Home-work exercises for week 3

Introduction to Contact Topology, Fall 2014

Exercise 1. Let (W, ω) be a $2n$-dimensional symplectic manifold and $H : W \to \mathbb{R}$ a smooth function. If 0 is a regular value of H, the level set $M = H^{-1}(0)$ is a codimension 1 submanifold of W. The Hamiltonian vector field X_H is tangent to M at all its points. If we further assume that there exists a Lioville vector field Y, defined in a neighborhood of M and everywhere transverse to M, it follows that the restriction to M of $\alpha = i_Y\omega$ is a contact form on M. Denote the associated Reeb vector field by R_α. What is the relationship between X_H and R_α?

Exercise 2. Let (B, g) be a Riemannian manifold of dimension n. Let T^*B be its cotangent bundle, with the standard symplectic form

$$\omega = \sum_{i=1}^{n} dp_i \wedge dq_i,$$

where $q_1, \ldots, q_n$ are local coordinates on B and $p_1, \ldots, p_n$ are the induced cotangent fiber coordinates. Denote by ST^*B the unit cotangent bundle (with respect to the bundle metric induced on T^*B by g). Prove that there exists a Liouville vector field Y on T^*B which is everywhere transverse to ST^*B and such that $i_Y\omega$ is the tautological 1-form.

Exercise 3. Let $(M_i, \xi_i = \ker \alpha_i)$, $i = 1, 2$, be two contact manifolds and $f : M_1 \to M_2$ a strict contactomorphism. Denote by $R_i \in \Gamma(TM_i)$ the Reeb vector field of α_i. Prove that

$$df(R_1) = R_2.$$