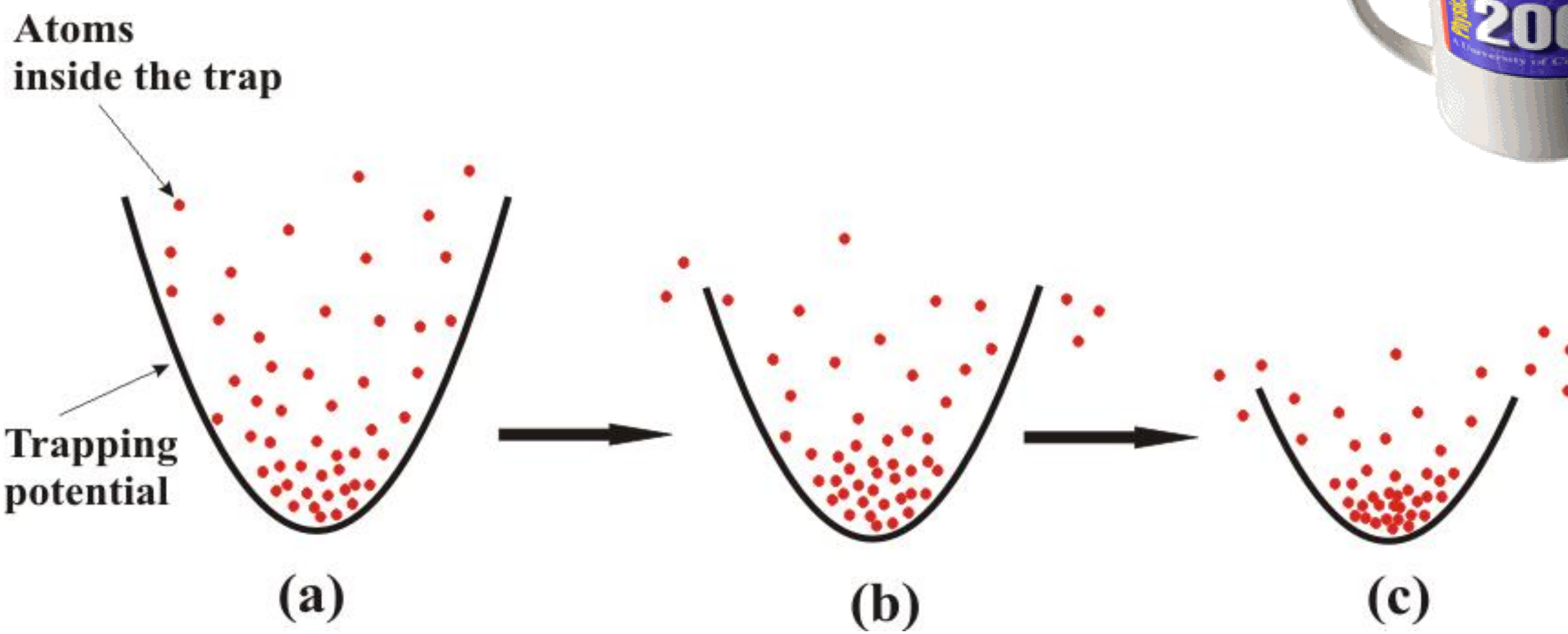
Lecture 2

Evaporative cooling

Bose-Einstein condensation

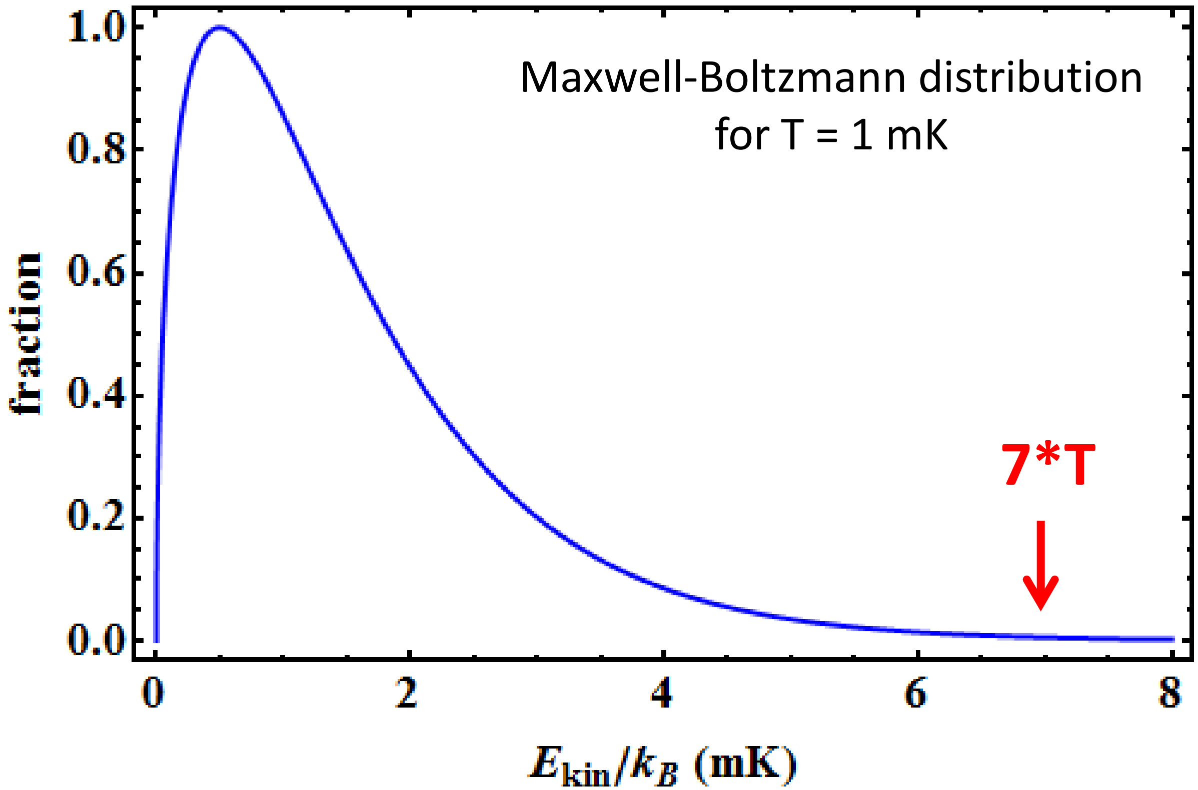
Evaporative cooling



Plain evaporative cooling

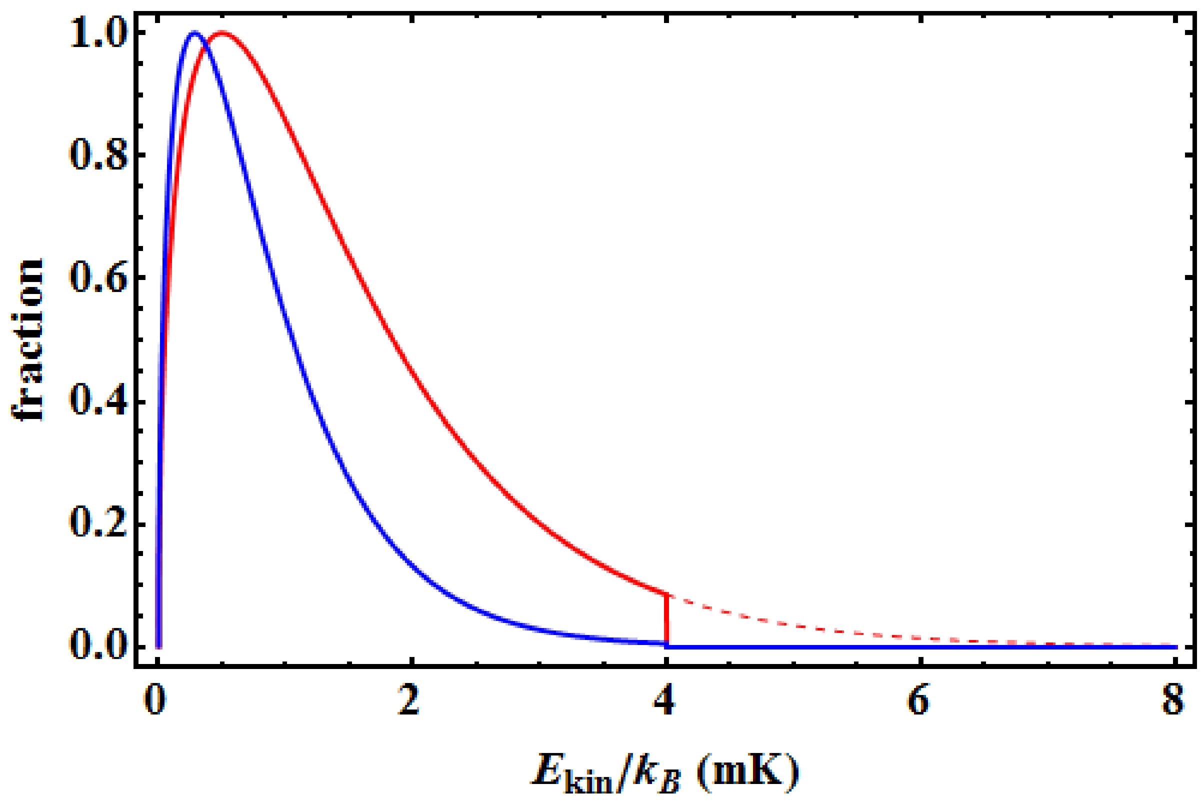
allow the gas to thermalize at a fixed trap depth

$$T \approx U_0/7$$



Forced evaporative cooling

dynamically lowering the trap depth



Temperature vs. atom loss

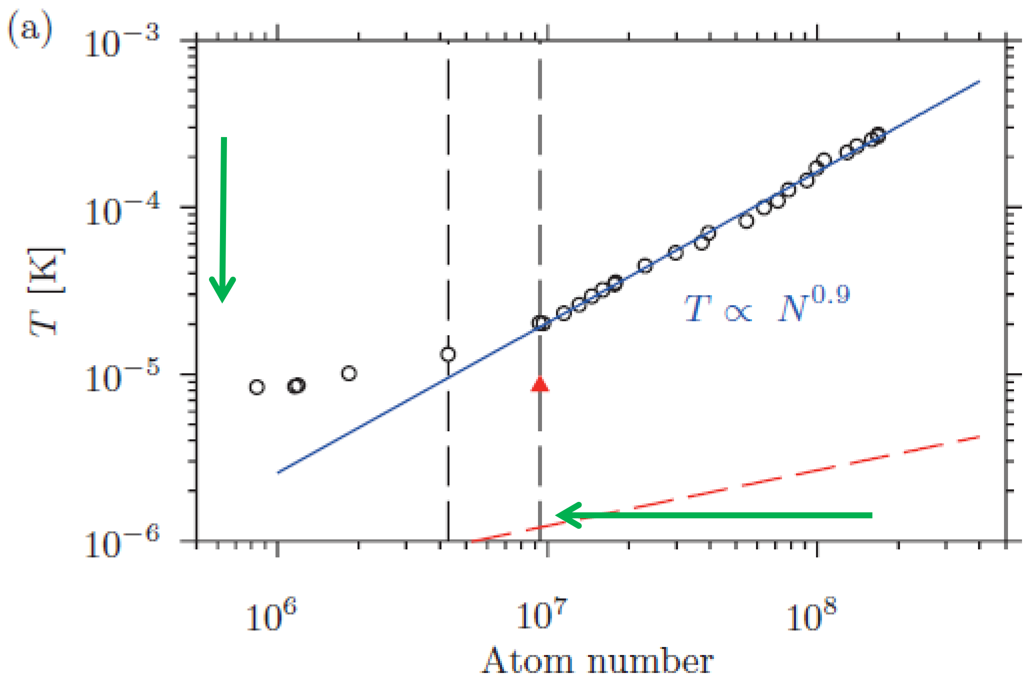
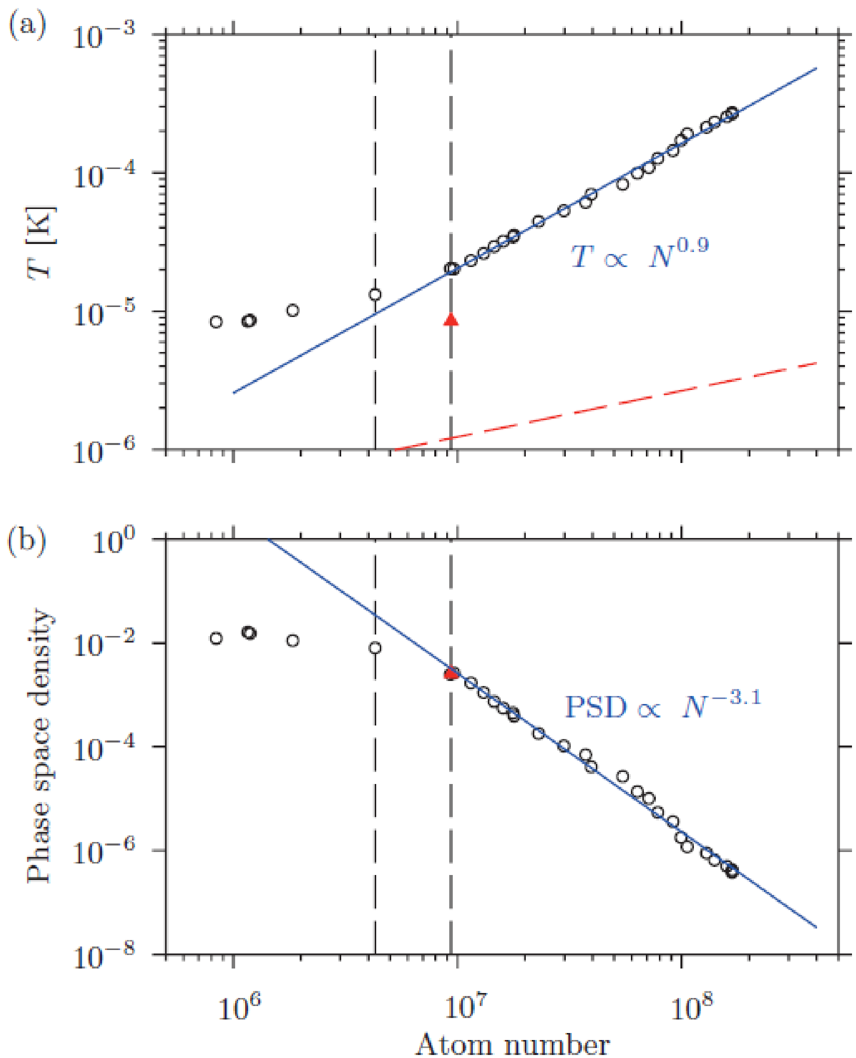


Figure of merit:

$$\alpha = \frac{\ln(T'/T)}{\ln(N'/N)}$$

typically a factor 10 lower temperature “costs” a factor 10 in atom number: $\alpha = 1$

Phase-space density vs. atom loss



More important figure of merit:

$$\gamma = -\frac{\ln\left(D'/D\right)}{\ln\left(N'/N\right)}$$

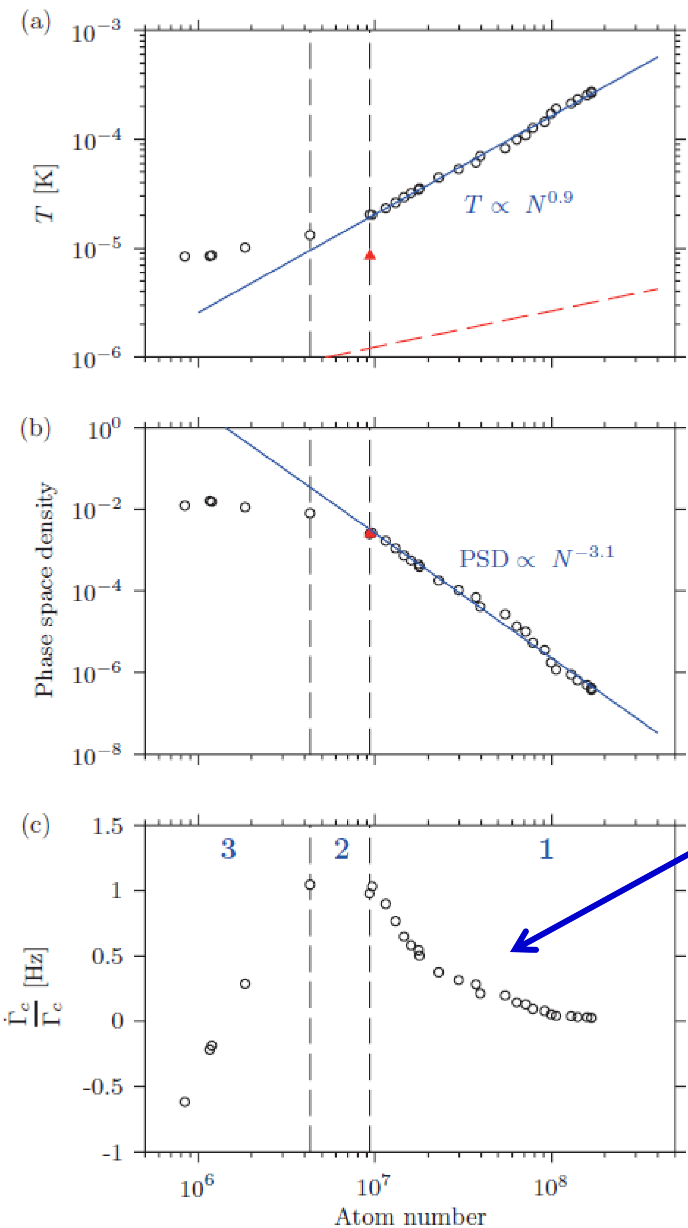
$$D = n\lambda_{\mathrm{dB}}^3$$ phase-space density

$$\lambda_{\mathrm{dB}} = \frac{h}{\sqrt{2\pi m k_B T}}$$ De Broglie wavelength

$$n_0 = \frac{N}{4\pi}\left(\frac{\mu\alpha}{k_B T}\right)^3 \qquad n_0 = N\left(\frac{m\overline{\omega}^2}{2\pi k_B T}\right)^{3/2}$$

QMT | harmonic trap

Speed of evaporation



$$\Gamma_c = n\langle \sigma \mathrm{v} \rangle$$ collision rate

$$\langle \mathrm{v} \rangle = \sqrt{\frac{16 k_B T}{\pi m}}$$ mean relative velocity

$$\sigma = 8\pi a^2$$ elastic cross section (T->0 limit)

a = scattering length

if Γ_c increases:
"runaway evaporation"

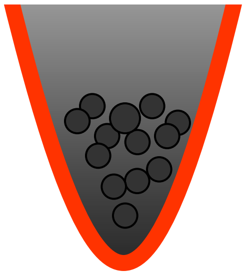
good collisions / bad collisions

elastic two-body collisions

- collisions with background gas
- inelastic two-body collisions
- inelastic three-body collisions

Dubessy et al, PRA 2012

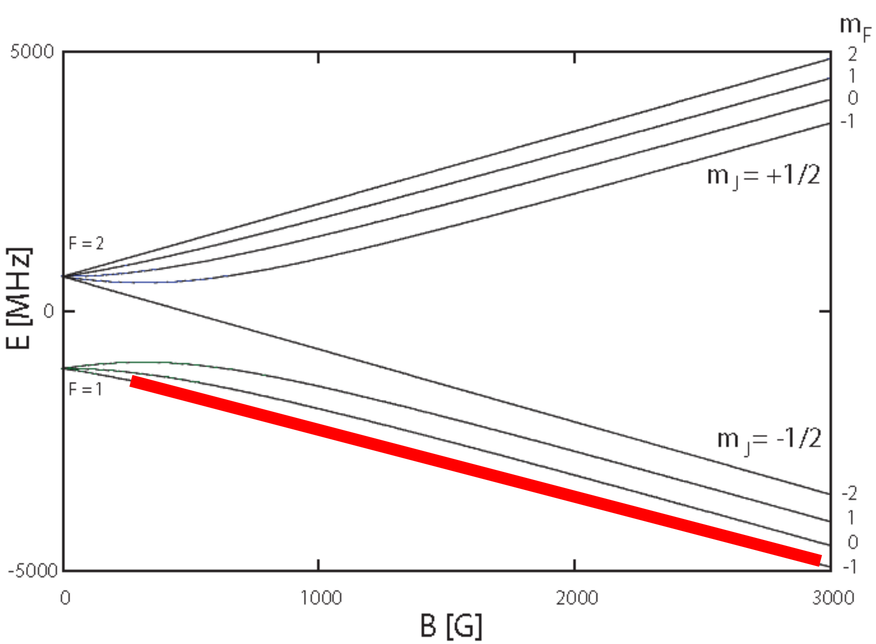
Lifetime of trapped ultracold gas

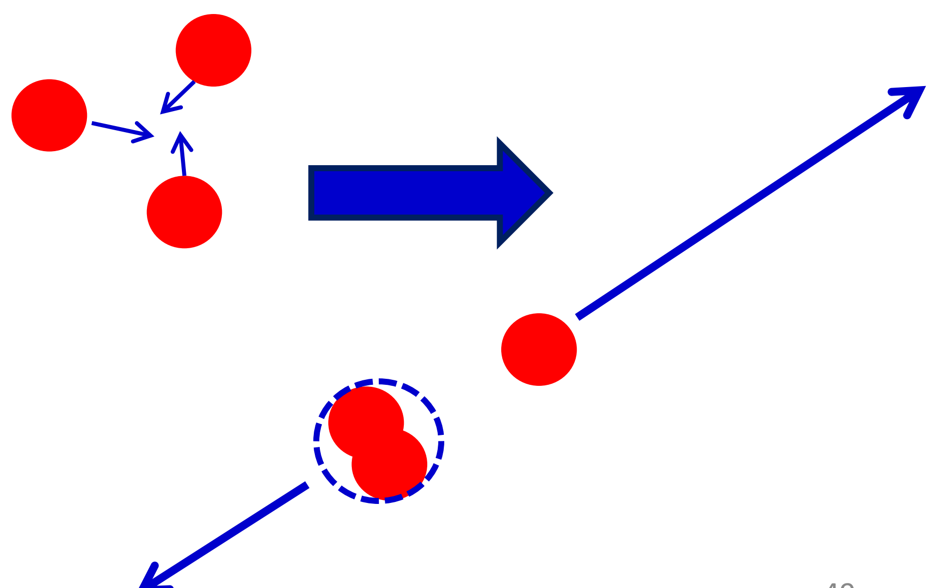


$$\dot{n} = -L_1 n - L_2 n^2 - L_3 n^3 - \ldots$$

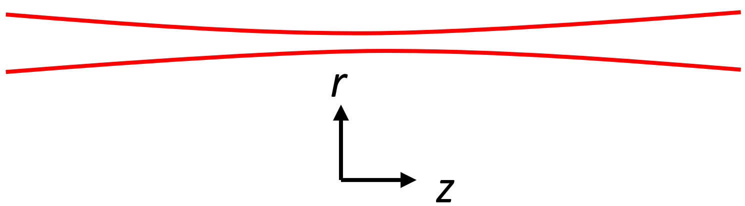
Inelastic collisions in which release of energy >> trap depth

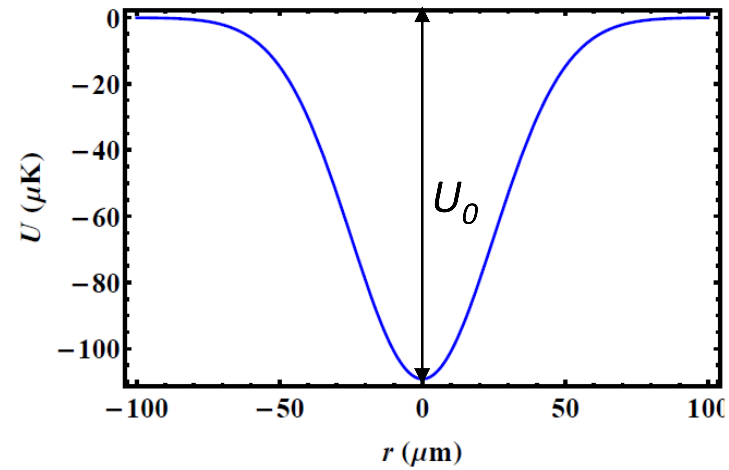
- L_1 : background collisions
- L_2 : spin exchange & spin relaxation
- L_3 : three-body recombination





Evaporative cooling in ODT





$$U_0 \propto P$$

Forced evaporative cooling by lowering the power of optical dipole beam ...

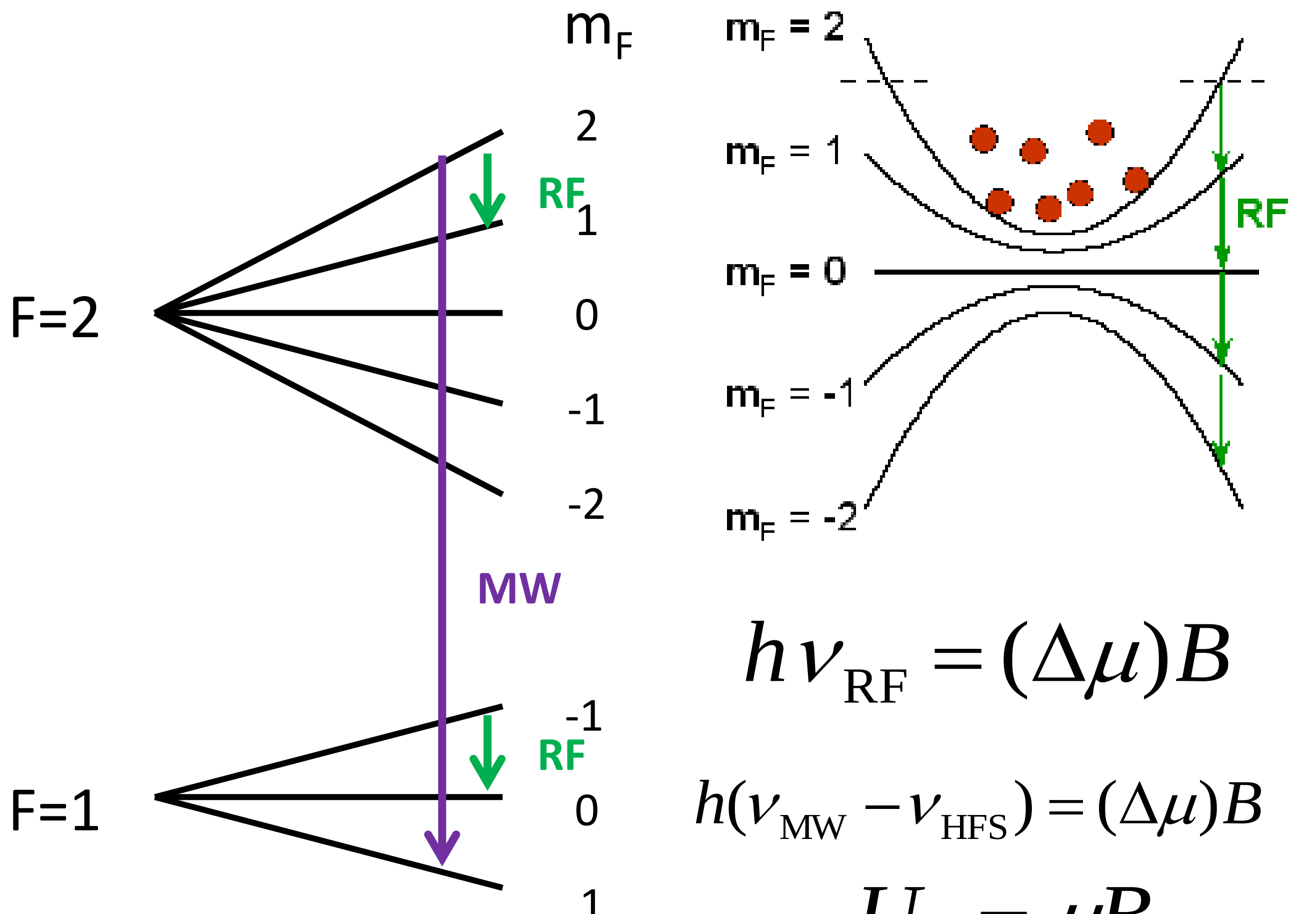
... also reduction of trap frequencies

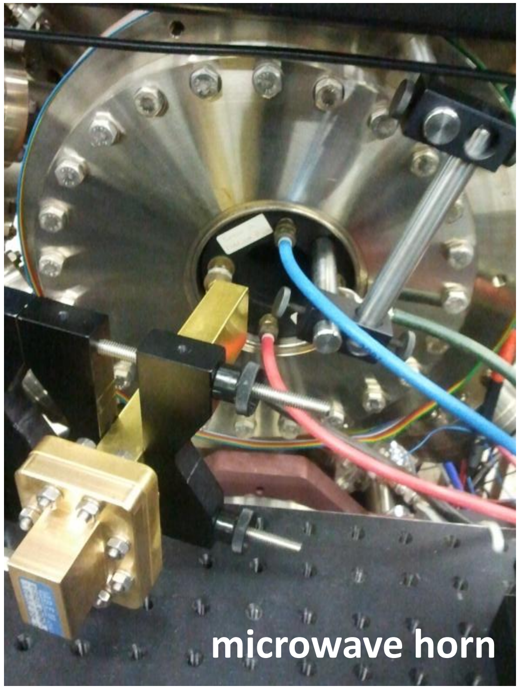
$$\omega_r = \sqrt{\frac{4U_0}{mw_0^2}}$$

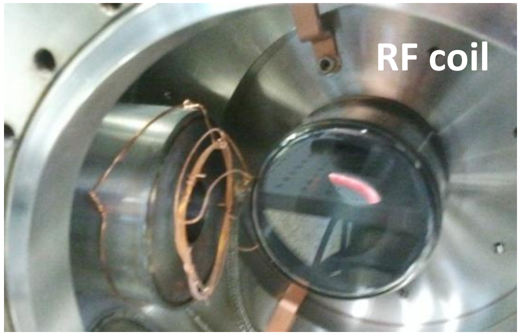
$$\omega_z = \sqrt{\frac{2U_0}{mz_R^2}}$$

$$n_0 = N\left(\frac{m\overline{\omega}^2}{2\pi k_B T}\right)^{3/2}$$

Evaporative cooling in magnetic trap







$$h\nu_{RF} = (\Delta\mu)B$$

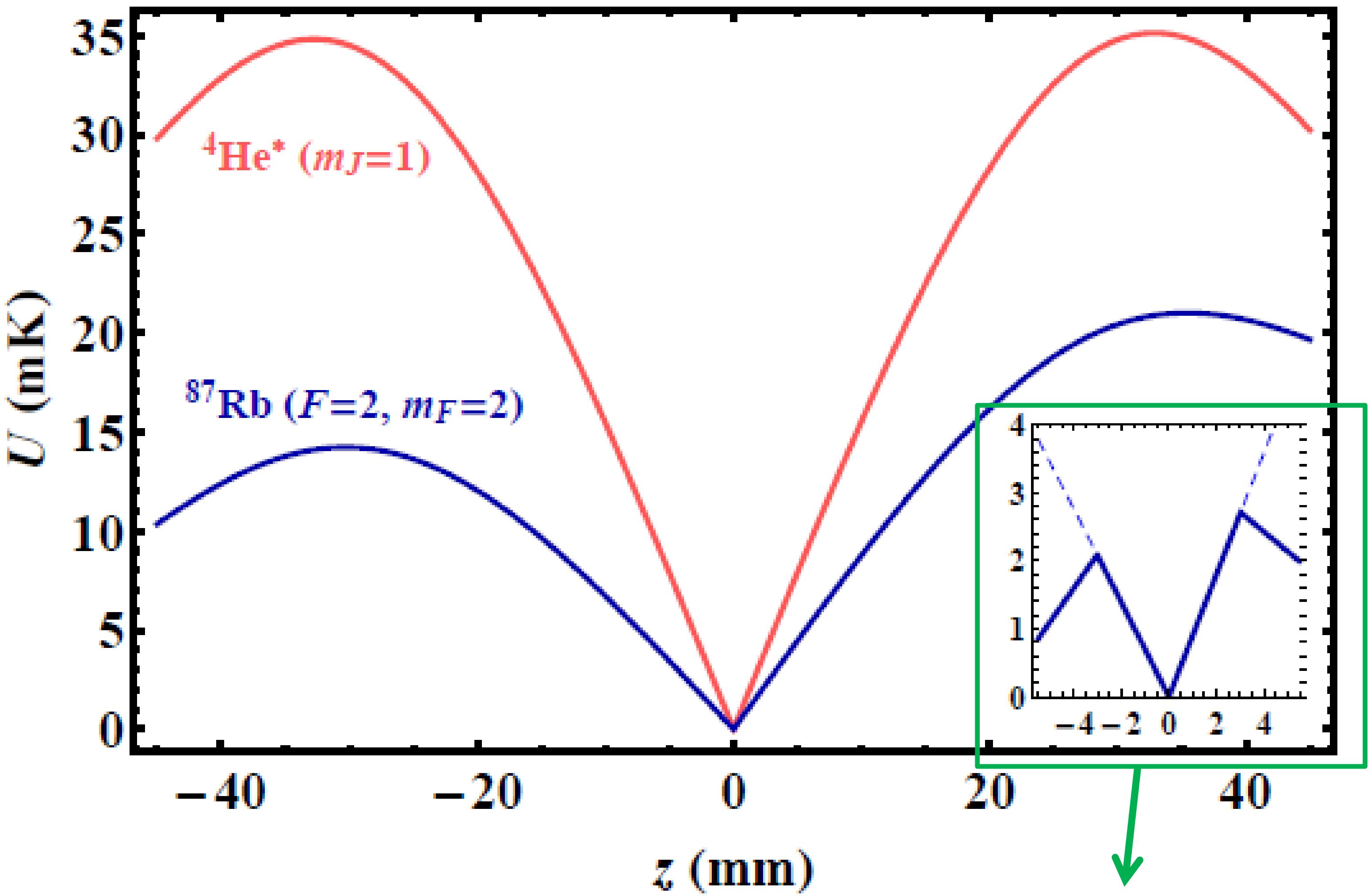
$$h(\nu_{MW} - \nu_{HFS}) = (\Delta\mu)B$$

$$U_0 = \mu B$$

$$\boxed{U_0 = \frac{\mu}{\Delta\mu} h\nu_{RF} \text{ or } \frac{\mu}{\Delta\mu} h(\nu_{MW} - \nu_{HFS})}$$

RF or MW "knife"

Evaporative cooling in magnetic trap



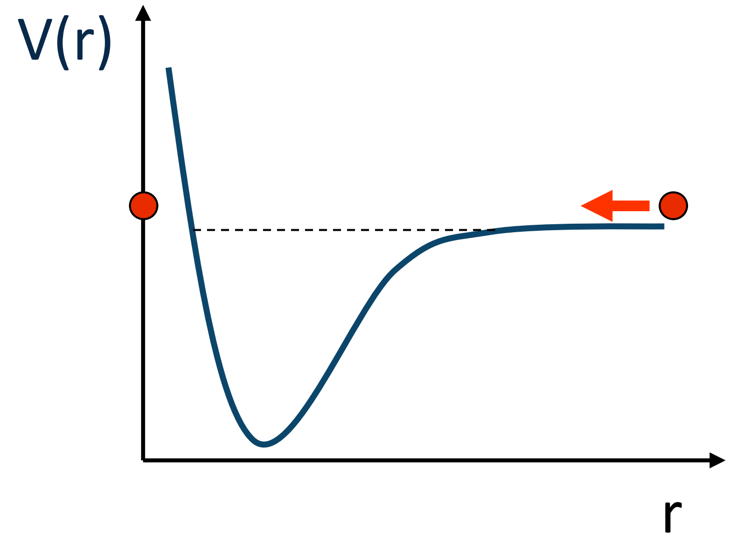
MW knife on F=2, m_F=2 -> F=1 ,m_F=1

... no reduction of trap frequencies

Elastic cross section

$$\frac{d^2u_l}{dr^2}+\left[k^2-\frac{2\mu V(r)}{\hbar^2}-\frac{l(l+1)}{r^2}\right]u_l(r)=0$$

$$k=\sqrt{2\mu E}/\hbar$$



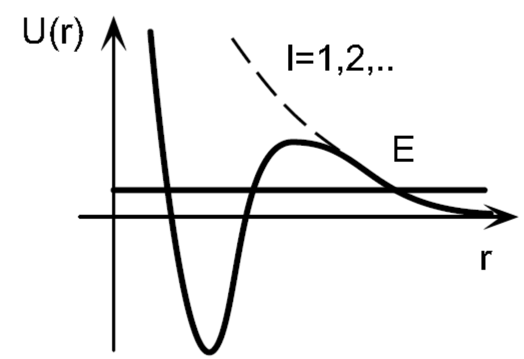
$$u_l(r)\sim A_l\sin\left(kr-l\frac{\pi}{2}+\delta_l\right)$$

$$\sigma(E)=\frac{4\pi}{k^2}\sum_{l=0}^{\infty}(2l+1)\sin^2\delta_l$$

B+B: even *l*

F+F: odd *l*

X+X': all *l*



$$\lim_{k\to 0}\delta_l(k)\propto k^{2l+1}$$

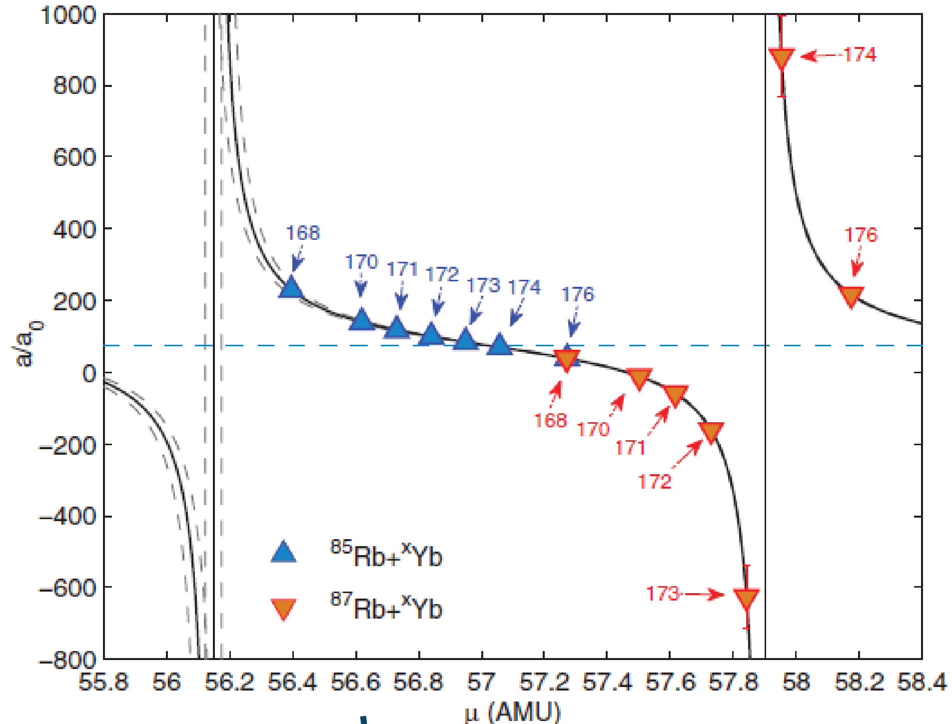
$$\lim_{k\to 0}\sigma(E)=\lim_{k\to 0}\frac{4\pi}{k^2}\sin^2\delta_0(k)=(2)4\pi a^2$$

a: scattering length

$$a=-\lim_{k\to 0}\frac{\tan\delta_0(k)}{k}$$

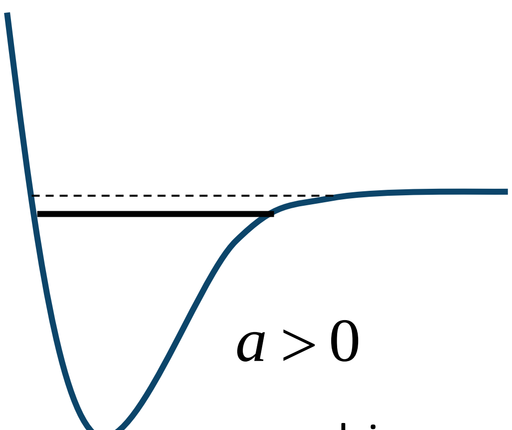
Scattering length

$$\frac{d^2 u_l}{dr^2} + \left[k^2 - \frac{2\mu V(r)}{\hbar^2} - \frac{l(l+1)}{r^2} \right] u_l(r) = 0$$

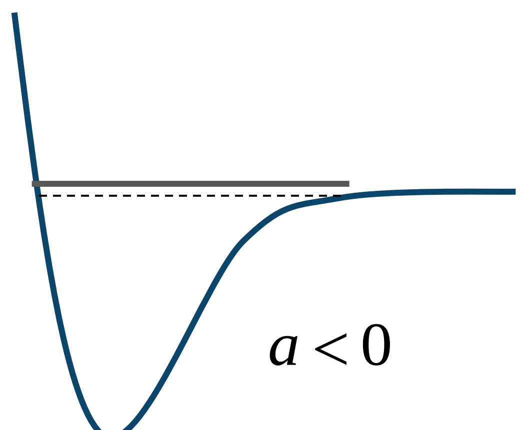


Borkowski et al, PRA 2013

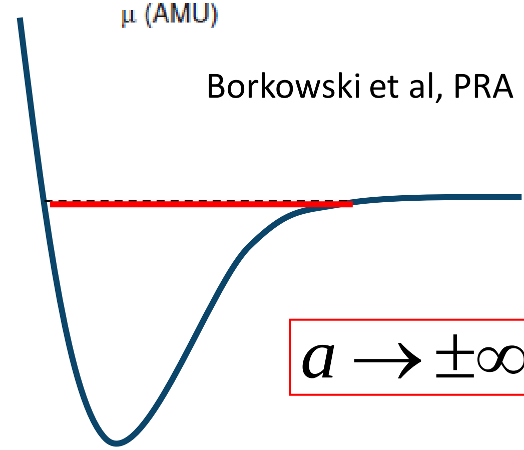
$E_b = -\hbar^2/(2\mu a^2)$



repulsive



attractive



Bosons vs. Fermions for T->0

$$\sigma(E) = \frac{4\pi}{k^2} \sum_{l=0}^{\infty} (2l+1) \sin^2 \delta_l$$

B+B: even *l*

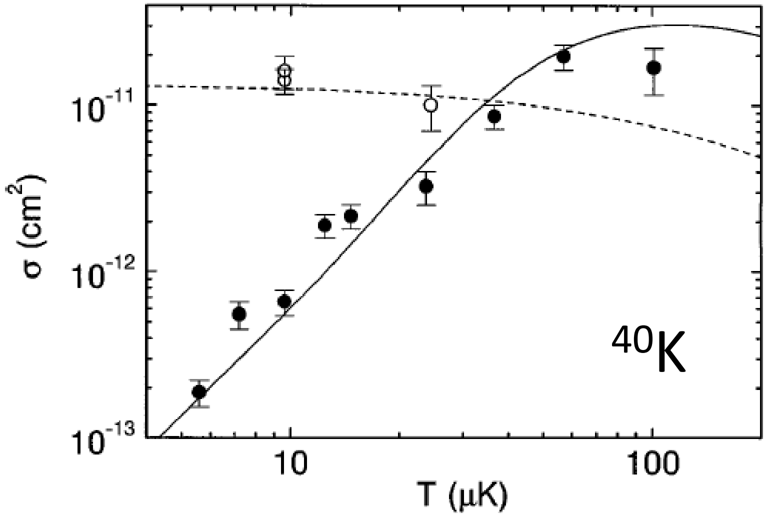
F+F: odd *l*

X+X': all *l*

- identical bosons: s-wave scattering (*l=0*) $\sigma = 8\pi a^2$
- distinguishable particles: s-wave scattering *(l=0)* $\sigma = 4\pi a^2$
- identical fermions: p-wave scattering (*l=1*) $\sigma \propto T^2$

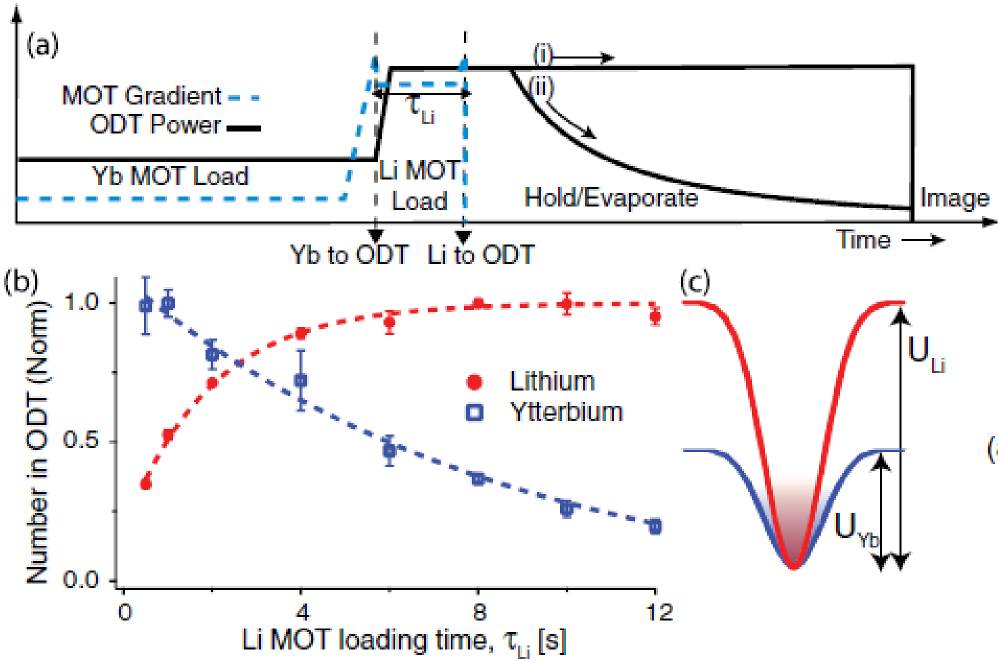
- How to do evaporative cooling with fermions?
  - Different spin states (Zeeman or hyperfine)
  - Different isotopes (e.g. ^{6}Li with ^{7}Li)
  - Different species (e.g. ^{6}Li with Na)

sympathetic cooling



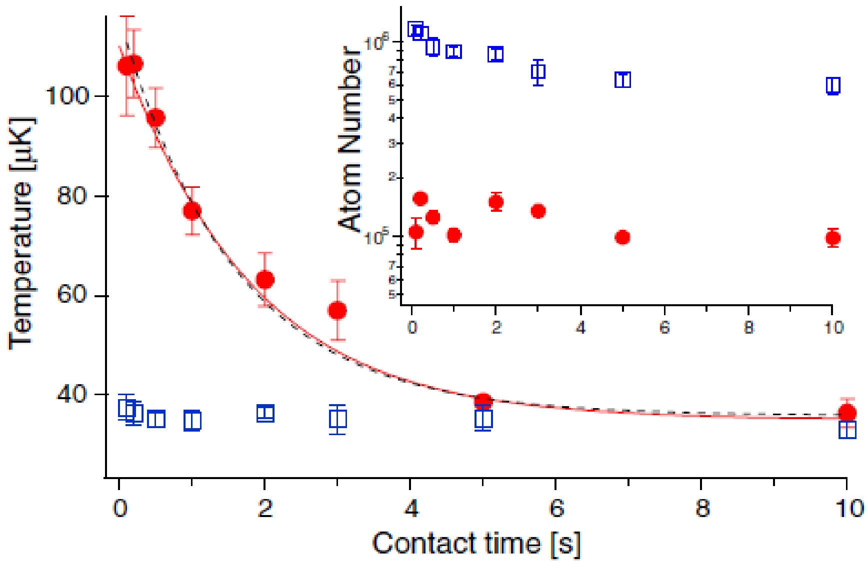
DeMarco et al, PRL 1999

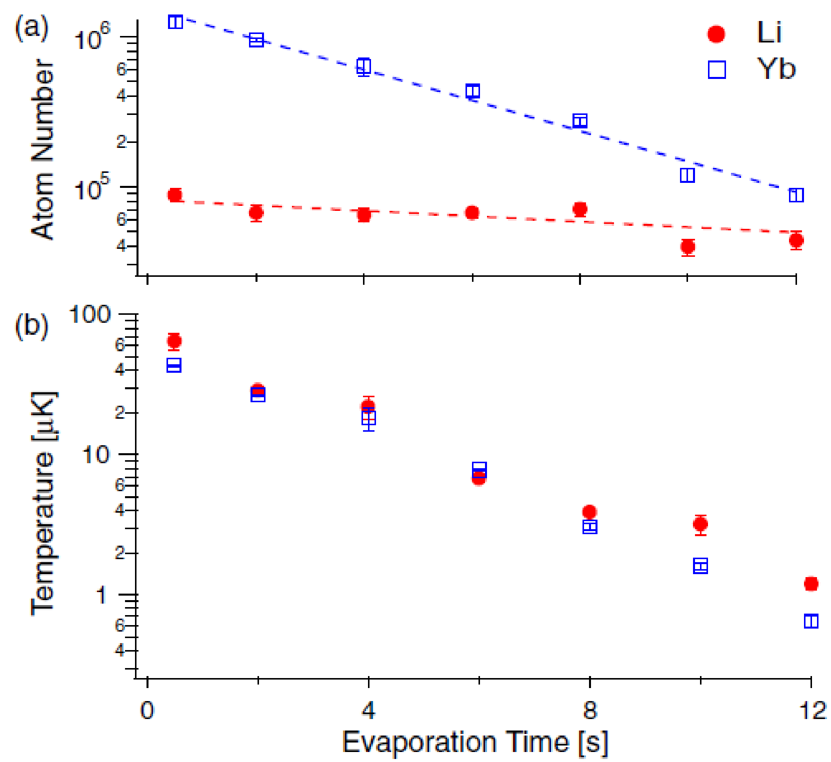
Sympathetic cooling



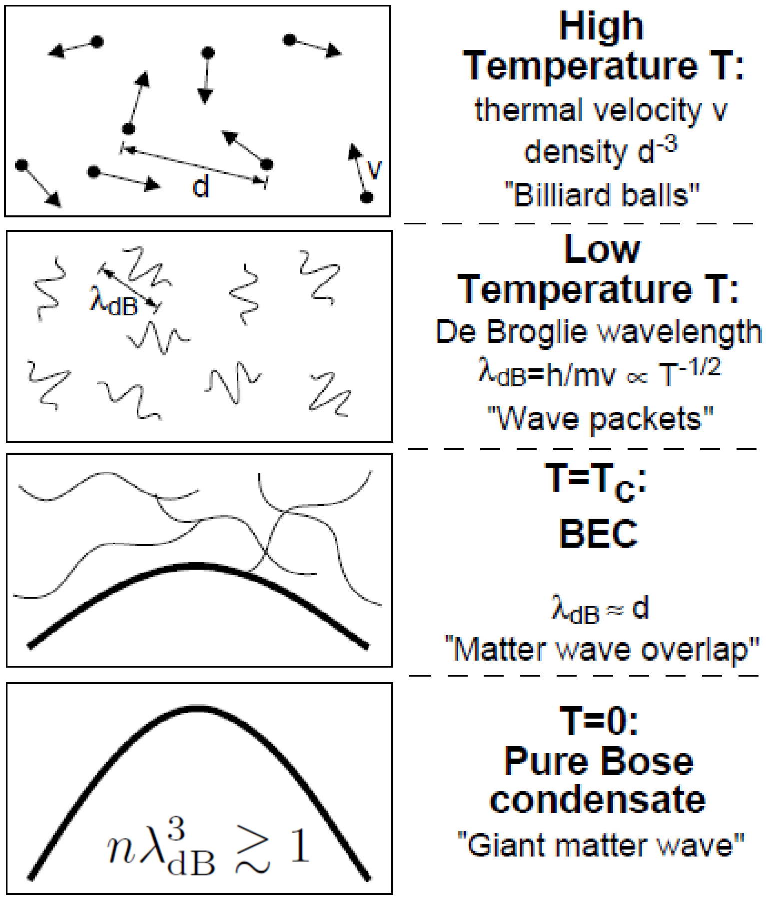
Example : ^{6}Li+^{174}Yb
(Ivanov et al, PRL 2011)

$$-\frac{1}{\Delta T}\frac{d(\Delta T)}{dt} = \gamma_{\mathrm{th}} = \frac{\xi}{\alpha}\bar{n}\sigma_{\mathrm{LiYb}}\bar{v}$$

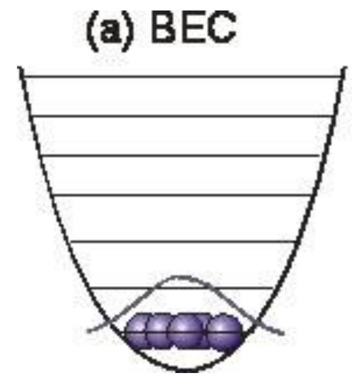




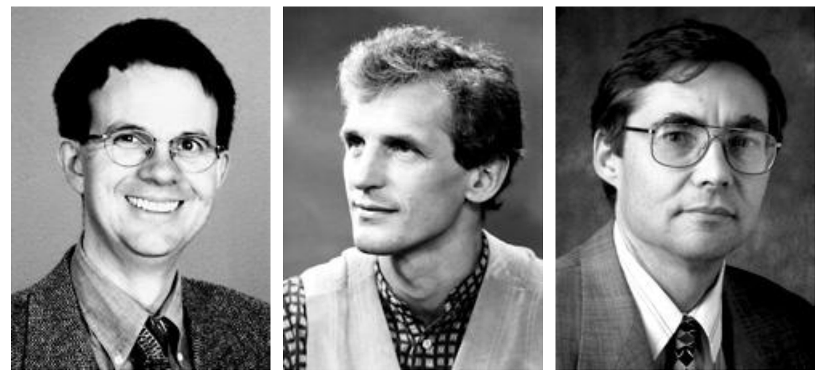
Bose-Einstein condensation



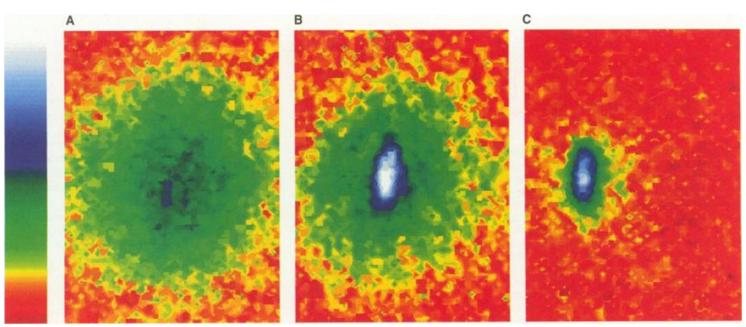
Bose 1924, Einstein 1925:
macroscopic occupation of the lowest energy level

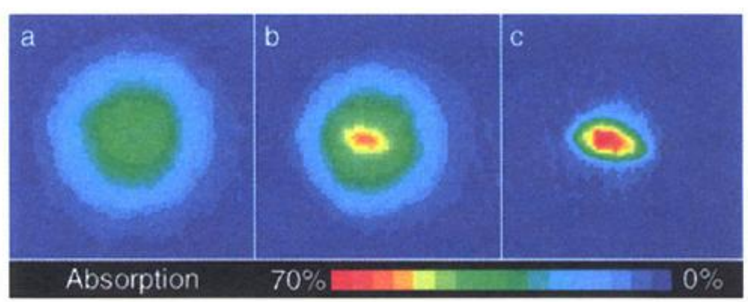


Nobelprize 2001



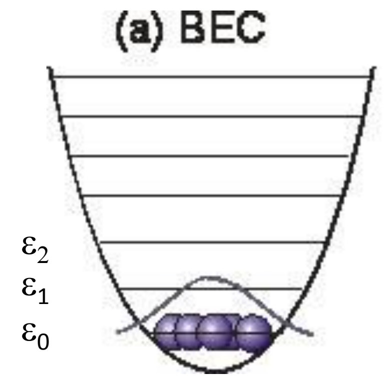
Cornell Ketterle Wieman





Anderson et al, Science 1995
Davis et al, PRL 1995

Theory of BEC (Ideal gas)



$$N = \sum_i N_i = \sum_i \frac{1}{e^{(\varepsilon_i - \mu)/k_B T} - 1}$$

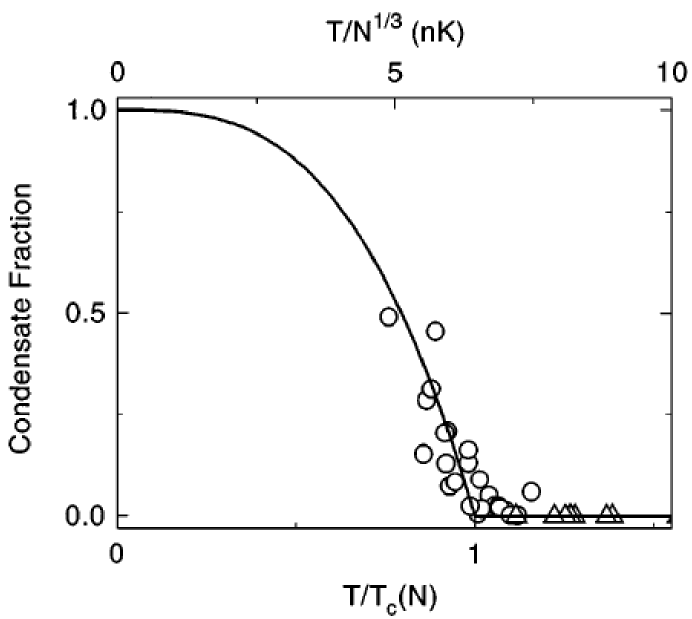
μ: chemical potential $\quad \mu < \varepsilon_0$

$$T \to 0, \quad \mu \to 0, \quad N_0 \to N$$

(trivial)

but for $\quad k_B T >> \varepsilon_1 - \varepsilon_0$

$$N_{exc} = \sum_{i=1} \frac{1}{e^{(\varepsilon_i - \mu)/k_B T} - 1} < N\,?$$



harmonic trap $\quad n_0 \lambda_{dB}^3 (T_C) \approx 2.612$

$$k_B T_C \approx 0.94 \hbar \overline{\omega} N^{1/3} \qquad \frac{N_0}{N} = 1 - \left(\frac{T}{T_C}\right)^3$$

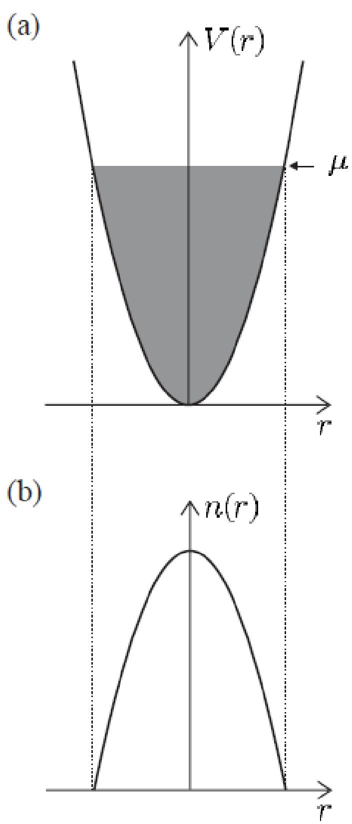
Mewes et al, PRL 1996

Interacting gas

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + U(\vec{r}) + g|\psi|^2\right)\psi = \mu\psi$$

Gross-Pitaevskii equation

$$n(r) = N|\psi|^2 \qquad U(\vec{r}) = \frac{1}{2}m\left(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2\right) \qquad g = \frac{4\pi\hbar^2 a}{m}$$



$$\left(U(\vec{r}) + g|\psi|^2\right)\psi = \mu\psi$$

Thomas-Fermi approximation
(neglect kinetic energy)

$$|\psi|^2 = \frac{\mu - U(\vec{r})}{g}$$

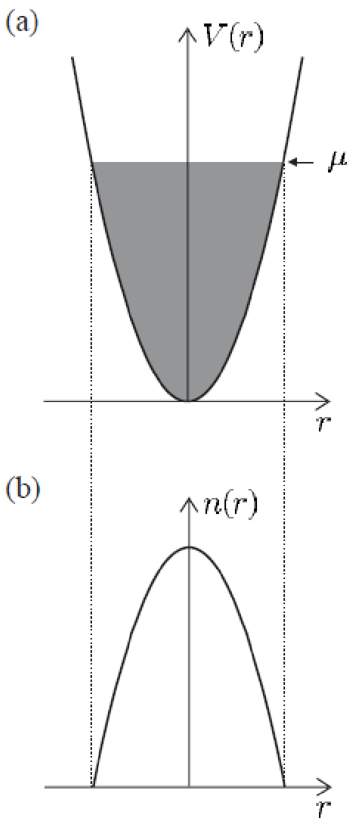
$$n(r) = n_0\left(1 - \frac{x^2}{R_x^2} - \frac{y^2}{R_y^2} - \frac{z^2}{R_z^2}\right)$$

inverted parabola

$$\frac{1}{2}m\omega_i^2 R_i^2 = \mu$$

$$n_0 = \frac{N\mu}{g}$$

Interacting gas



$$n(r) = n_0\left(1 - \frac{x^2}{R_x^2} - \frac{y^2}{R_y^2} - \frac{z^2}{R_z^2}\right)$$

$$n_0 = \frac{N\mu}{g}$$

$$\frac{1}{2} m\omega_i^2 R_i^2 = \mu$$

inverted parabola

$$\int n(\vec{r})\mathrm{d}\vec{r} = N = \frac{N\mu}{g}\frac{8\pi}{15} R_x R_y R_z$$

$$n_0 = \left(\frac{m\overline{\omega}}{\hbar}\right)^{6/5} \frac{(15N)^{2/5}}{8\pi a^{3/5}}$$

$$\overline{\omega} = \left(\omega_x\omega_y\omega_z\right)^{1/3}$$

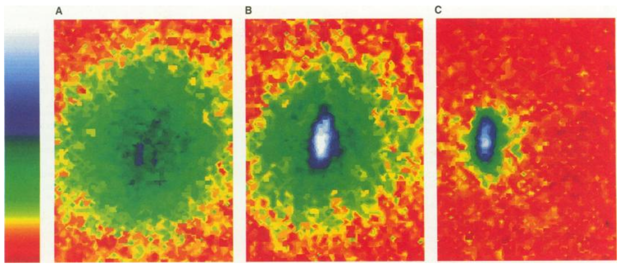
compare with thermal gas:

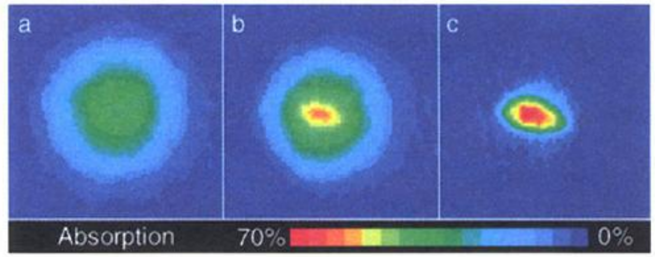
$$n(r) = n_0 e^{-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2} - \frac{z^2}{2\sigma_z^2}}$$

$$n_0 = N\left(\frac{m\overline{\omega}^2}{2\pi k_B T}\right)^{3/2} \quad \sigma_i = \sqrt{\frac{k_B T}{m\omega_i^2}}$$

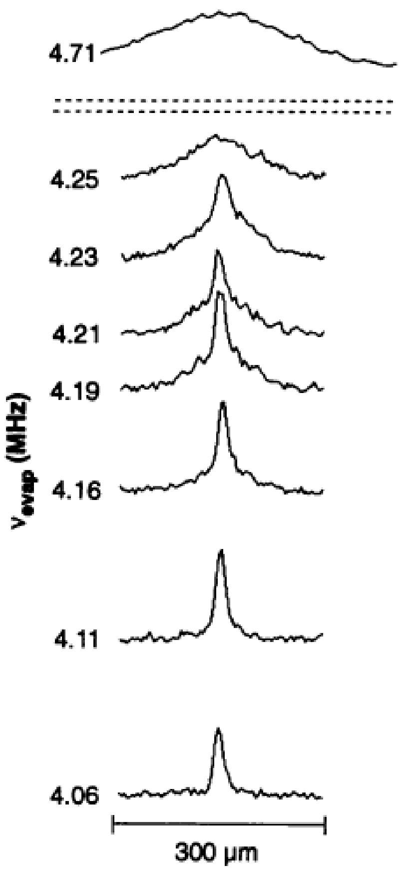
Gaussian distribution

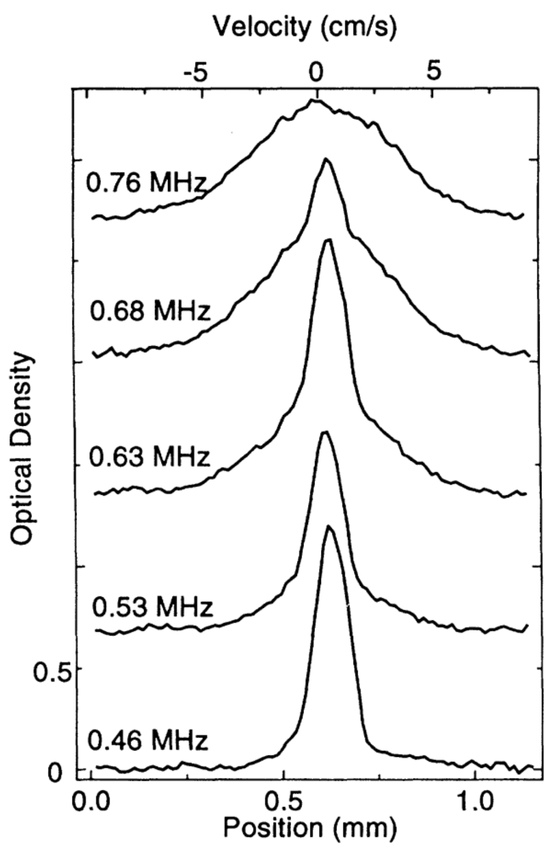
Experimental signature of BEC



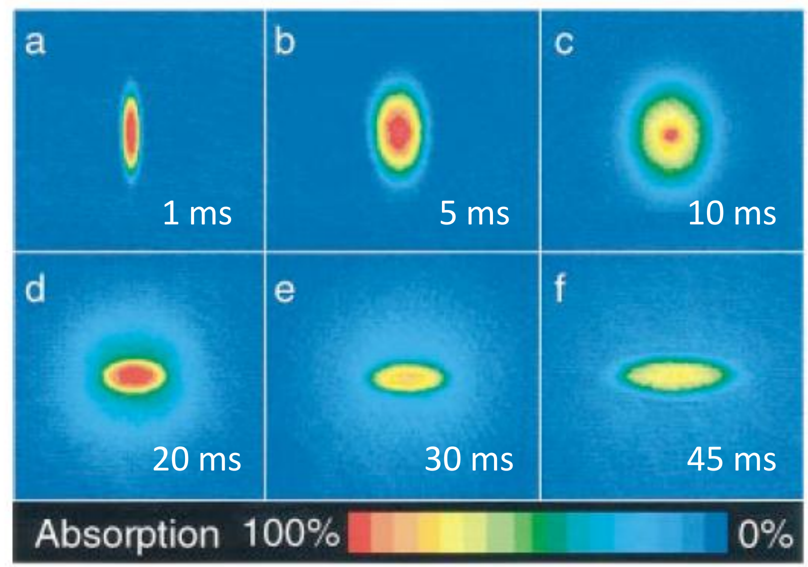


Bimodal distribution



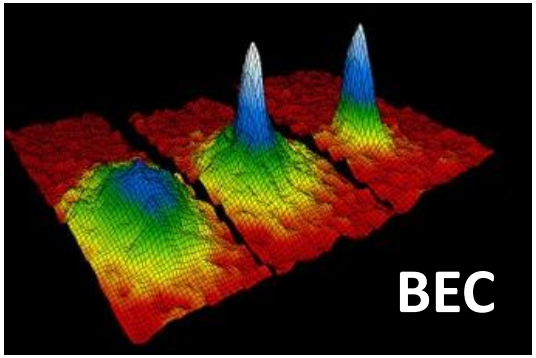


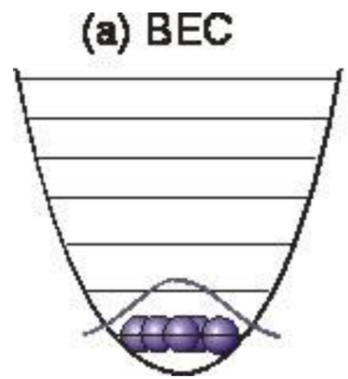
Anistropic expansion

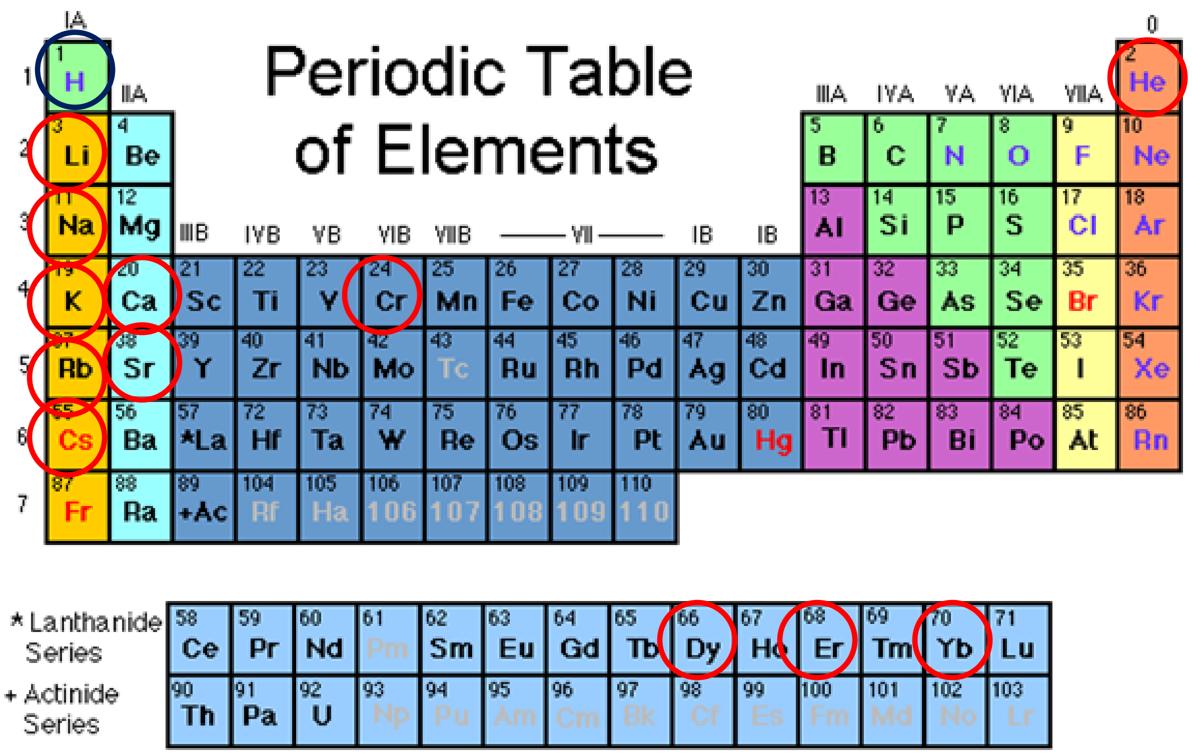


Mewes et al, PRL 1996

BEC‘s







BEC:
H, ^{4}He*, ^{7}Li, ^{23}Na, ^{39,41}K, ^{40}Ca, ^{52}Cr, ^{85,87}Rb, ^{84,86,88}Sr, ^{133}Cs, ^{162,164}Dy, ^{168}Er, ^{168,170,174,176}Yb

first: Rb, Na (1995)
latest: Er (2012)

BEC for a>0 and a<0

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + U(\vec{r}) + g|\psi|^2\right)\psi = \mu\psi \qquad g = \frac{4\pi\hbar^2 a}{m}$$

$$|\psi|^2 = \frac{\mu - U(\vec{r})}{g} \qquad n(r) = N|\psi|^2$$

BEC with *a<0* not stable (only for small atom number)

^{7}Li	*$-28a_0$*
^{23}Na	*$+55a_0$*
^{85}Rb	*$-388a_0$*
^{87}Rb	*$+98a_0$*

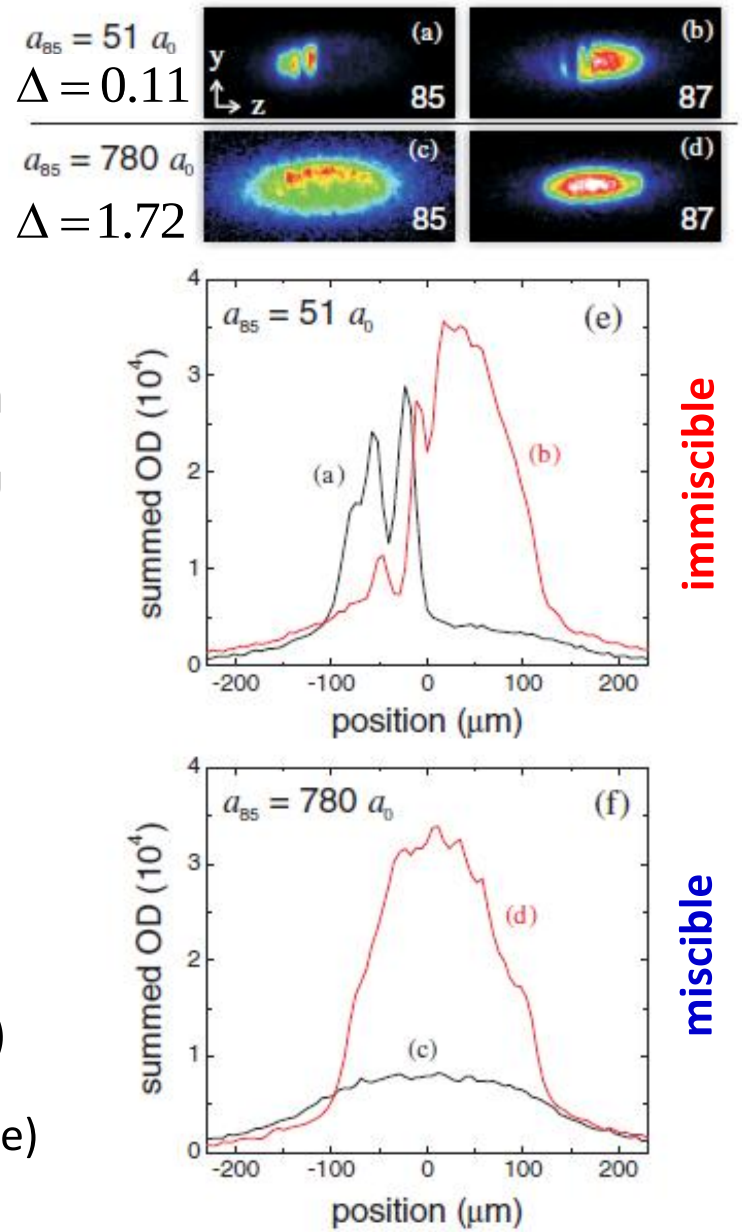
Quantum degenerate mixtures

Bose-Bose mixture:

three scattering lengths: a_{11}, a_{22}, a_{12}

$$g_{ij} = 2\pi\hbar^2 a_{ij}\left(\frac{m_i + m_j}{m_i m_j}\right)$$

$$g_{11} > 0,\ g_{22} > 0,\ g_{11}g_{22} > g_{12}^2 \qquad \boxed{\Delta = \frac{g_{12}}{\sqrt{g_{11}g_{22}}} > 1}$$

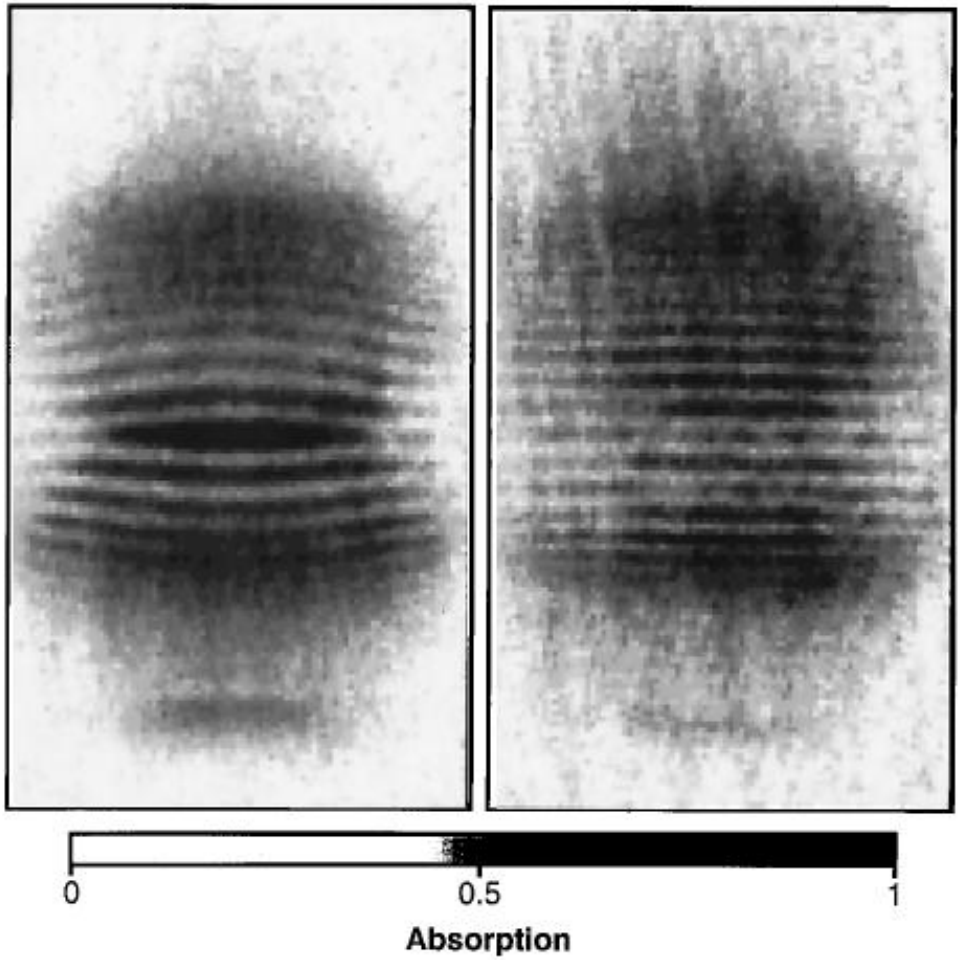


Example: ^{85}Rb+^{87}Rb (Papp, Pino, Wieman, PRL 2008)

a_{87}=+99a_0; $a_{85\text{-}87}$=+213a_0; a_{85} varied (Feshbach resonance)

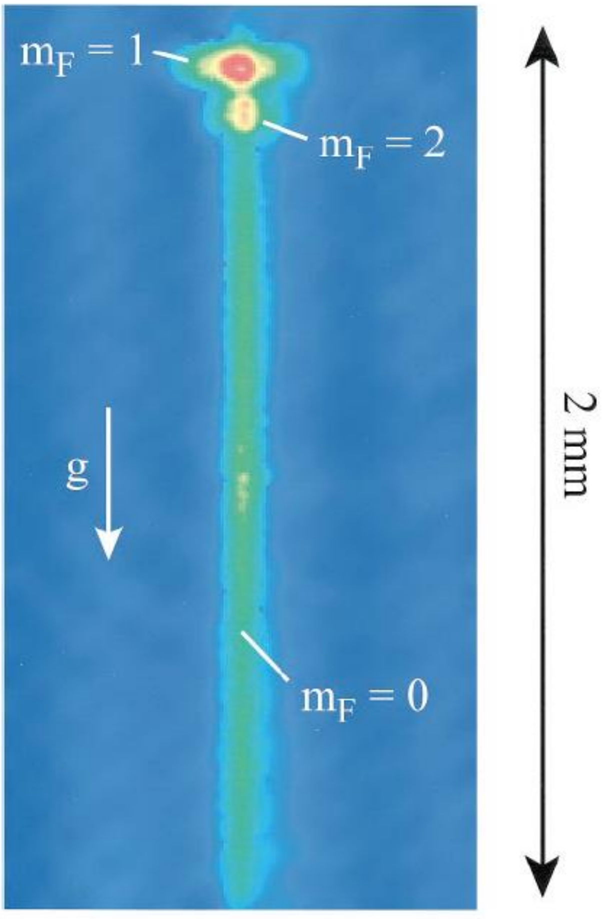
Early key experiments BEC

Interference between two BEC's



Andrew et al, Science 1997

Atom laser

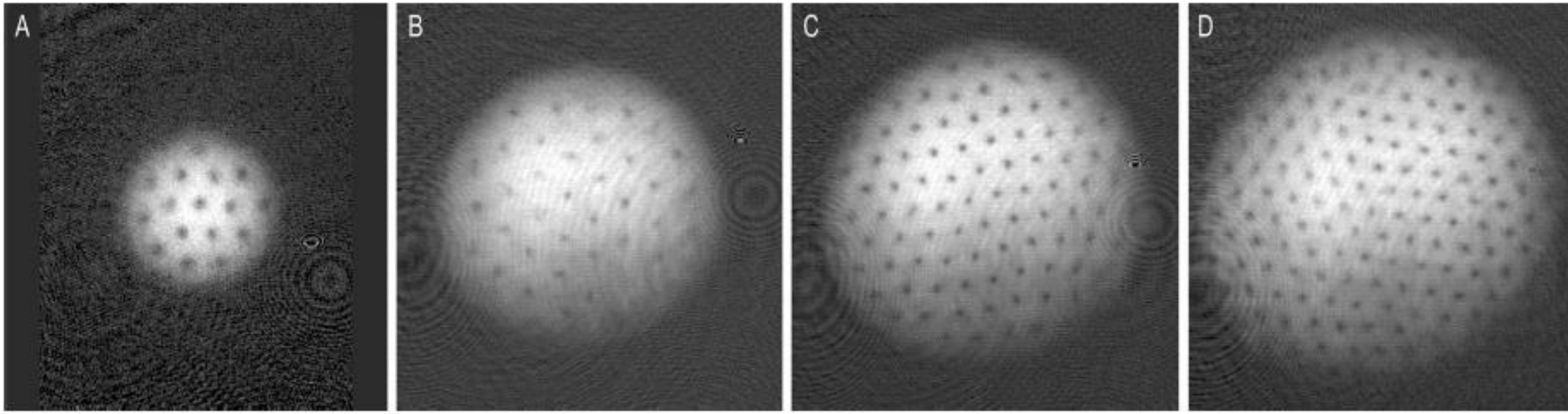


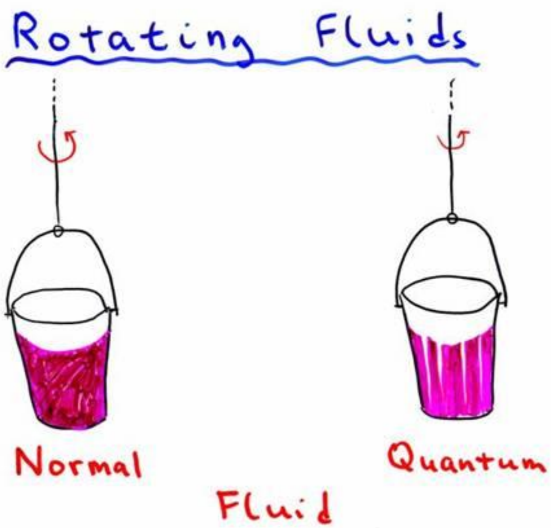
Bloch et al, PRL 1999

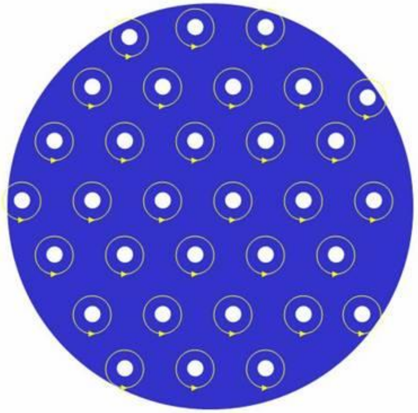
Early key experiments BEC

Vortices in rotating BEC's

Abo-Shaeer et al, Science 2001



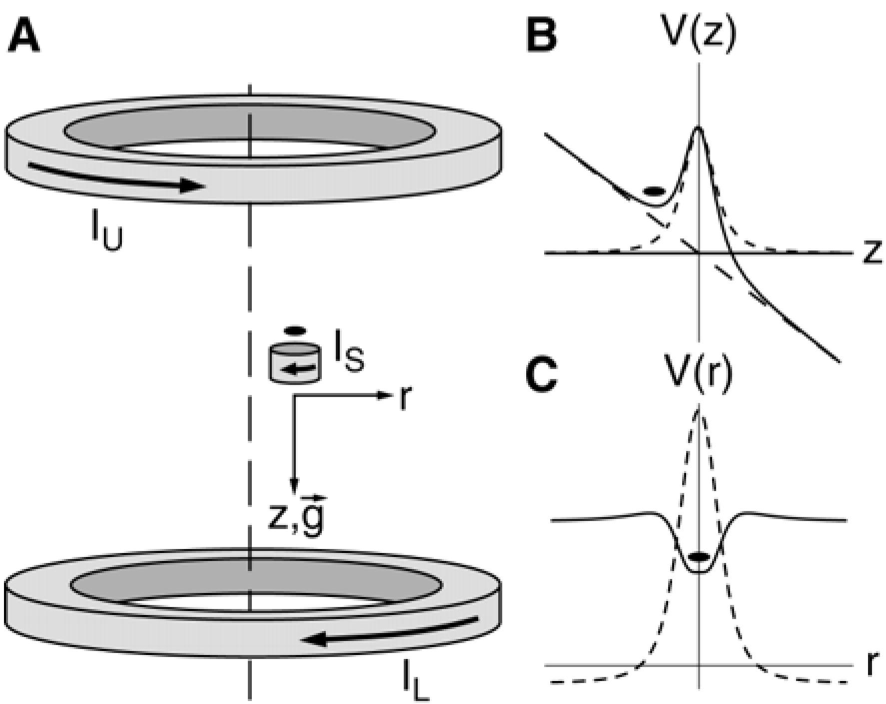


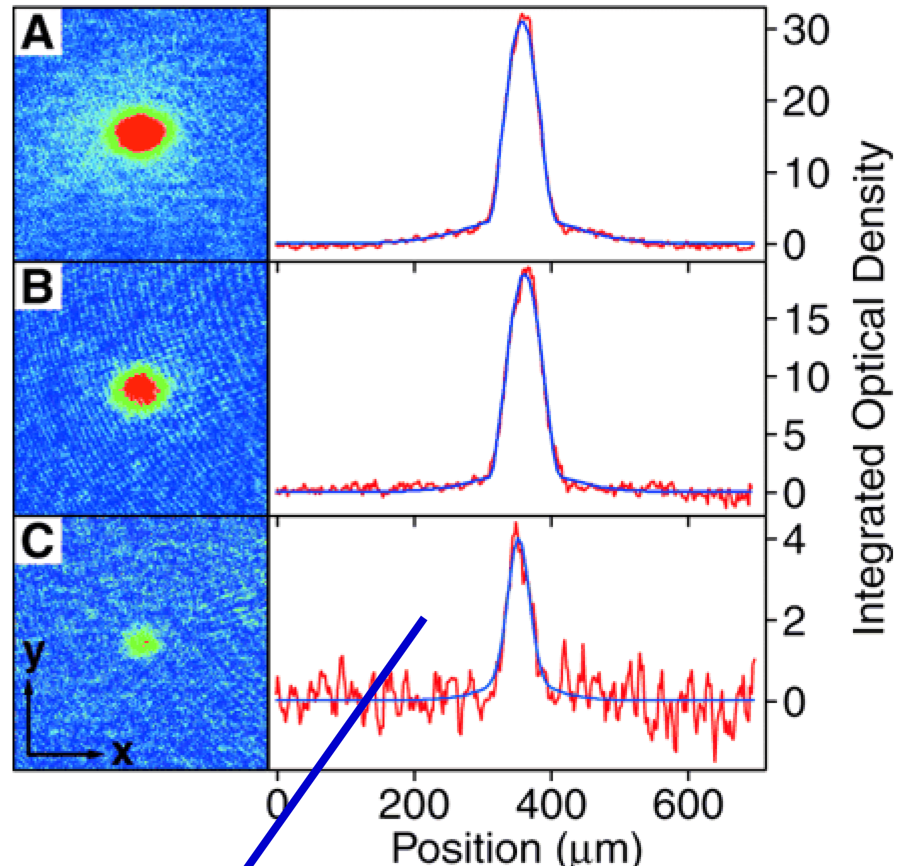


superfluid

website Ketterle group

Lowest temperature





$$k_B T_C \approx 0.94\hbar\overline{\omega}N^{1/3}$$

$$\frac{N_0}{N} = 1 - \left(\frac{T}{T_C}\right)^3$$

450±80 pK !!!

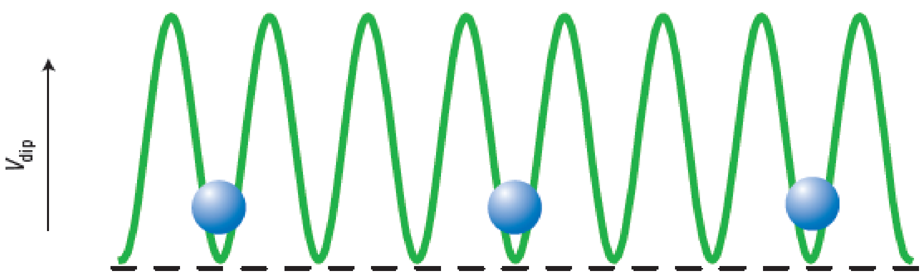
Leanhardt et al, Science 2003

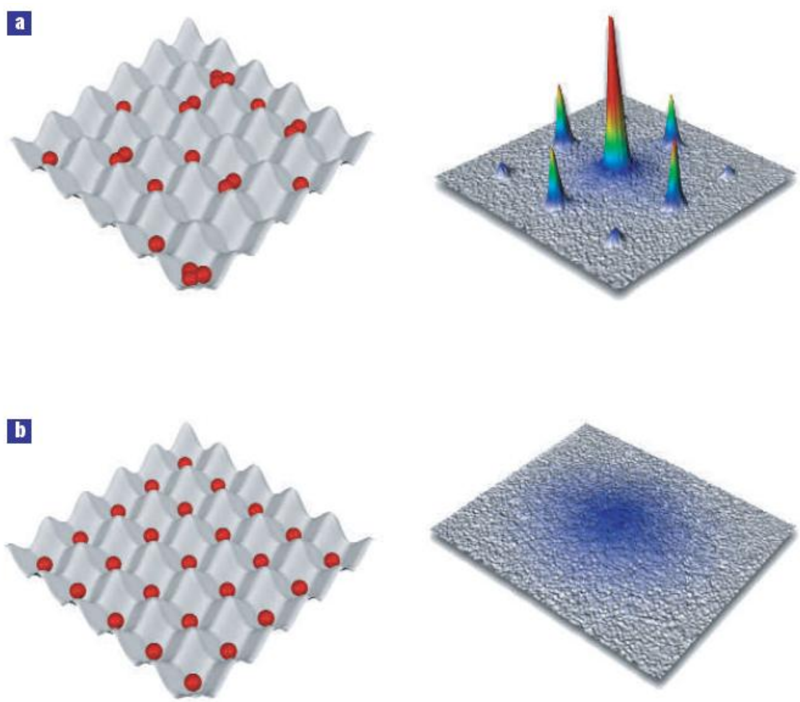
Early key experiments BEC

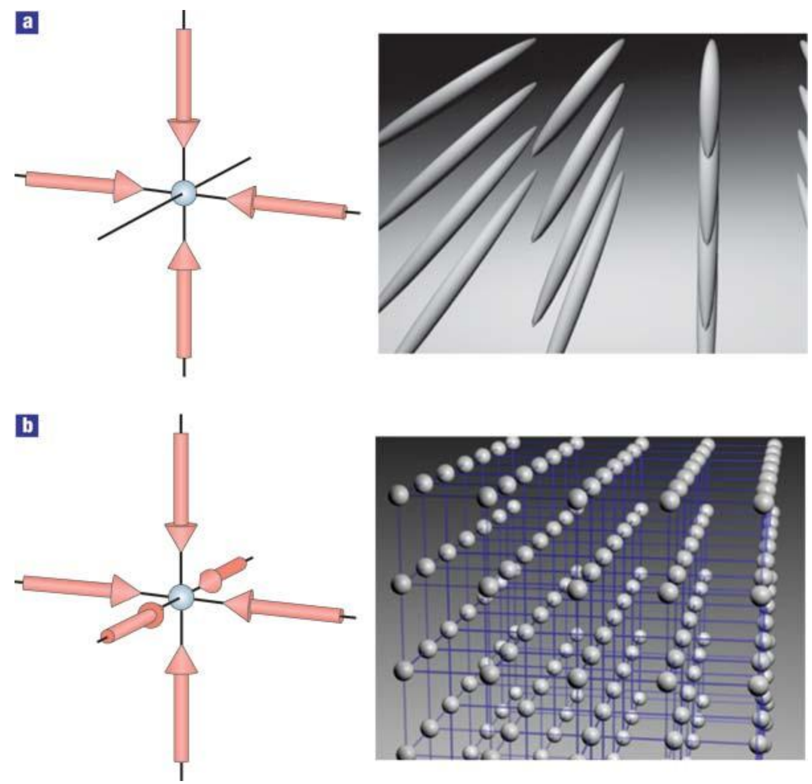
Superfluid to Mott insulator transition for BEC in optical lattice

$$V_{\mathrm{lat}}(x) = V_0 \sin^2(k_L x) \qquad k_L = 2\pi/\lambda_L$$

Greiner et al, Nature 2002



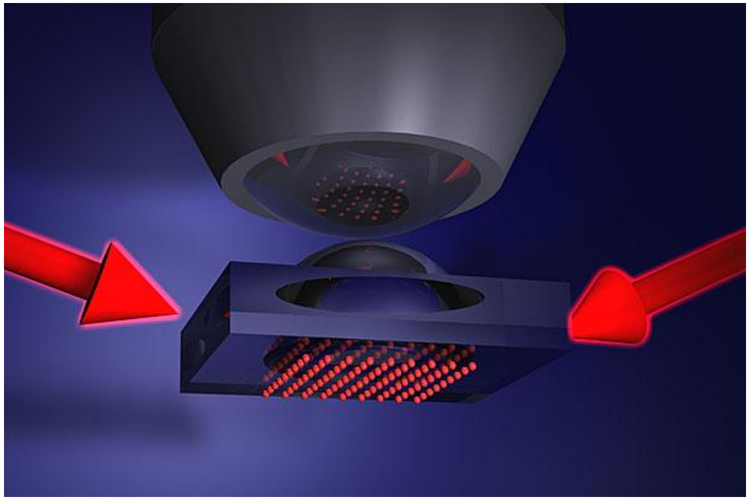


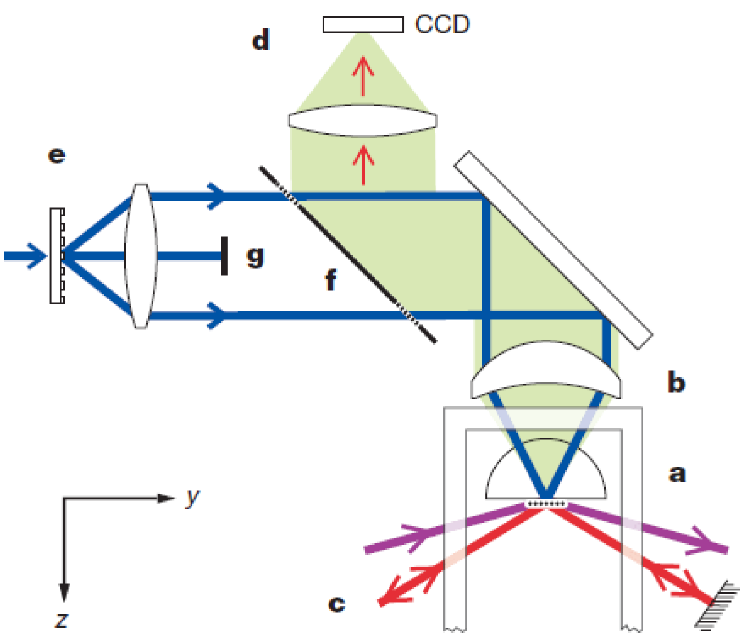


$$\hat{H} = -J \sum_{<i,j>} \hat{a}_i^{+} \hat{a}_j + \frac{U}{2} \sum_i \hat{n}_i (\hat{n}_i - 1)$$

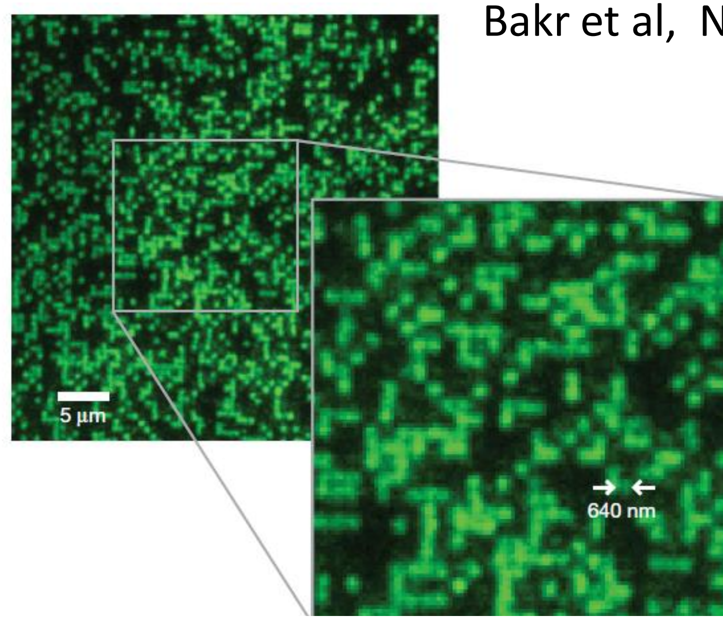
Bose-Hubbard model

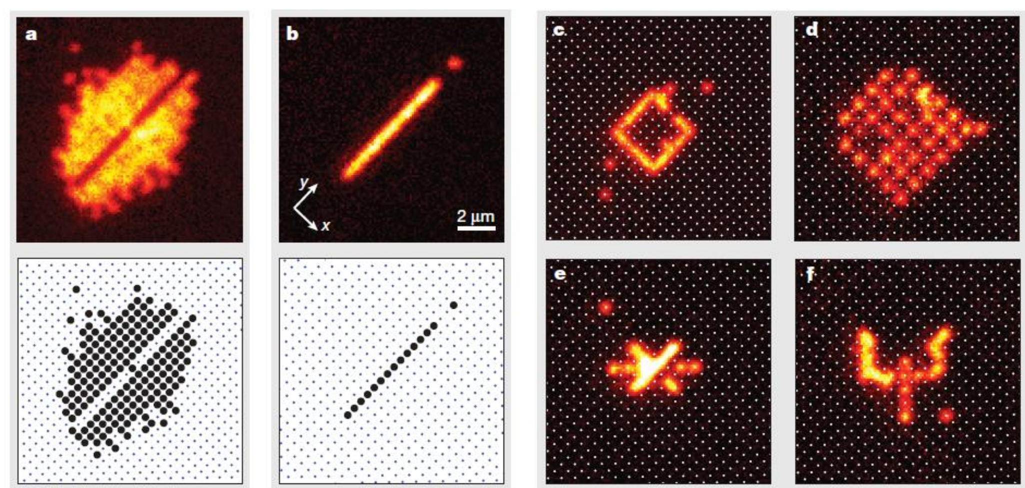
Quantum gas microscope





Bakr et al, Nature 2009





Weitenberg et al, Nature 2011

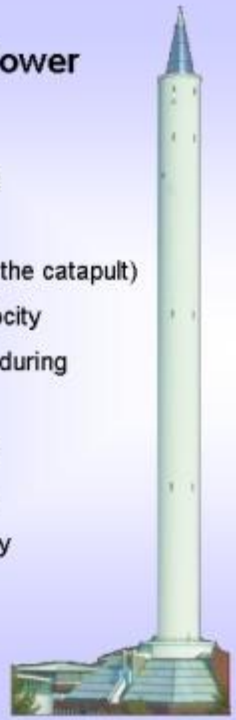
BEC in micro gravity

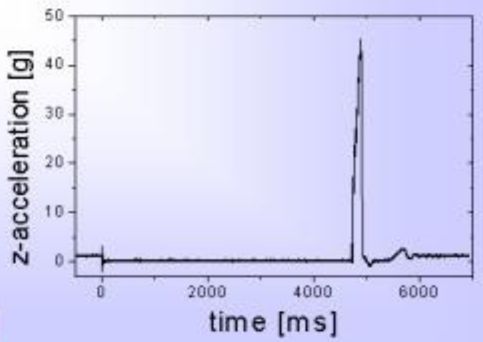
van Zoest et al, Science 2010
QUANTAS

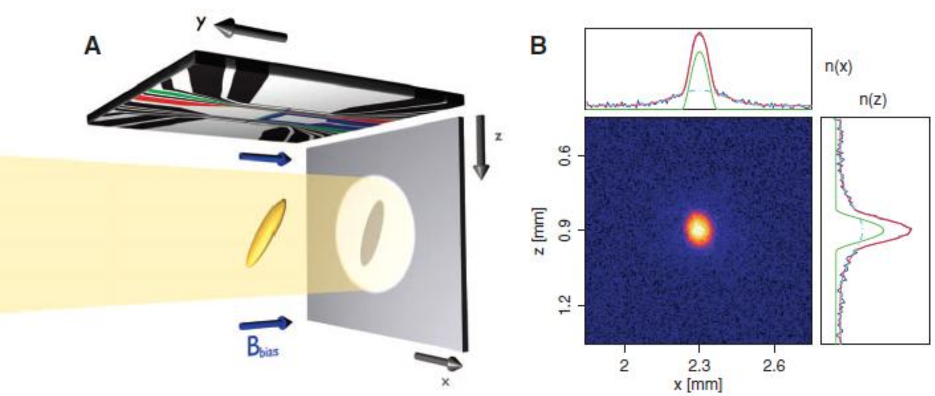
BEC on atom chip

ZARM Drop Tower

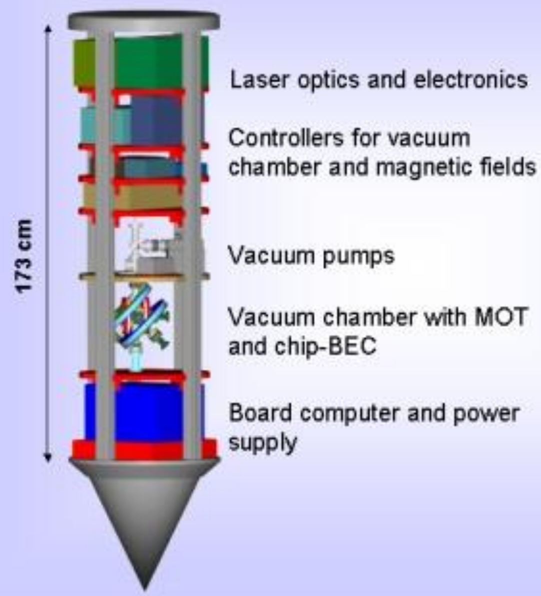
- 110 m drop height
- 4.5 s drop time
  (>9 s planned for the catapult)
- 170 km/h end velocity
- 50 g deceleration during recapture
- Micro-g level: 10^{-6}
- 10^{-1} mbar vacuum
- up to 3 flights / day







Drop capsule

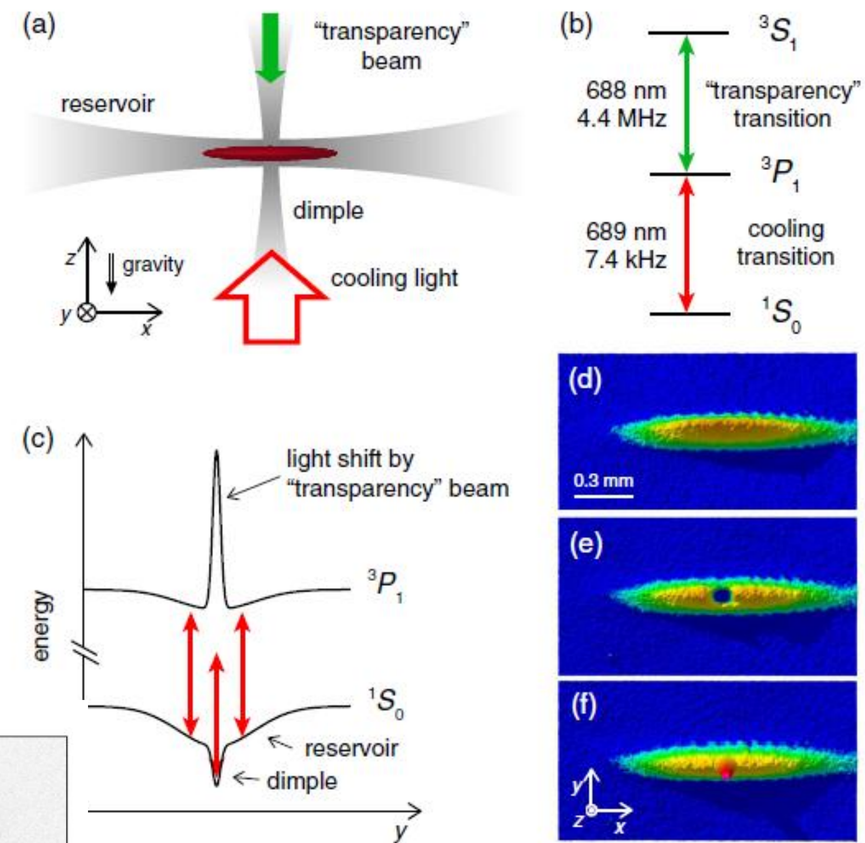


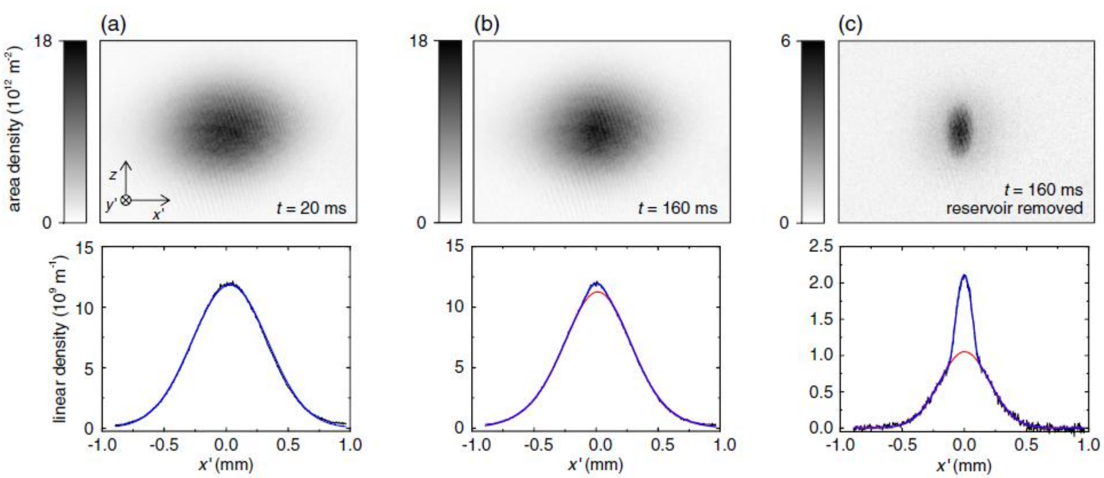
Laser cooling to BEC

Stellmer et al, PRL 2013

- strontium
- Doppler cooling below 1μK on narrow transition

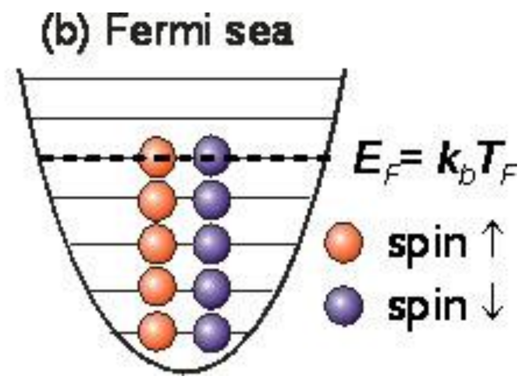
- dimple trap, protected from laser cooling beam by "transparency" beam
- no evaporative cooling!

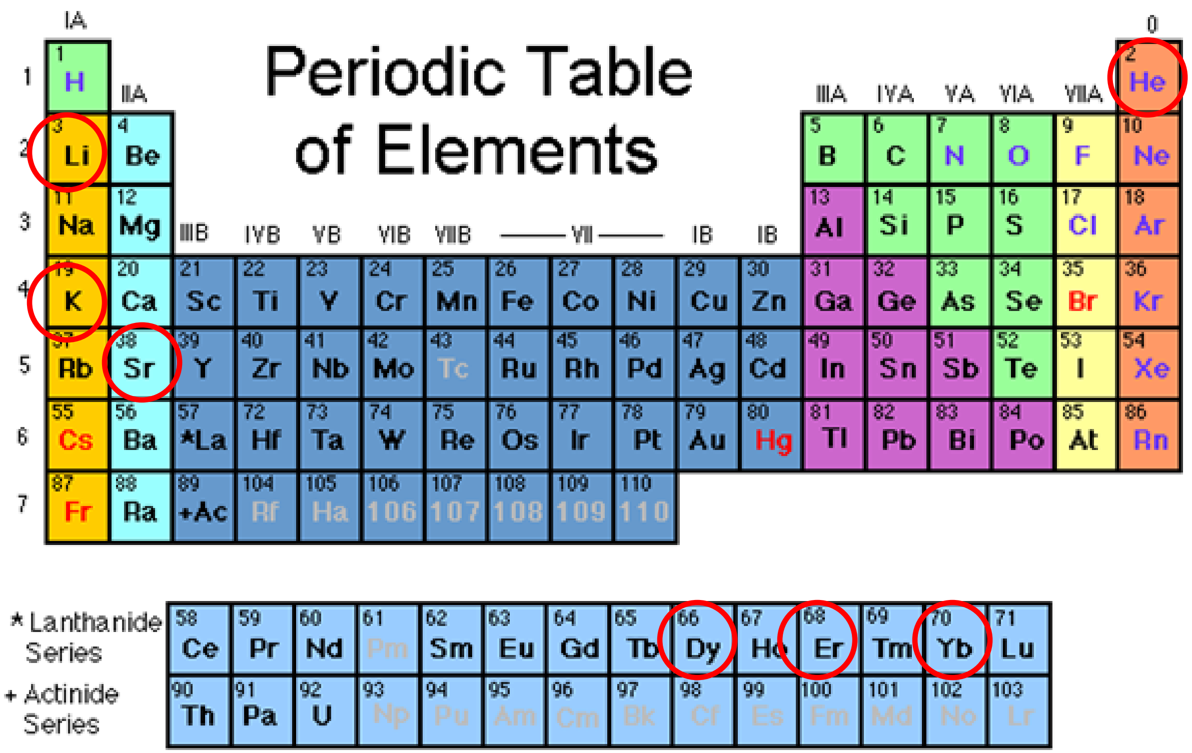




$$k_B T_D = \frac{\hbar \Gamma}{2}$$

Degenerate Fermi gases (DFG)

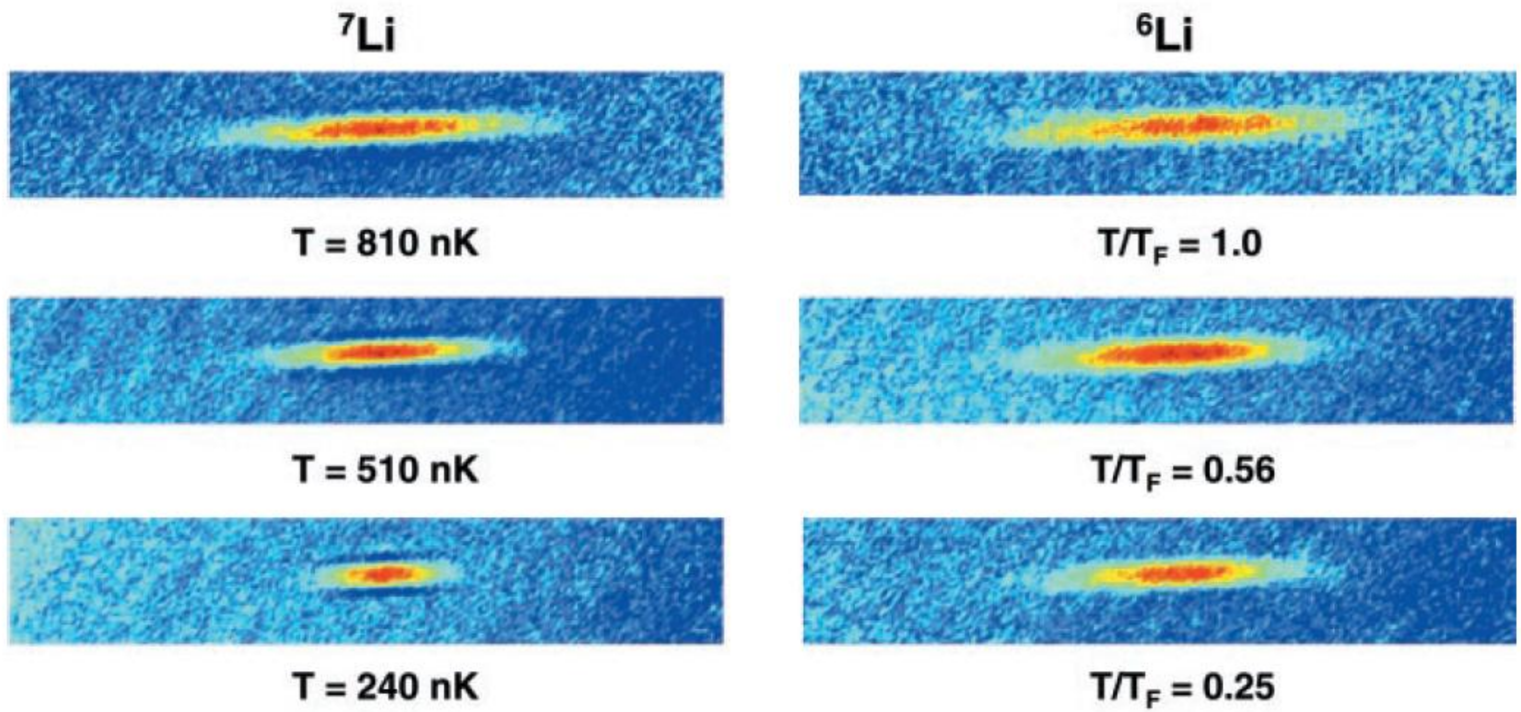




DFG:

^{3}He*, ^{6}Li, ^{40}K, ^{87}Sr, ^{161}Dy, ^{167}Er, ^{171,173}Yb

Experimental signature of DFG



Truscott et al, Science 2001