Hamiltonian Mechanics

Course objectives
This course deals with the analysis of the ordinary differential equations and the dynamical systems involved in classical mechanics. The basic concepts and analytical techniques of the Lagrange's and Hamilton's formulations are presented.
The aim is to provide the students with a good working knowledge of the formalisms and their applications in classical systems. As examples, the central force motion and the restricted three-body problem will be discussed.
The analytical concepts and techniques developed serve as the springboard for various branches of moder physics, for instance, quantum mechanics.

Course content
Classical mechanics is concerned with the physical laws and differential equations governing the motion of bodies under the influence of system of forces. Examples are a falling body, the planetary motion, the swinging of a pundulum but also the motion of a charged particle in a electromagnetic force field. In this course we review the mathematical apparatus developed to study classical mechanics, providing a unified formulation for a large class of mechanical problems.
We start our discussion from the Lagrange's formulation and the variational principle. We study the D' Alambert's Principle and the Euler-Lagrange equations. Then we move into the Hamiltonian mechanics, a even more unified and powerful formalism for the analysis of mechanical systems. We deal with fundamental concepts like canonical transformations, Poisson brackets, action-angles variables, integrable systems. We also sketch some modern developments, like perturbation theory and the KAM results.
Several examples will be used to clarify the theory. Particular emphasis is given to celestial mechanics related problems: we perform a detailed analysis of the central force motion and of the circular restricted three body problem, a model widely adopted in applications, for instance in space mission design.

Teacher
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Course Literature
Goldstein, Poole, Safko; *Classical Mechanics*, 3rd ed., Addison Wesley, 2002.
Hand out material

Program
Still preliminary

Argument	Book section
Equations of motion in classical mechanics. Newton's laws. Some examples.	1.1-1.4
D' Alambert's Principle, Lagrange's equations	
Variational principle. Euler-Lagrange equations, Energy function, symmetries	2.1-2.7
Central force problem. First integrals, orbit classifications	3.1-3.8
Power-law potentials, Bertrand's theorem, Kepler problem	
Three-body problem	3.12
Hamilton equation, Legendre transform, Phase space, Conservation theorems	8.1-8.2
Recurrence theorem Variational principles	8.5-8.6
Canonical transformation, Poisson brackets	9.1-9.3, 9.5
Hamilton-Jacobi theory, Action-angle variables	10.1-10.8
Periodic motion, introduction to KAM theory	11.1-11.2
Perturbation theory, adiabatic invariants	12.1 - 12.5

Other topics that maybe covered are the Restricted Three body problem, Elliptic-Parabolic coordinate systems, N-body problem formulation, Henon-Heiles system.

Schedule
One class per week, 3 hours each time. At each class 2 hours of lecturing and 1 hour of discussion/exercise/questioning.

Prerequisite
Standard courses in Bachelor of mathematics

Target group
MSc Mathematics, MSc Stochastics and Financial Mathematics, MSc Physics

Form of tuition
Teaching consists of Lectures and Exercises. The lectures (2 hours approx. every time) will present the topics of the course. Exercise class (1 hour week) is intended to practise. Time by time, the students are expected to present some exercises at the blackboard.

Assessment

1. Homework exercises: to be handed in every two weeks.
2. In class assessment: students discuss at the blackboard some of the homework
3. Oral exam.

Partial grades on points (1), (2), (3) contribute to the final grade as 20% (1) + 20 %(2) + 60 % (3)