Home-work exercises for week 9

Introduction to Contact Topology, Fall 2014

Exercise 1. Let (M, ξ) be a contact 3-manifold and let S be an embedded surface with area form Ω.

1. The *characteristic foliation* of S, denoted by S_ξ, is the singular foliation determined by the vector field X which satisfies $i_X\Omega = \alpha|_S$. Prove that $X_p = 0$ if and only if $\xi_p = T_pS$ and that at each nonsingular point p, X_p spans the line $\xi_p \cap T_pS$.

2. We have seen that a vector field X defines the characteristic foliation S_ξ if and only if $X_p = 0$ implies $\mathrm{div}_\Omega X \neq$ at p. Prove that this statement is independent of the choice of area form Ω on S.

Exercise 2. Consider $\mathbb{T}^3 = S^1 \times S^1 \times S^1$ with the contact form

$$\alpha_n = \cos(n\theta)dx - \sin(n\theta)dy, \quad (x, y) \in \mathbb{R}^2/\mathbb{Z}^2, \quad \theta \in \mathbb{R}/2\pi\mathbb{Z}.$$

1. Prove that taking $n = 1$ gives the canonical contact structure on $\mathbb{T}^3$, regarded as the unit cotangent bundle in the symplectic manifold $T^*(\mathbb{T}^2)$.

2. Describe the characteristic foliation of a torus $\{\theta = \text{constant}\}$ in $(\mathbb{T}^3, \xi_n = \ker(\alpha_n))$. How many closed orbits (leaves) are there?