A Comparative Analysis on Adaptive Modelling of Induced Feelings

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Abstract. Stimuli and activations of mental states usually induce emotional responses that are experienced by their associated feelings. This paper concentrates on how the the strengths by which such emotional responses are induced, depend on previous experiences. It presents a comparative analysis of three adaptive modelling approaches addressing how these induction strengths are adapted over time: Hebbian learning, temporal discounting and memory traces. Example simulation results are shown and commonalities and differences between the models are analysed.

Keywords: Induced feelings, adaptive, memory traces, temporal discounting, Hebbian learning

1 Introduction

For many responses to certain (external or internal) circumstances, an important role is played by experiences for similar circumstances in the past. How such circumstances are experienced does not only depend on the circumstances themselves but also on the extent to which emotional responses are induced and felt. For this paper it is assumed that such an induction process of experienced emotional responses takes the form of triggered preparations for body states that are in a recursive as-if body loop with certain feelings; e.g., [4], [7], [15]. The strengths by which stimuli or activations of mental states induce certain preparations for body states or (other) actions occurring as emotional responses, might be innate, but are often considered to be acquired, strengthened and/or adapted during lifetime; e.g. [6], [12]. These induction strengths of responses based on experiences from the past are assumed to play an important role in a variety of behaviors, for example, involving decision making according to the Somatic Marker Hypothesis e.g. [6], [8].

To explain or model such a development of induction strength of a response, in the literature different perspectives can be found. In this paper three different alternatives for modelling approaches are considered. One of them is a *Hebbian learning* approach e.g. [1], [9], [10]. A second alternative is based on a *temporal discounting* approach as often is used in modelling intertemporal decision making or in modelling trust dynamics e.g. [5], [11]. The third alternative considered is a case-based memory modelling approach based on *memory traces* e.g. [13], [14]. Each of the three approaches is briefly presented and some simulation results for a common case study are discussed and compared to each other to find out under which circumstances the approaches coincide, and when they differ.

As a starting point, in Section 2 it is shown how elements from neurological theories on generation of emotion and feeling were adopted, and based on them first a computational model was set up that models the effect of stimuli on emotional responses and feelings. In accordance with such literature a converging recursive as-if body loop to generate a feeling is taken as a point of departure. Next, as the main step, in Section 3 the three different adaptation models for the strength of the emotional response were integrated in this model. In Section 4 simulation results are presented. Section 5 shows how the three resulting models were evaluated and compared based on simulation experiments. Finally, Section 6 is a discussion.

2 Dynamics of Emotional Responses and Feelings

As any mental state in a person, a sensory representation state induces emotions felt within this person, as described by [7], [8]; for example: '…Through either innate design or by learning, we react to most, perhaps all, objects with emotions, however weak, and subsequent feelings, however feeble.' [8] p. 93. In some more detail, emotion generation via an as-if body loop roughly proceeds according to the following causal chain; see Damasio [7], [8]: sensory representation of stimulus → preparation for body state → sensory representation of body state → feeling. The as-if body loop is extended to a recursive as-if body loop by assuming that the preparation of the bodily response is also affected by the state of feeling the emotion: feeling → preparation for body state as an additional causal relation. Such recursiveness is also assumed by Damasio [8], as he notices that what is felt by sensing is actually a preparation for a body state which is an internal object, under control of the person: 'The object at the origin on the one hand, and the brain map of that object on the other, can influence each other in a sort of reverberative process that is not to be found, for example, in the perception of an external object.' [8] p. 91. Thus the obtained model for emotion generation is based on reciprocal causation relations between emotion felt and preparations for body states. Within the model presented in this paper both the preparation for the bodily response and the feeling are assigned an (activation) level or gradation, expressed by a number. The cycle is modelled as a positive feedback loop, triggered by a sensory representation and converging to a level of feeling and preparation for body state.

Informally described theories in scientific disciplines, for example, in biological or neurological contexts, often are formulated in terms of causal relationships or in terms of dynamical systems. To adequately formalise such a theory the hybrid dynamic modelling language LEADSTO has been developed that subsumes qualitative and quantitative causal relationships, and dynamical systems; cf. [2]. This language has been proven successful in a number of contexts, varying from biochemical processes that make up the dynamics of cell behaviour to neurological and cognitive processes e.g. [3], [4]. Within LEADSTO the *dynamic property* or temporal relation $a \twoheadrightarrow_D b$ denotes that when a state property a occurs, then after a certain time delay (which for each relation instance can be specified as any positive real number *D*), state property b will occur. Below, this *D* will be taken as the time step Δt, and usually not be mentioned explicitly. In LEADSTO both logical and numerical calculations can be specified in an integrated manner, and a dedicated software environment is available to support specification and simulation.

An overview of the basic model for the generation of emotional responses and feelings is depicted in Fig. 1. This picture also shows representations from the detailed specifications explained below. However, note that the precise numerical relations between the indicated variables V shown are not expressed in this picture, but in the detailed specifications below, labeled by LP1 to LP5 in the picture.

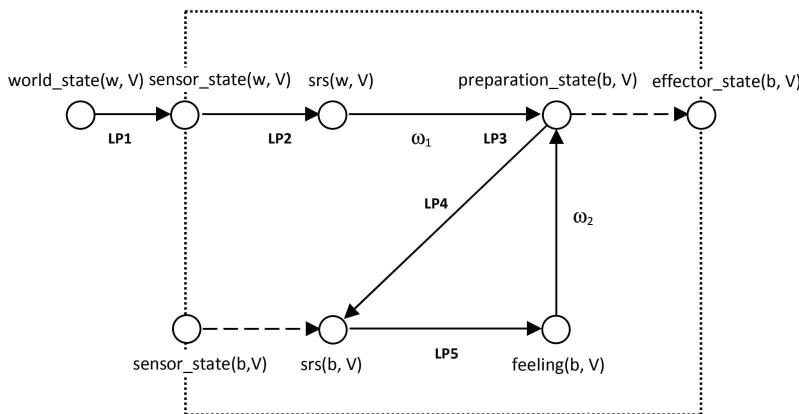

Fig. 1. Overview of the connections in the model for induced emotional responses and feelings

Note that the sensor and effector state for body states and the dashed arrows connecting them to internal states are not used in the model considered here. In the dynamic properties below capitals are used for variables (assumed universally quantified). First the part is presented that describes the basic mechanisms to generate a belief state and the associated feeling, starting with how the world state is sensed.

LP1 Sensing a world state
If world state property W occurs of strength *V*
then the sensor state for W will have strength *V*.
world_state(W, V) $\twoheadrightarrow$ sensor_state(W, V)

From the sensor states, sensory representations are generated according to the dynamic property LP2.

LP2 Generating a sensory representation for a sensed world state
If the sensor state for world state property W has strength *V*,
then the sensory representation for W with have strength *V*.
sensor_state(W, V) $\twoheadrightarrow$ srs(W, V)

Dynamic property LP3 describes the emotional response a sensory representation of a stimulus in the form of the preparation for a specific bodily reaction.

LP3 From sensory representation and feeling to preparation of a body state

If a sensory representation for w with strength V_1 occurs
and feeling the associated body state b has strength V_2
and the preparation state for b has strength V_3
and the connection from sensory representation of w to preparation for b has strength ω_1
and the connection from feeling to preparation for b has strength ω_2
and β is the person's orientation for emotional response
and γ is the person's flexibility for bodily responses
then after Δt the preparation state for body state b will have strength $V_3 + \gamma(h(\beta, \omega_1, \omega_2, V_1, V_2) - V_3)\, \Delta t$.

srs(w, V_1) & feeling(b, V_2) & preparation_state(b, V_3) &
has_connection_strength(srs(w), preparation(b), ω_1) &
has_connection_strength(feeling(b), preparation(b), ω_2)
↠ preparation(b, $V_3 + \gamma(h(\beta, \omega_1, \omega_2, V_1, V_2) - V_3)\, \Delta t$)

The resulting level for the preparation is calculated based on a function $h(\beta, \omega_1, \omega_2, V_1, V_2)$ of the original levels. For the function $h(\beta, \omega_1, \omega_2, V_1, V_2)$ the following was taken:

$$h(\beta, \omega_1, \omega_2, V_1, V_2) = \beta(1-(1-\omega_1 V_1)(1-\omega_2 V_2)) + (1-\beta)\, \omega_1 \omega_2 V_1 V_2$$

Note that this formula describes a weighted sum of two cases. The most positive case considers the two source values as strengthening each other, thereby staying under *1*: combining the imperfection rates $1-\omega_1 V_1$ and $1-\omega_2 V_2$ of them provides a decreased rate of imperfection expressed by $1-(1-\omega_1 V_1)(1-\omega_2 V_2)$. The most negative case considers the two source values in a negative combination: combining the imperfections of them provides an increased imperfection. This is expressed by $\omega_1 \omega_2 V_1 V_2$. The parameter β can be used to model a characteristic that expresses the person's orientation for emotional response (from *0* as weakest response to *1* as strongest response). Dynamic properties LP4 and LP5 describe the as-if body loop.

LP4 From preparation to sensory representation of a body state

If preparation state for body state B occurs with strength *V*,
then the sensory representation for body state B with have strength *V*.

preparation(B, V) ↠ srs(B, V)

LP5 From sensory representation of body state to feeling

If a sensory representation for body state B with strength *V* occurs,
then B will be felt with strength *V*.

srs(B, V) ↠ feeling(B, V)

3 Integrating Adaptation Models for the Induction Strengths

Three adaptation models for the induction strength ω_1 of the connection from sensory representation to preparation have been integrated. As a scenario it is assumed that over time different sensory representations occur in a repeated fashion. The first adaptation model presented follows a *Hebbian approach*. By this model the induction strength ω_1 of the connection from sensory representation to preparation is adapted using the following Hebbian learning rule. It takes into account a maximal connection strength *1*, a learning rate η, and an extinction rate ζ.

LP6 Hebbian learning rule for connection from sensory representation of stimulus to preparation

If the connection from sensory representation of w to preparation of b has strength ω_1
and the sensory representation for w has strength V_1
and the preparation of b has strength V_2
and the learning rate from sensory representation of w to preparation of b is η
and the extinction rate from sensory representation of w to preparation of b is ζ
then after Δt the connection from sensory representation of w to preparation of b will have strength $\omega_1 + (\eta V_1 V_2 (1 - \omega_1) - \zeta\omega_1)\,\Delta t$.

has_connection_strength(srs(w), preparation(b), ω_1) & srs(w, V_1) & preparation(b, V_2) &
has_learning_rate(srs(w), preparation(b), η) & has_extinction_rate(srs(w), preparation(b), ζ)
$\twoheadrightarrow$ has_connection_strength(b, w, $\omega_1 + (\eta V_1 V_2 (1 - \omega_1) - \zeta\omega_1)\,\Delta t$)

A similar Hebbian learning rule can be found in [9], p. 406. As a next model a *temporal discounting principle* is used to adapt the induction strength ω_1 of the connection from sensory representation to preparation.

LP7a Temporal discounting learning rule for sensory representation of stimulus

If the connection from sensory representation of w to preparation of b has strength ω_1
and the sensory representation for w has strength V and $V>0$
and the discounting rate from sensory representation of w to preparation of b is α
and the extinction rate from sensory representation of w to preparation of b is ζ
then after Δt the connection from sensory representation of w to preparation of b will have strength $\omega_1 + (\alpha(V - \omega_1) - \zeta\omega_1)\,\Delta t$.

has_connection_strength(srs(w), preparation(b), ω_1) & srs(w, V) & V>0 &
has_discounting_rate(srs(w), preparation(b), α) & has_extinction_rate(srs(w), preparation(b), ζ)
$\twoheadrightarrow$ has_connection_strength(srs(w), preparation(b), $\omega_1 + (\alpha(V - \omega_1) - \zeta\omega_1)\,\Delta t$)

LP7b Temporal discounting learning rule for sensory representation of stimulus

If the connection from sensory representation of w to preparation of b has strength ω_1
and the sensory representation for w has strength 0
and the extinction rate from sensory representation of w to preparation of b is ζ
then after Δt the connection from sensory representation of w to preparation of b will have strength $\omega_1 + (\alpha(V - \omega_1) - \zeta\omega_1)\,\Delta t$.

has_connection_strength(srs(w), preparation(b), ω_1) & srs(w, 0) &
has_extinction_rate(srs(w), preparation(b), ζ)
$\twoheadrightarrow$ has_connection_strength(srs(w), preparation(b), $\omega_1 - \zeta\omega_1\,\Delta t$)

The third model integrated is based on *memory traces*. Suppose is_followed_by(γ, a, b) indicates that within memory trace with identification label γ state a is followed by state b. The states addressed are the sensory representation state and the subsequent preparation state. It is assumed that each new pair of events <sensory representation, preparation> gets a new unique identification label γ, for example, based on a time stamp. The idea is then that for given states a and b, the strength of the induction from a to b is extracted in a case-based manner by the fraction of all represented traces in which a occurs with b as a next state from all traces in which a occurs:

$$\#\{\gamma \mid \text{is\_followed\_by}(\gamma, a, b)\} \,/\, \#\{\gamma \mid \exists c\ \text{is\_followed\_by}(\gamma, a, c)\}$$

A temporal element can be incorporated by giving more weight to more recently represented memory traces. This was modelled by using temporal discounting when extracting the induction strength from the represented memory traces. Moreover, also levels of activations of both states were taken into account. In this approach traces get weights depending on their time label where traces that occurred further back in time have lower weights. The following rules represent how information is extracted from the time-labeled representations by counting the discounted numbers of occurrences.

LP8 Discounting memory traces
If the sensory representation for w has strength V_1
and the preparation of b has strength V_2
and the discounted number of memory traces with state srs(w) is X
and the discounted number of memory traces with state srs(w) and successor state preparation(b) is Y
and the discounting rate from sensory representation of w to preparation of b is α
then the discounted number of memory traces with state srs(w) is $\alpha X+(1-\alpha)\ V_1$
and the discounted number of memory traces with state srs(w) and successor state preparation(b) is $\alpha Y+(1-\alpha)\ V_1V_2$

srs(w, V_1) & preparation(b, V_2) & has_discounting_rate(srs(w), preparation(b), α) &
memory_traces_including(srs(w), X) & memory_traces_including_both(srs(w), preparation(b), Y)
↠ memory_traces_including(srs(w), αX+(1-α) V_1) &
memory_traces_including_both(srs(w), preparation(b), αY+(1-α) V_1V_2)

Given these numbers the induction strength of the connection from sensory representation to preparation state is determined as Y/X.

LP9 Generation of preparations based on discounted memory traces
zzIf the discounted number of memory traces with state srs(w) is X
and the discounted number of memory traces with state srs(w) and successor state preparation(b) is Y
then the connection strength from srs(w) to preparation(b) is Y/X

memory_traces_including(srs(w), X) & memory_traces_including_both(srs(w), preparation(b), Y)
↠ has_connection_strength(srs(w), preparation(b), Y/X)

4 Example Simulation Results

Based on the computational model described in the previous section, a number of simulations have been performed. Some example simulation traces were included in this section as an illustration; see Fig. 2, 3 and 4, for the Hebbian learning, temporal discounting and memory traces approach, respectively (here the time delays within the temporal LEADSTO relations were taken 1 time unit). Note that only a selection of the relevant nodes (represented as state properties in LEADSTO) is shown. In all of these figures, where time is on the horizontal axis, and the activation levels of the different state properties are on the vertical axis, quantitative information for state properties values for the different time periods are shown (by the dark lines). The activation levels of the state properties gradually increase while the sensory representation of the stimulus occurs, following the recursive feedback loop discussed in Section 2. These levels sharply decrease after the sensory representation of the stimulus stops occurring, as described by the temporal relationship LP3 in Section 2. Moreover, except for some decrease due to extinction, the induction strength of the connection from sensory representation to preparation state keeps its value in the phases without stimulus, until the sensory representation of the stimulus (again) occurs, as described by temporal relationship LP6 in case of Hebbian learning, LP7a and LP7b in case of temporal discounting and by LP8a and LP8b in case of memory traces. Further comparison of the three adaptation models are discussed in Section 5.

Fig. 2 shows the adaptation model following the Hebbian approach. As can be seen in this figure, for sensory representation activation level *0.6* and initial level of preparation state *0*, during the phases of the stimulus the activation levels of the preparation and feeling states progressively increase over time until they reach levels close to *0.9*.

The induction strength which initially was set to *0.01*, gradually increases to attain a strength around *0.8*. The occurrence of this pattern is in line with the mathematical analysis which is discussed in the next section.

Fig. 3 shows the temporal discounting approach. For sensory representation activation level *0.6* and initial level of preparation state *0*, the activation levels of the preparation and feeling states gradually increase over the time until they reach values close to *0.8*. The induction strength state initially set at *0.01*, gradually increases to attain a strength around 0.55. Also the occurrence of this pattern is in line with the mathematical analysis discussed in the next section.

Fig. 4 shows the adaptation model following the memory traces approach. As can be seen in the figure, for the sensory representation activation level *0.6* and initial level of preparation *0*, the activation levels of preparation and feeling states gradually increases over time until levels close to *0.9* are reached. The induction strength gradually increases from initial value *0.01* to values around *0.7*. The occurrence of this pattern also is in line with results from the next section.

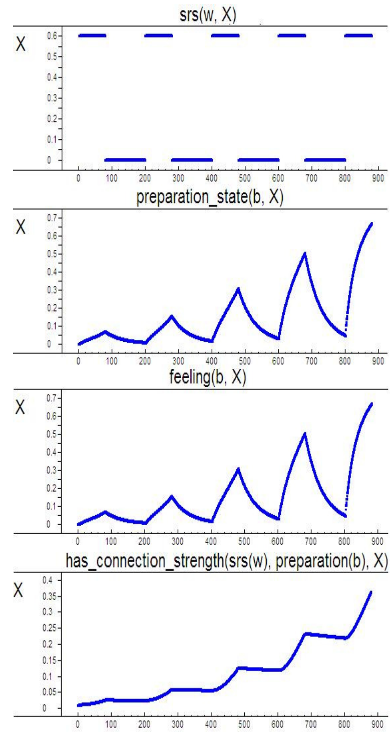

Fig. 2. Adaptation by Hebbian Learning (V_1=0.6, β=0.9, γ=0.3, η=0.01, ζ=0.0005)

5 Comparative Analysis of the Three Adaptation Models

This section compares the results of simulation for the three adaptive dynamic modeling approaches and presents some of the results of a mathematical analysis of the model that has been undertaken.

Comparison of simulation results

For a brief overview of a comparison of simulation results, see Table 1. The adaptation speed of the induction strength in the temporal discounting approach is faster as compared to other two approaches (see Fig. 2 to 4), even though the discounting rate for memory traces approach was set at a higher value. Notice also that the pattern of the learning curves differs: Hebbian learning shows a slow start, but later on gets more speed, while the other cases show a more or less opposite pattern. Moreover, the activation levels of the preparation and feeling states in the temporal discounting and memory traces approach increase faster as compared to the Hebbian approach.

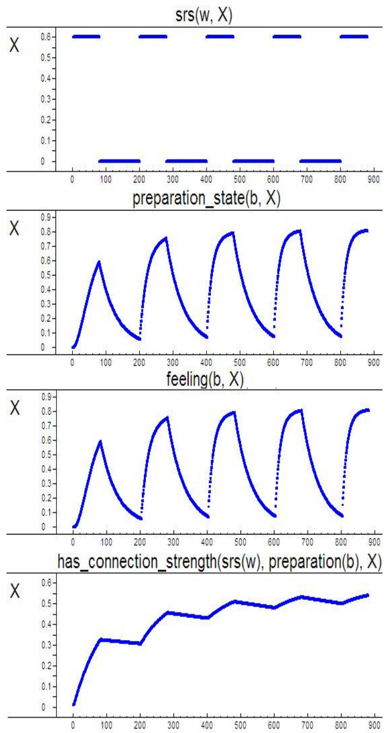

Fig. 3: Adaptation by Temporal Discounting (V=0.6, β=0.9, γ=0.3, α=0.01, ζ=0.0005)

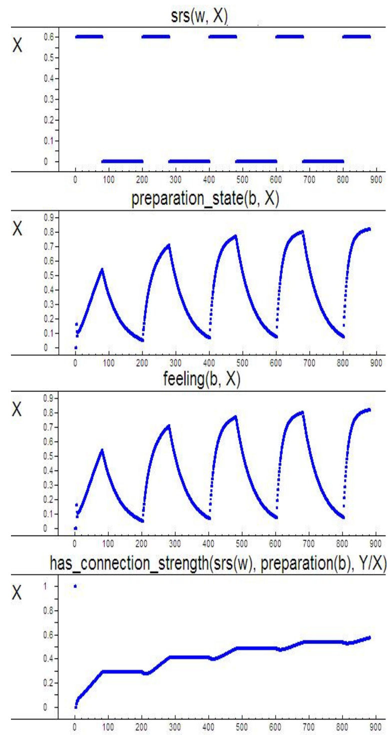

Fig. 4: Adaptation by Memory Traces (V_1=0.6, β=0.9, γ=0.3, α=0.5, ζ=0.0005)

The memory traces approach persist the strength more as compared to the temporal discounting approach, and the temporal discounting approach persist the strength more as compared to the Hebbian approach.

Below, some of the results of a mathematical analysis of possible equilibria of the model that has been undertaken are discussed. For an overview see also Table 2. Note that an equilibrium of the model involves constant values both for activation levels and connection strengths and it is also assumed that the stimulus is constant. Moreover, to avoid too many exceptional cases, it is assumed that the values for parameters γ, η, ζ are nonzero.

Table 1. Overview of outcomes of the example simulations for the three approaches

	Hebbian Learning	**Temporal Discounting**	**Memory Traces**
Maximal induction strength reached	*0.8*	*0.55*	*0.7*
Adaptation speed	lowest	highest	middle
Adaptation pattern	slow start – fast finish	fast start – slow finish	fast start – slow finish
Extinction speed	highest	middle	lowest
Maximal preparation and feeling levels	*0.9*	*0.8*	*0.9*
Speed in preparation and feeling levels	lowest	highest	middle

Equilibrium for activation level of preparation state

First the equilibrium for the activation level of the preparation state has been investigated, expressed in the following relation derived from LP3 (note that due to LP4 the feeling level V_2 and preparation level V_3 are equal in an equilibrium; see):

$$\gamma(\beta(1-(1-\omega_1 V_1)(1-\omega_2 V_2)) + (1-\beta)\, \omega_1 \omega_2 V_1 V_2 - V_2) = 0$$

Assuming $\gamma \neq 0$, this equation can be solved by expressing V_2 into the other variables among which V_1 that denotes the activation level of the sensory representation.

$$\beta(1-(1-\omega_1 V_1)(1-\omega_2 V_2)) + (1-\beta)\, \omega_1 \omega_2 V_1 V_2 - V_2 = 0 \;\Leftrightarrow$$

$$\beta(\omega_1 V_1 + \omega_2 V_2 - \omega_1 \omega_2 V_1 V_2) = V_2 - \omega_1 \omega_2 V_1 V_2 + \beta\, \omega_1 \omega_2 V_1 V_2 \;\Leftrightarrow$$

$$\beta\omega_1 V_1 = V_2 - \omega_1 \omega_2 V_1 V_2 + \beta\, \omega_1 \omega_2 V_1 V_2 - \beta\omega_2 V_2 + \beta\omega_1 \omega_2 V_1 V_2 \;\Leftrightarrow$$

$$\beta\omega_1 V_1 = V_2 - \beta\omega_2 V_2 + (2\beta-1)\omega_1 \omega_2 V_1 V_2 \;\Leftrightarrow\; \beta\omega_1 V_1 = (1 - \beta\omega_2 + (2\beta-1)\omega_1 \omega_2 V_1) V_2 \;\Leftrightarrow$$

$$V_2 = \beta\omega_1 V_1 / (1 - \beta\omega_2 + (2\beta-1)\omega_1 \omega_2 V_1) \qquad \textbf{(1)}$$

For 3 example values of β the equation $\beta\omega_1 V_1 = (1 - \beta\omega_2 + (2\beta-1)\omega_1\omega_2 V_1)V_2$ reduces to

$$\beta = 0 \qquad 0 = (1 - \omega_1 \omega_2 V_1) V_2 \qquad \Leftrightarrow\; V_2 = 0 \quad \text{OR} \quad \omega_1 = \omega_2 = V_1 = 1$$

$$\beta = 0.5 \qquad 0.5\, \omega_1 V_1 = (1 - 0.5\omega_2) V_2 \quad \Leftrightarrow\; \omega_1 V_1 = (2 - \omega_2) V_2 \Leftrightarrow V_2 = \omega_1 V_1 / (2 - \omega_2)$$

$$\beta = 1 \qquad \omega_1 V_1 = (1 - \omega_2 + \omega_1 \omega_2 V_1) V_2 \Leftrightarrow\; V_2 = \omega_1 V_1 / (1 - \omega_2 + \omega_1 \omega_2 V_1)$$

For $V_1 = 1$ equation (1) is reduced to

$$V_2 = \beta\omega_1 / (1 - \beta\omega_2 + (2\beta-1)\omega_1 \omega_2)$$

For $\omega_2 = 1$ this is

$$V_2 = \beta\omega_1 V_1 / (1 - \beta + (2\beta-1)\omega_1 V_1)$$

For $\omega_2 = 1$ this is for the three values of β

$$\beta = 0 \qquad V_2 = 0 \quad OR \quad \omega_1 = \omega_2 = V_1 = 1$$

$$\beta = 0.5 \qquad V_2 = \omega_1 V_1$$

$$\beta = 1 \qquad V_2 = \omega_1 V_1 / (\omega_1 V_1) = 1$$

For both $V_1 = 1$ and $\omega_2 = 1$ the equation is

$$V_2 = \beta\omega_1 / (1 - \beta + (2\beta-1)\omega_1) \;\Leftrightarrow\; V_2 = \beta / ((1 - \beta)/\omega_1 + 2\beta - 1) \qquad \textbf{(2)}$$

Hebbian approach

Next the equilibrium for the connection strength ω_1 from sensory representation to preparation is analyzed for the Hebbian approach; this is expressed in the following relation derived from LP6:

$$\eta V_1 V_2 (1 - \omega_1) - \zeta\omega_1 = 0$$

For cases that V_1 and V_2 are nonzero, this can be used to express ω_1 as follows

$$\eta V_1 V_2 = (\zeta + \eta V_1 V_2)\, \omega_1 \Leftrightarrow\; \omega_1 = \eta V_1 V_2 / (\zeta + \eta V_1 V_2) \Leftrightarrow$$

$$\omega_1 = 1 / (\zeta/(\eta V_1 V_2) + 1) \qquad \textbf{(3)}$$

In principle the two equations (1) and (3) in ω_1 and V_2 can be explicitly solved, but for the general case this provides rather complex expressions for ω_1 and V_2. Therefore only the specific case $V_1 = 1$ and $\omega_2 = 1$ is pursued further.

Case $V_1 = 1$ and $\omega_2 = 1$ for the Hebbian approach

For $V_1 = 1$ equation (3) can be rewritten into

$$\omega_1 = 1 / (\zeta/(\eta V_2) + 1) \;\Leftrightarrow\; 1/\omega_1 = (\zeta/(\eta V_2) + 1)$$

Substituting the above equation in the expression (2) for V_2 provides:

$$V_2 = \beta / ((1 - \beta)/\omega_1 + 2\beta - 1) \Leftrightarrow ((1 - \beta)/\omega_1 + 2\beta - 1)\, V_2 = \beta \Leftrightarrow$$

$$((1 - \beta)(\zeta/(\eta V_2) + 1) + 2\beta - 1)\, V_2 = \beta \Leftrightarrow (1 - \beta)(\zeta/\eta + V_2) + 2\beta V_2 - V_2 = \beta \Leftrightarrow$$

$$(\zeta/\eta + V_2) - \beta\, (\zeta/\eta + V_2) + 2\beta V_2 - V_2 = \beta \Leftrightarrow\; \zeta/\eta - \beta\, (\zeta/\eta + V_2) + 2\beta V_2 = \beta \Leftrightarrow$$

$$\zeta/\eta - \beta\, \zeta/\eta + \beta V_2 = \beta \Leftrightarrow \beta V_2 = \beta - \zeta/\eta + \beta\, \zeta/\eta \;\Leftrightarrow$$

$$V_2 = 1 - (1 - \beta)\zeta/\beta\eta = 1 - (1/\beta - 1)\zeta/\eta$$

From this an expression for ω_1 can be determined:

$\omega_1 = 1 / (\zeta/(\eta V_2) + 1) \Leftrightarrow \omega_1 = 1 / (\zeta/(\eta(1 - (1/\beta - 1)\zeta / \eta) + 1) \Leftrightarrow$
$\omega_1 = 1 / (\zeta/((\eta - (1/\beta - 1)\zeta) + 1) \Leftrightarrow \omega_1 = (\eta - (1/\beta - 1)\zeta) / (\zeta + (\eta - (1/\beta - 1)\zeta)) \Leftrightarrow$
$\omega_1 = (\eta - (1/\beta - 1)\zeta) / (2\zeta + (\eta - (1/\beta)\zeta))$

For the three example values of β the equation $\beta V_2 = \beta - \zeta/\eta + \beta \; \zeta/\eta$ reduces to

$\beta = 0$ This is impossible for nonzero ζ, V_1 and V_2
$\beta = 0.5$ $0.5 V_2 = 0.5 - \zeta/\eta + 0.5 \; \zeta/\eta$
$V_2 = 1 - \zeta/\eta$
$\beta = 1$ $V_2 = 1 - \zeta/\eta + 1 \; \zeta/\eta = 1$

Temporal discounting approach

For the temporal discounting approach the variable V_2 does not play a role in the adaptation method. Temporal relation LP7a implies that for an equilibrium it holds:

$\alpha(V_1 - \omega_1) - \zeta\omega_1 = 0 \Leftrightarrow \alpha(V_1 - \omega_1) = \zeta\omega_1 \Leftrightarrow \alpha V_1 = \zeta\omega_1 + \alpha\omega_1 = (\zeta + \alpha)\,\omega_1 \Leftrightarrow$
$\omega_1 = \alpha V_1 / (\zeta + \alpha) = V_1 / (\zeta/\alpha + 1)$ **(4)**

Note that since $(\zeta/\alpha + 1) \geq 1$ it follows that always $\omega_1 \leq V_1$ which indeed was observed in the simulations.

Case $V_1 = 1$ and $\omega_2 = 1$ for the temporal discounting approach

For $V_1 = 1$ expression (4) becomes $\omega_1 = 1 / (\zeta/\alpha + 1)$. By equation (2) for V_2 it follows

$V_2 = \beta / ((1 - \beta)/ \omega_1 + 2\beta - 1) \Leftrightarrow V_2 = \beta / ((1 - \beta)(\zeta/\alpha + 1) + 2\beta - 1) \Leftrightarrow$
$V_2 = \beta / ((\zeta/\alpha + 1) - \beta(\zeta/\alpha + 1) + 2\beta - 1) \Leftrightarrow V_2 = \beta / (\zeta/\alpha - \beta(\zeta/\alpha + 1) + 2\beta) \Leftrightarrow$
$V_2 = \beta / (\zeta/\alpha - \beta(\zeta/\alpha) + \beta) \Leftrightarrow V_2 = \beta / (\zeta/\alpha\,(1 - \beta) + \beta) \Leftrightarrow$
$V_2 = 1 / (\zeta/\alpha\,(1/\beta - 1) + 1)$

For the three example values of β the equation $V_2 = \beta / (\zeta/\alpha\,(1 - \beta) + \beta)$ reduces to

$\beta = 0$ $V_2 = 0$
$\beta = 0.5$ $V_2 = 0.5 / (\zeta/\alpha\, 0.5 + 0.5) = 1 / (\zeta/\alpha + 1)$
$\beta = 1$ $V_2 = 1$

Memory traces approach

For the memory traces approach in an equilibrium the expression $\omega_1 = Y/X$ should remain the same, although in principle X and Y still may change. So the criterion is

$Y + \Delta Y / X + \Delta X = Y/X$

which can be rewritten as

$(Y + \Delta Y)\, X = (X + \Delta X)\, Y \Leftrightarrow YX + \Delta Y\, X = XY + \Delta X\, Y \Leftrightarrow X\, \Delta Y = Y\, \Delta X \Leftrightarrow$
$\Delta Y / \Delta X = Y / X$

Therefore according to temporal relation LP8a for an equilibrium it holds:

$(\alpha V_1 V_2 - \zeta Y) / (\alpha V_1 - \zeta X) = Y/X$

This can be rewritten as:

$(\alpha V_1 V_2 - \zeta Y)\, X = (\alpha V_1 - \zeta X)\, Y \Leftrightarrow \alpha V_1 V_2 X - \zeta YX = \alpha V_1 Y - \zeta XY \Leftrightarrow$
$\alpha V_1 V_2 X = \alpha V_1 Y \Leftrightarrow$
$V_2 X = Y$ OR $V_1 = 0 \Leftrightarrow$
$\omega_1 = Y / X = V_2$ OR $V_1 = 0$

This can be used to obtain a value for both ω_1 and V_2 as follows:

$\beta\omega_1 V_1 = (1 - \beta\omega_2 + (2\beta - 1)\omega_1\omega_2 V_1)V_2 \Leftrightarrow \beta V_2 V_1 = (1 - \beta\omega_2 + (2\beta - 1) V_2\omega_2 V_1)V_2 \Leftrightarrow$
$\beta V_1 = 1 - \beta\omega_2 + (2\beta - 1) V_2\omega_2 V_1$ OR $V_2 = 0 \Leftrightarrow$
$\beta V_1 - 1 + \beta\omega_2 = (2\beta - 1) V_2\omega_2 V_1$ OR $V_2 = 0 \Leftrightarrow$
$\omega_1 = V_2 = (\beta V_1 - 1 + \beta\omega_2) / (2\beta - 1)\omega_2 V_1$ OR $V_2 = 0$ OR
$\beta = 0.5 \;\&\; V_1 = 2 - \omega_2 \Leftrightarrow$

$$\omega_1 = V_2 = (\beta V_1 - 1 + \beta\omega_2) / (2\beta\text{-}1)\omega_2 V_1 \quad \text{OR} \quad V_2 = 0 \quad \text{OR} \quad \beta = 0.5 \;\&\; V_1 = \omega_2 = 1 \qquad (5)$$

For the three specific example values of β the following is obtained.

$\beta = 0$	$\omega_1 = V_2 = 0$	OR	$\omega_1 = \omega_2 = V_1 = V_2 = 1$
$\beta = 0.5$	$\omega_1 = V_2 = \omega_1 V_1 / (2 - \omega_2) \Leftrightarrow$		
	$\omega_1 = V_2 = 0$	OR	$1 = V_1 / (2 - \omega_2) \Leftrightarrow$
	$\omega_1 = V_2 = 0$	OR	$V_1 = 2 - \omega_2 \Leftrightarrow$
	$\omega_1 = V_2 = 0$	OR	$V_1 = \omega_2 = 1 \;\&\; \omega_1 = V_2$
$\beta = 1$	$\omega_1 = V_2 = \omega_1 V_1 / (1 - \omega_2 + \omega_1\omega_2 V_1) \Leftrightarrow$		
	$\omega_1 = V_2 = 0$	OR	$1 = V_1 / (1 - \omega_2 + \omega_1\omega_2 V_1) \Leftrightarrow$
	$\omega_1 = V_2 = 0$	OR	$V_1 = 1 - \omega_2 + \omega_1\omega_2 V_1 \Leftrightarrow$
	$\omega_1 = V_2 = 0$	OR	$(1\text{-}\omega_1\omega_2)V_1 = 1 - \omega_2 \Leftrightarrow$
	$\omega_1 = V_2 = 0$	OR	$V_1 = (1 - \omega_2)/(1\text{-}\omega_1\omega_2) \;\&\; \omega_1 = V_2 \Leftrightarrow$
	$\omega_1 = V_2 = 0$	OR	$V_1 = (1 - \omega_1\omega_2 + \omega_1\omega_2 - \omega_2)/(1\text{-}\omega_1\omega_2) \;\&\; \omega_1 = V_2$
	$\omega_1 = V_2 = 0$	OR	$V_1 = 1\text{-} (1\text{-}\omega_1)\, \omega_2/(1\text{-}\omega_1\omega_2) \;\&\; \omega_1 = V_2$

Case $V_1 = 1$ and $\omega_2 = 1$ for the memory traces approach

For $\omega_2 = 1$ this can be simplified as

$$\omega_1 = V_2 = (\beta V_1 - 1 + \beta) / (2\beta\text{-}1)V_1 \quad \text{OR} \quad V_2 = 0 \quad \text{OR} \quad \beta = 0.5 \;\&\; V_1 = 1$$

and when also $V_1 = 1$ it becomes:

$$\omega_1 = V_2 = (\beta - 1 + \beta) / (2\beta\text{-}1) = 1 \quad \text{OR} \quad V_2 = 0 \quad \text{OR} \quad \beta = 0.5 \;\&\; V_1 = 1$$

Table 2: Overview of expressions for a number of possible equilibria

	General case	*Case $V_1 = 1$ & $\omega_2 = 1$*
Hebbian Learning	$V_2 = \beta\omega_1 V_1 / (1 - \beta\omega_2 + (2\beta - 1)\omega_1\omega_2 V_1)$ $\omega_1 = \eta V_1 V_2 / (\zeta + \eta V_1 V_2) = 1 / (\zeta/(\eta V_1 V_2) + 1)$	$\omega_1 = (((\eta - (1/\beta - 1)\zeta) / (2\zeta + (\eta - (1/\beta)\zeta))$ $V_2 = 1 - (1 - \beta)\zeta/\beta\eta = 1 - (1/\beta - 1)\zeta/\eta$
Temporal Discounting	$V_2 = \beta\omega_1 V_1 / (1 - \beta\omega_2 + (2\beta\text{-}1)\omega_1\omega_2 V_1)$ $\omega_1 = V_1 / (\zeta/\alpha + 1) \leq V_1$	$\omega_1 = 1 / (\zeta/\alpha + 1)$ $V_2 = 1 / (\zeta/\alpha\,(1/\beta\text{-} 1) + 1)$
Memory Traces	$\omega_1 = V_2 = (\beta V_1 - 1 + \beta\omega_2) / (2\beta\text{-}1)\omega_2 V_1$	$\omega_1 = V_2 = 1$

5 Discussion

In this paper a number of learning models for the induction strength of an emotional response on a stimulus were analysed and compared. The introduced models on the one hand describe more specifically how a stimulus generates an emotional response that is felt, and on the other hand how the induction strength of the experienced emotional response is adapted over time. For feeling the emotion, a converging recursive body loop was used, based on elements taken from [4], [7], [8]. One of the adaptation models was based on a Hebbian learning rule cf. [1], [9], [10], [16]. Another one was based on temporal discounting, and the third one was based on memory traces. The models were specified in the hybrid dynamic modelling language LEADSTO, and simulations were performed in its software environment; cf. [2]. Moreover, a mathematical analysis was made to determine possible equilibria. In the comparison differences in adaptation speed and pattern have been found, and in the maximal value of the induction strength.

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