Two-body approximations in the design of low-energy transfers between Galilean moons

Elena Fantino and Roberto Castelli

Abstract Over the past two decades, the robotic exploration of the Solar System has reached the moons of the giant planets. In the case of Jupiter, a strong scientific interest towards its icy moons has motivated important space missions (e.g., ESAs' JUICE and NASA's Europa Mission). A major issue in this context is the design of efficient trajectories enabling satellite tours, i.e., visiting the several moons in succession. Concepts like the Petit Grand Tour and the Multi-Moon Orbiter have been developed to this purpose, and the literature on the subject is quite rich. The models adopted are the two-body problem (with the patched conics approximation and gravity assists) and the three-body problem (giving rise to the so-called low-energy transfers, LETs). In this contribution, we deal with the connection between two moons, Europa and Ganymede, and we investigate a two-body approximation of trajectories originating from the stable/unstable invariant manifolds of the two circular restricted three body problems, i.e., Jupiter-Ganymede and Jupiter-Europa. We develop *ad-hoc* algorithms to determine the intersections of the resulting elliptical arcs, and the magnitude of the maneuver at the intersections. We provide a means to perform very fast and accurate evaluations of the minimum-cost trajectories between the two moons. Eventually, we validate the methodology by comparison with numerical integrations in the three-body problem.

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1 Introduction

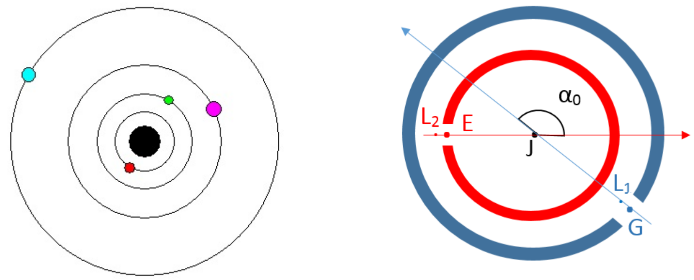
The justification of the work resides in the strong scientific interest towards the close-up observation of the moons of the giant planets. The arrival of Cassini at Saturn in 2004 marked the beginning of the era of exploration of the outer planetary systems. Concerning Jupiter's system, Europa is a very attractive target and Quite a number of space missions have been proposed, in particular ESA/JUICE [1], that will be launched in 2022 and will perform flybys of Europa, Ganymede and Callisto. A capture at a moon presents formidable challenges to traditional conic analysis since the dynamics in its vicinity is strongly influenced by Jupiter. Hence, three-body techniques are required if the transfer is to end up with a temporary or permanent capture at the moon. In the case of a multi-moon exploration mission, the cost of the transfer between moons is an issue of fundamental importance. For example, a traditional Hohmann transfer between Ganymede and Europa costs 2.8km/s. The use of three-body dynamics may provide low-fuel solutions. Additionally, LETs add flexibility and versatility to the mission. The specialized literature on the use of three-body dynamics for trajectories among the moons of Jupiter can be divided into two main research lines: the Petit Grand Tour [2] and the Multi-Moon Orbiter concept [3]. The former consists in coupling three-body problems and seeking intersections in phase space between the corresponding invariant manifolds. This approach requires longer transfer times than Hohmann maneuvers, but allows ΔV savings at the 40% level with respect to the classical solution. The Multi-Moon Orbiter concept consists in integrating low-energy trajectories with resonant flybys. The resulting transfers are necessarily longer than traditional conic solutions, but the costs are greatly reduced.

In this contribution, we present a method to determine the minimum-cost, direct low-energy trajectories between two moons of Jupiter. The bodies considered are Ganymede and Europa. The starting and end points of these trajectories are planar Lyapunov orbits around collinear libration points of each moon. Sections 2 and 3 illustrate the statement of the problem and the dynamical model, respectively. This is followed by a description of the approach (Sect. 4). The application is dealt with in Sect. 5 and concluding remarks are made in Sect. 6.

2 Problem statement

The study is carried out in planar approximation, with the orbits of the moons assumed circular (Fig. 1a). This allows the adoption of the circular approximation for the Jupiter-moon-spacecraft restricted three-body problem (CR3BP). Table 1 reports the values of orbital radii, orbital periods and masses of the four Galilean moons. The transfers sought connect planar Lyapunov orbits of Jupiter-Ganymede's and Jupiter-Europa's CR3BPs. Specifically, the Europa-to-Ganymede connection consists in traveling from a planar Lyapunov orbit around the L_2 point of Jupiter-Europa (JEL_2 from now on) to a planar Lyapunov orbit around the L_1 point of

Jupiter-Ganymede (JGL_1). The reverse holds for the Ganymede-to-Europa connection, which starts close to JGL_1 and ends at an orbit around JEL_2.



(a) (b)

Fig. 1 (a) The orbits of the Galilean moons; (b) Schematic representation of the coupled CR3BP made up of Jupiter, Ganymede and Europa: α_0 is the angle between the x-axes of the two synodical reference frames.

Table 1 Relevant data of the four Galilean moons: orbital period (days), orbital radius (km), mass (kg). The mass of Jupiter is $1898.13 \cdot 10^{24}$kg

Moon	Period	Radius	Mass
Io	1.77	$4.218 \cdot 10^5$	$8.93 \cdot 10^{22}$
Europa	3.55	$6.711 \cdot 10^5$	$4.80 \cdot 10^{19}$
Ganymede	7.15	$1.070 \cdot 10^6$	$1.48 \cdot 10^{23}$
Callisto	16.69	$1.883 \cdot 10^6$	$1.08 \cdot 10^{23}$

3 Dynamical model

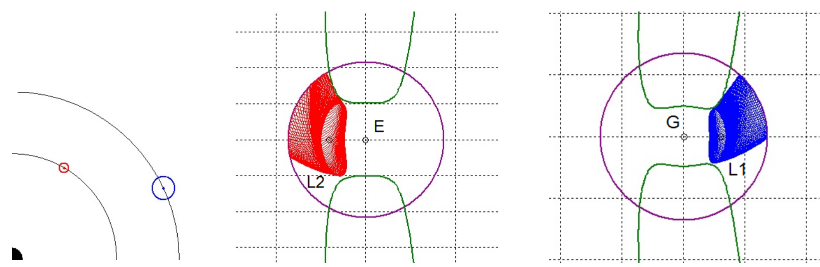
The design is carried out in the following two models:

1. The coupled CR3BP, which results from the kinematical link between the CR3BP with Jupiter and Ganymede as primaries, and the CR3BP in which the primaries are Jupiter and Europa. The coupling is possible because the two models have a common primary, i.e., Jupiter. The kinematical connection requires the definition of a relative angle α_0 between the two synodical frames at some reference time t_0 (see Fig. 1b)
2. The Jupiter-spacecraft two-body problem.

The coupled CR3BP is used in the vicinity of the moons, i.e., when the spacecraft-to-moon distance is smaller than 3 times the radius of the moon's sphere of influence calculated with the Tisserand formula. At the boundary of such domain (that we called circle of influence), the two-body approximation takes over and is used to deal with the spacecraft dynamics far from the moons. This is justified by the fact that the masses of the moons are 4 to 5 orders of magnitude smaller than Jupiter's.

4 Solution method

The solution is constructed upon a database of planar Lyapunov orbits around the two previously-mentioned libration points. The Jacobi constant levels range from 3.0024 and 3.0036 for Europa and from 3.0061 to 3.0075 for Ganymede. The y-amplitudes reach 13600km and 23500km, respectively in the two cases. Fig. 2a shows the circles of influence, the Tisserand circles and a portion of the orbits of the two moons, all scale drawn. The selected ratio between the circles's of influence radius and the Tisserand radius allows to include the largest Lyapunov orbits of the database. The appropriate branch of the suitable (unstable or stable) invariant

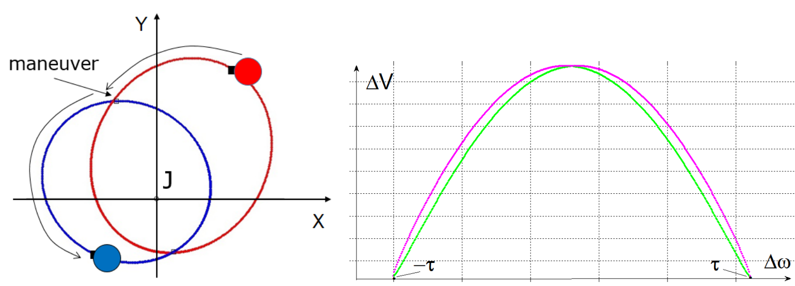


(a) (b) (c)

Fig. 2 (a) Europa (red), Ganymede (blue) and the respective circles of influence; (b) the circle of influence around Europa and the trajectories of an unstable invariant manifold originating from L_2; (c) the circle of influence around Ganymede and the trajectories of a stable invariant manifold originating from L_1.

manifold is propagated to the circle of influence (Figs. 2b and 2c). At the intersection, the state vector of the spacecraft is transformed from the synodical frame to an inertially-oriented Jupiter-centered frame denoted (J,X,Y,Z). The transformation consists in a change of origin, a rotation around the z-axis and a change of scale (from synodical normalized units to physical units). The state vector is used to determine the orbital elements of the osculating ellipse. Specifically, since the model is two-dimensional, the semi-major axis a, the eccentricity e and the argument of pericenter ω uniquely identify the ellipse geometrically. ω is defined as the angle from the X-axis to the pericenter of the ellipse. The position on the elliptical orbit is given by the true anomaly θ. The procedure is applied to all the trajectories in a manifold for a given discretization on the original Lyapunov orbit, and to all the

Lyapunov orbits of the database. In this way, a set of Keplerian orbits is obtained. Then, we form all the possible pairs of Keplerian orbits from the departure CR3BP and the arrival CR3BP, and we compute the geometrical intersections within each pair. Since the two ellipses of each pair have a common focus, the number of points of intersection cannot be higher than two. The difference $\Delta\mathbf{V}$ in velocity at each intersection represents the impulse to be applied by the propulsion system of the spacecraft to transfer from one orbit to the other. In the case of two intersections, the magnitude of the impulse is as shown by the two curves of Fig. 3b (by changing semi-major axis and eccentricity of the two ellipses, the shape of the curves and the difference between them may vary but the pattern followed is similar). The total transfer time is different on the two solutions. One typically chooses the shorter transfer (Fig. 3a). In principle, the procedure should be repeated for every choice of α_0 obtained by scanning the domain (from 0 to 2π) according to a selected angular resolution. However, it is straightforward to prove that two different values of α_0 provide the same ellipses rotated by α_0. Hence, there is no need to recompute the ellipses upon varying α_0, because performing the correct rotation gives the same result. Rotating an ellipse changes its argument of pericenter and the relative orientation $\Delta\omega$ with the other ellipse of the pair. Given two ellipses, we determine the



(a) (b)

Fig. 3 (a) Schematic representation of the solution method; (b) example of the relationship between the magnitude of the impulses needed to pass from one elliptical arc to the other and the relative orientation $\Delta\omega$ between the ellipses.

interval of values of $\Delta\omega$ for which intersections exist. This is done by setting up the system of polar equations for the two curves

$$\begin{cases} r = \dfrac{a_1(1-e_1^2)}{1+e_1\cos\theta_1} \\ r = \dfrac{a_2(1-e_2^2)}{1+e_2\cos(\theta_1+\Delta\omega)} \end{cases} \tag{1}$$

and determining the values of $\Delta\omega$ for which the discriminant of the associated second-degree algebraic equation is positive or zero (the latter corresponding to the situation in which the two ellipses are tangent to one another and the two intersections reduce to one point). This range is symmetrical around $\Delta\omega = 0$, in which the two pericenters are on the same side of the same line through the focus. Let us call $-\tau$, and τ the limits of the interval, with $\tau \geq 0$. When τ is strictly positive, the two limits correspond to tangency configurations. By analysing the relationship ΔV versus $\Delta\omega$, we observed that, at least when the two orbital eccentricities are small (< 0.2), the dependency has the pattern shown in Fig. 3b, i.e., ΔV is minimum at the two limits $-\tau$ and τ. Hence, when aiming at minimum-cost connections, only the tangency configuration need to be computed, which drastically reduces the number of intersections to be tried.

5 Application: from Europa to Ganymede and viceversa

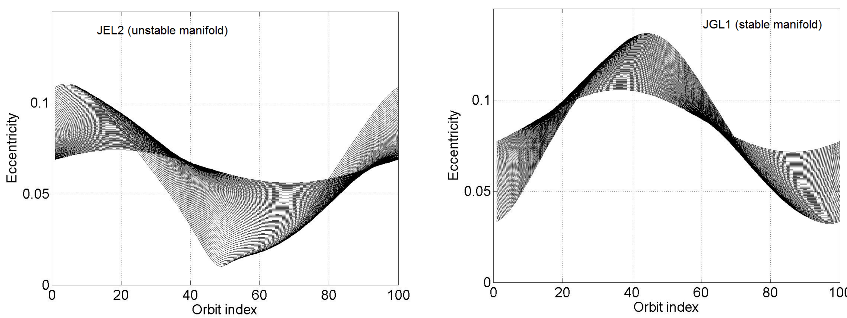
Our database of Keplerian orbits includes 7000 orbits on each side of the transfer and for each type of stability (resulting from 70 energy levels and a discretization of 100 points on each Lyapunov orbit). This produces 49 million pairs of ellipses in each transfer direction. Fig. 4 illustrates the orbital eccentricities, which are always smaller than 0.14. Fig. 5 maps the pericenter and apocenter radii, which are important parameters in the *a priori* exclusion of non-intersecting pairs: as a matter of fact, for an intersection to exist the pericenter radius of the orbits originating from Ganymede must be smaller than the apocenter radius of the orbits originating from Europa. The number of candidate pairs left is close to 3.5 million, i.e., 7% of the total. This subset is processed by the algorithm illustrated in Sect. 4. All combinations exhibit strictly positive τ, thus they all include the tangency, minimum-cost configurations. The minimum-cost ΔV's range from 0.97km/s to 1.27km/s in both directions (see Fig. 6). Eventually, by taking the absolute minima in the two directions, we have been able to fully characterize the corresponding trajectories (Fig. 7). These minimum-cost connections originate from invariant manifolds of Lyapunov orbits with Jacobi constant values of 3.0024 and 3.0061, respectively in the case of JEL_2 and JGL_1. Note that the exact symmetry of the model is responsible for the two solutions having the same cost and transfer time.

A verification of the physical validity of the trajectories obtained has been carried out by propagating the state vector, taken respectively at the beginning and at the end of each transfer, in the corresponding CR3BP up to the maneuver point (Fig. 8).

6 Conclusions

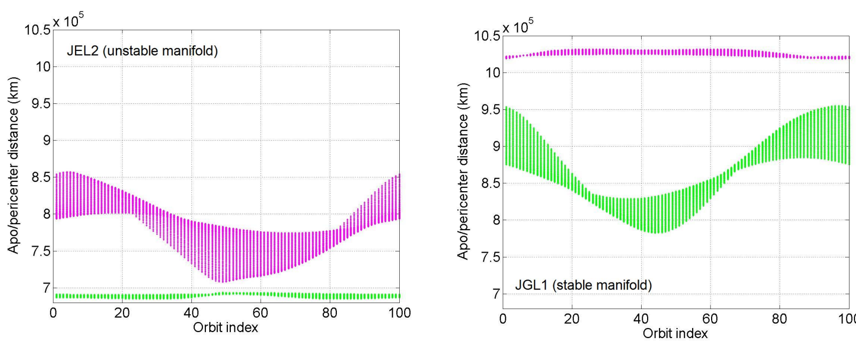
In this paper a new strategy for the preliminary design of low-energy transfers between Galilean moons is presented. The advantage of adopting the two-body approximation in the region where the influence of Jupiter is largely predominant is twofold. Since the two-body problem is completely integrable, we do not need to perform costly and unnecessary numerical simulations and we can *a-priori* determine the set of physical parameters associated to the minimum-ΔV transfer. This allows to deal with a huge amount of possible connections in a very short time. The validity of the method is proved in the design of transfers between Europa and Ganymede. Our results are close to those published by other authors ([2] find LETs requiring an impulsive maneuver at the level of 1.2km/s). Furthermore, the trajectory obtained is in very good agreement with the result of the numerical integration of the same initial conditions in the CR3BP.

The method is problem-independent and can be applied in different scenarios, anytime part of the force field is dominated by the gravitational influence of one body and the model is parametrized by a relative phase angle.



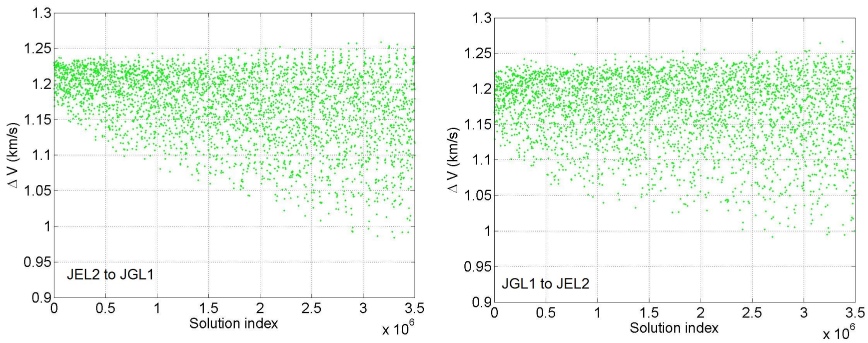
(a) (b)

Fig. 4 Orbital eccentricities of the ellipses originating from JEL_2 (a) and JGL_1 (b).



(a) (b)

Fig. 5 Apocenter (purple) and pericenter (green) radii of the ellipses originating from JEL_2 (a) and JGL_1 (b).



(a) (b)

Fig. 6 Minimum-cost intersections between elliptical orbits originating from JEL_2 and JGL_1: from Europa to Ganymede (a) and from Ganymede to Europa (b).

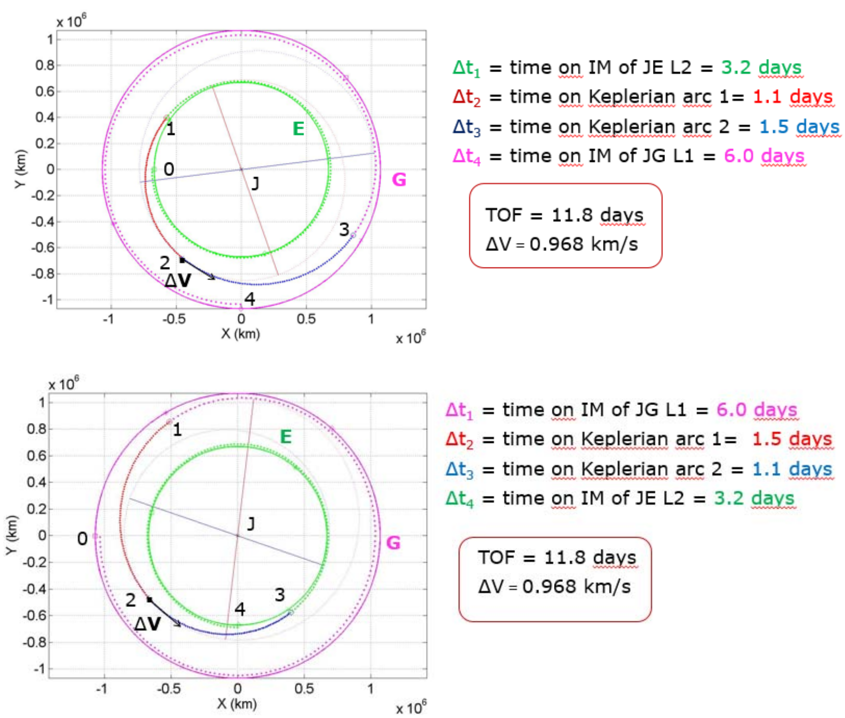


Fig. 7 Illustration of the minimum-cost Europa-to-Ganymede (top) and Ganymede-to-Europa (bottom) low-energy transfer.

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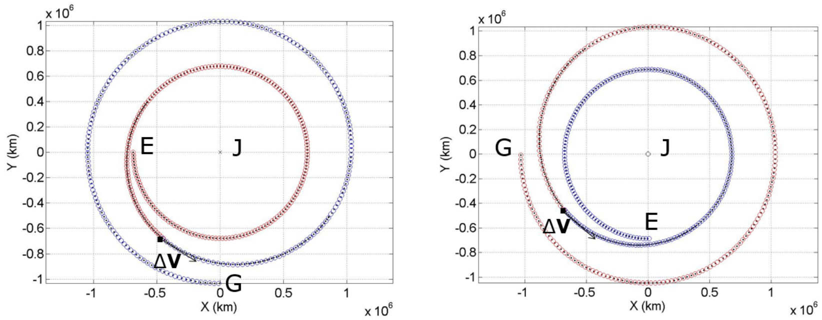

Fig. 8 The minimum-cost Europa-to-Ganymede (left) and Ganymede-to-Europa (right) low-energy transfers as obtained by the proposed approach (red and blue dots) and by numerical integration in the CR3BPs (black open circles).

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