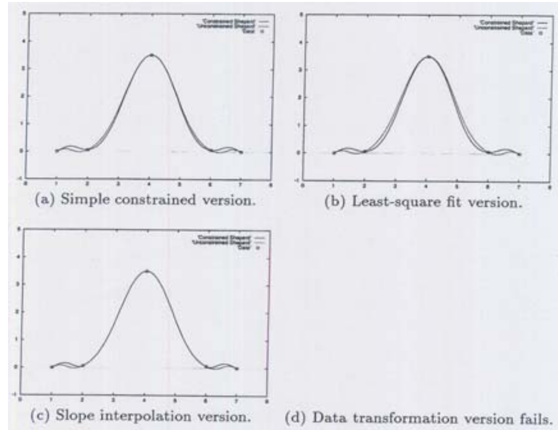

Figure 2: Graphs due to various versions of Modified Shepard's Method for data in Table 2

instead of raising the minimum value- to zero. The addition of a least-square fitting condition again requires extra flexibility, which we get by using a quadratic spline with one knot. This case is a bit simpler in that the knot can be placed at x_s.

Suppose $x_i < x_s$, and suppose the original basis function is negative at x_N. We create a constrained basis function which is a concave quadratic spline, with S_l interpolating (x_i, f_i) and S_2 interpolating $(x_N, 0)$. This gives us:

$$Q_i(x) = S_1(x) \text{ if } x_1 \le x < x_s$$
$$Q_i(x) = S_2(x) \text{ if } x_s \le x \le x_N \qquad (10)$$

such that

$$S_1(x) = y_s + (f_i - y_s)(x - x_s)^2 / (x_i - x_s)^2 \qquad (11)$$

$$S_2(x) = y_s - y_s(x - x_s)^2 / (x_N - x_s)^2 \qquad (12)$$

with freedom to vary y_s to provide a best least-square fit. We need to minimize:

$$\sum_{x_k \neq x_i} w_i(x_k)\,[Q_i(x_k) - f_k]^2 \qquad (13)$$

subject to $Q_i(x_1) \geq 0$. The concavity of the spline is ensured by restricting $y_s > f_i$ and $Q_i(x_1) > 0$ is guaranteed by taking

$$y_s < [f_i(x_1 - x_s)^2]/[x_1 - x_s)^2 - (x_i - x_s)^2] = \theta f_i$$

Since $(x_1-x_s)^2 > (x_i-x_s)_2$, $\theta > 1$. In other words finally we need to restrict y_s by

$y_s \in [f_i, \theta f_i]$

Similar discussion follows for the case $x_i > x_s$, where the original basis function is negative at x_1.

A special case arises when $(x_i, f_i) = (x_s, y_s)$ or (x_i, f_i) and (x_s, y_s) are very very close to each other. We can not lower the maximum point in this case and are thus constrained to change the curvature for the two pieces of the spline

$$Q_i(x) = S_1(x) \text{ if } x_1 \le x < x_s$$

$$Q_i(x) = S_2(x) \text{ if } x_s \le x \le x_N \qquad (14)$$

such that

$$S_1(x) = f_i + a_1 (x-x_s)^2 \qquad (15)$$

with $a_l \ge -f_i / (x_1 - x_s)^2$ and

$$S_2(x) = f_i + a_N (x-x_s)^2 \qquad (16)$$

with $a_N \ge -f_i / (x_N - x_s)^2$ It may be mentioned here that if x_l and x_N are equidistant from x_s, then only one quadratic covers both sides.

3.4 Slope Interpolating Positive ID Modified Quadratic Shepard Method

The action in this case is somewhat similar to the one we took in the previous subsection except that the freedom due to spline is availed to incorporate slope interpolation instead of least-square fit. The details of the brief description of the method given here can be seen in Asim and Brodlie [6]. We proceed to deal with convex and concave cases one after the other.

Convex Case

Let the basis function be convex but negative and $x_i < x_s$, in which case we must have $b_i < 0$. A knot in interval, $[x_i, x_s]$, is inserted to construct quadratic spline, $Q_i(x)$, such that

$$Q_i(x) = S_1(x) \text{ if } x_1 \le x < x_{knot}$$

$$Q_i(x) = S_2(x) \text{ if } x_{knot} \le x \le x_N \qquad (17)$$

such that,

$S_1(x) = f_{knot} + b_{knot}(x - x_{knot}) + a_{knot}(x - x_{knot})^2$ and

$S_2(x) = a_s(x - x_s)^2$.

We have conditions on S_1 and S_2 as follows:

$$S_1(x_i) = f_i \qquad (18)$$

$$\acute{S}_1(x_i) = b_i \qquad (19)$$

$$S_1(x_{knot}) = S_2(x_{knot}) = f_{knot} \qquad (20)$$

$$\acute{S}_1(x_{knot}) = \acute{S}_2(x_{knot}) = b_{knot} \qquad (21)$$

$$a_s > 0 \qquad (22)$$

The second case, for which $xi > x_s$, is analogous where now $b_i > 0$ and

- quadratics S_1 and S_2 swap their domains of definition,
- the knot is inserted in the interval $[x_s, x_i]$.

Concave Case

Suppose the original quadratic basis function is concave but negative at either or both end-points and $x_i < x_s$. If $Q_i(x_1) < 0$, we select a point x_L, midway between xi and the point at which $Q_i(x)$ becomes negative on the left hand side. We then define a quadratic spline, with two quadratic pieces $Q_L(x)$ and $Q_i(x)$, such that:

$$Q_L(x_1) = 0,\ Q'_L(x_1) = 0,\ Q_L(x_L) = Q_i(x_L) \text{ and } Q'_L(x_L) = Q'_i(x_L).$$

Since the spline in question is a convex fit between the point $(x_L, Q_i(x_L))$ with a positive slope on it, and the point $(x_1, 0)$ where slope is zero, the problem reduces to the one already solved in the convex case. As such similar action will do.

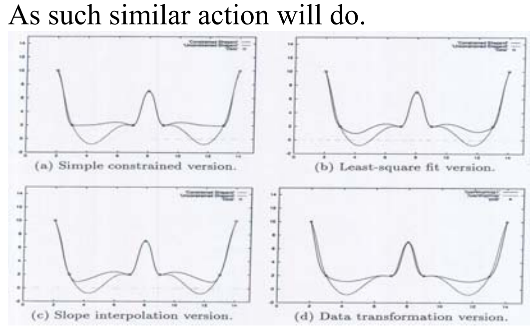

Figure 3: Graphs due to various versions of Modified Shepard's Method for data in Table 3

Similar argument works if $Q_i(x_N) < 0$. However, if a concave quadratic basis function is negative at both ends we will need a three piece quadratic spline such that a quadratic is provided on each side of the original quadratic.

x	2	3	7	8	9	13	14
y	10	2	2	7	2	2	10

Table 3: Data for Figures 3.

3.5 The Modified Quadratic Shepard Method in Con junction with Data Transformation Technique

In data transformation technique the data values $\{f_i\}^N_{i=1}$ are transformed to log space to map onto $\{ f^*_i \}^N_{i=1}$ using, say the natural log function such that:

$$f^*_i = \ln(f_i),\ i = 1, 2, \ldots, N$$

For this we need min $\{f_i\}^N_{i=1} > 0$, because log of zero is not defined. So we preprocess the data such that if:

$\boldsymbol{f_i = 0,}$

we abandon this technique. Otherwise we apply original modified quadratic Shepard method to the data transformed to the log space to construct a model function, F*. The interpolated data thus obtained is finally transformed back to the original space using F(x) = exp(F*(x)). Note that F* may be negative in log space due to possible negativity of *f**. Since exponential function returns positive values to the normal space, the possible negativity in log space is no problem.

Regarding extension of this data transformation technique to a more general linear constraint G(x), as a lower bound, we firstly need to ensure that $f_i > G(x_i)$ for all i. Secondly we work out the differences $f_i - G(x_i)$ for all i. This reduces the problem to the one solved above. If however, $f_i = G(x_i)$, for some i we bracket it with the impossible case.

This simpler scheme is also slope interpolating because

$\boldsymbol{d^*_i = d_i / f_i}$ ***for all i***

is now applicable to strictly positive data without any complication.

4 Discussion

In this section we summarize the findings of the review undertaken by us. In principal we have studied 1D modified quadratic Shepard method in general. But in practice our study is subject to positivity constraint. Since constrained versions are variants of the original modified quadratic Shepard method, the original modified quadratic Shepard method is also discussed for a comparison. For simplicity and convenience we label the original modified quadratic Shepard method as Method-0, the simple constrained Modified Quadratic Shepard method as Method-1, the least-squares fitting based constrained Modified Quadratic Shepard method as Method-2, slope interpolating constrained Modified Quadratic Shepard method as Method-3 and the data transformation based Modified Quadratic Shepard method as Method-4.

We provide the slope data for Method-3 by calculating the slopes of the unconstrained Modified Quadratic Shepard interpolant. This turns out to be a viable approach in itself, providing a constrained curve not only to illustrate slope interpolation but also as a method to compare with other versions of the Shepard method. Parameters of judgment in this discussion are simplicity, cost, visual appearance and extendibility of the method to higher dimensions. Looking at pictures in Figures 1 through 4 we observe that Method-l performed the worst, Method-4 performed the best, while the remaining two proved to be comparable. The pleasing visual appearance of picture due to Method-l in Figure 4(a) and a bit odd one due to Method 4 in Figure 4(d) as compared to

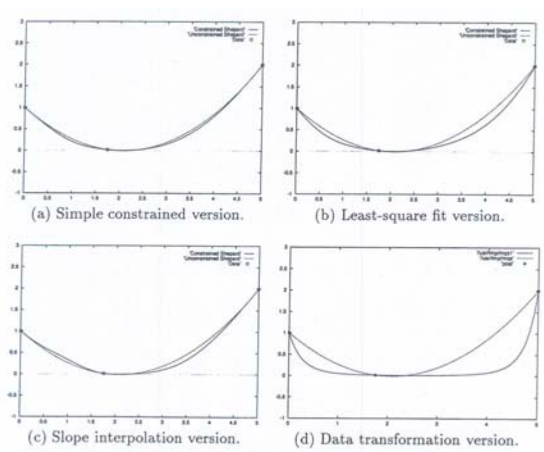

Figure 4: Graphs due to various versions of Modified Shepard's Method for data in Table 4

X	0	1.75	5
Y	1	.015	2

Table 4: Data for Figures 4.

their counterparts due to Method-2 and Method-3 are exceptions. The odd behavior of Method-4 in Figure 4(d) seems to be a result of near zero data value at the second point. This odd behavior, on a near zero data value, tends to a failure of Method-4 as data value tends to zero. The detailed discussion on this type of failure is available in Asim [9] and Figure 2(d) makes a practical example. To reach an overall conclusion regarding visual appearance we have another look at pictures due to all methods and conclude that nothing can be said in absolute terms but that Method-4 is the best, Method-2 and Method-3 are comparable and Method-l is inferior to them. Since Method-3 performs poorly for steep slopes at near zero data values as in Figures 1 (c) and 4(c) Method- 2 has a slight edge over Method-3 in this regard. So far as simplicity and extendibility to higher dimensions are concerned Method-4 is the best and Method-l is the second best. Method-2 and Method-3 are once again comparable in simplicity and extendibility but Method-2 may be more easily extendible to higher dimensions as compared to Method-3. If the circumstances compel the user to preserve certain slopes then Method-3 and Method-4 can be used. The cost of a program depends upon its timing and storage allocation requirements. The storage allocation may not be as important as timing but anyhow, Method-2 and Method-3 involve the storage of some extra arrays. Method-4 has the highest timing and Method-0 the lowest, Method-l the second lowest and the other two methods are at par with each other. Since different methods involve very many different operations and library functions, counting of number of operations in each version could not give true comparison. So we have found and recorded the time for thirty executions of each program using data in Table 1.

Methods	**Timing**	**Extendibility/ Simplysity**	**Robustness**	**Slope Interplt'n**	**Shape Preservt'n**
Method-0	0.9s	Easy	Yes	Yes	No
Method-1	1.1s	Easy	Yes	No	Positivity
Method-2	1.2s	Hard	Yes	No	Positivity
Method-3	1.2s	Hard	Yes	Yes	Positivity
Method-4	28.1s	Trivial	No	Yes	Positivity

Table5: Summary table for comparison of various methods on a networked Silicon Graphics O_2 workstation with MIPS R5000

Normally speaking Method-2 is the overall best of all methods. However, if we ignore the cost and robustness Method-4 is the best. For those who prefer a cheaper method Method-0 is the best if no shape preservation is required otherwise Method-l should be used. The summary of our findings in this research is given in table 5.

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