Application of the Circular Restricted Three-Body problem to mission design.

Roberto Castelli

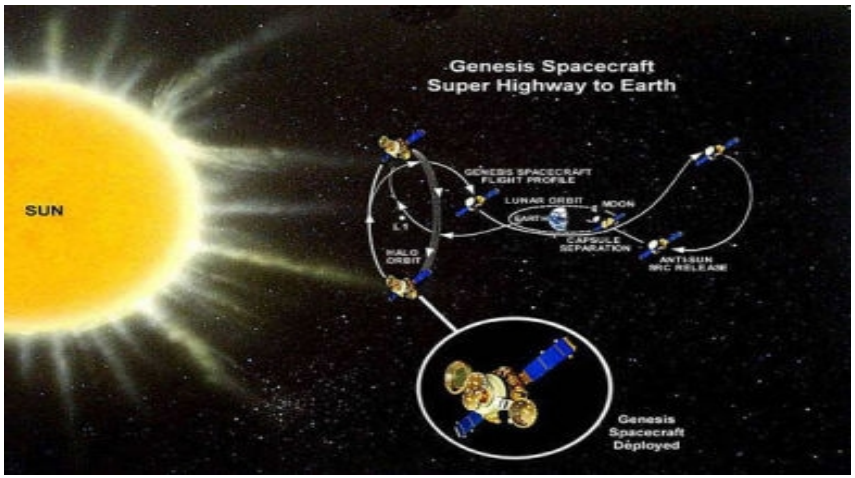
BCAM - Basque Center for Applied Mathematics

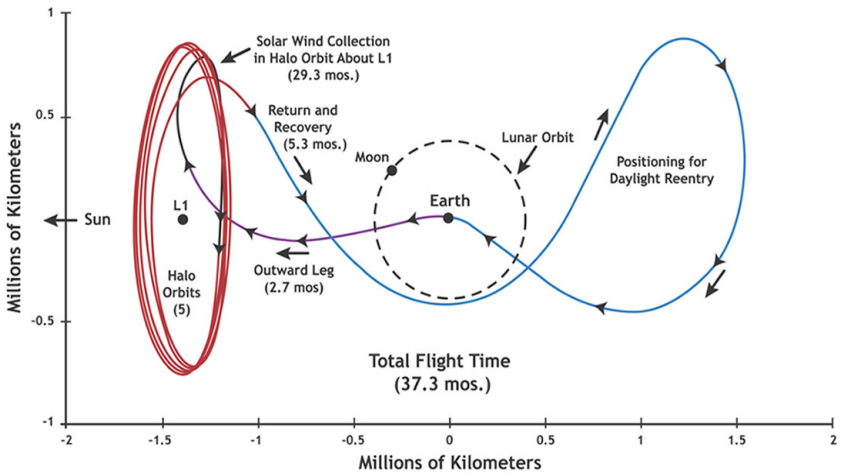
Universitá del Salento, Lecce, 9th March 2011

Introduction

In space mission design

- Consider the Force Field acting on the Spacecraft
- Consider Physical and Technical constraints
- Satisfy some mission requirements
- Take care of the fuel consumption and the travelling time
- ....





Genesis Mission

Introduction

N-BODY PROBLEM

First guess trajectories designed in simplified model

- Two-body model
- Restricted Three-body problem
- Bicircular model
- ...

Numerical Optimisation in Full system

- Direct/Indirect methods
- Multiple shooting technique
- Multiobjective optimisation

Different type of Propulsion (Electric - Chemical)

Low thrust propulsion – Impulsive manoeuvre

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OUTLINE

Dynamical model CRTBP

- Tube Dynamics
- Patched CRTBP approximation

Examples of mission design

- Earth to Halo
- Regions of prevalence
- Sun-Earth DPO to Earth-Moon DPO

Conclusion

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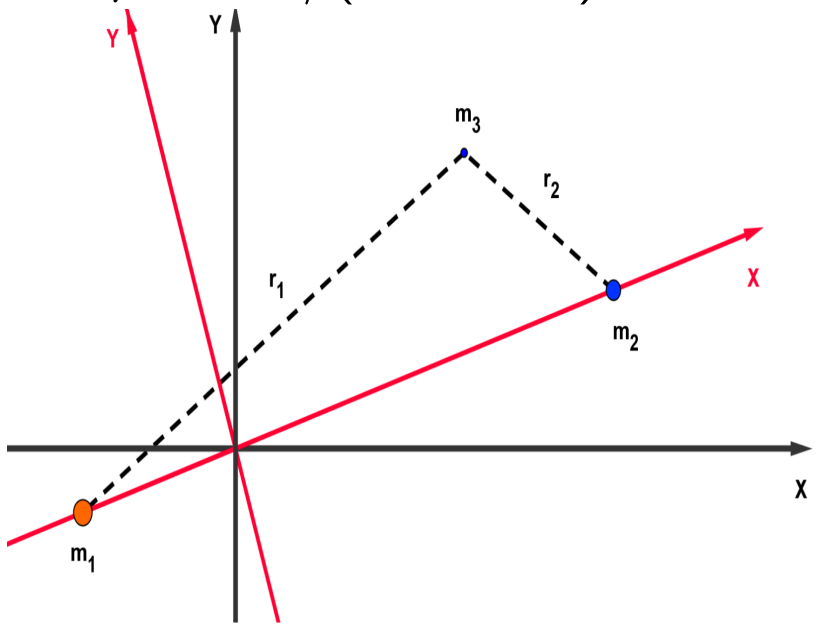
Circular Restricted Three-Body problem

- Two Primaries move in circular orbits under the mutual gravitational attraction
- Massless particle moves under the gravitational influence of two primaries

In a rotating, adimensional reference frame, $\mu = m_2/(m_1 + m_2)$,

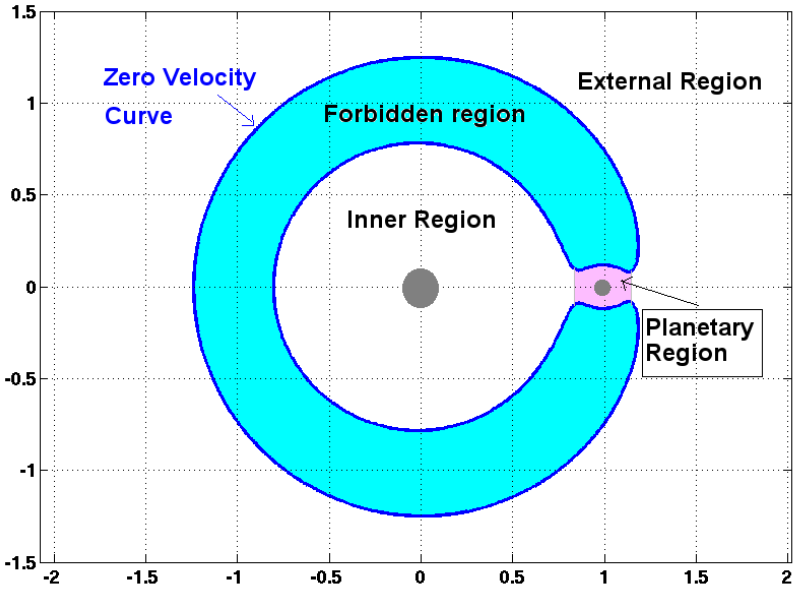
$$(CRTBP) \quad \begin{cases} \ddot{x} - 2\dot{y} = \Omega_x \\ \ddot{y} + 2\dot{x} = \Omega_y \\ \ddot{z} = \Omega_z \end{cases}$$

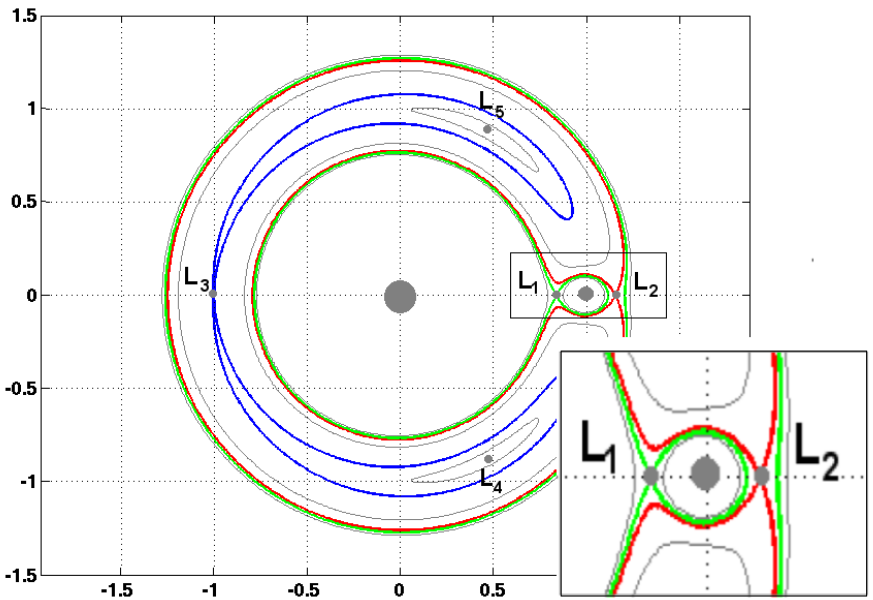
$$\Omega(x, y, z) = \frac{1}{2}(x^2+y^2)+\frac{1-\mu}{r_1}+\frac{\mu}{r_2}+\frac{1}{2}\mu(1-\mu)$$



Properties of CRTBP

- Non integrable Autonomous Hamiltonian System
- Symmetry $(x, y, z, \dot{x}, \dot{y}, \dot{z}; t) \rightarrow (x, -y, z, -\dot{x}, \dot{y}, -\dot{z}; -t)$
- Jacobi Integral: $C = 2\Omega(x, y, z) - (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) = -2E$
- Equilibrium points: Lagrangian Points L_j , $j = 1, ..., 5$.
- Hill's Region: $\mathcal{H}(C) = \{(x, y, z) : 2\Omega(x, y, z) - C \geq 0\}$





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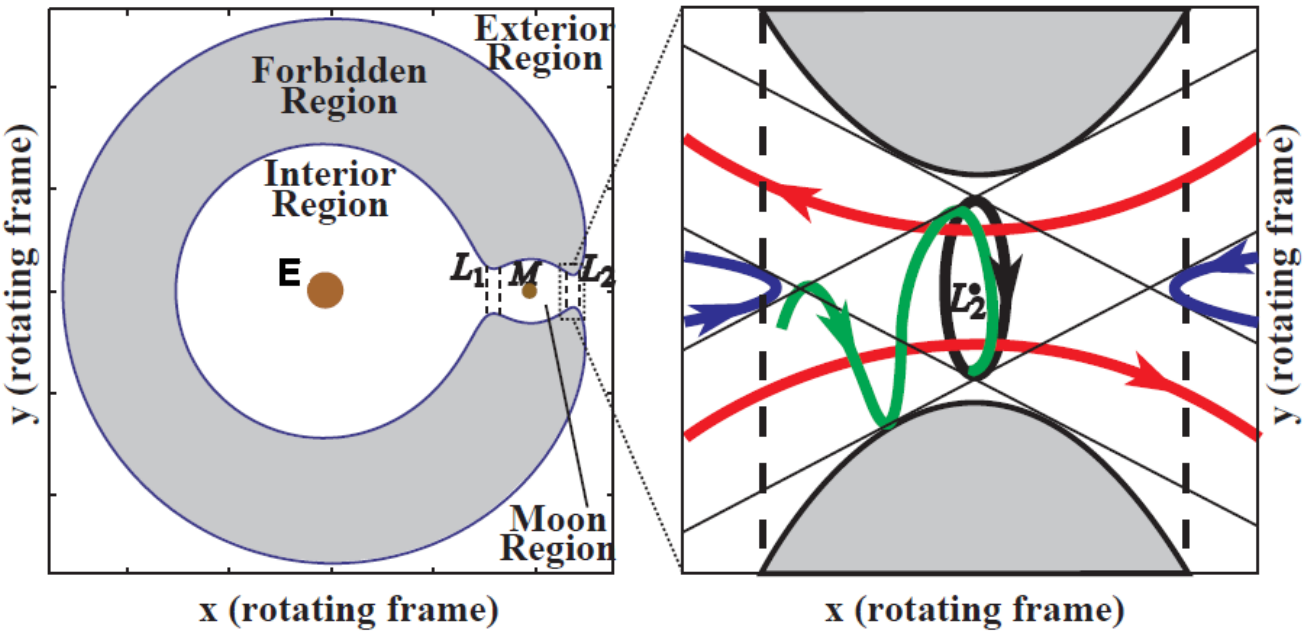
Dynamics near periodic orbits

- The periodic orbits separates two necks in the Hill's region
- Linear Dynamics: saddle $\times$ center
- 3 types of orbits: asymptotic, transit, non-transit

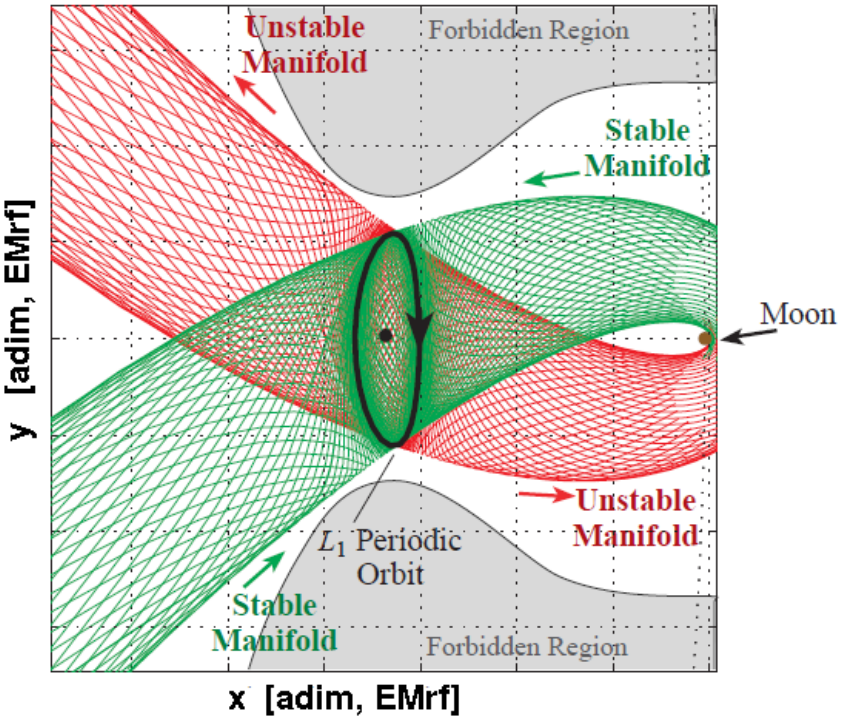


[W.S. Koon et al.]

Invariant manifolds

The Stable/Unstable Invariant manifolds

Set of orbits asymptotic to the periodic orbit for $t \to \pm\infty$



[G. Gomez at al.]

- are topologically equivalent to $N-2$ dimensional cylinders in the $N-1$ dim. energy manifold
- act as separatrices in the phase space between transit and non-transit orbit

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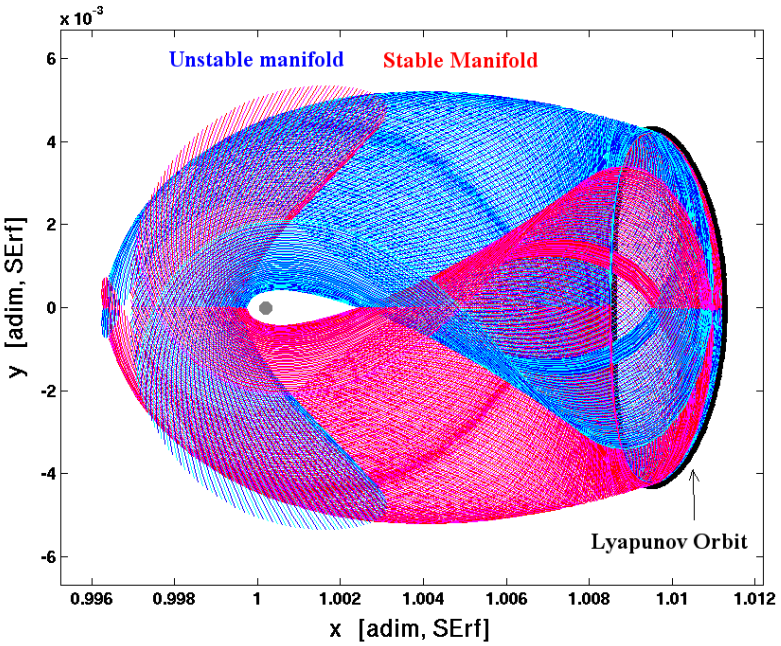
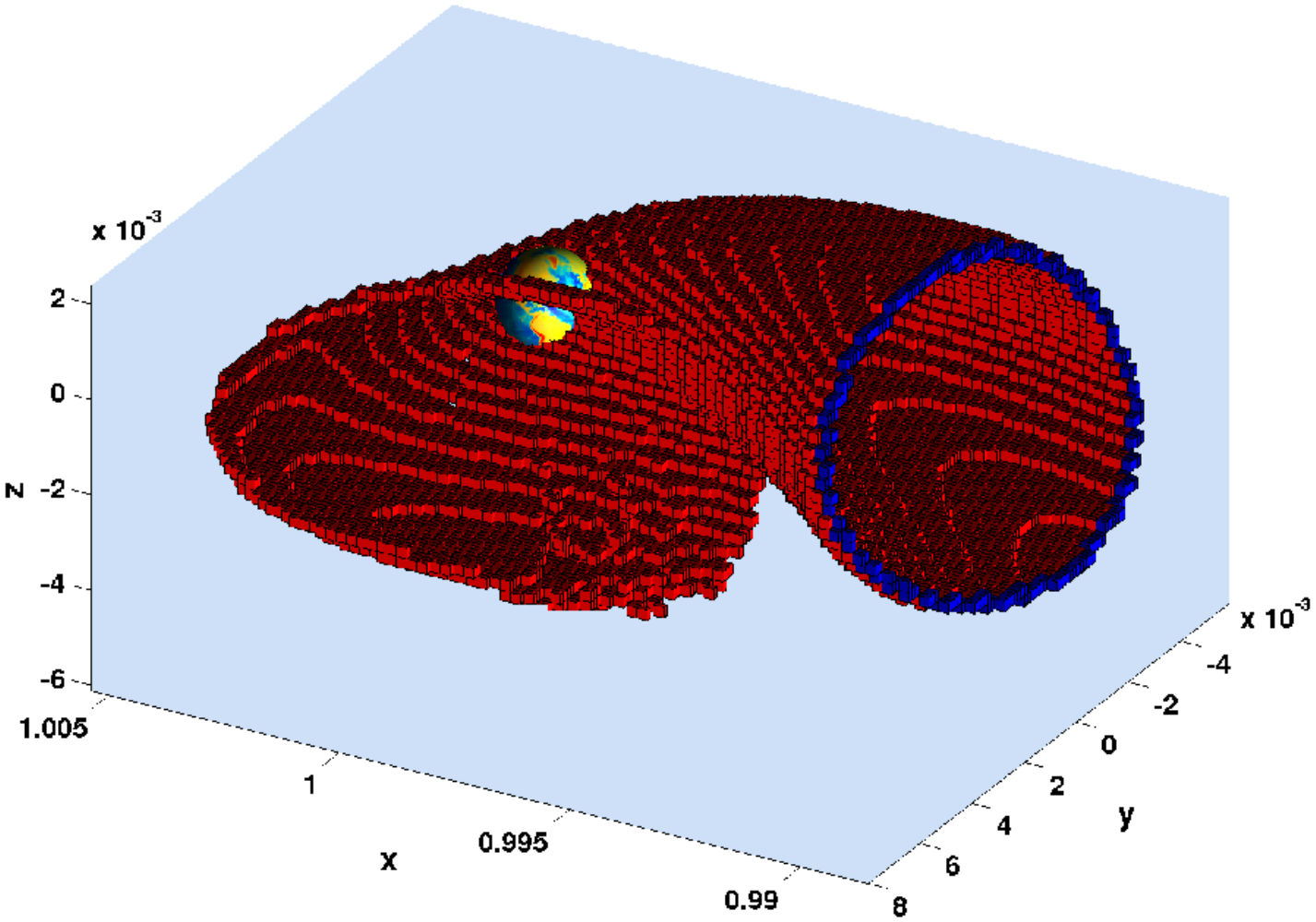


- are topologically equivalent to $N-2$ dimensional cylinders in the $N-1$ dim. energy manifold
- act as separatrices in the phase space between transit and non-transit orbit
- approach the smaller primary
- tangent to the eigenspace of the linearized system (monodromy matrix)



Box covering of the part of unstable manifold of an Halo orbit in the Sun-Earth CRTBP.

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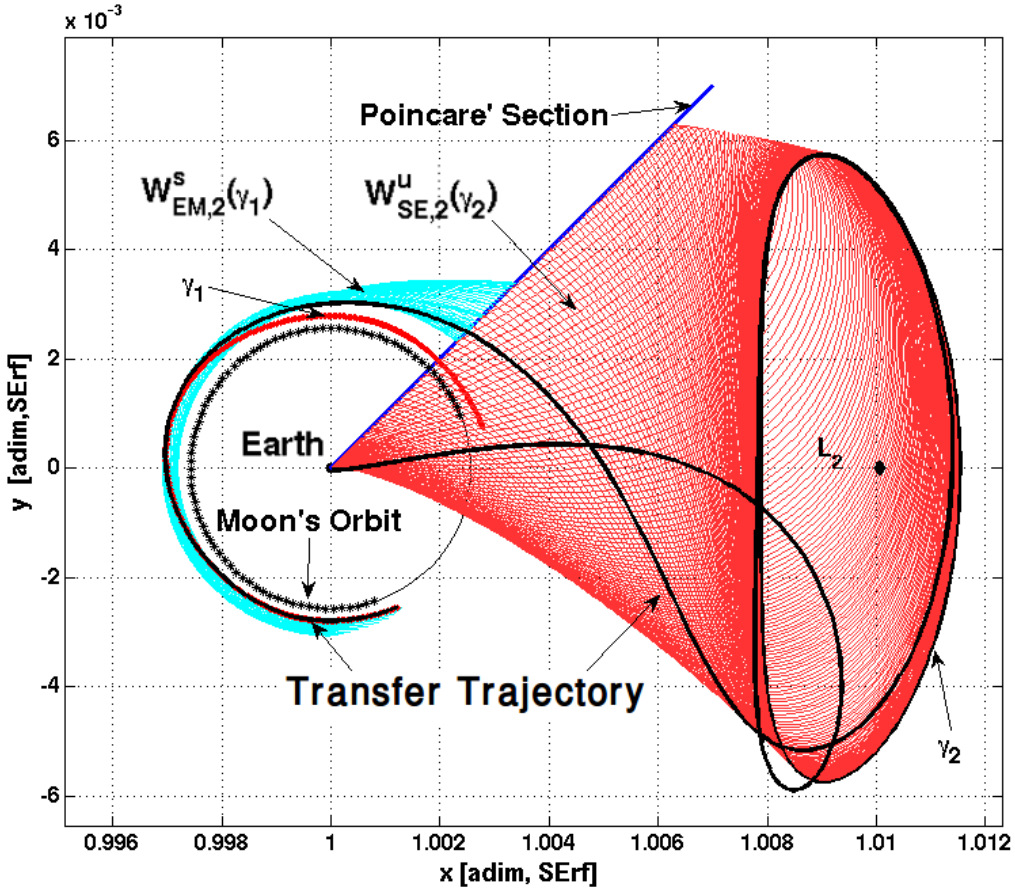
Mission Design: dynamical system theory

Dynamical system theory in low energy trajectory design

Patched 3-body problem

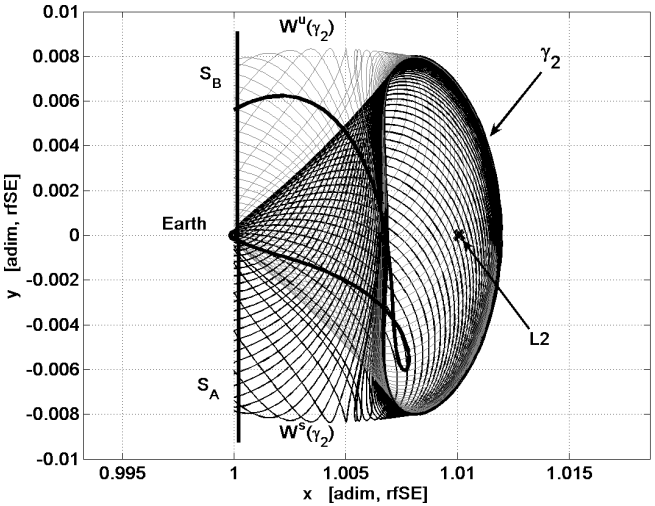
- The 4-Body system is approximated with the superpositions of two Restricted Three-Body problems
- The invariant manifold structures are exploited to design legs of trajectory
- The design restricts to the selection of a connection point on a suitable Poincaré section

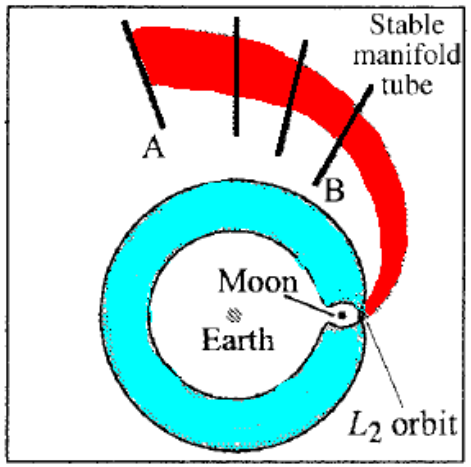


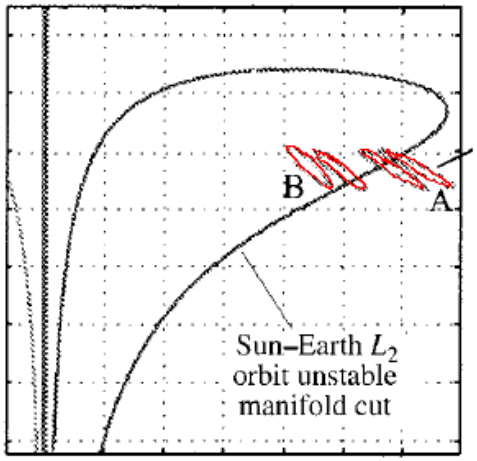
Trajectory in the Coupled CR3BP Approximation in SE Rotating Frame

Some examples

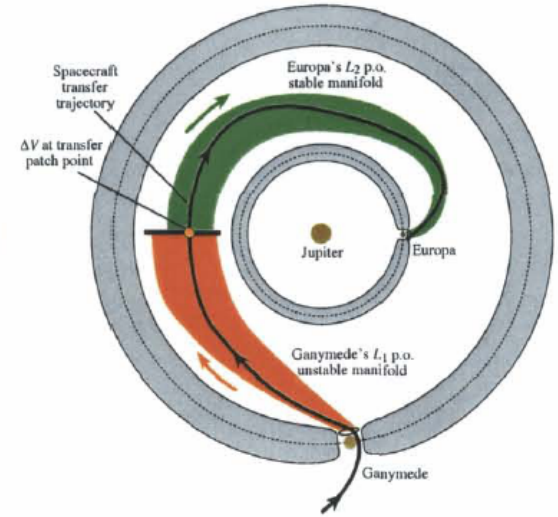
Low energy transfer to the Moon (Fig. from [W.S. Koon et al.])

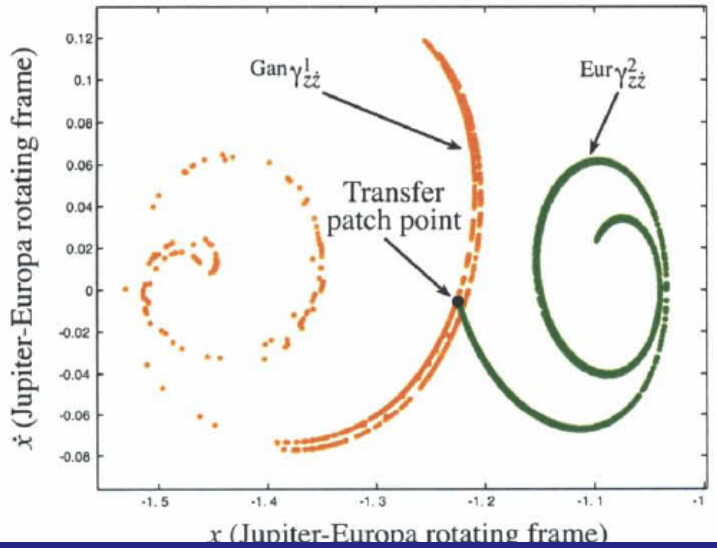






Petit Grand Tour of the moons of Jupiter, (Fig. from [G. Gomez at al.])





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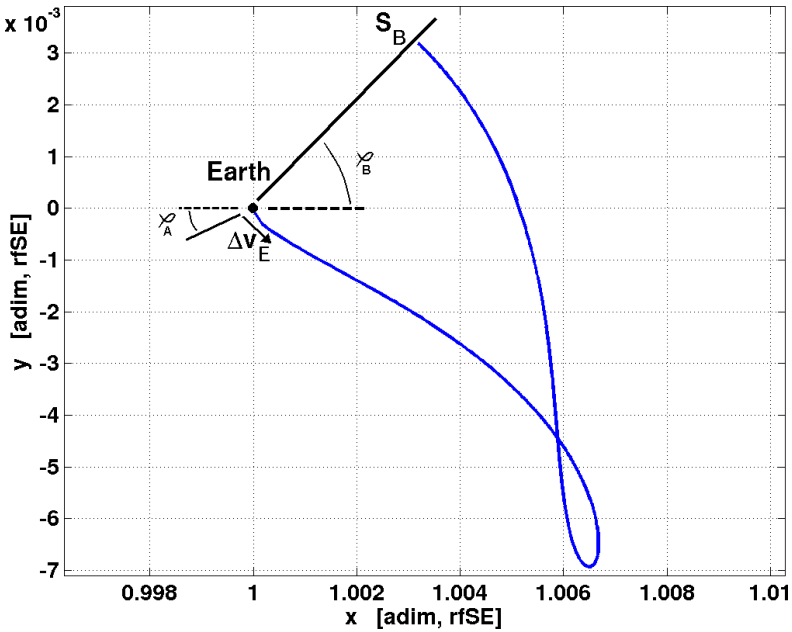
Leo to Halo mission design

- Scientific purposes: Solar observer [ISEE, SOHO, Genesis], Lunar far-side data relay
- Low energy ballistic transfers made up of impulsive manoeuvres.
- two coupled Restricted Three-Body Problem Planar + Spatial
- Statement of the problem: Optimisation theory, with dynamics described by the Restricted Four-Body model - bicircular, spatial - with the Sun gravitational influence (Sun perturbed CRTBP).

[R. Castelli et al.]

Mission Design

Earth escape stage:

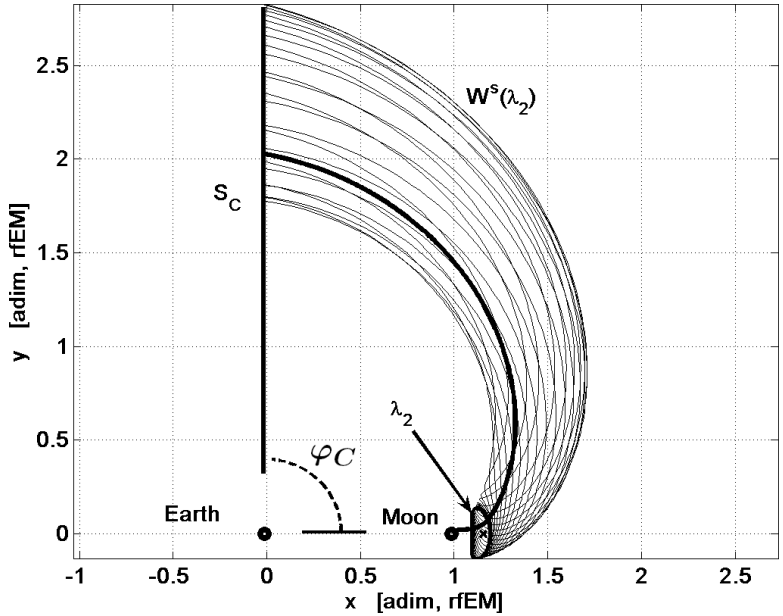


Planar Sun-Earth model

Launch point on LEO (167 km)

Tangential manoeuvre (ΔV)

Halo orbit arrival



Spatial Earth-Moon model

Stable manifold

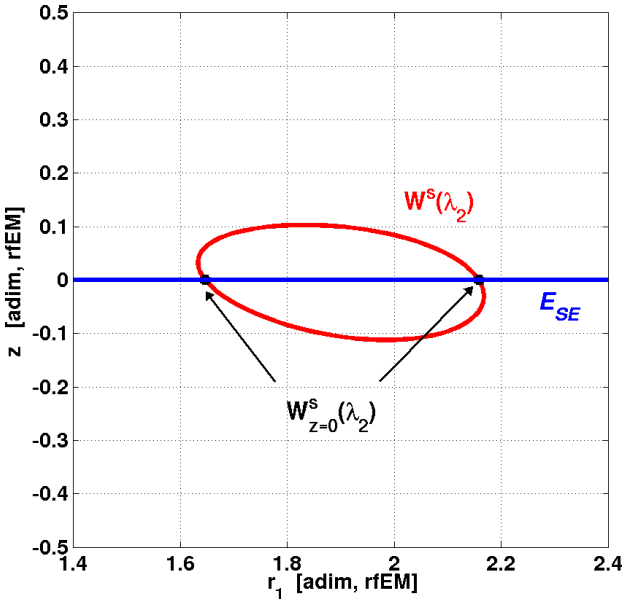
Ballistic capture to the Halo

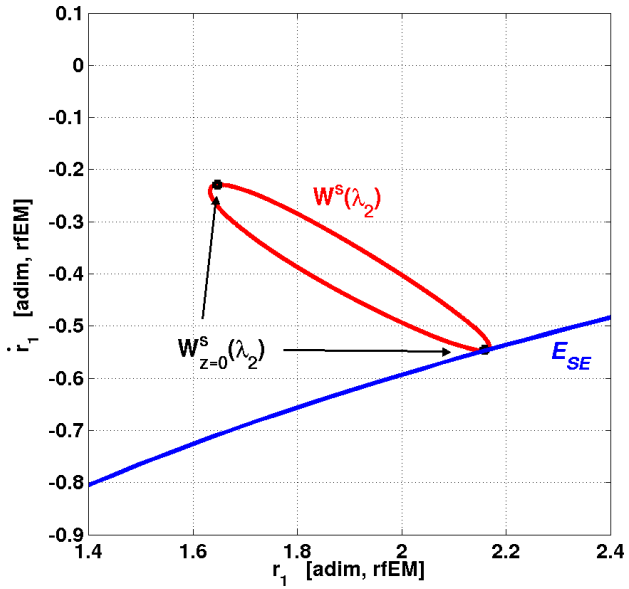
Poincaré section along a line in configuration space

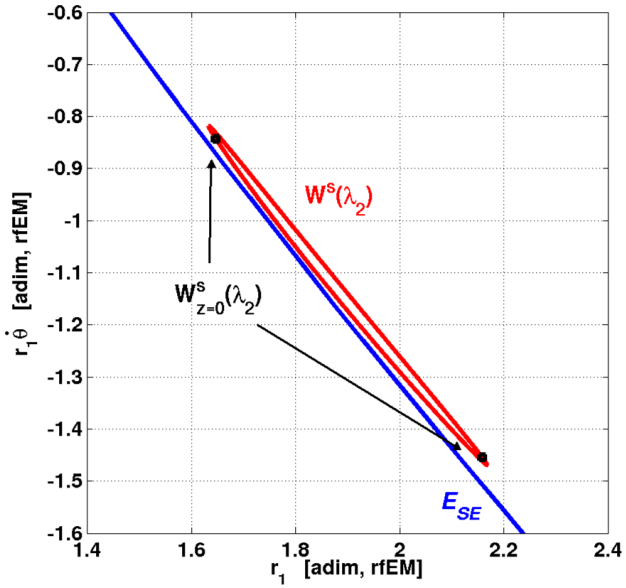
Transfer Points

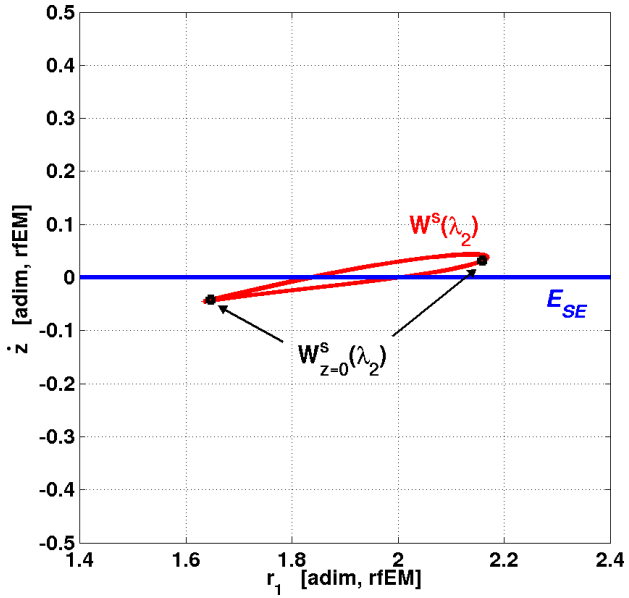
- Properties of transfer points:
  - Necessary condition for a feasible transfer:
    the pair of points on the section must have
    the same location in configuration space.
  - The discontinuity in terms of Δv has to be small.

Poincaré maps $\longrightarrow$ Transfer Points

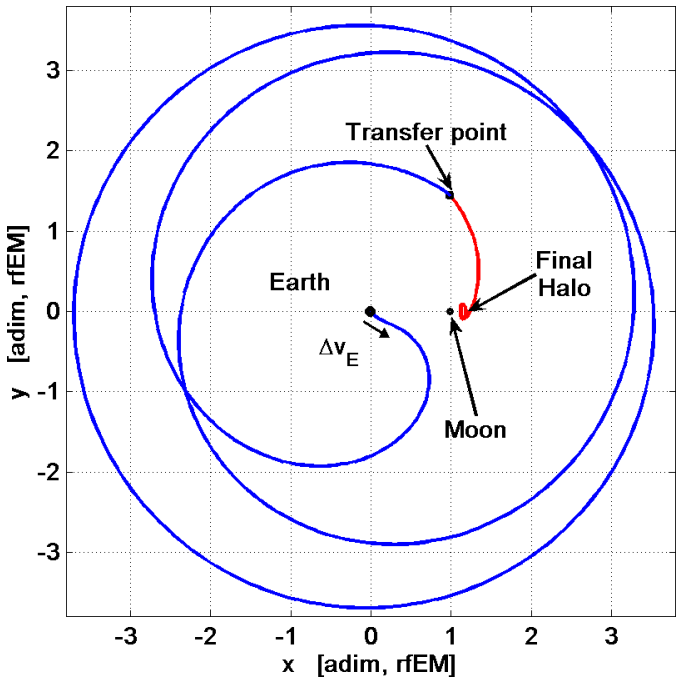


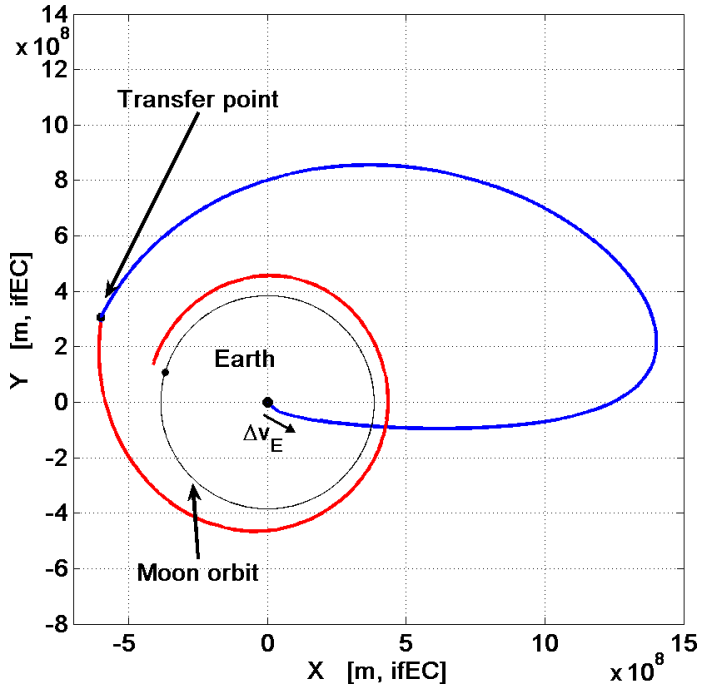






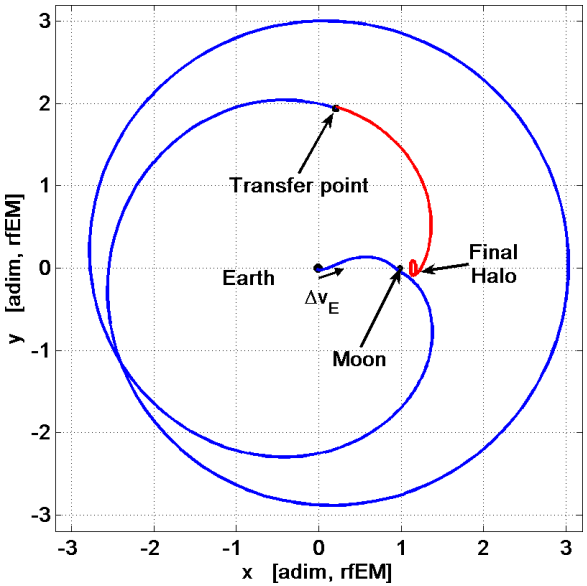
Sample first guess trajectory

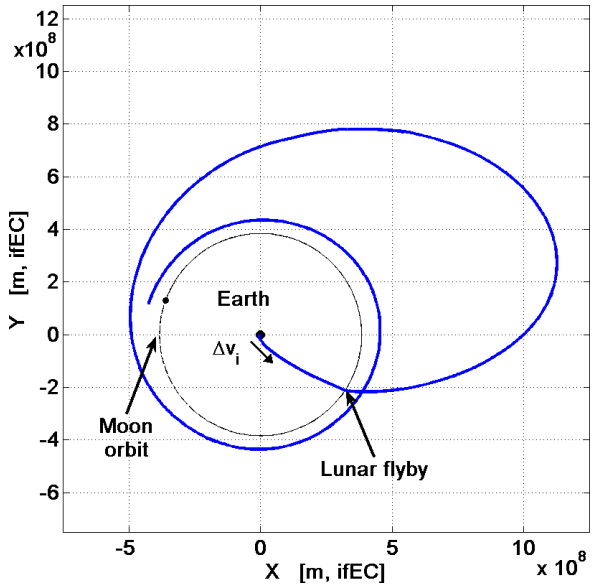


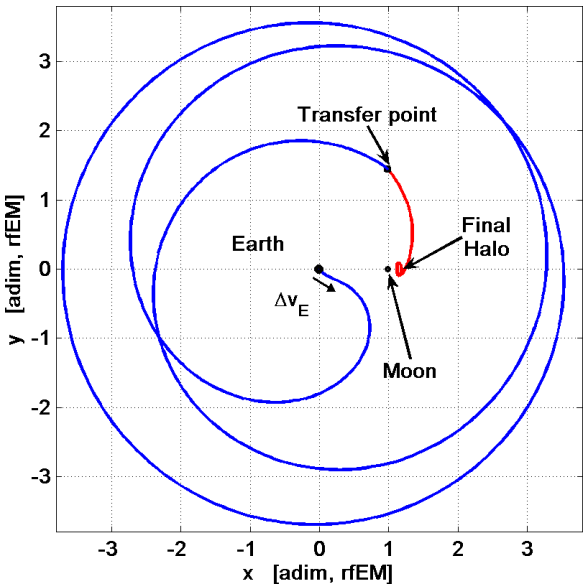


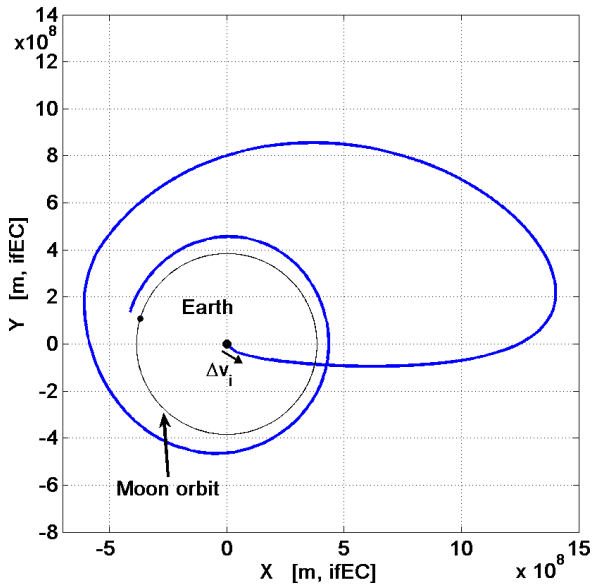
- First guess trajectories with $J_{EM} = 3.159738$ (Az = 8000 km) and $J_{EM} = 3.161327$ (Az = 10000 km) are later optimized in the bicircular Sun-perturbed EM model.

Designed trajectories









SOLUTION PERFORMANCES

Name	Type	Δv_i [m/s]	Δv_f [m/s]	Δv_t [m/s]	Δt [days]
sol.1.1	Two-Imp.	3110	214	3324	106
sol.1.2	Sing-Imp.	3161	0	3161	105
sol.2.1	Two-Imp.	3150	228	3378	128
sol.2.2	Sing-Imp.	3201	0	3201	134
Mingotti	Two-Imp.	–	–	3676	65
Parker	Two-Imp.	3132	618	3750	–
Parker	Sing-Imp.	3235	–	3235	–
Mingtao	Three-Imp.	3120	360	3480	17

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Choice of the Poincaré section

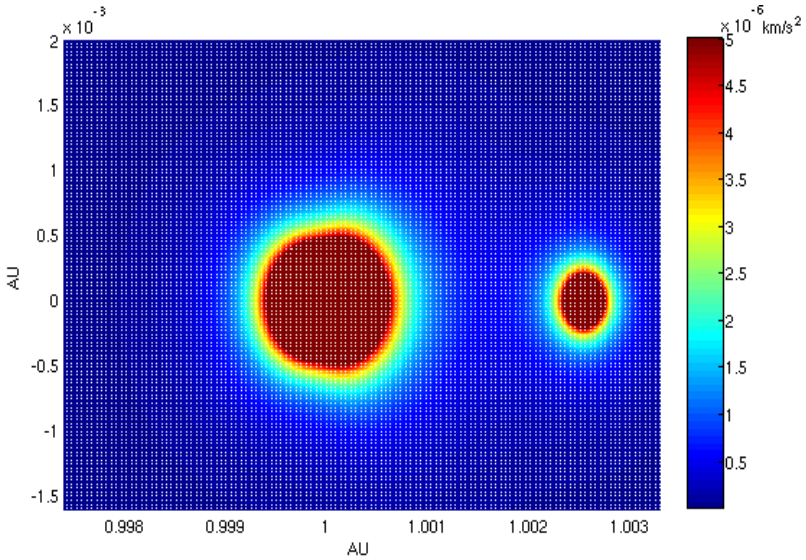
How to chose the Poincaré section in the Patched CRTBP approximation?

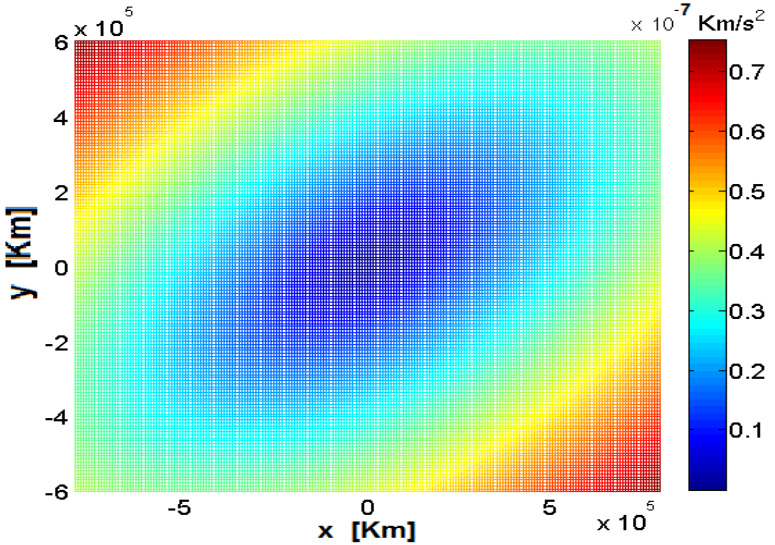
- Usually: on a straight line
- Sometimes: on the boundary of the sphere of influence
- Here: The PS is set according with the prevalence of each CRTBP [R. Castelli]

The regions of prevalence

Comparison: Bicircular model $\Leftrightarrow$ two CRTBP

$$\Delta_{SE}(z) = \| BCP - CR3BP_{SE} \|, \quad \Delta_{EM}(z) = \| BCP - CR3BP_{EM} \|$$





The regions of prevalence

The Regions of Prevalence of each CR3BP is defined according with by the sign of

$$\Delta E(z) = (\Delta_{SE} - \Delta_{EM})(z)$$

For a choice of the relative phase of the primaries θ

$$\mathcal{RP}_{EM}(\theta) = \{z \in \mathbb{C} : \Delta E(z) > 0\} \quad \text{EM Region of Prevalence}$$

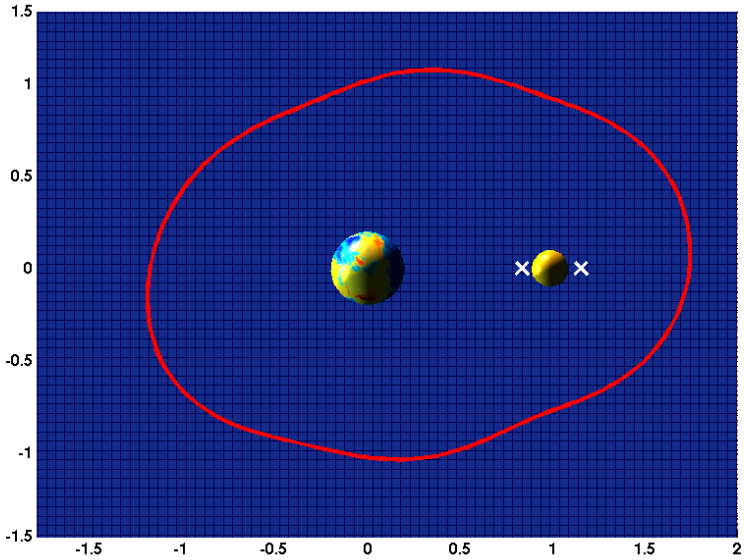
$$\mathcal{RP}_{SE}(\theta) = \{z \in \mathbb{C} : \Delta E(z) < 0\} \quad \text{SE Region of Prevalence}$$

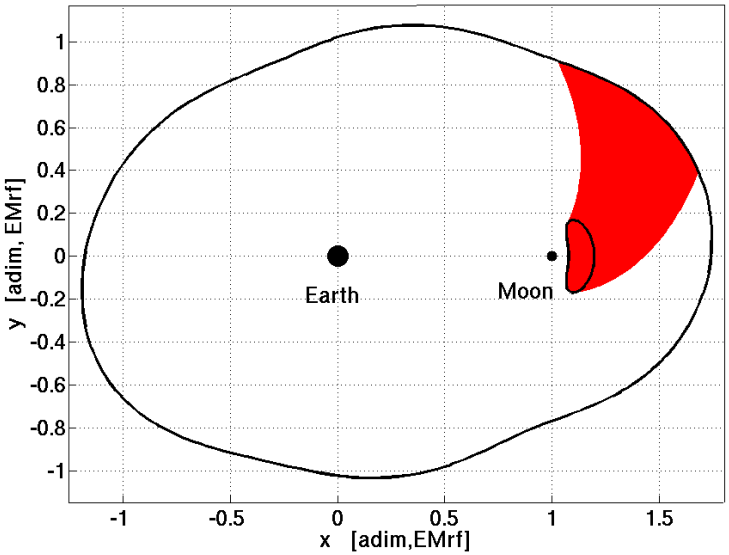
The curve $\Gamma(\theta) = \{z \in \mathbb{C} : \Delta E(z) = 0\}$

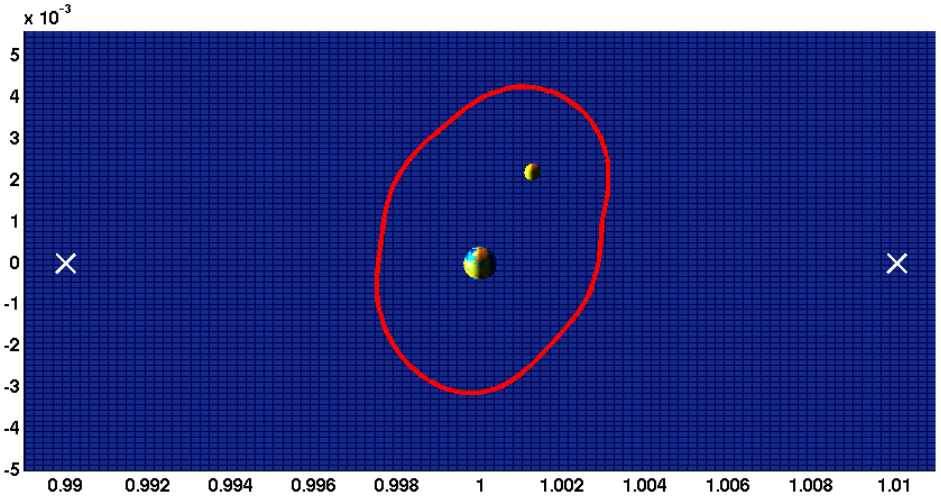
- is a closed, simple curve
- is defined implicitly as a function of (x, y)
- depends on $\theta \longrightarrow$ changes in time.

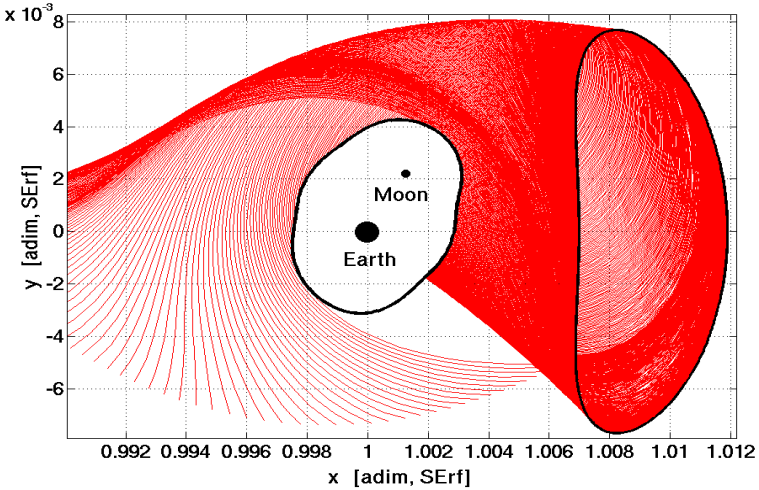
Coupled CR3BP Approximation

Choice of the Poincaré section as the boundary of the *Regions of Prevalence*





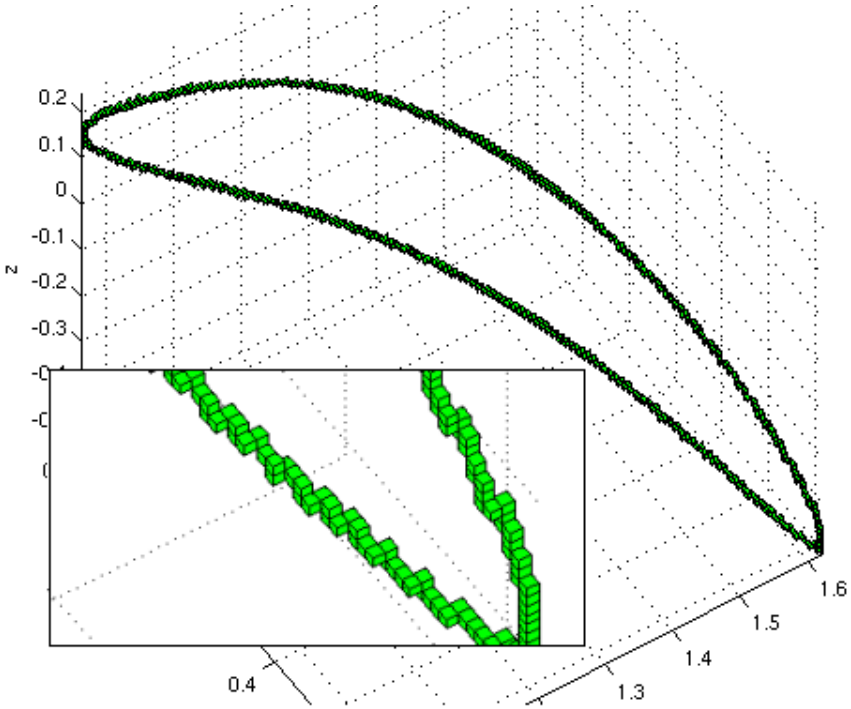


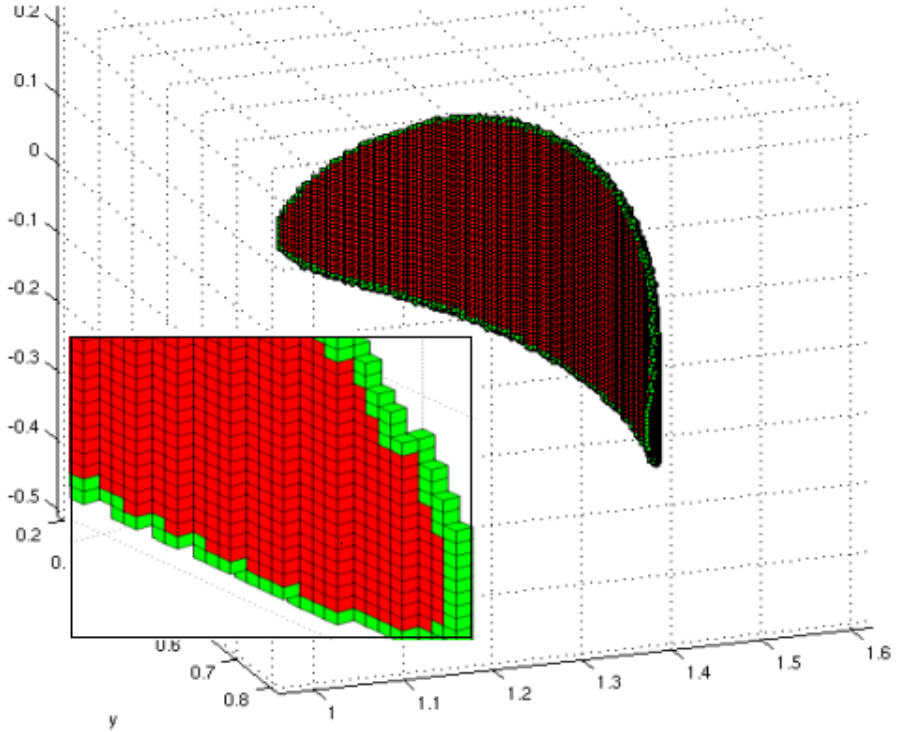


Detection of the connection points

Box Covering Approach, *GAIO*

- Compute the Poincaré map $W^s_{EM,2}(\gamma_1) \cap \Gamma(\theta)$ and cover it with *Box Structures*
- Cover the *transfer* region





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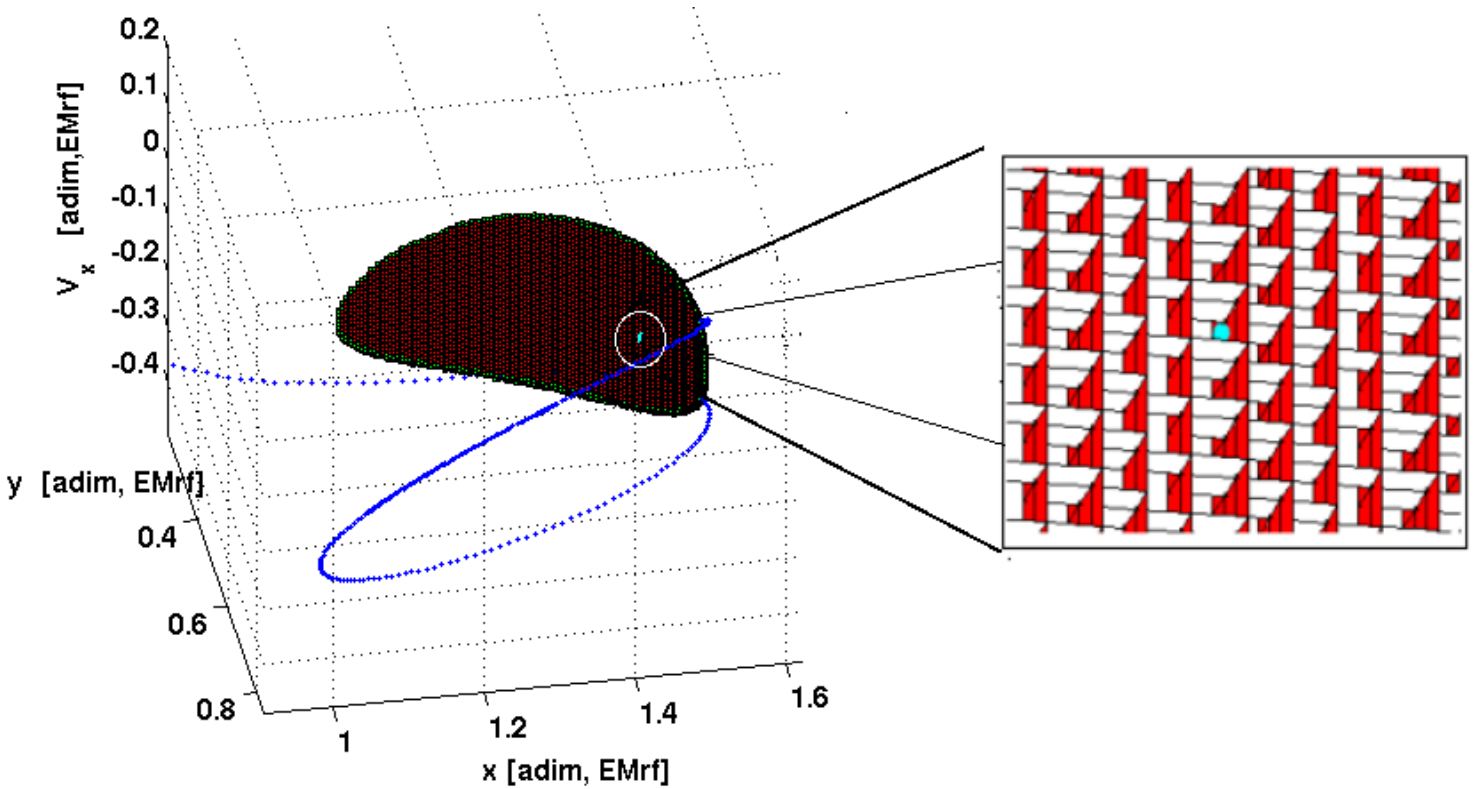
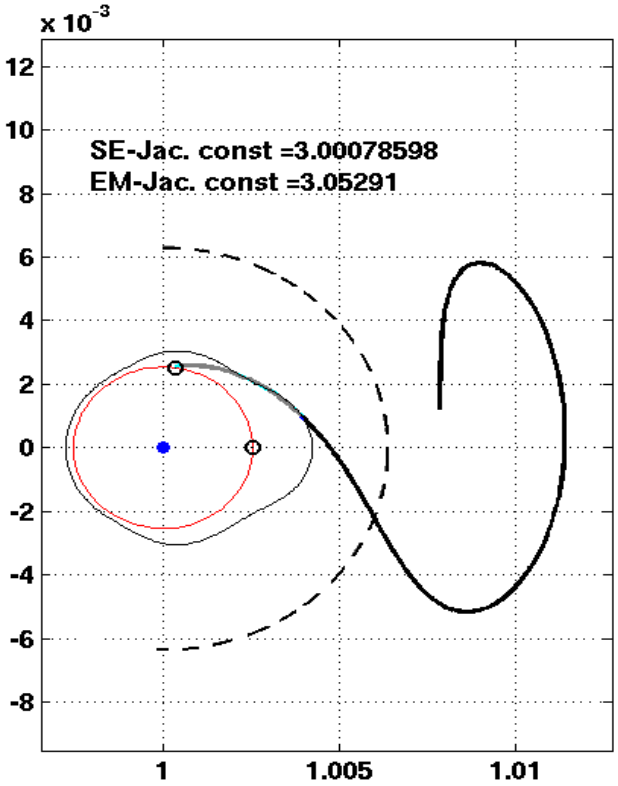
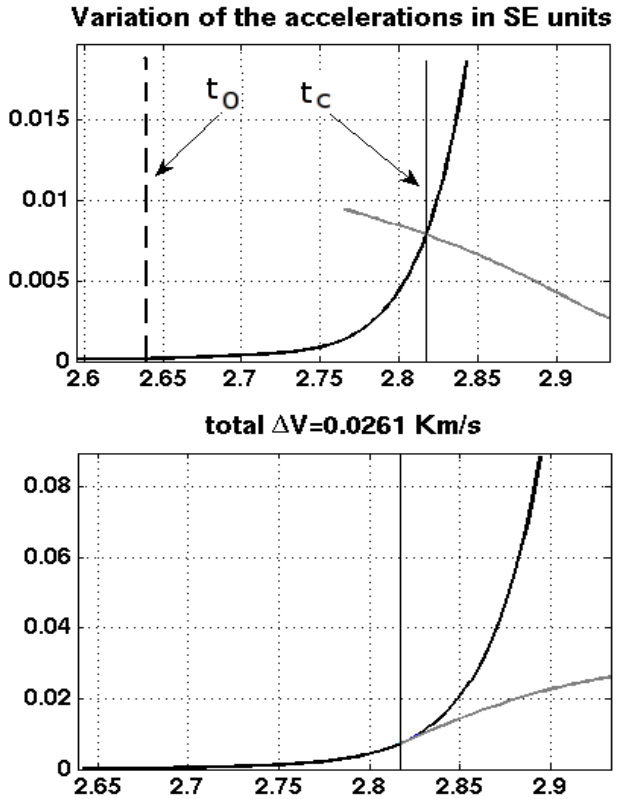
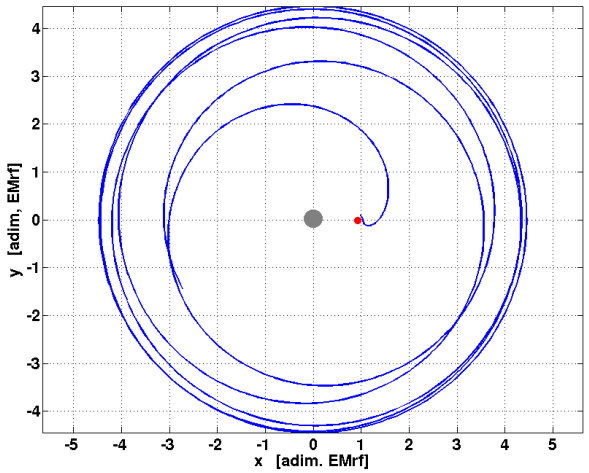
- Cover the *transfer* region
- Intersect the Box Covering with $W^u_{SE,1,2}(\gamma_2)$




x 10^-3
SE-Jac. const =3.00078598
EM-Jac. const =3.05291


Variation of the accelerations in SE units
t0
tc
total ΔV=0.0261 Km/s


y [adim. EMrf]
x [adim. EMrf]

OUTLINE

Dynamical model CRTBP
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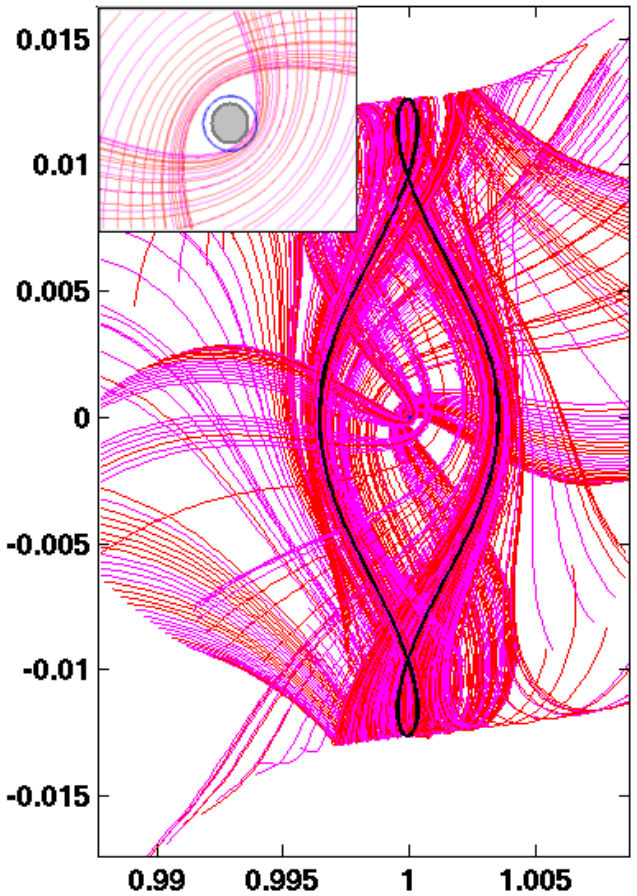
Conclusion

First stage: Leo to SE-DPO

Look for impulsive manoeuvre transfer from Leo to DPO in SE-CRTBP

- Integrate backwards the stable manifold
- Intersect the manifold with Leo
- Select those intersections that are tangent to Leo w.r.t geocentric coordinates

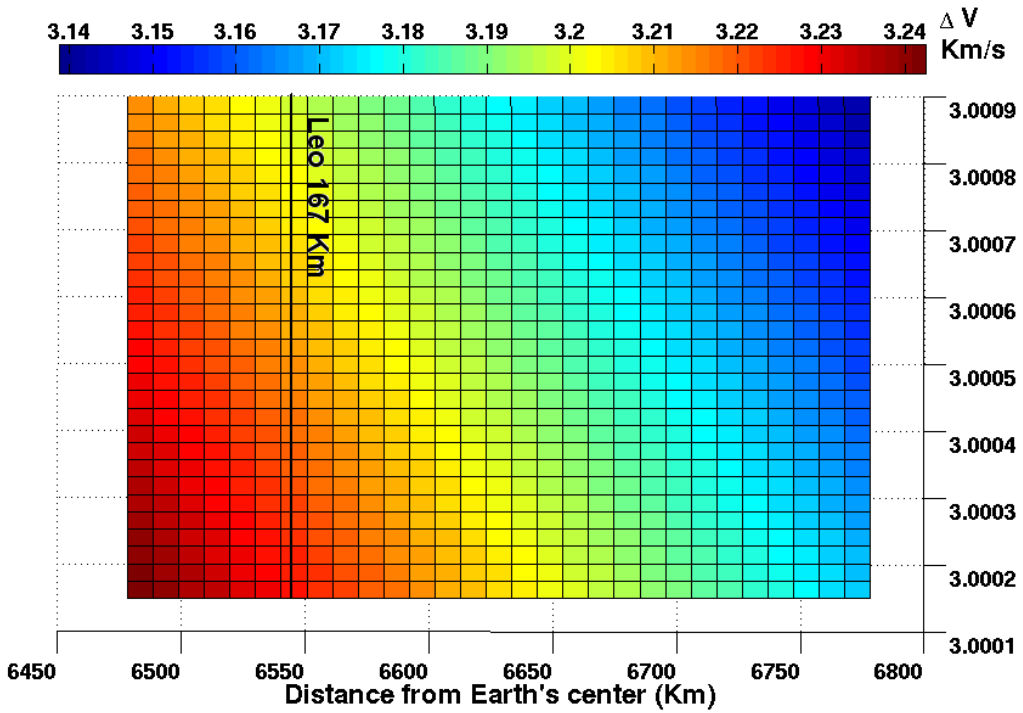


ΔV at Leo

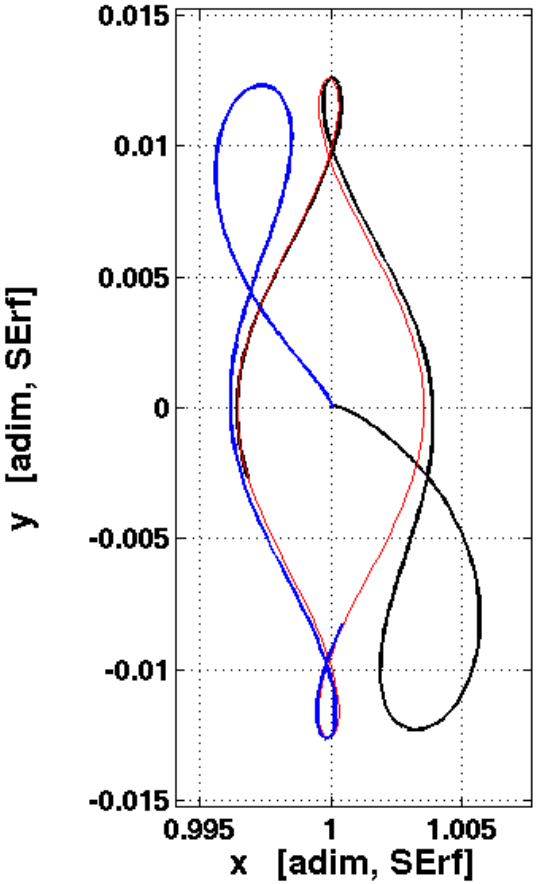
Remark A manoeuvre $|v|$ applied in the same direction of the motion produces the maximal change of Jacobi constant. It holds

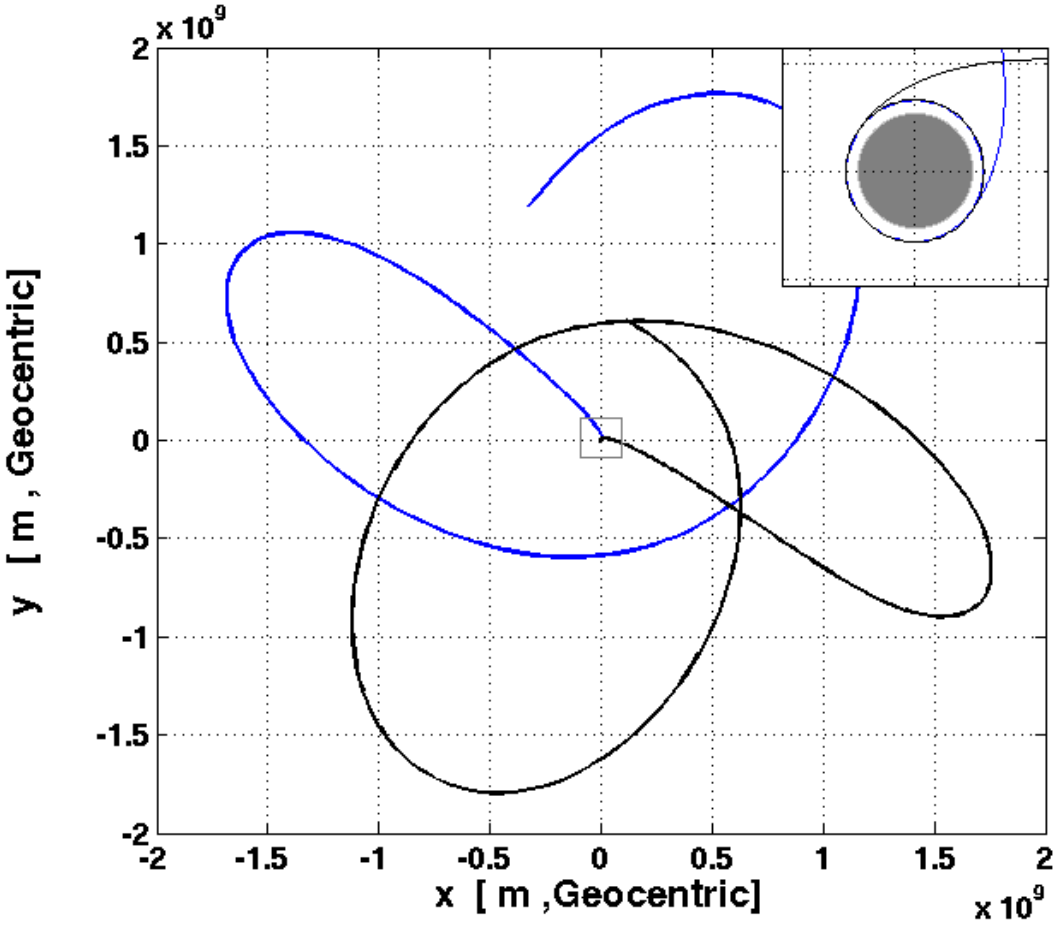
$$\Delta J = |v|^2 + 2|C||v||z|$$

where $C = \frac{V_t}{|z|} - 1$ depends on the Leo altitude and $|z|$ is the geocentric distance, V_t orbital velocity



The Jac.const on a Leo (167 Km alt.) is about 3.070352
The Jac.const. of family g is in the range [3.00014;3.00092]



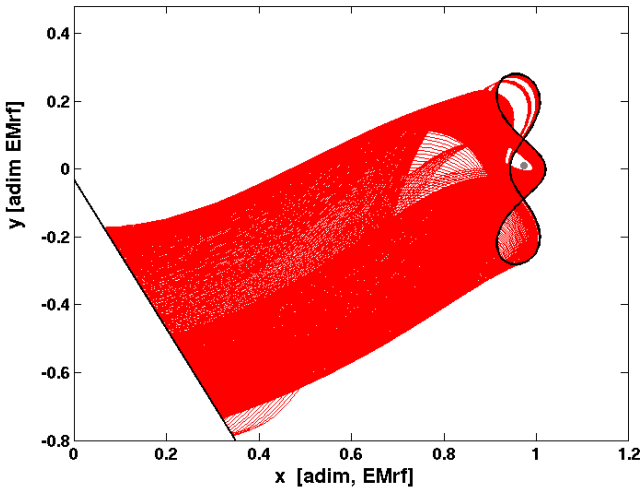


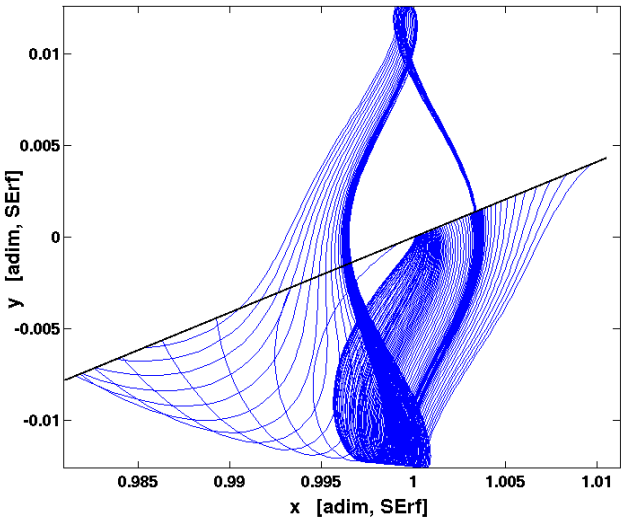
Jacobi- DPO	h-LEO (Km)	ΔV (m/s)
3.000464798057	220	3190
3.000464798057	160	3212

Second stage: SE-DPO to EM-DPO

Procedure for design the transfer:

1) Select two DPOs
2) Compute the Poincaré map of (un)-stable manifold on a section (line through the Earth with slope θ_{SE} and θ_{EM}).





Left: Stable manifold in the interior region for a DPO in the EM-CRTBP.

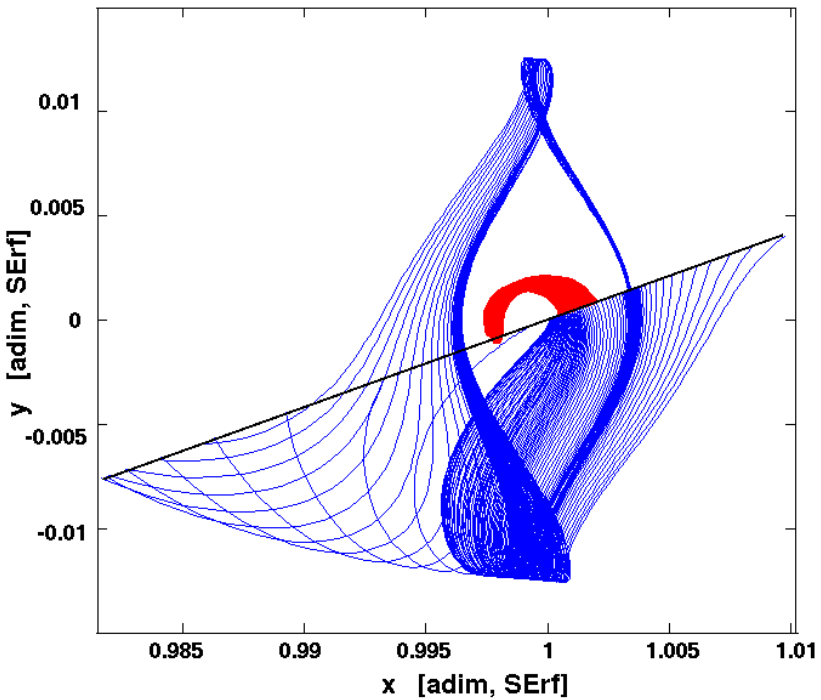
Right: Unstable manifold of a DPO in SE-CRTBP

Design

Procedure for design the transfer:

- Select two DPOs
- Compute the Poincaré map of (un)-stable manifold on a section

3) Write the two maps in the same system of coordinates, being $\theta = \theta_{SE} - \theta_{EM}$ the relative phase of the primaries at the *transfer time*

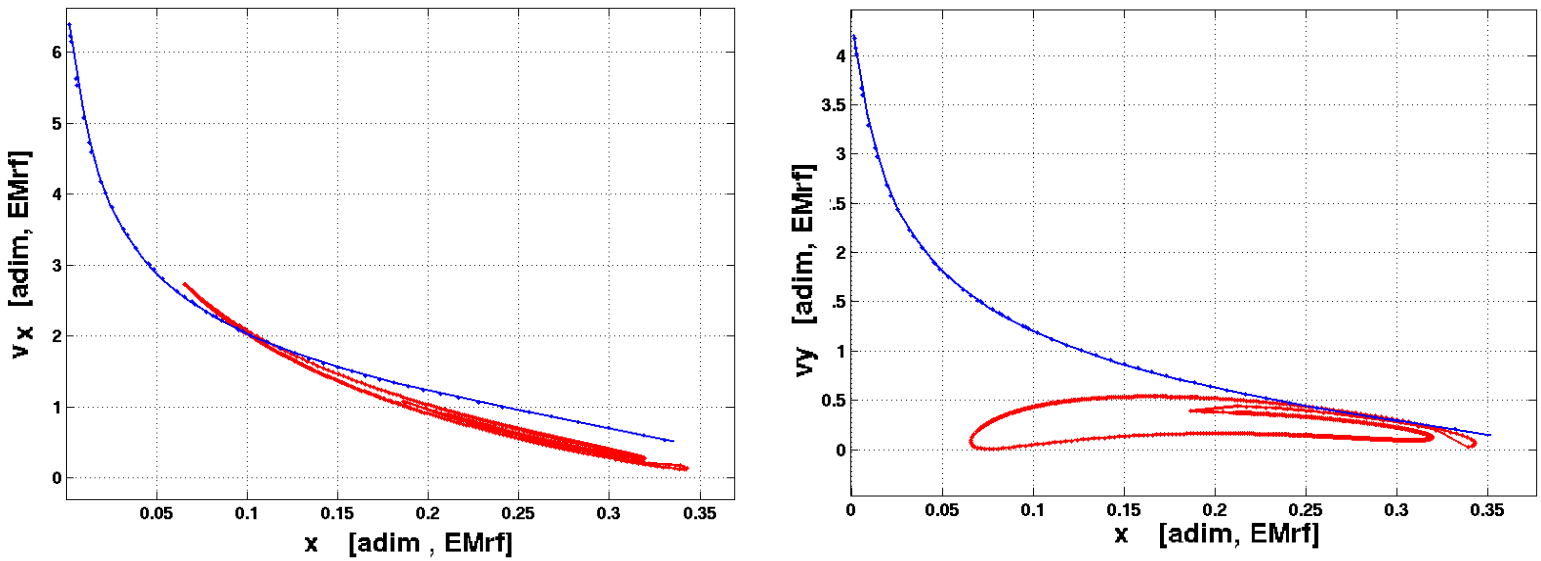


Design

Procedure for design the transfer:

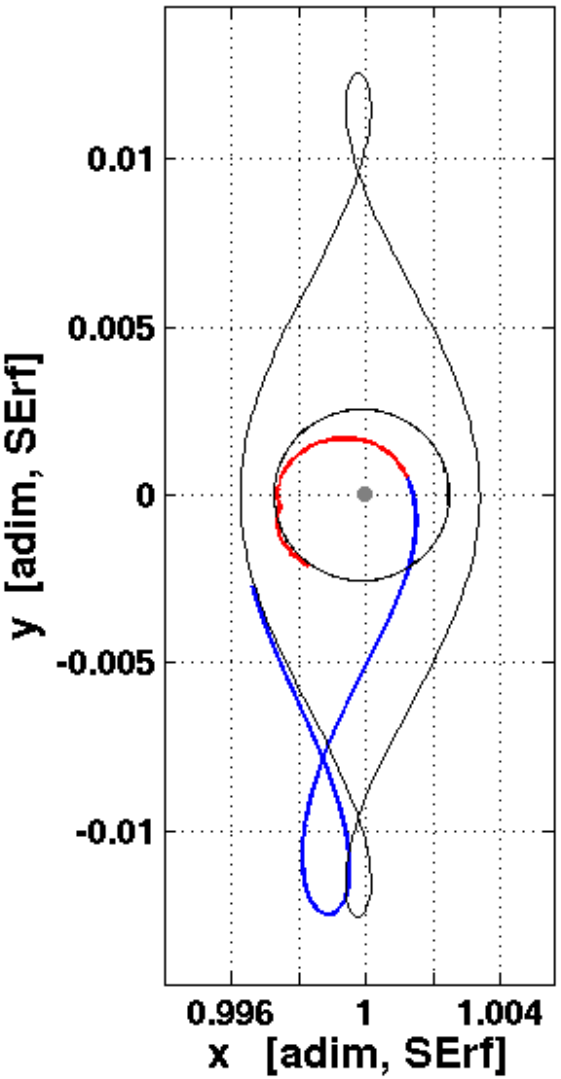
- Select two DPOs
- Compute the Poincaré map of (un)-stable manifold on a section
- Write the two maps in the same system of coordinates, being $\theta = \theta_{SE} - \theta_{EM}$ the relative phase of the primaries

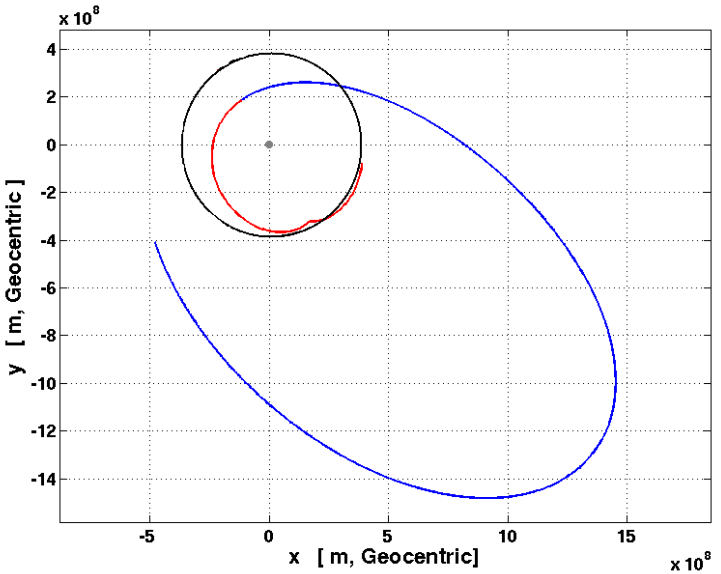
4) Look for possible connections on the Poicaré section

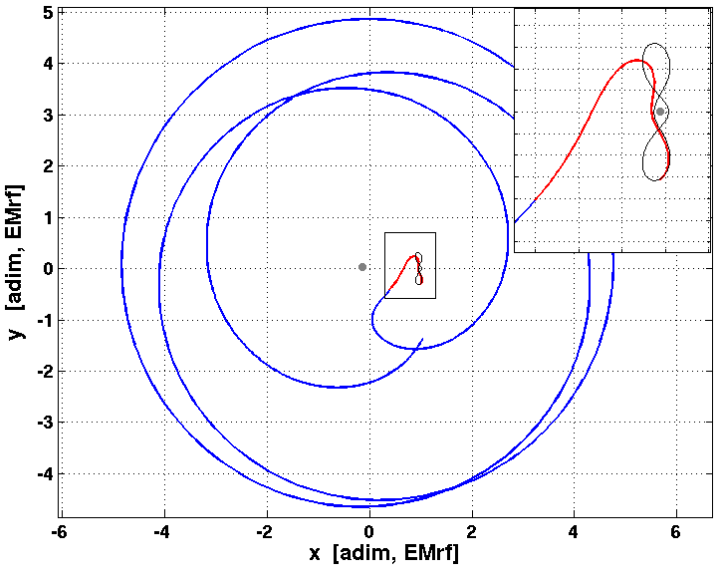


Projection of the Poincaré maps onto the (x, v_x) plane and (x, v_y) plane, in EM-rf

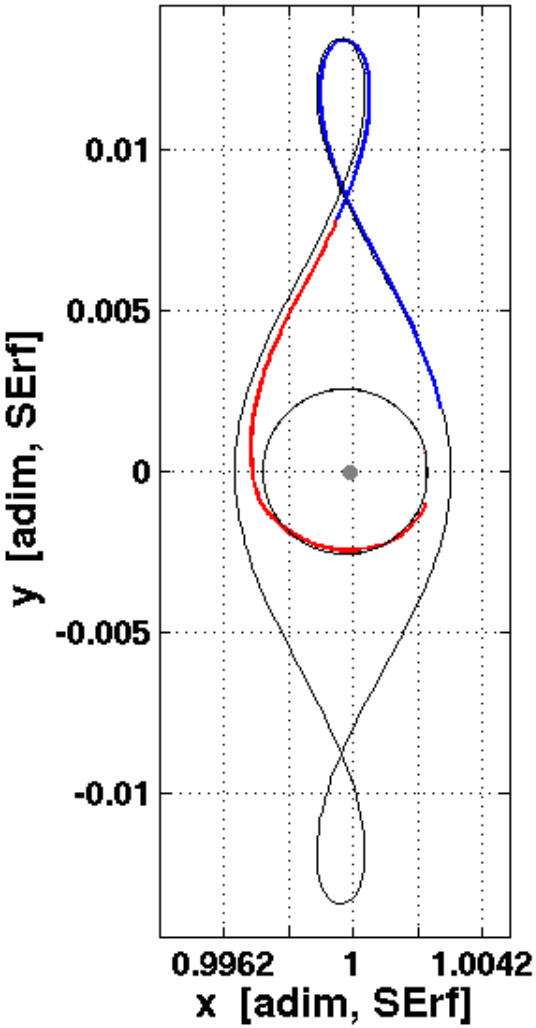
Results: Interior connection

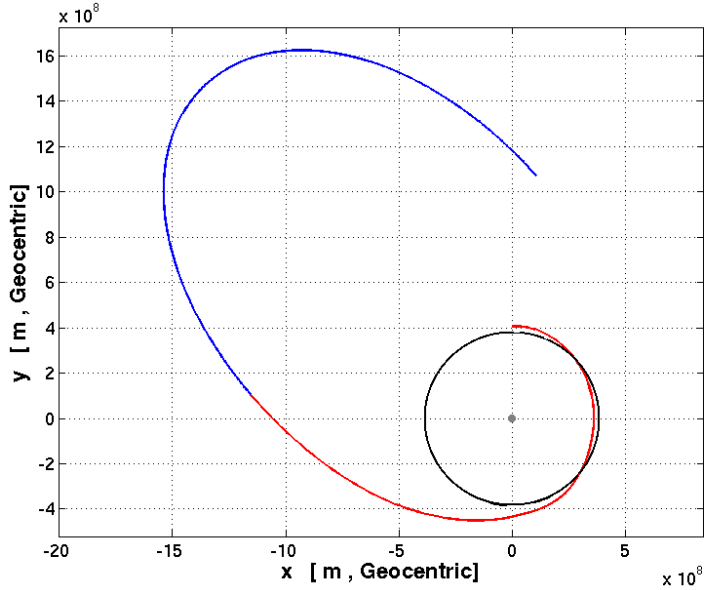


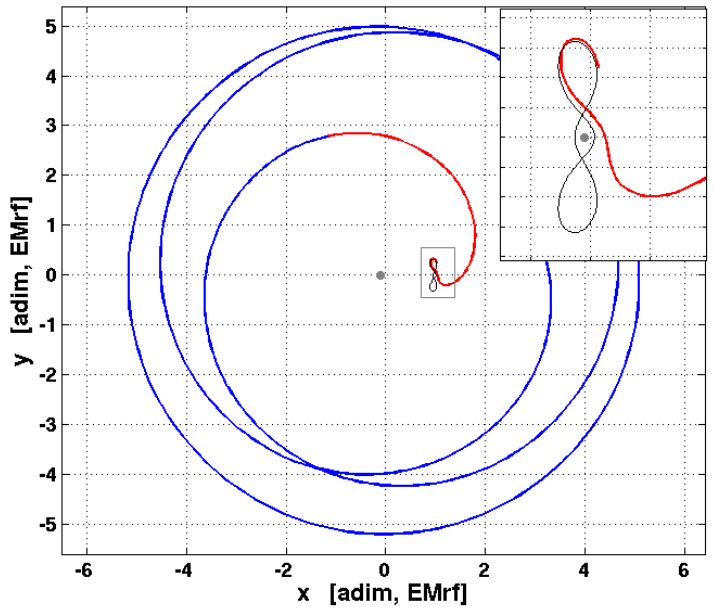




Results: Exterior connection







	SE-Jacobi	EM-Jacobi	Time, (days)	$\Delta V(m/s)^{\star}$
Interior	3.0004647980	3.02599	115	339
Exterior	3.00043012418	3.026764	116	8

$\star$ Connection between two DPOs

	Type	Start	Target	Time	ΔV (m/s)
Mingotti	Ext- Optim.	LEO	DPO Jac=?	90	3160
Ming	Exterior	LEO	Retr. DPO	101	3207
Ming	Interior	LEO	Retr. DPO	33	3802

[G. Mingotti et al.] : Earth to EM-DPO with low thrust propulsion
[X. Ming at el.]: Earth to retrograde stable orbit around the Moon

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Conclusion

Conclusion

- Dynamical model
  - The CRTBP is introduced to model the dynamics
  - The invariant manifolds provide low energy transfers in the phase space
- Design technique:
  - The patched CRTBP approximation has been formalized.
  - Immediate definition of the transfer points in the phase space through the box covering approach.
  - A technique to design impulsive transfers has been developed.
- Designed trajectory:
  - Efficient trajectories in terms of Δv have been designed in the planar and spatial case
  - A non-classical Poincaré section has been presented in terms of Regions of prevalence.

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W. S. Koon, M. W. Lo, J. E. Marsden and S. D. Roos, – *Low energy transfer to the Moon* – Celestial Mech. Dynam. Astronom, Vol 81, pp 63-73, 2001

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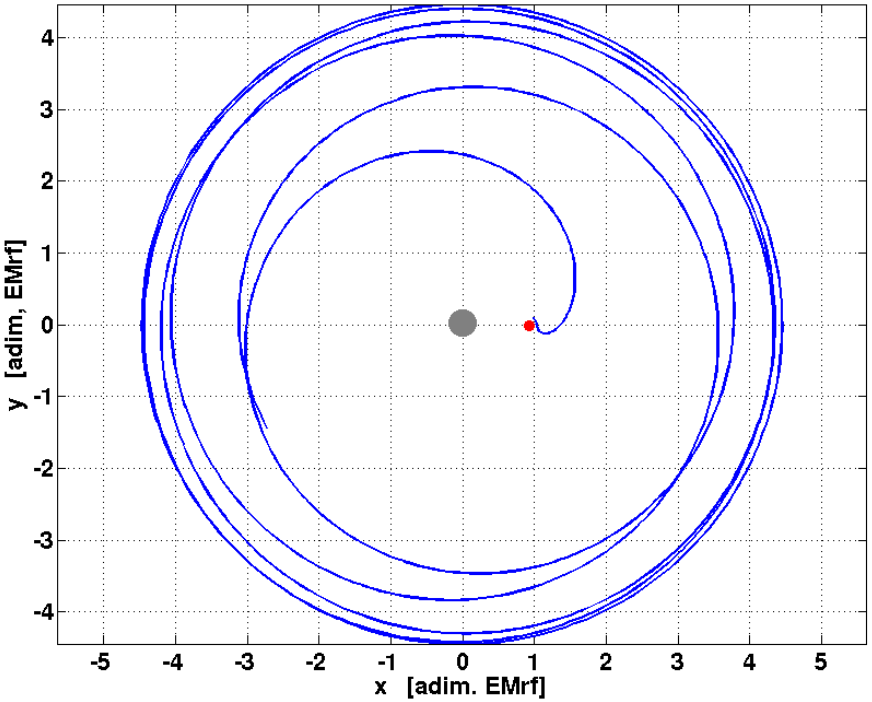
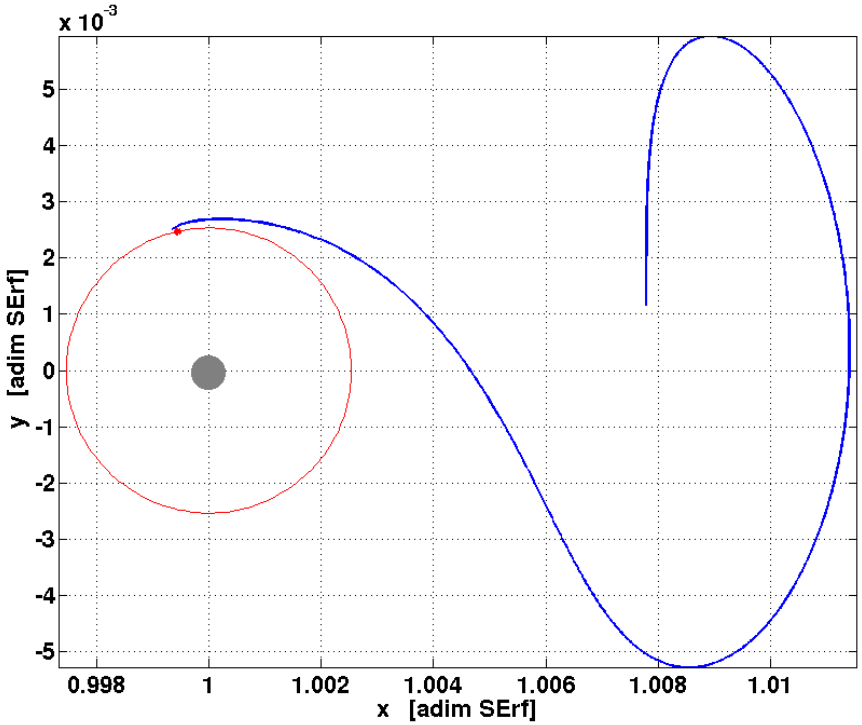
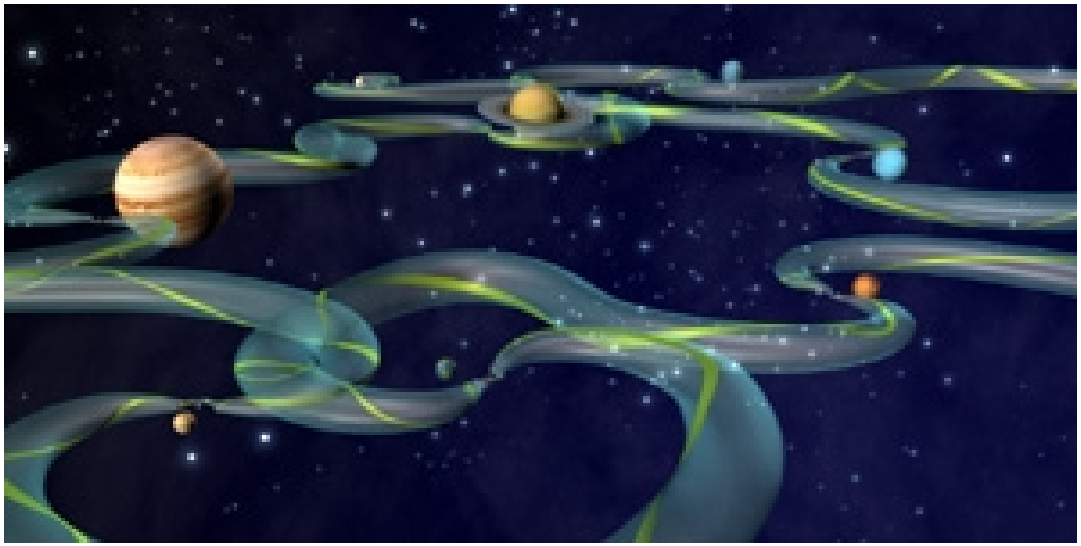
- Ming X. and Shijie X. *Exploration of distant retrograde orbits around Moon*, Acta Astronautica, Vol.65, pp. 853–850, 2009





Thank you



$\Rightarrow$ Dynamical model

$\Rightarrow$ Mission Design

$\Rightarrow$ References