# Home-work exercises for week 2

## Introduction to Contact Topology, Fall 2014

**Exercise 1.** Let $S^{2n+1}$ denote the unit sphere in the Euclidean space $\mathbb{R}^{2n+2}$. Let $\alpha_0$ be the 1-form

$$\alpha_0 = \sum_{j=1}^{n+1} (x_j\, dy_j - y_j\, dx_j),$$

with $(x_1, y_1, \ldots, x_{n+1}, y_{n+1})$ denoting cartesian coordinates on $\mathbb{R}^{2n+2}$.

(a) Show that $\alpha_0$ defines a contact structure on the sphere.

(b) Compute the Reeb flow of $\alpha_0$.

**Exercise 2.** Let $M$ be a manifold of dimension $2n+1$ and $\xi$ a co-orientable hyperplane field.

(a) Let $\alpha$ be a 1-form with $\ker \alpha = \xi$. Prove that $d\alpha$ is non-degenerate on $\xi$ if and only if

$$\alpha \wedge (d\alpha)^n \neq 0.$$

(b) Let $\alpha$ and $\alpha'$ be 1-forms with $\xi = \ker\, \alpha = \ker\, \alpha'$. Check that $\alpha$ satisfies the contact condition if and only if $\alpha'$ does.

**Exercise 3.** Let $(B, g)$ be a Riemannian manifold and denote by $STB$ and $ST^*B$ the unit sphere bundle in the tangent and cotangent bundle, respectively, of $B$. Prove that the geodesic flow on $STB$ and the Reeb flow of the Liouville form on $ST^*B$ coincide under the metric isomorphism.