Dynamical system theory and numerical methods applied to Astrodynamics

Roberto Castelli

Institute for Industrial Mathematics
University of Paderborn

BCAM, Bilbao, 20th December, 2010

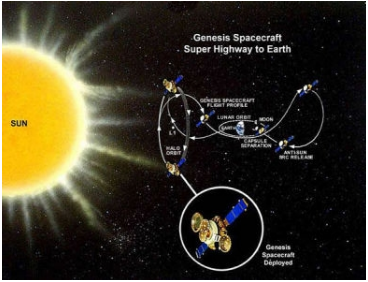
UNIVERSITÄT PADERBORN
Die Universität der Informationsgesellschaft

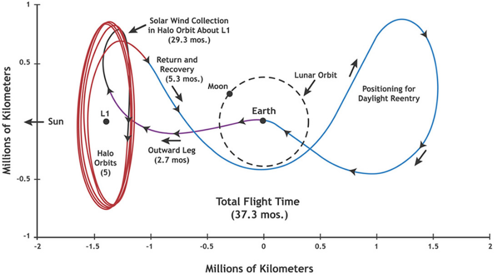
IFIM

Introduction

In designing a space mission, one has to

- Consider the Force Field acting on the Spacecraft
- Consider some Physical and Technical constraints
- Satisfy some mission requirements
- Take care of the fuel consumption and the travelling time
- ....





Introduction

N-BODY PROBLEM

$\Downarrow$

First guess trajectories designed in simplified model

- Two-body sections
- Restricted Three-body problem
- Bicircular model
- ...

Numerical Optimisation in Full system

- Direct/Indirect methods
- Multiple shooting technique
- Multiobjective optimisation

Different type of Propulsion (Electric - Chemical)

$\Downarrow$

Low thrust – Impulsive manoeuvre

Introduction

N-BODY PROBLEM

$\Downarrow$

First guess trajectories designed in simplified model

- Two-body sections
- Restricted Three-body problem
- Bicircular model
- ...

Numerical Optimisation in Full system

- Direct/Indirect methods
- Multiple shooting technique
- Multiobjective optimisation

Different type of Propulsion (Electric - Chemical)

$\Downarrow$

Low thrust – Impulsive manoeuvre

Introduction

N-BODY PROBLEM

$\Downarrow$

First guess trajectories designed in simplified model

- Two-body sections
- Restricted Three-body problem
- Bicircular model
- ...

Numerical Optimisation in Full system

- Direct/Indirect methods
- Multiple shooting technique
- Multiobjective optimisation

Different type of Propulsion (Electric - Chemical)

$\Downarrow$

Low thrust – Impulsive manoeuvre

Introduction

N-BODY PROBLEM

$\Downarrow$

First guess trajectories designed in simplified model

- Two-body sections
- Restricted Three-body problem
- Bicircular model
- ...

Numerical Optimisation in Full system

- Direct/Indirect methods
- Multiple shooting technique
- Multiobjective optimisation

Different type of Propulsion (Electric - Chemical)

$\Downarrow$

Low thrust – Impulsive manoeuvre

OUTLINE

Dynamical model CRTBP

- Properties
- Periodic orbits
- Tube Dynamics-Interplanetary Superhighway
- Patched CRTBP approximation

Computational methods: Set Oriented Numerics

- Covering of Invariant Sets
- GAIO implementation

Examples of mission design

- Earth to Halo
- Trajectory Connection

OUTLINE

Dynamical model CRTBP
- Properties
- Periodic orbits
- Tube Dynamics-Interplanetary Superhighway
- Patched CRTBP approximation

Computational methods: Set Oriented Numerics
- Covering of Invariant Sets
- GAIO implementation

Examples of mission design
- Earth to Halo
- Trajectory Connection

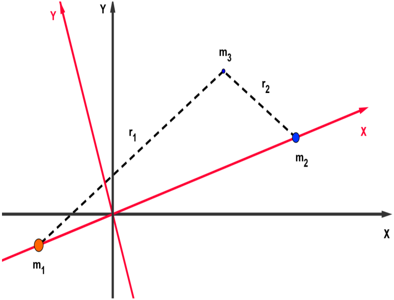
Circular Restricted Three-Body problem

- Two Primaries move in circular orbits under the mutual gravitational attraction
- Massless particle moves under the gravitational influence of two primaries

In a rotating, adimensional reference frame, $\mu = m_2/(m_1 + m_2)$,

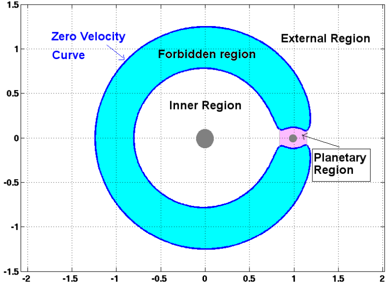
$$(CR3BP) \quad \begin{cases} \ddot{x} - 2\dot{y} = \Omega_x \\ \ddot{y} + 2\dot{x} = \Omega_y \\ \ddot{z} = \Omega_z \end{cases}$$

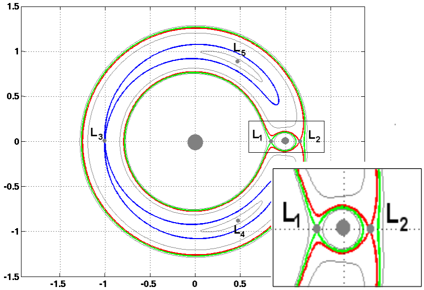
$$\Omega(x,y,z) = \frac{1}{2}(x^2+y^2) + \frac{1-\mu}{r_1} + \frac{\mu}{r_2} + \frac{1}{2}\mu(1-\mu)$$



Properties CRTBP

- Non integrable Autonomous Hamiltonian System
- Symmetry $(x, y, z, \dot{x}, \dot{y}, \dot{z}; t) \to (x, -y, z, -\dot{x}, \dot{y}, -\dot{z}; -t)$
- Jacobi Integral: $C = 2\Omega(x, y, z) - (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) = -2E$
- Equilibrium points: Lagrangian Points L_j , $j = 1, ..., 5$.
- Hill's Region: $\mathcal{H}(C) = \{(x, y, z) : 2\Omega(x, y, z) - C \geq 0\}$



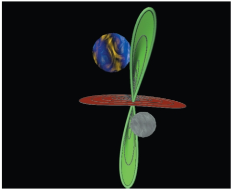


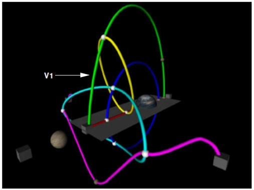
Families of Periodic Orbits.

Hamiltonian system

$\Downarrow$

continuous families of periodic orbits.





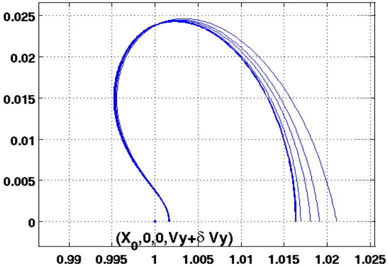
E.J.Doedel et al. [?]

Simple Symmetric periodic orbits

Differential correction scheme, based on the variational eq.. Find δVy such that the first x-axis crossing of

$$\phi_t(X_0, 0, 0, Vy + \delta Vy)$$

is perpendicular

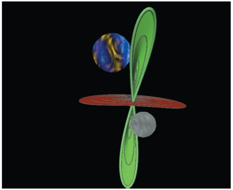


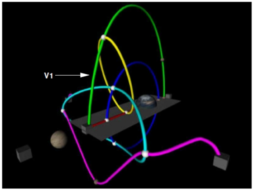
Families of Periodic Orbits.

Hamiltonian system

$\Downarrow$

continuous families of periodic orbits.





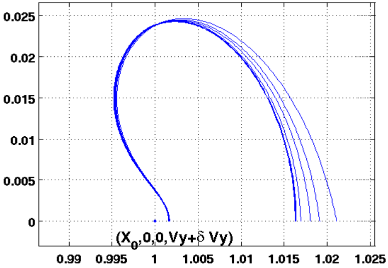
E.J.Doedel et al. [?]

Simple Symmetric periodic orbits

Differential correction scheme, based on the variational eq.. Find δVy such that the first x-axis crossing of

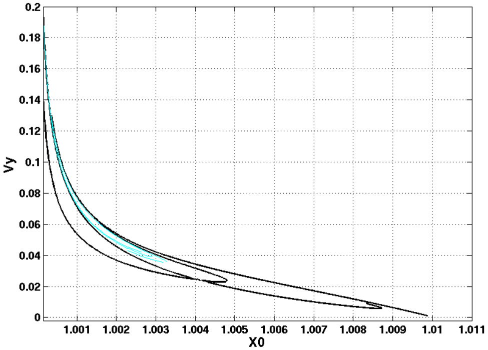
$$\phi_t(X_0, 0, 0, Vy + \delta Vy)$$

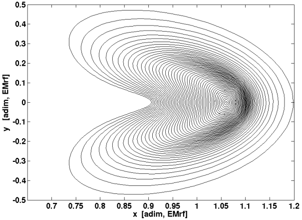
is perpendicular

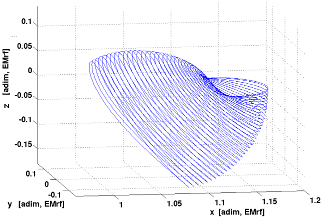


Periodic Orbits Diagram

Diagram of Simple Symmetric PO in SE system.



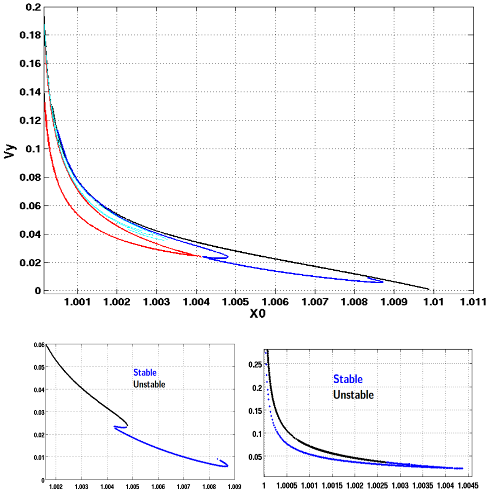


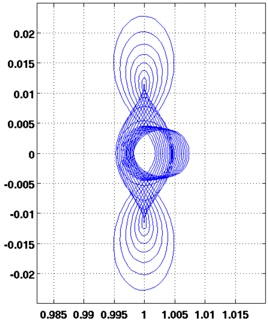


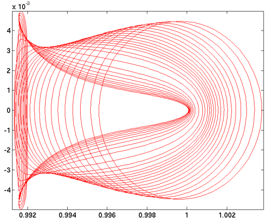
$(X_0, Vy) \rightarrow (X_0, 0, 0, V_y)$,

Earth $< X_0 < L_2$, $Vy > 0$

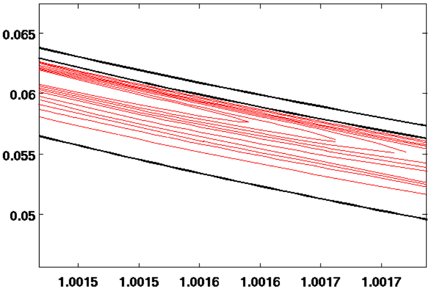
Periodic orbits: DPO

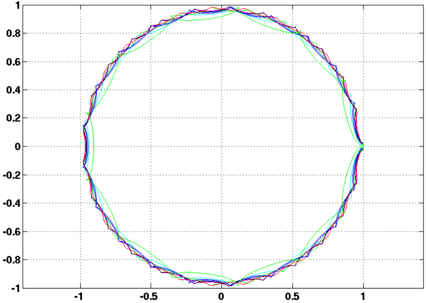






Resonant Orbits

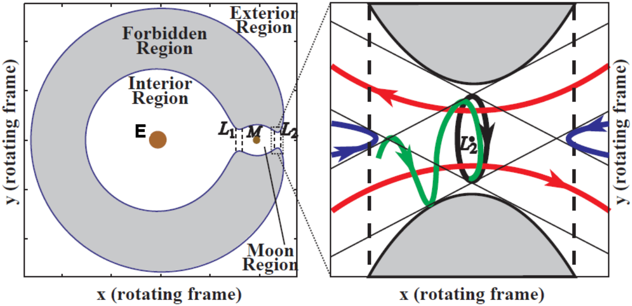




For which resonances there exist families of periodic orbits?

Dynamics near periodic orbits

- The periodic orbits separates two necks in the Hill's region
- Linear Dynamics: saddle $\times$ center
- 3 types of orbits: asymptotic, transit, non-transit

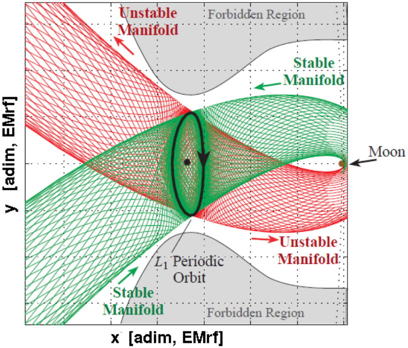


Koon et al [?]

Invariant manifolds

The Stable/Unstable Invariant manifolds

Set of orbits asymptotic to the periodic orbit for $t \to \pm\infty$



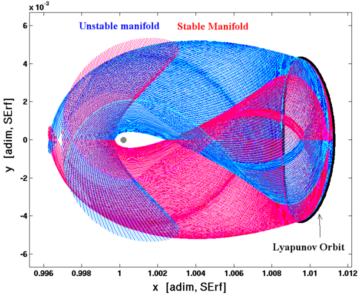
Gomez et al. [?]

- are topologically equivalent to $N-2$ dimensional cylinders in the $N-1$ dim. energy manifold
- act as separatrices in the phase space between transit and non-transit orbit

Invariant manifolds

The Stable/Unstable Invariant manifolds

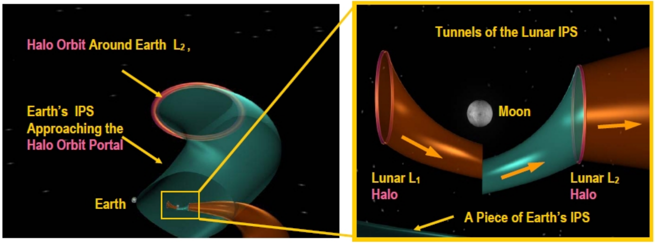
Set of orbits asymptotic to the periodic orbit for $t \to \pm\infty$

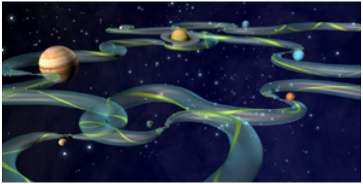


- are topologically equivalent to $N-2$ dimensional cylinders in the $N-1$ dim. energy manifold
- act as separatrices in the phase space between transit and non-transit orbit
- approach the smaller primary
- tangent to the eigenspace of the linearized system (monodromy matrix)

Interplanetary Superhighway

The Tube Dynamics provides Low Energy paths, natural transfers, ballistic capture.
The union of all the Tube Channels $\rightsquigarrow$ Highway network in the space



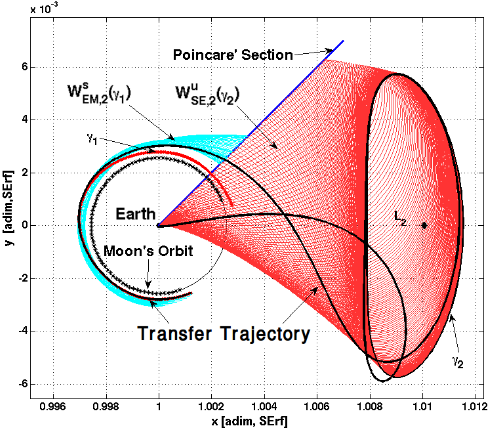


Mission Design: dynamical system theory

Dynamical system theory in low energy trajectory design

Patched 3-body problem

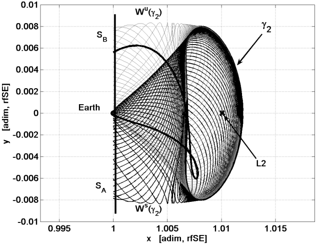
- The 4-Body system is approximated with the superpositions of two Restricted Three-Body problems
- The invariant manifold structures are exploited to design legs of trajectory
- The design restricts to the selection of a connection point on a suitable Poincaré section

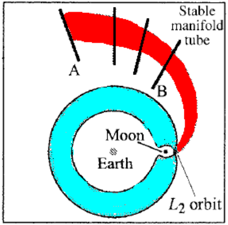


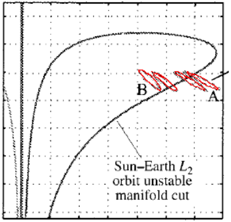
Trajectory in the Coupled CR3BP Approximation in SE Rotating Frame

Some examples

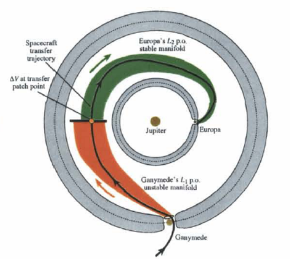
Low energy transfer to the Moon (Figures from [?])

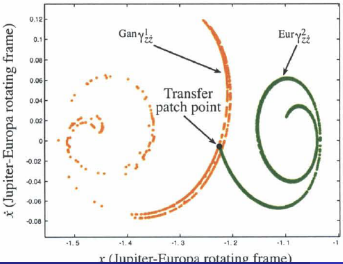






Petit Grand Tour of the moons of Jupiter, (Fig. from [?])





OUTLINE

Dynamical model CRTBP

- Properties
- Periodic orbits
- Tube Dynamics-Interplanetary Superhighway
- Patched CRTBP approximation

Computational methods: Set Oriented Numerics

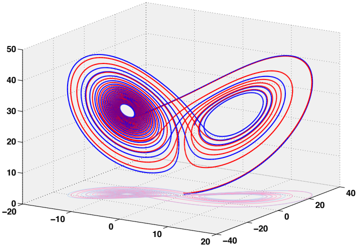
- Covering of Invariant Sets
- GAIO implementation

Examples of mission design

- Earth to Halo
- Trajectory Connection

Set Oriented Numerics

Complicated Dynamical Systems:
Chaos – Sensitive Dependence on initial Data – The long term behavior of a trajectory may give misleading result



Set Oriented Numerics

- Computation of several short term trajectories instead of single long term trajectory
- Approximation of Global structure
  - Invariant Sets : global attractors, Invariant manifolds
  - Invariant measures, almost invariant sets
  - Transport operators

Methodology

Consider a descrete dynamical system

$$x_{k+1} = f(x_k), \quad k = 0, 1, 2, \ldots, \quad f : \mathbb{R}^n \to \mathbb{R}^n$$

Aim: Approximation of a structure within a bounded set Q.
Method: Generate a sequence of collections $\mathcal{B}_1, \mathcal{B}_2 \ldots$ of subsets of Q s.t $\mathcal{B}_0 = \{Q\}$, and iteratively $\mathcal{B}_k$ from $\mathcal{B}_{k-1}$

- Subdivision: define a new collection $\tilde{\mathcal{B}}_k$ such that

$$\bigcup_{B \in \tilde{\mathcal{B}}_k} B = \bigcup_{B \in \mathcal{B}_{k-1}} B$$

$$\mathrm{diam}(\tilde{\mathcal{B}}_k) \leq \theta_k \mathrm{diam}(\mathcal{B}_{k-1}), \quad \theta_k \in (0, 1)$$

- Selection: define $\mathcal{B}_k$ as

$$\mathcal{B}_k = \{B \in \tilde{\mathcal{B}}_k : \text{ such that } \quad g(B) \cap \tilde{B} \neq \emptyset \text{ for some } \tilde{B} \in \tilde{\mathcal{B}}_k\}$$

Relative Global Attractor

Definition Relative Global attractor

Let $Q \subset \mathbb{R}^n$ be a compact set. Define **global attractor relative to** Q by

$$A_Q = \bigcap_{j \geq 0} f^j(Q)$$

Properties

$A_Q \subset Q$

$f^{-1}(A_Q) \subset A_Q$ but not necessarly $f(A_Q) \subset A_Q$.

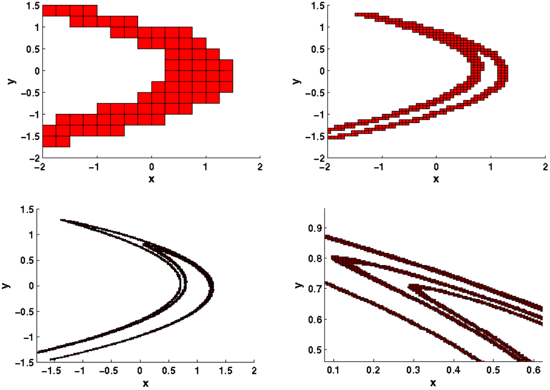
A_Q is a subset of the global attractor A but, in general $A_Q \neq A \cap Q$

Selection step: define $\mathcal{B}_k$ as

$$\mathcal{B}_k = \{B \in \tilde{\mathcal{B}}_k : \text{ such that } \quad f^{-1}(B) \cap \tilde{B} \neq \emptyset \text{ for some } \tilde{B} \in \tilde{\mathcal{B}}_k\}$$

Hénon map

$$\begin{cases} x_{k+1} = 1 - a x_k^2 + b y_k & a = 1.4, b = 0.3 \\ y_{k+1} = x_k \end{cases}$$



Successive finer covering of the global attractor relative to $[-2, 2]^2$ of the Henon map, $k = 8, 12, 16, 20$

Computing the unstable manifold

Initialization - Continuation Algorithm

Aim: Compute the unstable manifold of a point p into a (large) compact set Q, ($p \in Q$)

1 Given Q, compute $\mathcal{P}_0, \mathcal{P}_1, \ldots \mathcal{P}_l$ nested sequence of fine partitions of Q. Select the element $C \in \mathcal{P}_l$ such that $p \in C$ and $A_C = W^u_{loc}(p) \cap C$

2 Initialization Starting from $\mathcal{B}_0 = \{C\}$, refine the approximation of $W^u_{loc}(p) \cap C$ by subdivision, yielding $\mathcal{B}_k^{(0)} \subset \mathcal{P}_{l+k}$

3 Continuation From $\mathcal{B}_k^{(j-1)}$ compute

$$\mathcal{B}_k^{(j)} = \{B \in \mathcal{P}_{l+k} : f(B') \cap B \neq \emptyset, \text{for some } B' \in \mathcal{B}_k^{(j-1)}\}$$

Computing the unstable manifold

Initialization - Continuation Algorithm

Aim: Compute the unstable manifold of a point p into a (large) compact set Q, ($p \in Q$)

1. Given Q, compute $\mathcal{P}_0, \mathcal{P}_1, \dots \mathcal{P}_l$ nested sequence of fine partitions of Q. Select the element $C \in \mathcal{P}_l$ such that $p \in C$ and $A_C = W^u_{loc}(p) \cap C$
2. Initialization Starting from $\mathcal{B}_0 = \{C\}$, refine the approximation of $W^u_{loc}(p) \cap C$ by subdivision, yielding $\mathcal{B}^{(0)}_k \subset \mathcal{P}_{l+k}$
3. Continuation From $\mathcal{B}^{(j-1)}_k$ compute

$$\mathcal{B}^{(j)}_k = \{B \in \mathcal{P}_{l+k} : f(B') \cap B \neq \emptyset, \text{for some } B' \in \mathcal{B}^{(j-1)}_k\}$$

Computing the unstable manifold

Initialization - Continuation Algorithm

Aim: Compute the unstable manifold of a point p into a (large) compact set Q, ($p \in Q$)

1 Given Q, compute $\mathcal{P}_0, \mathcal{P}_1, \ldots \mathcal{P}_l$ nested sequence of fine partitions of Q. Select the element $C \in \mathcal{P}_l$ such that $p \in C$ and $A_C = W^u_{loc}(p) \cap C$

2 Initialization Starting from $\mathcal{B}_0 = \{C\}$, refine the approximation of $W^u_{loc}(p) \cap C$ by subdivision, yielding $\mathcal{B}_k^{(0)} \subset \mathcal{P}_{l+k}$

3 Continuation From $\mathcal{B}_k^{(j-1)}$ compute

$$\mathcal{B}_k^{(j)} = \{B \in \mathcal{P}_{l+k} : f(B') \cap B \neq \emptyset, \text{for some } B' \in \mathcal{B}_k^{(j-1)}\}$$

Computing the unstable manifold

Initialization - Continuation Algorithm

Aim: Compute the unstable manifold of a point p into a (large) compact set Q, ($p \in Q$)

1. Given Q, compute $\mathcal{P}_0, \mathcal{P}_1, \dots \mathcal{P}_l$ nested sequence of fine partitions of Q. Select the element $C \in \mathcal{P}_l$ such that $p \in C$ and $A_C = W^u_{loc}(p) \cap C$
2. Initialization Starting from $\mathcal{B}_0 = \{C\}$, refine the approximation of $W^u_{loc}(p) \cap C$ by subdivision, yielding $\mathcal{B}_k^{(0)} \subset \mathcal{P}_{l+k}$
3. Continuation From $\mathcal{B}_k^{(j-1)}$ compute

$$\mathcal{B}_k^{(j)} = \{B \in \mathcal{P}_{l+k} : f(B') \cap B \neq \emptyset, \text{for some } B' \in \mathcal{B}_k^{(j-1)}\}$$

Computing the unstable manifold

Initialization - Continuation Algorithm

Aim: Compute the unstable manifold of a point p into a (large) compact set Q, ($p \in Q$)

1. Given Q, compute $\mathcal{P}_0, \mathcal{P}_1, \dots \mathcal{P}_l$ nested sequence of fine partitions of Q. Select the element $C \in \mathcal{P}_l$ such that $p \in C$ and $A_C = W^u_{loc}(p) \cap C$
2. Initialization Starting from $\mathcal{B}_0 = \{C\}$, refine the approximation of $W^u_{loc}(p) \cap C$ by subdivision, yielding $\mathcal{B}^{(0)}_k \subset \mathcal{P}_{l+k}$
3. Continuation From $\mathcal{B}^{(j-1)}_k$ compute

$$\mathcal{B}^{(j)}_k = \{B \in \mathcal{P}_{l+k} : f(B') \cap B \neq \emptyset, \text{for some } B' \in \mathcal{B}^{(j-1)}_k\}$$

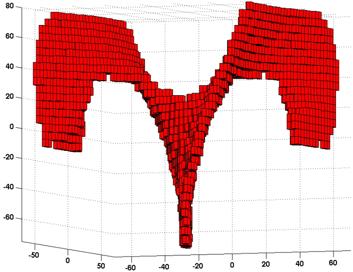
Remark: The sets $C^{(j)}_k = \bigcup_{B \in \mathcal{B}^{(j)}_k} B \subset \mathbb{R}^n$ form a nested sequence in k:

$$C^{(j)}_0 \supset C^{(j)}_1 \supset C^{(j)}_2 \dots \text{ for } j = 0, 1, 2, \dots$$

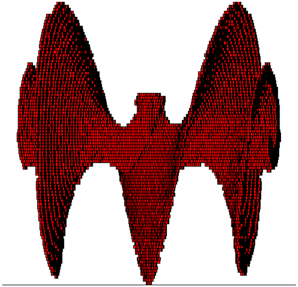
Lorenz system

$$\dot{x} = \sigma(y - x)$$
$$\dot{y} = \rho x - y - xz$$
$$\dot{z} = -\beta z + xy$$

$(0, 0, 0)$ is fixed point
$\sigma = 10,\ \rho = 28,\ \beta = 8/3$

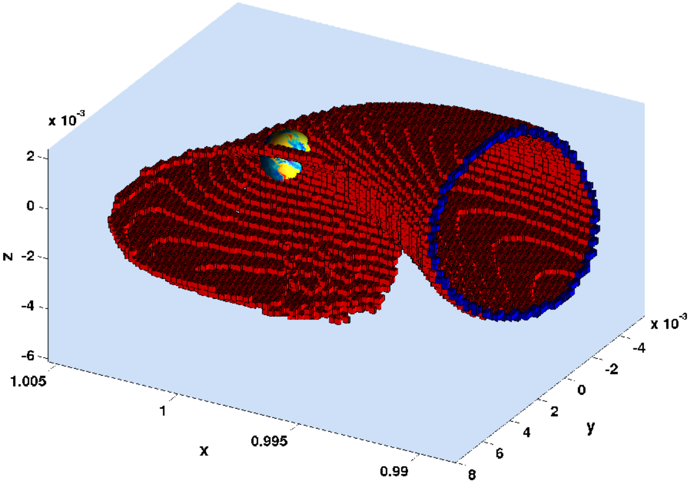


Covering for the two-dimensional stable manifold of the fixed point (0,0,0) in Lorenz System



Covering of the two-dimensional stable manifold of the fixed point (0,0,0) for the Lorenz System

Covering of the two-dimensional stable manifold of the origin
Left: $l = 9, k = 6, j = 4$ initial box $Q = [-70, 70] \times [-70, 70] \times [-80, 80]$,
Right: $l = 21, k = 0, j = 10$, initial box $Q = [-120, 120] \times [-120, 120] \times [-160, 160]$



Box covering of the part of unstable manifold of an Halo orbit in the Sun-Earth CRTBP.

Implementation – GAIO package

GAIO: Global Analysis Invariant Object [?]

Boxes : Generalized rectangle $B(c, r) \subset R^n$

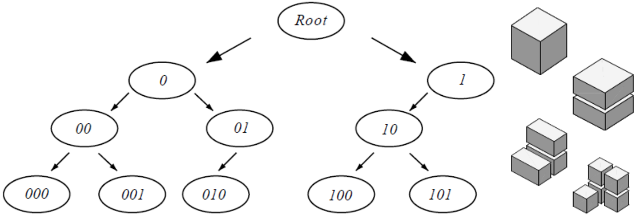
$$B(c, r) = \{y \in \mathbb{R}^n : |y_i - c_i| \leq r_i,\ \ i : 1 \ldots n\}$$

identified by a centre and vector of radii, $c, r \in \mathbb{R}^n$.

Subdivision : by bisection along one of the coordinate direction: from $B(c, r)$ to $B_1(c_+, r^1).B_2(c_-, r^1)$

$$r_i^1 = \begin{cases} r_i & i \neq j \\ r_i/2 & i = j \end{cases}, c_i^{\pm} = \begin{cases} c_i & i \neq j \\ c_i \pm r_i/2 & i = j \end{cases} \tag{1}$$

Storage of boxes: Binary tree



In Fig: representation of three subdivision steps in three dimension.
The spatial coordinates of a collection are completely determined by the tree structure of the boxes and the initial rectangle $\mathcal{B}_0$.

Image of a Box

The image of a box B in the collection $\mathcal{B}$ is defined

$$\mathcal{F}_{\mathcal{B}}(B) = \{B' \in \mathcal{B} : f(B) \cap B' \neq \emptyset\}$$

Definition of $f(B)$

Choose a finite set $T \subset B$ of test points and define

$$\tilde{\mathcal{F}}_{\mathcal{B}}(B) = \{B' \in \mathcal{B} : f(T) \cap B' \neq \emptyset\}$$

Clearly

$$\tilde{\mathcal{F}}_{\mathcal{B}}(B) \subset \mathcal{F}_{\mathcal{B}}(B)$$

but possibly

$$\tilde{\mathcal{F}}_{\mathcal{B}}(B) \neq \mathcal{F}_{\mathcal{B}}(B)$$

Some of the boxes in the image could be missed.

Image of a Box

The image of a box B in the collection $\mathcal{B}$ is defined

$$\mathcal{F}_{\mathcal{B}}(B) = \{B' \in \mathcal{B} : f(B) \cap B' \neq \emptyset\}$$

Definition of $f(B)$

Choose a finite set $T \subset B$ of test points and define

$$\tilde{\mathcal{F}}_{\mathcal{B}}(B) = \{B' \in \mathcal{B} : f(T) \cap B' \neq \emptyset\}$$

Clearly

$$\tilde{\mathcal{F}}_{\mathcal{B}}(B) \subset \mathcal{F}_{\mathcal{B}}(B)$$

but possibly

$$\tilde{\mathcal{F}}_{\mathcal{B}}(B) \neq \mathcal{F}_{\mathcal{B}}(B)$$

Some of the boxes in the image could be missed.

Image of a Box

The image of a box B in the collection $\mathcal{B}$ is defined

$$\mathcal{F}_{\mathcal{B}}(B) = \{B' \in \mathcal{B} : f(B) \cap B' \neq \emptyset\}$$

Definition of $f(B)$

Choose a finite set $T \subset B$ of test points and define

$$\tilde{\mathcal{F}}_{\mathcal{B}}(B) = \{B' \in \mathcal{B} : f(T) \cap B' \neq \emptyset\}$$

Clearly

$$\tilde{\mathcal{F}}_{\mathcal{B}}(B) \subset \mathcal{F}_{\mathcal{B}}(B)$$

but possibly

$$\tilde{\mathcal{F}}_{\mathcal{B}}(B) \neq \mathcal{F}_{\mathcal{B}}(B)$$

Some of the boxes in the image could be missed.

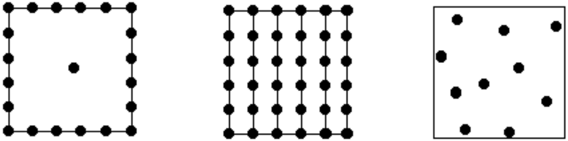
Choice of test-points

In low dimensional phase space ($d \leq 3$)

- N points on the edges of the boxes + the center
- on uniform grid within the box

In higher dimension

- randomly distribuited



Remark: Rigorous choice of test point in such a way that no boxes are lost due to the discretization could be done if the Lipschitz costant of the map f is known.

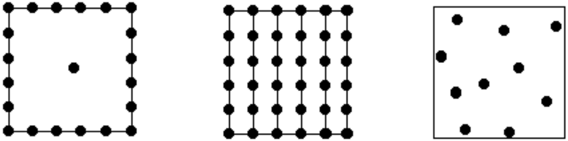
Choice of test-points

In low dimensional phase space ($d \leq 3$)

- N points on the edges of the boxes + the center
- on uniform grid within the box

In higher dimension

- randomly distribuited



Remark: Rigorous choice of test point in such a way that no boxes are lost due to the discretization could be done if the Lipschitz costant of the map f is known.

OUTLINE

Dynamical model CRTBP
- Properties
- Periodic orbits
- Tube Dynamics-Interplanetary Superhighway
- Patched CRTBP approximation

Computational methods: Set Oriented Numerics
- Covering of Invariant Sets
- GAIO implementation

Examples of mission design
- Earth to Halo
- Trajectory Connection

Leo to Halo mission design

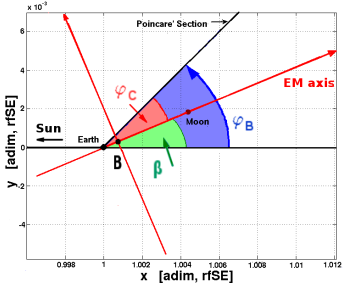
- Trajectory from Leo to Halo
- Low energy ballistic transfers made up of impulsive manoeuvres.
- two coupled Restricted Three-Body Problem Planar + Spatial
- Statement of the problem: Optimisation theory, with dynamics described by the Restricted Four-Body model - bicircular, spatial - with the Sun gravitational influence (Sun perturbed CRTBP).

Leo to Halo mission design

- Trajectory from Leo to Halo
- Low energy ballistic transfers made up of impulsive manoeuvres.

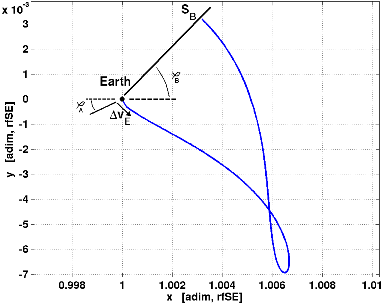
- two coupled Restricted Three-Body Problem Planar + Spatial
- Statement of the problem: Optimisation theory, with dynamics described by the Restricted Four-Body model - bicircular, spatial - with the Sun gravitational influence (Sun perturbed CRTBP).

- φ_B blue angle: S_B surface.
- φ_C red angle: S_C surface.
- $\beta = \varphi_B - \varphi_C$ green angle: Relative phase.



Mission Design

Earth escape:

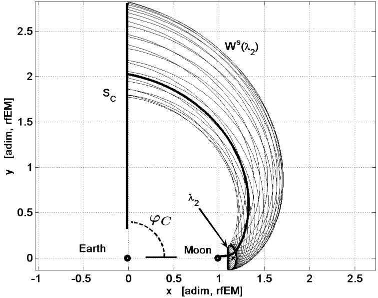


Planar Sun-Earth model

Launch point on LEO (167 km)

Tangential manouevre (ΔV)

Halo orbit arrival



Spatial Earth-Moon model

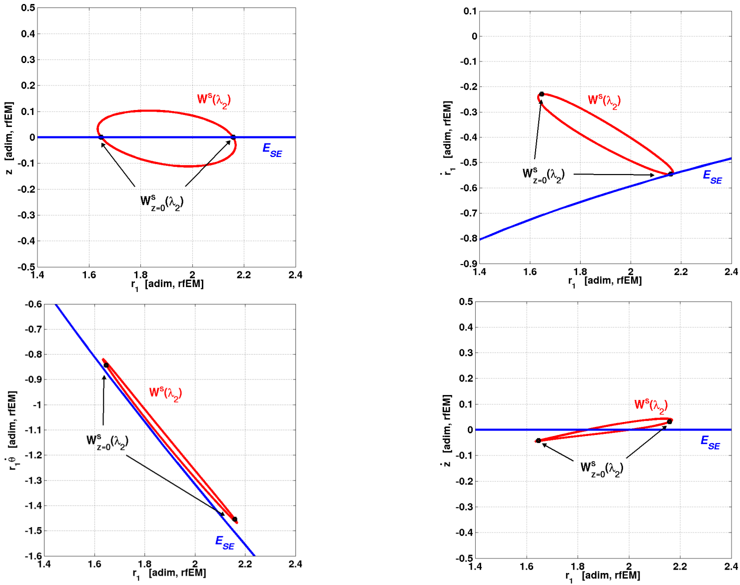
Stable manifold

Ballistic capture to the Halo

Transfer Points

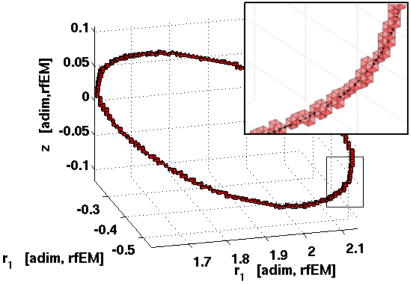
- Properties of transfer points:
  - Necessary condition to obtain a feasible transfer is that the pair of points $\in P_{SE}, P_{EM}$ have the same location in configuration space.
  - The discontinuity in terms of Δv has to be small.

Poincaré maps →Transfer Points

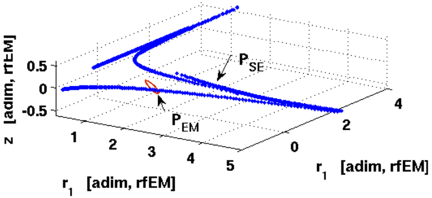


Technique: Box approach

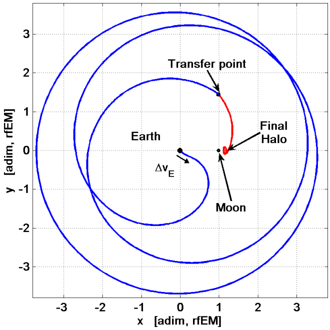
Box Covering of the EM Poincaré section

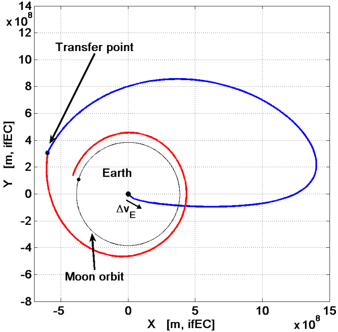


Intersection with the SE Poincaré section



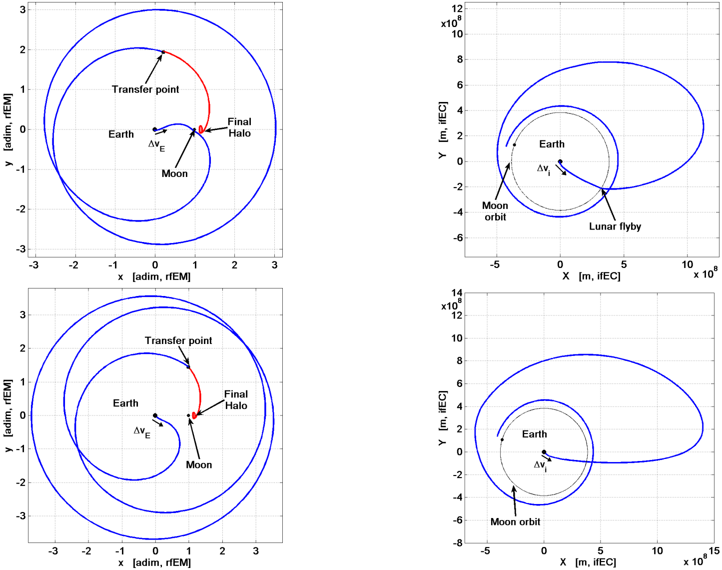
Sample first guess trajectory





- First guess trajectories with $J_{EM} = 3.159738$ (Az = 8000 km) and $J_{EM} = 3.161327$ (Az = 10000 km) are later optimized in the bicircular Sun-perturbed EM model.

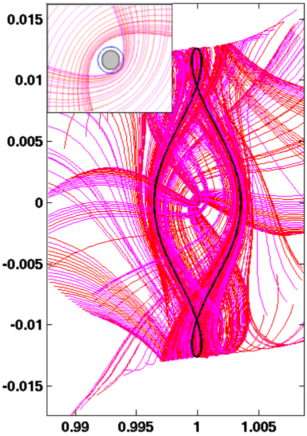
Designed trajectories



First stage: Leo to SE-DPO

Look for impulsive manoeuvre transfer from Leo to DPO in SE-CRTBP

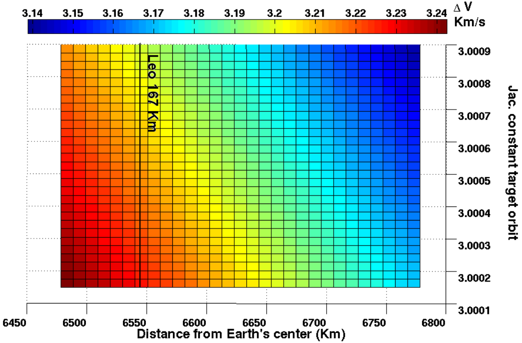
- Integrate backwards the stable manifold
- Intersect the manifold with Leo
- Select those intersections that are tangent to Leo w.r.t geocentric coordinates



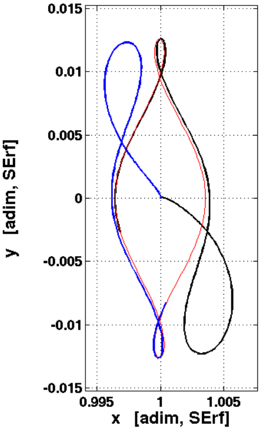
Remark A manoeuvre $|v|$ applied in the same direction of the motion produces the maximal change of Jacobi constant. It holds

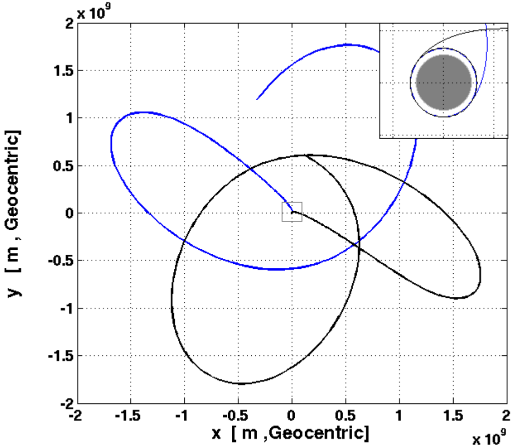
$$\Delta J = |v|^2 + 2|C||v||z|$$

where $C = \frac{V_t}{|z|} - 1$ depends on the Leo altitude and $|z|$ is the geocentric distance, V_t orbital velocity



The Jac.const on a Leo (167 Km alt.) is about 3.070352
The Jac.const. of family g is in the range [3.00014;3.00092]





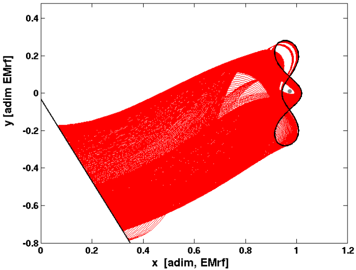
Jacobi- DPO	h-LEO (Km)	ΔV (m/s)
3.000464798057	220	3190
3.000464798057	160	3212

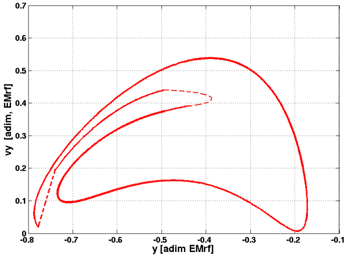
Transfer between DPOs

Design connection between SE-DPO and EM-DPO

- Dynamical system : Patched Restricted Three-Body Problem
- Low energy , ballistic transfer with eventually impulsive maneouvre
- Intersect invariant manifolds on Poincaré sections
- Consider *interior* and *exterior* connections

Invariant Manifolds





Left: Stable manifold in the interior region for a DPO in the EM-CRTBP.

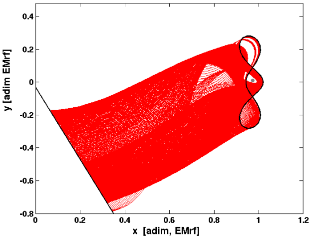
Right: Projection of the Poincaré map

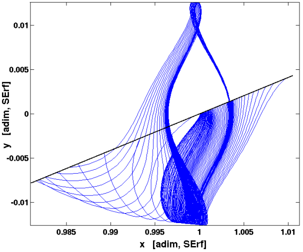
- The Invariant manifold are topologically equivalent to cylinder $\mathbb{S}^1 \times \mathbb{R}$
- Initial data on a Invariant Manifold approach the periodic orbit for $t \to \pm\infty$

Design

Procedure for design the transfer:

1) Select two DPOs
2) Compute the Poincaré map of (un)-stable manifold on a section (line through the Earth with slope θ_{SE} and θ_{EM}).





Left: Stable manifold in the interior region for a DPO in the EM-CRTBP.

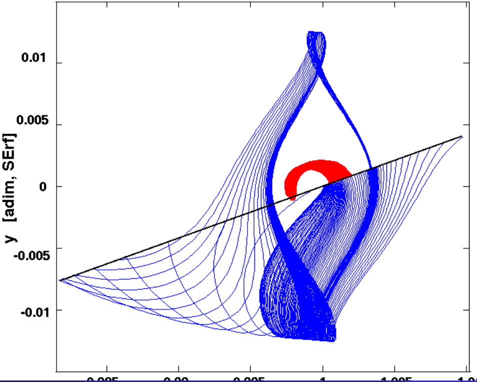
Right: Unstable manifold of a DPO in SE-CRTBP

Design

Procedure for design the transfer:

- Select two DPOs
- Compute the Poincaré map of (un)-stable manifold on a section

3) Write the two maps in the same system of coordinates, being $\theta = \theta_{SE} - \theta_{EM}$ the relative phase of the primaries at the *transfer time*

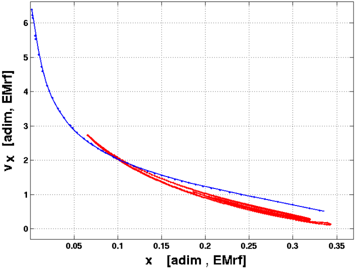


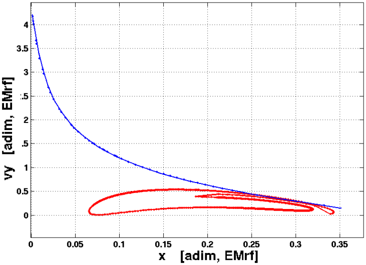
Design

Procedure for design the transfer:

- Select two DPOs
- Compute the Poincaré map of (un)-stable manifold on a section
- Write the two maps in the same system of coordinates, being $\theta = \theta_{SE} - \theta_{EM}$ the relative phase of the primaries

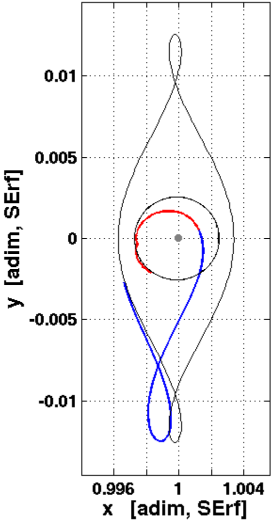
4) Look for possible connections: points on the two maps with the same position and small ΔV

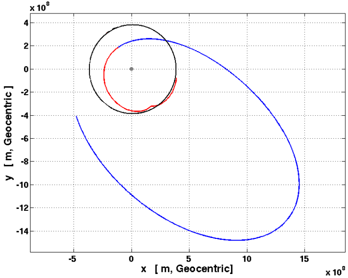


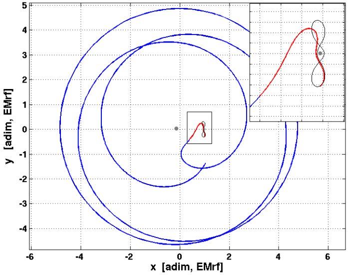


Projection of the Poincaré maps onto the (x, v_x) plane and (x, v_y) plane, in EM-rf

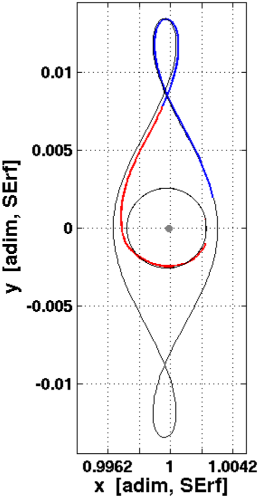
Results: Interior connection

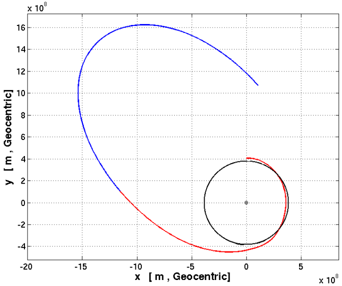


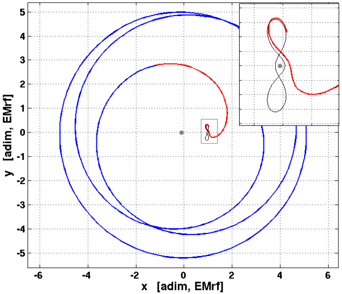




Results: Exterior connection







	SE-Jacobi	EM-Jacobi	Time, (days)	ΔV (m/s)
Interior	3.0004647980	3.02599	115	339
Exterior	3.00043012418	3.026764	116	8

	Type	Start	Target	Time	ΔV (m/s)
Mingotti	Ext- Optim.	LEO	DPO Jac=?	90	3160
Ming	Exterior	LEO	Retr. DPO	101	3207
Ming	Interior	LEO	Retr. DPO	33	3802

[?]
[?]