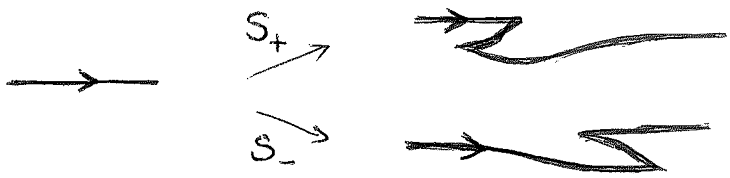
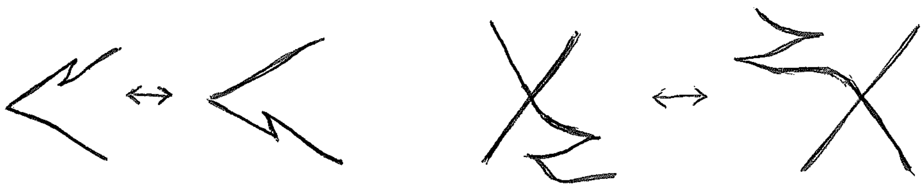
Home-work exercises for week 8: Please hand in before or on Wednesday, November 12

Introduction to Contact Topology, Fall 2014

Exercise 1. Let L be an oriented Legendrian knot in $(\mathbb{R}^3, \xi_{\mathrm{st}})$. The operation of adding two successive cusps, a *zigzag*, in the front projection of the knot L is called a *stabilization*. One distinguishes between positive and negative stabilizations and denotes the new knots by $S_+(L)$ and $S_-(L)$, respectively.



1. Prove that a zigzag can be passed through a cusp or a crossing, i.e., there exists Legendrian isotopies



2. Prove that the stabilization operation is well-defined (i.e, independent of the location where the zigzag is added).

Exercise 2. Describe the characteristic foliation on the tori $S^1 \times S^1 \times \{0\}$ and $S^1 \times S^1 \times \{\pi/2\}$ in $(S^1 \times S^1 \times S^1, \xi = \ker(\cos\theta_1\, d\theta_2 + \sin\theta_1\, d\theta_3))$.

Exercise 3. An *open book* decomposition of a closed, oriented 3-manifold M consists of a link K in M (the *binding*) and a smooth fibration $p : M - K \to S^1$ which coincides with the angular coordinate of D^2 in a neighborhood $K \times D^2$ of $K = K \times \{0\}$. The fibers of p are called *pages.* A contact structure ξ on M is called *adapted* to the open book (K, p) if there is a contact form α such that

- $d\alpha$ is a symplectic form on each page F;
- K is transverse to ξ, and the orientation on K given by α coincides with the boundary orientation induced from F (and coming from the symplectic structure).

Consider the standard contact structure on $S^3 \subset \mathbb{C}^2$ and show that the following data describe an open book decomposition adapted to the contact structure:

$$K = \{(z_1, z_2) \,\|\, z_1 = 0\} \quad \text{and} \quad p : S^3 - K \to S^1 \subset \mathbb{C}\,, \quad (z_1, z_2) \mapsto \frac{z_1}{\|z_1\|}.$$