Home-work exercises for week 12: Please hand in before or on Thursday, December 11

Introduction to Contact Topology, Fall 2014

Exercise 1. Let $T^3 = \mathbb{R}^3/\mathbb{Z}^3$, with coordinates (x, y, z) and n a positive integer number. Consider the form

$$\alpha_n = \sin(2\pi nz)dx + \cos(2\pi nz)dy.$$

1. Prove that α_n is a positive contact form on T^3 and find two vector fields spanning the contact distribution $\xi_n = \ker \alpha_n$ at each point.
2. Describe the behaviour of the contact planes along the circles
$$\{x = y = \text{const}\}.$$
3. What kind of structure would you get for $n = 0$ or n negative?
4. Let L be the Legendrian knot $\{x = y = \text{const}\}$: compute its Thurston-Bennequin invariant (for instance, as $lk(L, L')$, where L' is the push-off of L in the direction of a vector field v along L transverse to ξ_n).
5. Consider a torus $\{x = \text{const}\}$ inside (T^3, ξ_n): how does the characteristic foliation on this torus look like? In particular, describe the singularities (if present) and dividing set.
6. Is the above torus a convex surface for ξ_n?
7. Prove that the contact manifolds (T^3, ξ_n) are weakly symplectically fillable. Hint: consider the family of 1-forms $\beta_t = (1 - t)dz + t\alpha_n$.