Home-work exercises for week 6

Introduction to Contact Topology, Fall 2014

Exercise 1. Let $\gamma : (a, b) \to (\mathbb{R}^3, \xi_{\text{st}} = \ker(dz + x\, dy))$, $s \mapsto (x(s), y(s), z(s))$ be a Legendrian immersion and let γ_L be its Lagrange-projection. Then:

1. γ_L is also an immersion;

2. γ is completely determineed by γ_L (up to translation in the z-direction);

3. if γ_L is a closed curve, then $\oint_{\gamma_L} x\, dy = 0$.

Exercise 2. Let $\gamma : (a, b) \to (\mathbb{R}^3, \xi_{\text{st}})$, $s \mapsto (x(s), y(s), z(s))$ be a positively transverse curve (i.e., $(dz + x\, dy)(\dot{\gamma}(s)) > 0$ for all $s \in (a, b)$). Convince yourself that the only forbidden crossing in the front projection of γ is the one shown in the picture below and all other crossings are possible.

Exercise 3. Work out Example 3.5.7 in the book.