

# On Rationality of Decision Models Incorporating Emotion-Related Valuing and Hebbian Learning

Jan Treur<sup>1</sup>, Muhammad Umair<sup>1,2</sup>

<sup>1</sup>VU University Amsterdam, Agent Systems Research Group  
De Boelelaan 1081, 1081 HV Amsterdam, The Netherlands

<sup>2</sup>COMSATS Institute of Information Technology, Lahore, Pakistan  
Email: <sup>1</sup>{j.treur, m.umair}@vu.nl, <sup>2</sup>mumair@ciitlahore.edu.pk  
URL: <http://www.few.vu.nl/~{treur, mumair}>

**Abstract.** In this paper an adaptive decision model based on predictive loops through feeling states is analysed from the perspective of rationality. Four different variations of Hebbian learning are considered for different types of connections in the decision model. To assess the extent of rationality, a measure is introduced reflecting the environment's behaviour. Simulation results and the extents of rationality of the different models over time are presented and analysed.

**Keywords:** decision making, cognitive agent model, emotion, Hebbian learning

## 1 Introduction

In decision making tasks different options are compared in order to make a reasonable choice out of them. Options usually have emotional responses associated to them relating to a prediction of a rewarding or aversive consequence. In decisions such an emotional valuing often plays an important role. In recent neurological literature this has been related to a notion of value as represented in the amygdala [1, 2, 14, 15, 17]. In making decisions experiences with the environment (from the past) play an important role. By learning processes the decision making mechanism is adapted to these experiences, so that the decision choices made are reasonable or in some way rational, given the environment reflected in these past experiences. In this sense the emotion-related valuing in the amygdala as a basis for decision making may be expected to satisfy some rationality criterion. The question to which extent this indeed is the case for certain biologically plausible learning models is the focus of this paper.

The decision model considered involves predictive as-if body loops through feeling states in order to reach decisions for selections of actions (e.g., [3, 6, 8]). The type of learning considered is Hebbian learning (cf. [10, 12]), in four different variations by applying it to different types of connections in the decision model. To assess their extent of rationality, a rationality measure is introduced reflecting the environment's behaviour.

In this paper, in Section 2 the decision model and the different variants of adaptivity considered are introduced. Section 3 presents a number of simulation results. In Section 4 measures for rationality are discussed, and the different models are evaluated. Finally, Section 5 is a discussion.

## 2 The Adaptive Decision Models Addressed

Traditionally an important function attributed to the amygdala concerns the context of fear. However, in recent years much evidence on the amygdala in humans has been collected showing a function beyond this fear context. In humans many parts of the prefrontal cortex (PFC) and other brain areas such as hippocampus, basal ganglia, and hypothalamus have extensive, often bidirectional connections with the amygdala [11, 15, 18]. A role of amygdala activation has been found in various tasks involving emotional aspects [16]. Usually emotional responses are triggered by stimuli for which a prediction is possible of a rewarding or aversive consequence. Feeling these emotions represents a way of experiencing the value of such a prediction: to which extent it is positive or negative. This idea of value is also the basis of work on the neural basis of economic choice in neuroeconomics. In particular, in decision-making tasks where different options are compared, choices have been related to a notion of value as represented in the amygdala [1, 2, 14, 15, 17, 19].

Any mental state in a person induces emotions felt by this person, as described in [7, 8, 9]; e.g., [9], p. 93: ‘... few if any exceptions of any object or event, actually present or recalled from memory, are ever neutral in emotional terms. Through either innate design or by learning, we react to most, perhaps all, objects with emotions, however weak, and subsequent feelings, however feeble.’ More specifically, in this paper it is assumed that responses in relation to a sensory representation state roughly proceed according to the following causal chain for a *body loop* (based on elements from [4, 7, 8]):

sensory representation → preparation for bodily response → body state modification → sensing body state → sensory representation of body state → induced feeling

In addition, an *as-if body loop* uses a direct causal relation

preparation for bodily response → sensory representation of body state

as a shortcut in the causal chain; cf. [7]. This can be considered a prediction of the action effect by internal simulation (e.g., [13]). The resulting induced feeling is a valuation of this prediction. If the level of the feeling (which is assumed positive) is high, a positive valuation is obtained.

The body loop (or as-if body loop) is extended to a recursive (as-if) body loop by assuming that the preparation of the bodily response is also affected by the level of the induced feeling:

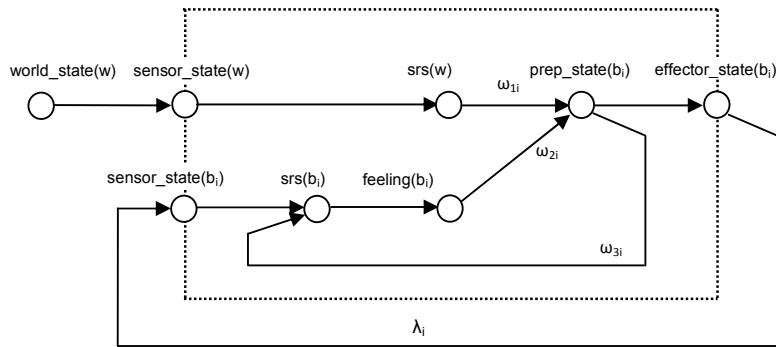
induced feeling → preparation for the bodily response

Such recursion is suggested in [8], pp. 91-92, noticing that what is felt is a body state which is under control of the person: ‘The brain has a direct means to respond to the object as feelings unfold because the object at the origin is inside the body, rather than external to it. The brain can act directly on the very object it perceives. (...) The object at the origin on the one hand, and the brain map of that object on the other, can influence each other in a sort of reverberative process that is not to be found, for example, in the perception of an external object.’ In this way the valuation of the prediction affects the preparation. A high valuation will strengthen activation of the preparation.

Informally described theories in scientific disciplines, for example, in biological or neurological contexts, often are formulated in terms of causal relationships or in

terms of dynamical systems. To adequately formalise such a theory the hybrid dynamic modelling language LEADSTO has been developed that subsumes qualitative and quantitative causal relationships, and dynamical systems; cf. [4]. This language has been proven successful in a number of contexts, varying from biochemical processes that make up the dynamics of cell behaviour to neurological and cognitive processes; e.g. [4, 5]. Within LEADSTO a *dynamic property* or temporal relation  $a \rightarrow_D b$  denotes that when a state property  $a$  occurs, then after a certain time delay (which for each relation instance can be specified as any positive real number  $D$ ), state property  $b$  will occur. Below, this  $D$  is the time step  $\Delta t$ . A dedicated software environment is available to support specification and simulation. A specification of the model in LEADSTO format can be found in Appendix A.

An overview of the basic decision model involving the generation of emotional responses and feelings is depicted in Fig. 1. This picture also shows representations from the detailed specifications explained below. However, note that the precise numerical relations are not expressed in this picture, but in the detailed specifications below, through local properties LP0 to LP6.



**Fig. 1.** Overview of the model for decision making evaluated from a rationality perspective

Note that the effector state for  $b_i$  combined with the (stochastic) effectiveness of executing  $b_i$  in the world (indicated by *effectiveness rate*  $\lambda_i$  between 0 and 1) activates the sensor state for  $b_i$  via body loop as described above. By a recursive as-if body loop each of the preparations for  $b_i$  generates a level of feeling for  $b_i$  which is considered a valuation of the prediction of the action effect by internal simulation. This in turn affects the level of the related action preparation for  $b_i$ . Dynamic interaction within these loops results in equilibrium for the strength of the preparation and of the feeling, and depending on these values, the action is actually activated with a certain intensity. The specific strengths of the connections from the sensory representation to the preparations, and within the recursive as-if body loops can be innate, or are acquired during lifetime. The computational model is based on such neurological notions as valuing in relation to feeling, body loop and as-if body loop. The adaptivity in the model is based on Hebbian learning. The detailed specification of the model is presented below starting with how the world state is sensed.

**LP0 Sensing a world state**

If world state property  $w$  occurs of level  $V_1$   
 and the sensor state for  $w$  occurs has level  $V_2$   
 then the sensor state for  $w$  will have level  $V_2 + \gamma[V_1 - V_2] \Delta t$ .

$$\frac{dsensor\_state(w)}{dt} = \gamma[world\_state(w) - sensor\_state(w)]. \quad (1)$$

From the sensor state, sensory representation is updated by dynamic property LP1.

**LP1 Generating a sensory representation for a sensed world state**

If the sensor state for world state property  $w$  has level  $V_1$ ,  
 and the sensory representation for  $w$  has level  $V_2$   
 then the sensory representation for  $w$  will have level  $V_2 + \gamma[V_1 - V_2] \Delta t$ .

$$\frac{dsrs(w)}{dt} = \gamma[sensor\_state(w) - srs(w)] \quad (2)$$

The combination function  $h$  to combine two inputs which activate a subsequent state uses the threshold function  $th$  thus keeping the resultant value in the range  $[0, 1]$  :

$$th(\sigma, \tau, V) = \left( \frac{1}{1+e^{-\sigma(V-\tau)}} - \frac{1}{1+e^{\sigma\tau}} \right) * (1 + e^{-\sigma\tau}) \quad (3)$$

where  $\sigma$  is the steepness and  $\tau$  is the threshold value. The combination function is:

$$h(\sigma, \tau, V_1, V_2, \omega_1, \omega_2) = th(\sigma, \tau, \omega_1 V_1 + \omega_2 V_2) \quad (4)$$

where  $V_1$  and  $V_2$  are the current activation level of the states and  $\omega_1$  and  $\omega_2$  are the connection strength of the links from these states.

Dynamic property LP2 describes the update of the preparation state for  $b_i$  from the sensory representation of  $w$  and feeling of  $b_i$ .

**LP2 From sensory representation and feeling to preparation of a body state**

If a sensory representation for  $w$  with level  $V$  occurs  
 and the feeling associated with body state  $b_i$  has level  $V_i$   
 and the preparation state for  $b_i$  has level  $U_i$   
 and  $\omega_{1i}$  is the strength of the connection from sensory representation for  $w$  to preparation for  $b_i$   
 and  $\omega_{2i}$  is the strength of the connection from feeling of  $b_i$  to preparation for  $b_i$   
 and  $\sigma_i$  is the steepness value for preparation of  $b_i$  and  $\tau_i$  is the threshold value for preparation of  $b_i$   
 and  $\gamma_i$  is the person's flexibility for bodily responses  
 then after  $\Delta t$  the preparation state for body state  $b_i$  will have level  $U_i + \gamma_i [h(\sigma_i, \tau_i, V, V_i, \omega_{1i}, \omega_{2i}) - U_i] \Delta t$ .

$$\frac{dpreparation(b_i)}{dt} = \gamma_i [h(\sigma_i, \tau_i, srs(w), feeling(b_i), \omega_{1i}, \omega_{2i}) - preparation(b_i)] \quad (5)$$

Dynamic property LP3 describes the update of the sensory representation of a body state from the respective preparation state and sensor state.

**LP3 From preparation and sensor state to sensory representation of a body state**

If preparation state for  $b_i$  has level  $X_i$   
 and sensor state for  $b_i$  has level  $V_i$   
 and the sensory representation for body state  $b_i$  has level  $U_i$   
 and  $\omega_{3i}$  is the strength of the connection from preparation state for  $b_i$  to sensory representation for  $b_i$   
 and  $\sigma_i$  is the steepness value for sensory representation of  $b_i$   
 and  $\tau_i$  is the threshold value for sensory representation of  $b_i$   
 and  $\gamma_2$  is the person's flexibility for bodily responses  
 then after  $\Delta t$  the sensory representation for  $b_i$  will have level  $U_i + \gamma_2 [h(\sigma_i, \tau_i, X_i, V_i, \omega_{3i}, 1) - U_i] \Delta t$ .

$$\frac{dsrs(b_i)}{dt} = \gamma [h(\sigma_i, \tau_i, preparation(b_i), sensor\_state(b_i), \omega_3, 1) - srs(b_i)] \quad (6)$$

Dynamic property LP4 describes update of feeling  $b_i$  from the sensory representation.

**LP4 From sensory representation of a body state to feeling**

If the sensory representation for body state  $b_i$  has level  $V_1$ ,  
and  $b_i$  is felt with level  $V_2$   
then  $b_i$  will be felt with level  $V_2 + \gamma[V_1 - V_2] \Delta t$ .

$$\frac{dfeeling(b_i)}{dt} = \gamma [srs(b_i) - feeling(b_i)] \quad (7)$$

LP5 describes how the effector state for  $b_i$  is updated from the preparation state.

**LP5 From preparation to effector state**

If the preparation state for  $b_i$  has level  $V_1$ ,  
and the effector state for body state  $b_i$  has level  $V_2$ .  
then the effector state for body state  $b_i$  will have level  $V_2 + \gamma[V_1 - V_2] \Delta t$ .

$$\frac{deffector\_state(b_i)}{dt} = \gamma [preparation\_state(b_i) - effector\_state(b_i)] \quad (8)$$

LP6 describes update of the sensor state for  $b_i$  from the effector state for  $b_i$ .

**LP6 From effector state to sensor state of a body state**

If the effector state for  $b_i$  has level  $V_1$ ,  
and  $\lambda_i$  is world preference/ recommendation for the option  $b_i$   
and the sensor state for body state  $b_i$  has level  $V_2$ ,  
then the sensor state for  $b_i$  will have level  $V_2 + \gamma[\lambda_i V_1 - V_2] \Delta t$

$$\frac{dsensor\_state(b_i)}{dt} = \gamma [\lambda_i effector\_state(b_i) - sensor\_state(b_i)] \quad (9)$$

For the considered case study it was assumed that three options are available to the agent and the objective is to see how rationally an agent makes its decisions using a given adaptive model: under constant as well as in stochastic world characteristics and in both cases static as well as changing worlds. The dynamic properties LP7 to LP9 describe a Hebbian learning mechanism for the connection strengths

- (A) from sensory representation for  $w$  to preparation for option  $b_i$
- (B) from feeling  $b_i$  to preparation for  $b_i$
- (C) from preparation for  $b_i$  to sensory representation of  $b_i$

These have been explored separately (A), (B), or (C), and in combination (ABC).

**LP7 Hebbian learning (A): connection from sensory representation of  $w$  to preparation of  $b_i$**

If the connection from sensory representation of  $w$  to preparation of  $b_i$  has strength  $\omega_{1i}$   
and the sensory representation for  $w$  has level  $V$   
and the preparation of  $b_i$  has level  $V_i$   
and the learning rate from sensory representation of  $w$  to preparation of  $b_i$  is  $\eta$   
and the extinction rate from sensory representation of  $w$  to preparation of  $b_i$  is  $\zeta$   
then after  $\Delta t$  the connection from sensory representation of  $w$  to preparation of  $b_i$  will have strength  $\omega_{1i} + (\eta V V_i (1 - \omega_{1i}) - \zeta \omega_{1i}) \Delta t$ .

$$\frac{d\omega_{1i}}{dt} = \eta srs(w) preparation(b_i) (1 - \omega_{1i}) - \zeta \omega_{1i} \quad (10)$$

**LP8 Hebbian learning (B): connection from feeling  $b_i$  to preparation of  $b_i$**

If the connection from feeling associated with body state  $b_i$  to preparation of  $b_i$  has strength  $\omega_{2i}$   
and the feeling for  $b_i$  has level  $V_i$

and the preparation of  $bi$  has level  $U_i$   
and the learning rate from feeling of  $bi$  to preparation of  $b_i$  is  $\eta$   
and the extinction rate from feeling of  $bi$  to preparation of  $b_i$  is  $\zeta$   
then after  $\Delta t$  the connection from feeling of  $bi$  to preparation of  $b_i$  will have  
strength  $\omega_{2i} + (\eta V_i U_i (1 - \omega_{2i}) - \zeta \omega_{2i}) \Delta t$ .

$$\frac{d\omega_{2i}}{dt} = \eta \text{feeling}(b_i) \text{preparation}(b_i) (1 - \omega_{2i}) - \zeta \omega_{2i} \quad (11)$$

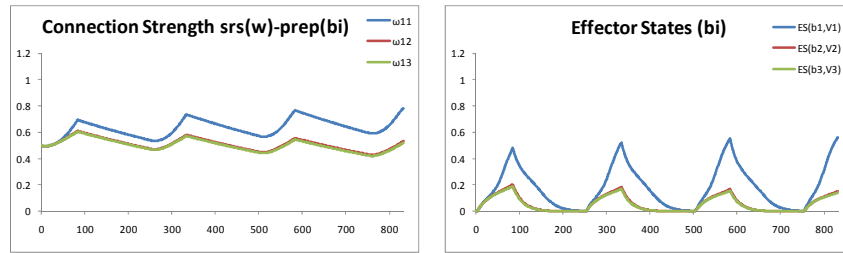
**LP9 Hebbian learning (C): connection from preparation of  $b_i$  to sensory representation of  $b_i$**

If the connection from preparation of  $b_i$  to sensory representation of  $b_i$  has strength  $\omega_{3i}$   
and the preparation of  $bi$  has level  $V_i$  and the sensory representation of  $bi$  has level  $U_i$   
and the learning rate from preparation of  $bi$  to sensory representation of  $b_i$  is  $\eta$   
and the extinction rate from preparation of  $bi$  to sensory representation of  $b_i$  is  $\zeta$   
then after  $\Delta t$  the connection from preparation of  $bi$  to sensory representation of  $bi$  will have  
strength  $\omega_{3i} + (\eta V_i U_i (1 - \omega_{3i}) - \zeta \omega_{3i}) \Delta t$ .

$$\frac{d\omega_{3i}}{dt} = \eta \text{preparation}(b_i) \text{srs}(b_i) (1 - \omega_{3i}) - \zeta \omega_{3i} \quad (12)$$

### 3 Simulation Results

In this section some of the simulation results, performed using numerical software, are described in detail. The simulation results address different scenarios reflecting different types of world characteristics, from constant to stochastic world, and from static to changing world. Moreover, learning the connections was done one at a time (A), (B), (C), and learning multiple connections simultaneously (ABC). Due to space limitation the graphs for only (A) are shown here. A summary of the results is given in Table 1. Results for the rationality factors are presented in the next section. For all simulation results shown, time is on the horizontal axis whereas the vertical axis shows the activation level of the different states. Step size for all simulations is  $\Delta t = 1$ . Fig. 2 shows simulation results for the model under constant, static world characteristics:  $\lambda_1 = 0.9$ ,  $\lambda_2 = 0.2$ , and  $\lambda_3 = 0.1$ . Other parameters are set as: learning rate  $\eta = 0.04$ , extinction rate  $\zeta = 0.0015$ , initial connection strength  $\omega_{2i} = \omega_{3i} = 0.8$ , speed factors  $\gamma = 1$ ,  $\gamma_1 = 0.5$ ,  $\gamma_2 = 1$ , steepness  $\sigma = 2$  and threshold  $\tau = 1.2$  for preparation state, and  $\sigma = 10$  and  $\tau = 0.3$  for sensory representation of  $b_i$ . For initial 80 time units the stimulus  $w$  is kept 1 and for next 170 time units it is kept 0 and same sequence of activation and deactivation for stimulus is repeated for rest of simulation.



**Fig. 2** Constant World: **(a)** Connection strengths (A) **(b)** Effector States for  $b_i$   
Initial values  $\omega_{11} = \omega_{12} = \omega_{13} = 0.5$ ;  $\eta = 0.04$ ,  $\zeta = 0.0015$

Moreover it depicts the situation in which only one type of links ( $\omega_{1i}$ ) is learned as specified in LP7 using the Hebbian approach (A) for the connection from sensory representation of  $w$  to preparation state for  $b_i$ . It is shown that the model adapts the connection strengths of the links  $\omega_{1i}$  according to the world characteristics given by  $\lambda_i$ . So  $\omega_{11}$  strengthens more and more over time, resulting in the higher activation level of the effector state for  $b_1$  compared to the activation level of the effector states for the other two options  $b_2$  and  $b_3$ .

Similar experiments were carried out for a stochastic world with four different cases as mentioned earlier. To simulate the stochastic world, probability distribution functions (PDF) were defined for  $\lambda_i$  according to a Normal Distribution. Using these PDFs, the random numbers were generated for  $\lambda_i$  limiting the values for the interval  $[0, 1]$  with  $\mu_1=0.9$ ,  $\mu_2=0.2$  and  $\mu_3=0.1$  for  $\lambda_i$  respectively. Furthermore the standard deviation for all  $\lambda_i$  was taken  $0.1$ . Fig. 3 shows the world state  $w$  and stochastic world characteristics  $\lambda_i$ . Fig. 4 shows the simulation results while learning is performed for the links (A) from sensory representation of  $w$  to preparation state for  $b_i$ .

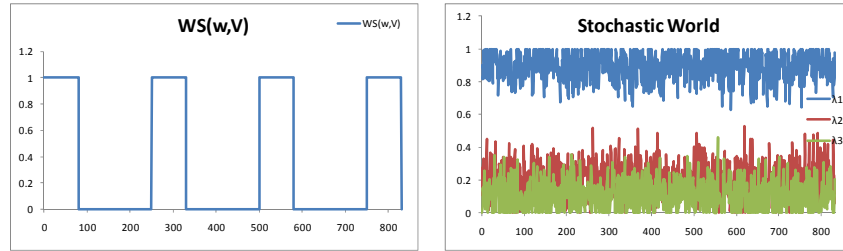


Fig. 3. Stochastic World

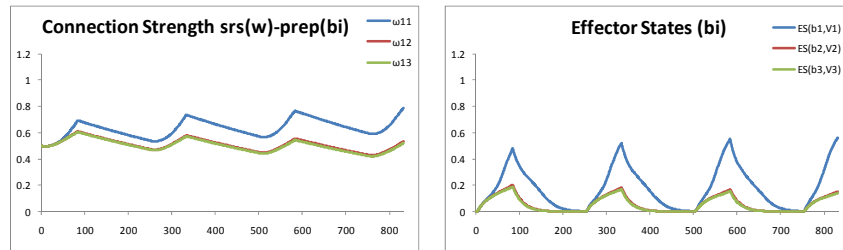


Fig. 4 Stochastic World: (a) Connection strengths (A) (b) Effector States  
Initial values  $\omega_{11}=\omega_{12}=\omega_{13}=0.5$ ;  $\eta=0.04$ ,  $\zeta=0.0015$

It can be seen from these results that also in a stochastic scenario the agent model successfully learnt the connections and adapted to the world characteristics rationally with results quite similar to the results for a static world.

Another scenario was explored in which the (stochastic) world characteristics were changing drastically from  $\mu_1=0.9$ ,  $\mu_2=0.2$  and  $\mu_3=0.1$  for  $\lambda_i$  respectively to  $\mu_1=0.1$ ,  $\mu_2=0.2$  and  $\mu_3=0.9$  for  $\lambda_i$  respectively with standard deviation of  $0.1$  for all. Fig. 5 and Fig. 6 show the results for such a scenario. The results show that the agent has successfully adapted to the changing world characteristics over time. The initial settings in this experiment were taken from the previous simulation results shown in

Figs. 3 and 4 to keep the continuity of the experiment. It can be observed that the connection strength for option 3 becomes higher compared to the other options, and consequently the value of the effector state for  $b_3$  becomes higher than for the other two by the end of experiment.

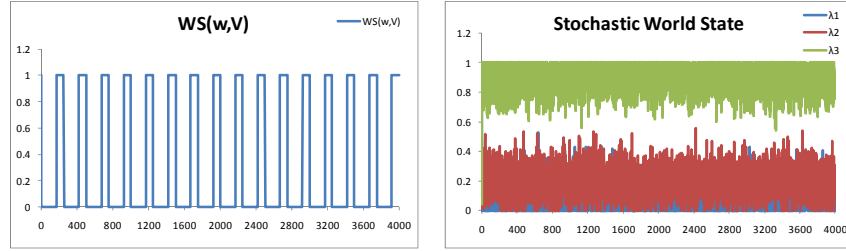


Fig. 5 World State

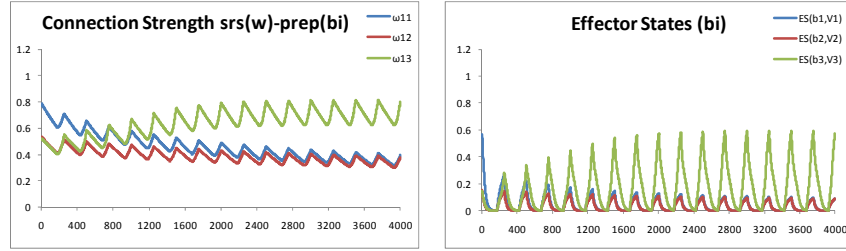


Fig. 6 Changing World: (a) Connection strengths (A) (b) Effector States  
Initial values  $\omega_{11}=0.78, \omega_{12}=0.53, \omega_{13}=0.52; \eta=0.04, \zeta=0.0015$

Table 1. Overview of the simulation results for all links (A), (B), (C) and (ABC)

Link	Scenario	$\omega_{x1}$	$\omega_{x2}$	$\omega_{x3}$	ES <sub>1</sub>	ES <sub>2</sub>	ES <sub>3</sub>
A	Static	0.78	0.53	0.52	0.56	0.15	0.14
	Stochastic	0.78	0.53	0.52	0.56	0.15	0.14
	Change World	0.40	0.38	0.80	0.09	0.09	0.58
B	Static	0.89	0.58	0.46	0.65	0.38	0.31
	Stochastic	0.89	0.59	0.47	0.65	0.39	0.32
	Change World	0.42	0.57	0.89	0.30	0.37	0.65
C	Static	0.88	0.29	0.23	0.63	0.28	0.26
	Stochastic	0.88	0.29	0.23	0.63	0.28	0.27
	Change World	0.04	0.08	0.87	0.25	0.26	0.63
ABC	Static	0.81	0.55	0.54	0.59	0.13	0.13
		0.85	0.30	0.29			
		0.85	0.30	0.29			
	Stochastic	0.80	0.55	0.54	0.57	0.13	0.13
		0.84	0.30	0.29			
		0.84	0.30	0.29			
	Changed World	0.64	0.64	0.94	0.16	0.16	0.75
		0.02	0.03	0.96			
		0.02	0.03	0.96			

Similar results were observed for all other cases (B), (C), and (ABC) as summarised in Table 1. Note that the table contains the values of different connection strengths and the activation level of effector states after the completion of simulation experiments. This shows a rational behavior of the agent in this particular scenario.

#### 4 Evaluating Agent Models on Rationality

In the previous section it was shown that the agent model behaves rationally in different scenarios. These scenarios and its different cases are elaborated in detail in the previous section, but the results were assessed with respect to their rationality in a qualitative and rather informal manner. For example, no attempt was made to assign an extent or level to the rationality observed during these experiments. The current section addresses this and to this end two different formally defined measures to assess the extent of the rationality are introduced; one rationality measure is based on a discrete scale and the other one on a continuous scale.

##### Method 1 (Discrete Rationality Measure)

The first method presented is based on the following point of departure: *an agent which has the same respective order of effector state activation levels for the different options compared to the order of world characteristics  $\lambda_i$  will be considered highly rational*. So in this method the rank of the average value  $\lambda_i$  at any given time unit is determined, and compared with the rank of the respective effector state levels. More specifically, the following formula is used to determine the irrationality factor  $IF$ .

$$IF = \sum_{i=1}^n abs(rank(es_i) - rank(\lambda_i)) \quad (13)$$

where  $n$  is the number of options available. This irrationality factor tells to which extent the agent is behaving rationally in the sense that the higher the irrationality factor  $IF$  is, the lower is the rationality of the agent. It is assumed that there is uniqueness in ranking and none of the two values assign a similar rank. To calculate the discrete rationality factor  $DRF$ , the maximum possible irrationality factor  $Max. IF$  can be determined as follows.

$$Max. IF = \frac{n(n+1)}{2} - ceiling(\frac{n}{2}) \quad (14)$$

Here  $ceiling(x)$  is the first integer higher than  $x$ . Note that  $Max. IF$  is approximately  $\frac{1}{2}n^2$ . As a higher  $IF$  means lower rationality, the discrete rationality factor  $DRF$  is calculated as:

$$DRF = 1 - \frac{IF}{Max. IF} \quad (15)$$

On this scale, for each  $n$  only a limited number of values are possible; for example, for  $n = 3$  three values are possible:  $0$ ,  $0.5$ , and  $1$ . In general  $\frac{1}{2} Max. IF + 1$  values are possible, which is approximately  $\frac{1}{4}n^2 + 1$ . As an example, suppose during a simulation average values of  $\lambda_1 = 0.107636$ ,  $\lambda_2 = 0.203044$ , and  $\lambda_3 = 0.888522$  are given, whereas the effector state values are  $ES_1 = 0.170554$ ,  $ES_2 = 0.12367$  and  $ES_3 =$

0.43477 at a given time point. So according to the given data the world's ranks will be 3, 2, 1 for  $\lambda_1, \lambda_2, \lambda_3$  and the agent's ranks 2, 3, 1 for  $ES_1, ES_2, ES_3$  respectively. So according to the given formulas  $IF = 2$ ,  $Max. IF = 4$  and  $DRF = 0.5$ . So in this particular case at this given time point the agent is behaving rationally for 50%.

### Method 2 (Continuous Rationality Measure)

The second method presented is based on the following point of departure: *an agent which receives the maximum benefit will be the highly rational agent. This is only possible if  $ES_i$  is 1 for the option whose  $\lambda_i$  is the highest.* In this method to calculate the continuous rationality factor  $CRF$ , first to account for the effort spent in performing actions, the effector state values  $ES_i$  are normalised as follows.

$$nES_i = \frac{ES_i}{\sum_{i=1}^n ES_i} \quad (16)$$

Here  $n$  is number of options available. Based on this the continuous rationality factor  $CRF$  is determined as follows, with  $Max(\lambda_i)$  the maximal value of the different  $\lambda_i$ .

$$CRF = \frac{\sum_{i=1}^n nES_i \lambda_i}{Max(\lambda_i)} \quad (17)$$

This method enables to measure to which extent the agent is behaving rationally in a continuous manner. For the given example used to illustrate the previous method  $CRF = 0.6633$ . So according to this method the agent is considered to behaving for 66.33% rationally in the given world.

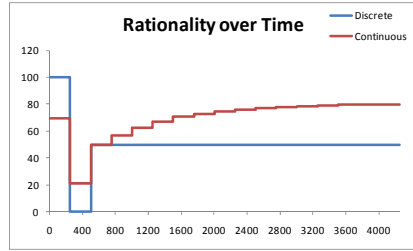


Fig. 7. Rationality during learning  $\omega_{1i}$  (A)

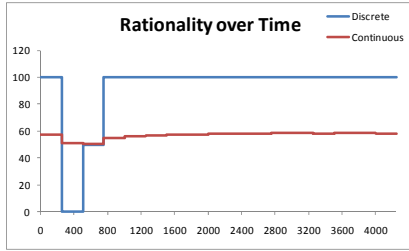


Fig. 8. Rationality during learning  $\omega_{2i}$  (B)

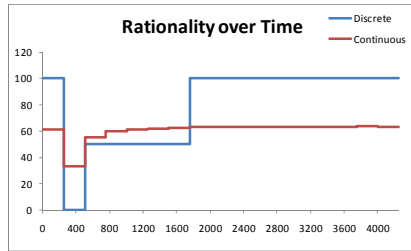


Fig. 9. Rationality during learning  $\omega_{3i}$  (C)

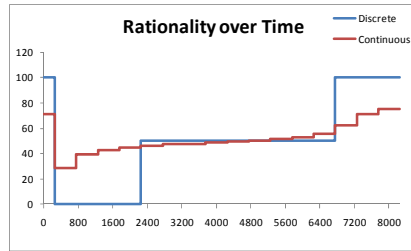


Fig. 10. Rationality for learning  $\omega_{1i}, \omega_{2i}, \omega_{3i}$  (ABC)

Fig. 7 to Fig. 10 show the two types of rationality (depicted as percentages) of the agent for the different scenarios with changing stochastic world. In these figures the

first 250 time points show the rationality achieved by the agent just before changing world characteristics drastically for the simulations shown from Fig. 4. From time point 250 onwards, it shows the rationality of the agent after the change has been made (see Fig. 6). It is clear from the results (Fig. 7 to Fig. 10) that the rationality factor of the agent in all four cases improves over time for the given world.

## 5 Discussion

This paper focused on how the extent of rationality of an adaptive decision model can be analysed. In particular, this was explored for variants of a decision model based on valuing of predictions involving feeling states generated in the amygdala; e.g., [1, 2, 6, 8, 14, 15, 17]. The adaptation was based on using four different variations of Hebbian learning; cf. [10, 12].

To assess the extent of rationality with respect to given world characteristics, two measures were introduced, and using these extents of rationality of the different models over time were analysed. It was shown how by the learning processes indeed a high level of rationality was obtained, and how after a major world change after some delay this rationality level is re-obtained. It turned out that emotion-related valuing of predictions in the amygdala as a basis for adaptive decision making according to Hebbian learning satisfies reasonable rationality measures.

## References

1. Bechara, A., Damasio, H., and Damasio, A.R.: Role of the Amygdala in Decision-Making. *Ann. N.Y. Acad. Sci.* 985, 356-369 (2003)
2. Bechara, A., Damasio, H., Damasio, A.R., and Lee, G.P., Different Contributions of the Human Amygdala and Ventromedial Prefrontal Cortex to Decision-Making. *Journal of Neuroscience* 19, 5473–5481 (1999)
3. Bosse, T., Hoogendoorn, M., Memon, Z.A., Treur, J., and Umair, M., An Adaptive Model for Dynamics of Desiring and Feeling based on Hebbian Learning. In: Yao, Y., Sun, R., Poggio, T., Liu, J., Zhong, N., and Huang, J. (eds.), *Proceedings of the Second International Conference on Brain Informatics, BI'10. Lecture Notes in Artificial Intelligence*, vol. 6334, pp. 14-28. Springer Verlag (2010)
4. Bosse, T., Jonker, C.M., and Treur, J. (2008). Formalisation of Damasio's Theory of Emotion, Feeling and Core Consciousness. *Consciousness and Cogn.* 17, 94–113 (2008)
5. Bosse, T., Jonker, C.M., Meij, L. van der, Treur, J.: A Language and Environment for Analysis of Dynamics by Simulation. *Intern. J. of AI Tools* 16, 435-464 (2007)
6. Damasio, A.: *Descartes' Error: Emotion, Reason and the Human Brain*. Papermac, London (1994)
7. Damasio, A.: *The Feeling of What Happens. Body and Emotion in the Making of Consciousness*. New York: Harcourt Brace (1999)
8. Damasio, A.: *Looking for Spinoza*. Vintage books, London (2004)
9. Forgas, J.P., Laham, S.M., and Vargas, P.T.: Mood effects on eyewitness memory: Affective influences on susceptibility to misinformation. *Journal of Experimental Social Psychology* 41, 574–588 (2005)

10. Gerstner, W., and Kistler, W.M.: Mathematical formulations of Hebbian learning. Biol. Cybern. 87, 404-415 (2002)
11. Ghahghaei, H.T., Hilgetag, C.C., Barbas, H.: Sequence of information processing for emotions based on the anatomic dialogue between prefrontal cortex and amygdala. Neuroimage 34, 905-923 (2007)
12. Hebb, D.O.: The Organization of Behaviour. New York: John Wiley & Sons (1949)
13. Hesslow, G.: Conscious thought as simulation of behaviour and perception. Trends Cogn. Sci. 6, 242-247 (2002)
14. Montague, P.R., Berns, G.S.: Neural economics and the biological substrates of valuation. Neuron 36, 265-284 (2002)
15. Morrison, S.E., and Salzman, C.D.: Re-valuing the amygdala. Current Opinion in Neurobiology 20, 221-230 (2010)
16. Murray EA: The amygdala, reward and emotion. Trends Cogn Sci 11, 489-497 (2007)
17. Rangel, A., Camerer, C., Montague, P.R.: A framework for studying the neurobiology of value-based decision making. Nat Rev Neurosci 9, 545-556 (2008)
18. Salzman, C.D., and Fusi, S.: Emotion, Cognition, and Mental State Representation in Amygdala and Prefrontal Cortex. Annu. Rev. Neurosci. 33, 173-202 (2010)
19. Sugrue, L.P., Corrado, G.S., Newsome, W.T.: Choosing the greater of two goods: neural currencies for valuation and decision making. Nat Rev Neurosci 6, 363-375 (2005)

## Appendix A Model Specifications in LEADSTO format

<p><b>LP0 Sensing a world state</b>  <math>\text{world\_state}(w, V1) \ \&amp; \ \text{sensor\_state}(w, V2) \rightarrow \text{sensor\_state}(w, V2 + \gamma [V1 - V2] \Delta t)</math></p> <p><b>LP1 Generating a sensory representation for a sensed world state</b>  <math>\text{sensor\_state}(w, V1) \ \&amp; \ \text{srs}(w, V2) \rightarrow \text{srs}(w, V2 + \gamma [V1 - V2] \Delta t)</math></p> <p><b>LP2 From sensory representation and feeling to preparation of a body state</b>  <math>\text{srs}(w, V) \ \&amp; \ \text{feeling}(b_i, V_i) \ \&amp; \ \text{preparation\_state}(b_i, U_i) \ \&amp; \ \text{has\_connection\_strength}(\text{srs}(w), \text{preparation}(b_i), \omega_{i1}) \ \&amp; \ \text{has\_connection\_strength}(\text{feeling}(b_i), \text{preparation}(b_i), \omega_{2i}) \ \&amp; \ \text{has\_steepness}(\text{prep\_state}(b_i), \sigma_i) \ \&amp; \ \text{has\_threshold}(\text{prep\_state}(b_i), \tau_i) \rightarrow \text{preparation}(b_i, U_i + \gamma_1 (h(\sigma_i, \tau_i, V, V_i, \omega_{1i}, \omega_{2i}) - U_i) \Delta t)</math></p> <p><b>LP3 From preparation and sensor state to sensory representation of a body state</b>  <math>\text{preparation\_state}(b_i, X_i) \ \&amp; \ \text{sensor\_state}(b_i, V_i) \ \&amp; \ \text{srs}(b_i, U_i) \ \&amp; \ \text{has\_connection\_strength}(\text{preparation}(b_i), \text{srs}(b_i), \omega_{3i}) \ \&amp; \ \text{has\_steepness}(\text{srs}(b_i), \sigma_i) \ \&amp; \ \text{has\_threshold}(\text{srs}(b_i), \tau_i) \rightarrow \text{srs}(b_i, U_i + \gamma_2 (h(\sigma_i, \tau_i, X_i, V_i, \omega_{3i}, 1) - U_i) \Delta t)</math></p> <p><b>LP4 From sensory representation of a body state to feeling</b>  <math>\text{srs}(b_i, V1) \ \&amp; \ \text{feeling}(b_i, V2) \rightarrow \text{feeling}(b_i, V2 + \gamma [V1 - V2] \Delta t)</math></p> <p><b>LP5 From preparation to effector state</b>  <math>\text{preparation\_state}(b_i, V) \ \&amp; \ \text{effector\_state}(b_i, V2) \rightarrow \text{effector\_state}(b_i, V2 + \gamma [V1 - V2] \Delta t)</math></p> <p><b>LP6 From effector state to sensor state of a body state</b>  <math>\text{effector\_state}(b_i, V1) \ \&amp; \ \text{effectiveness\_rate}(b_i, \lambda_i) \ \&amp; \ \text{sensor\_state}(b_i, V2) \rightarrow \text{sensor\_state}(b_i, V2 + \gamma [\lambda_i V1 - V2] \Delta t)</math></p> <p><b>LP7 Hebbian learning (A): connection from sensory representation of <math>w</math> to preparation of <math>b_i</math></b>  <math>\text{has\_connection\_strength}(\text{srs}(w), \text{preparation}(b_i), \omega_{i1}) \ \&amp; \ \text{srs}(w, V) \ \&amp; \ \text{preparation}(b_i, V_i) \ \&amp; \ \text{has\_learning\_rate}(\text{srs}(w), \text{preparation}(b_i), \eta) \ \&amp; \ \text{has\_extinction\_rate}(\text{srs}(w), \text{preparation}(b_i), \zeta) \rightarrow \text{has\_connection\_strength}(w, b_i, \omega_{i1} + (\eta V V_i (1 - \omega_{i1}) - \zeta \omega_{i1}) \Delta t)</math></p> <p><b>LP8 Hebbian learning (B): connection from feeling <math>b_i</math> to preparation of <math>b_i</math></b>  <math>\text{has\_connection\_strength}(\text{feeling}(b_i), \text{preparation}(b_i), \omega_{2i}) \ \&amp; \ \text{feeling}(b_i, V_i) \ \&amp; \ \text{preparation}(b_i, U_i) \ \&amp; \ \text{has\_learning\_rate}(\text{feeling}(b_i), \text{preparation}(b_i), \eta) \ \&amp; \ \text{has\_extinction\_rate}(\text{feeling}(b_i), \text{preparation}(b_i), \zeta) \rightarrow \text{has\_connection\_strength}(\text{feeling}(b_i), \text{preparation}(b_i), \omega_{2i} + (\eta V_i U_i (1 - \omega_{2i}) - \zeta \omega_{2i}) \Delta t)</math></p> <p><b>LP9 Hebbian learning (C): connection from preparation of <math>b_i</math> to sensory representation of <math>b_i</math></b>  <math>\text{has\_connection\_strength}(\text{preparation}(b_i), \text{srs}(b_i), \omega_{3i}) \ \&amp; \ \text{preparation}(b_i, V_i) \ \&amp; \ \text{srs}(b_i, U_i) \ \&amp; \ \text{has\_learning\_rate}(\text{preparation}(b_i), \text{srs}(b_i), \eta) \ \&amp; \ \text{has\_extinction\_rate}(\text{preparation}(b_i), \text{srs}(b_i), \zeta) \rightarrow \text{has\_connection\_strength}(\text{preparation}(b_i), \text{srs}(b_i), \omega_{3i} + (\eta V_i U_i (1 - \omega_{3i}) - \zeta \omega_{3i}) \Delta t)</math></p>
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