

# The energy market: From energy products to energy derivatives and in between

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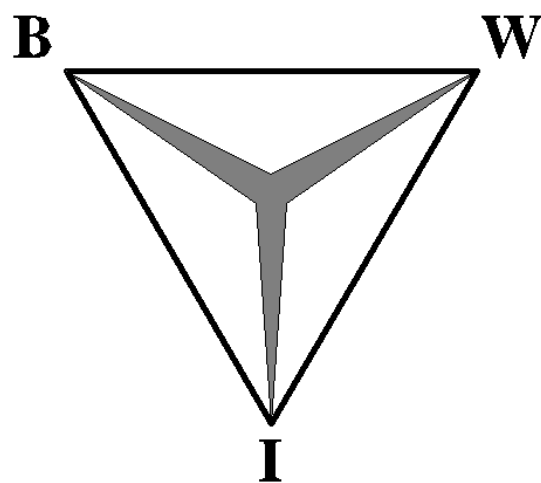


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# From energy products to energy derivatives and in between



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## Contents

Preface.....	7
Introduction.....	9
Chapter 1. Energy products.....	11
1.1. Oil .....	11
1.2. Gas .....	12
1.3. Electricity .....	13
1.4. Spreads .....	15
1.4.1. Spark spread .....	15
1.4.2. Crack spread .....	17
1.4.3. Spread options.....	17
Chapter 2. Characteristics of energy prices.....	19
2.1. Distribution.....	19
2.2. Spikes.....	20
2.3. Mean reversion .....	20
2.4. Volatility.....	22
Chapter 3. Modeling spot price processes.....	23
3.1. Brownian motion.....	24
3.2. Geometric Brownian motion.....	25
3.2.1. Parameter estimation.....	25
3.3. Jump Diffusion Processes .....	30
3.3.1. Jump diffusion processes with mean reversion .....	31
3.3.2. Completeness of the market .....	31
3.4. Regime switching models.....	32
Chapter 4. Energy derivatives .....	35
4.1. Basic products & Hedge examples .....	35
4.1.1. Swaps.....	36
4.1.2. Swaptions .....	38
4.2. Spread options.....	38
4.2.1. Margrabe formula .....	39
4.2.2. Adjusted Margrabe formula .....	40
4.2.3. The Bachelier model.....	40
4.2.4. Kirk's formula.....	40
4.3. Swings, Recalls, and Nominations .....	41

4.3.1	Swings.....	41
4.3.2	Recalls.....	42
4.3.3	Nominations .....	43
	Conclusion .....	45
	Bibliography.....	46

## Preface

This paper before you is one of the final compulsory parts of my Business Mathematics and Informatics (BMI) study at the VU University Amsterdam. It is mainly a literature study that has the character of including at least two of the three focus areas of the study, Business economics, Mathematics and Informatics.

This paper discusses the special characteristics of the energy market and how to incorporate these characteristics into the modeling process. It also looks at some special derivative products this market uses. As the title states we will go from energy products to energy derivatives and take a look in between. We will take a look at some common models in the financial world and see how they can be used in the energy market. The common models are chosen to make this rather complicated market a bit approachable for unfamiliar readers.

First of all, I would like to thank my Lord and Saviour Jesus Christ. Without Him I would certainly not be able to complete this paper. I would also like to thank my supervisor, Sandjai Bhulai, for his support and helpful comments and suggestions during the period I was working on this paper. Furthermore, I would like to thank my family, especially my aunt, without whom I would not be able to study abroad. Also, I would like to thank the vu11-group (you know who you all are) for the great fellowships we had at the VU and outside. And last but not least, I would like to thank my best friend, Monique Goeldjar, whom I dearly love and whose support I have greatly appreciated during our long distance study sessions and still appreciate as I am completing my studies.

*To the reader:*

*Thank you for taking the time to scroll through and read my work. It is my great pleasure to invite you to read and maybe take away some helpful insights into the subject that is discussed.*

Amsterdam, April 2008

Christel Nijman



## Introduction

Energy has become one of the most traded commodities after the deregulation in the oil and natural gas industries in the 1980s, followed by the deregulation in the electricity industry in the 1990s. Until then the prices were set by regulators, i.e., governments. Energy prices were relatively stable, but consumers had to pay high premia for inefficient costs, e.g., complex cross-subsidies from areas with surpluses to areas with shortages or inefficient technology. Due to the deregulation a free market with more competitive prices arose that revealed that energy prices are the most volatile among all commodities. This exposed both energy producers and consumers to many financial risks [13].

As the financial risks stem from the different interesting characteristics displayed in price processes of the energy market, we need to account for these characteristics when we are trying to model the consisting price processes. Worth knowing is that it is these characteristics that distinguish the energy market from others.

The characteristics that are being referred to are

- Spikes;
- Mean reversion;
- High Volatility.

It is also important to note that the price distributions portray pronounced skewness and kurtosis. This is helpful when choosing a mathematical model to describe a certain price process. Most common models tend to use the Normal distribution as the underlying distribution, but that leads to erroneous conclusions with respect to energy prices. We thus need to look for models with realistic price distributions that are able to capture the market's characteristics if we want to correctly model the price processes. We also need to distinguish between different products and analyze which characteristics hold for the prices of that particular product. As with all modeling attempts it is very important to closely look at and understand the process that we want to model in order to find the appropriate model. It is well known that the energy market is very complicated and hard to model correctly, referring to the price processes. New models are probably being developed at the very moment that you are reading this paper. In this paper the most common and intuitively clear mathematical models used in the financial world are discussed. The goal is to simply describe how these common models can be applied to the energy market and comment on which models better to use for spot price processes of certain energy products. The importance of this knowledge is useful if one wants to trade derivatives, options, futures, forwards, etc. for one of the discussed energy products. This is even a regular and daily task among stakeholders in the energy world. In the last section of this paper some of these derivatives will be discussed in the energy market framework.

So now let us look at the special characteristics the energy market displays and how to handle these characteristics and incorporate them into the modeling process as well as state some market specific derivative products.



## Chapter 1. Energy products

Let us first take a look at the energy products that we deal with in this paper. We will take a look at the three main products of the market and give a brief description. These products are: Oil, Gas and Electricity. The markets that consist of these products are ordered based on the historical pace at which they were opened. The literature [B1] describes them as follows.

1. Fuels: oil, gas, coal, and their derivatives and byproducts;
2. Electricity;
3. Weather, emissions, pulp and paper, and forced outage insurance.

The fuel markets opened up for competition in the 1980s, and the electricity markets in the early and mid-1990's on a wholesale level. Then the trading of new commodity types, the third group, followed in the late 1990's. These groups have some common parts, though, but might differ in these three activities: production, distribution, and consumption.

We also take a look at what is believed to be the most important structure in the energy market, namely spreads, both as a product and concept.

### 1.1. Oil

For the oil market the prices had been fixed until 1985 when the OPEC pricing regime collapsed. The crude oil market is the largest commodity market in the world. It is being traded at different locations but the most significant trading hubs are New York, London, and Singapore. Both crude oil and refined products are traded there. These refined products are Heating Oil, Gasoline, Fuel oil and Kerosene. We can distinguish different grades of crude oil which are determined by its weight (API gravity, the measure of how heavy or light a petroleum liquid is compared to water) and sulfur content. There have been three main regional crude oil benchmarks around on which the prices of the oil market have been concentrated for the last 20 years. These are also known as 'marker' crudes. These three are from the UK the Brent Blend, from the United States the West Texas Intermediate (WTI) crude oil and the Dubai or Fateh crude from the United Arab Emirates.

The spot price represents the balancing point of supply and demand. Crude oil forwards are traded over the counter (OTC) on, for instance, the New York Merchantile Exchange (NYMEX). The market deals with spreads on the input side and on the output side. So crude oil is cracked into refined products which produce a crack spread. Let us say for instance that 3 barrels of crude is cracked into 2 barrels of unleaded gasoline and 1 barrel of heating oil. This is called the 3:2:1 crack spread and is also the most widely used.

## 1.2. Gas

The gas markets were opened steadily for competition during the 1980s and early 1990s. Before then, prices were controlled by authorities such as the Federal Energy Regulatory Commission (FERC). In order 636 issued on April 9, 1992 the FERC mandated open pipeline systems for transporting natural gas.

There are different factors affecting demand such as consumption percentages and seasonal variation. In the winter months more gas is consumed for home heating which is reflected in the high winter prices as opposed to low summer prices. But this gap will probably become much smaller since the demand for air-conditioning and electricity consumption generated from gas is growing and the highest during the summer. So we will see a relative shift in prices from the winter to the summer.

The gas market knows five big market players [B1]:

- Gas producers;
- Pipeline companies;
- Local delivery companies;
- Consumers;
- Marketers.

Here the marketers suffice as intermediaries between the other four players. There are different transactions of natural gas possible. There is the physical market and the monthly market. The physical market trades daily and monthly and the monthly market trades during the bid-week, the last week of the month preceding the contract month. The benchmark or the index price is set by the bid-week transactions. So the bid-week gives the predictable part of the consumer demand and the daily trade nets out the balance. The price is an average over monthly contracts that have been transacted. The real price is determined by a telephone survey held by industry publications within the FERC's Gas Market Report.

The Natural gas market is first of all a collection of delivery and receipt locations. There are two services that the players sign up for, the delivery and receipt at a given location and also the transportation between two locations. The traded volumes are location dependent based on the position in the pipeline system.

So let us explore the two services a bit closer.

### **Delivery**

*The baseload firm:* The delivering party has to perform according to the contract under any condition.

Non-performance is imposed by financial penalties and liquidated damages.

*The baseload interruptible:* Delivery can be interrupted; this can be specified in the contract, baseload.

The baseload is the steady volume flow regardless of the demand.

*Swing*: The volume of delivered gas is adjusted daily at the buyer's discretion. These are used for daily pipeline volume balancing requirements.

## **Transportation**

*Firm transported (FTS)*: The highest priority service.

*Interruptible Transportation Contract*: A pipeline has the option to interrupt a service on short notice without a penalty. This can be done at peak-demand sessions because of demand from firm service customers.

## **1.3. Electricity**

One of the crucial features of the electricity market is the need for real-time balancing of location supply and demand. The none-storage ability makes it hard to design an efficiently functioning market. Supply and demand should always be in balance. So we have an additional set of services: balancing and reserve resources.

So instead of two services as with the Fuel Market we now have three services.

- Generation;
- Transmission;
- Ancillary services.

We can distinguish between two contracting structures in the cash market: pools and bilateral markets.

### **Pools**

This market has a formal market clearing price at which all cash energy transactions clear.

Some pools are the

- The Nordic Power Exchange (Nord Pool);
- The New England Power Pool (NEPOOL);
- The New York Intrastate Access Settlement Pool (NYPOOL);
- The California Independent System Operator (CAISO).

### **Bilateral Markets**

All transactions are entered into by two parties, hence bilateral, and are independent of other transactions in the market.

Some bilateral markets are the

- The Electric Reliability Council of Texas (ERCOT);
- The East Central Area Reliability Council (ECAR);
- The Southeastern Electric Reliability Council (SERC).

The power market has more than one cash market and is thus called a multi-settlement market. Let us take a look at the different energy cash markets:

- Day ahead: Contracts for generation of energy on the next day;
- Day-of: Contracts for generation of energy for the rest of the day;
- Hour-ahead: Contracts for generation of energy for the next hour;
- Ex-post (or Real-time): This market clears any deviations from the predicted schedules entered in the earlier market.

There are also different types of traded power. We distinguish between On-peak Power and Off-peak Power.

On-peak Power is power delivered during high (peak) demand hours and Off-peak Power is power delivered during low demand periods. Apart from these types there are many more products such as next-day power, next-week power, next-month power, etc.

A need for reserves in this market is very clear; as mentioned above there is a need to balance instantaneous supply and demand.

The way this is handled, depends on the type of market. Some markets use generators to preserve some of the energy and do not release it until called upon by the market administration (ISOs). Sometimes markets are created to provide these services. There are different types of reserves.

**Spinning reserves:** Resources synchronized to the system that are available immediately and that can be brought to full capacity within ten minutes.

**Non-spinning reserves:** Resources not synchronized to the system that are available immediately and can be brought to full capacity within ten minutes.

**Operating reserves:** The resources that can be brought to full capacity within 30 minutes.

**Energy Imbalance:** Resources needed for correcting supply/demand imbalances.

**Regulation:** Reactive energy to maintain the phase angle of the system. The phase angle measures how much generated energy is converted to heat and therefore lost. Thus by maintaining the phase angle, transmission between a power storage device and a load (utility network) is made controllable and efficient.

**Reactive Power supply:** Services to maintain voltage of transmission lines. At such, it is location specific.

Just as the fuel market, this market also knows spreads. Instead of crack spreads we deal with spark spreads. A spark spread is the theoretical difference between selling a unit of electricity and buying the fuel to produce this unit of electricity. This term only applies to gas-fired power plants. For coal-fired power plants we use the term dark spread which has the same definition as the spark spread.

## 1.4. Spreads

A spread is a price differential between two commodities. All aspects of energy delivery and transmission can be described using spreads.

We distinguish between different classes of spreads:

- The intracommodity spread or the quality spread: This gives the difference between the spot prices of different grades of a certain commodity. For instance sweet crude oil vs. sour crude oil.
- The geographical spread: This gives the difference between prices in two different locations.
- The time or calendar spread: This gives the difference between futures prices preset prices of certain goods to be sold or bought in the future [B3]) of different expiration.
- The intercommodity spread: This gives the price differential between two different but related commodities. It is also the most important class of spreads. The spark spread and crack spread mentioned in previous sections are part of this spread class.

The most important spreads are the intercommodity spreads. We can make the following distinction within this class:

- Spread between operational inputs;
- Spread between operational outputs;
- Spread between outputs and inputs (processing spread).

### 1.4.1. Spark spread

This is the price difference between electricity and one of its fuels, and falls under the “spread between outputs and inputs”. The spark spread between electricity and natural gas is the most common spread. It can give a short position in fuels and a long position in electricity, buy fuels and sell electricity, as is done by a power plant. Spark spreads are traded over the counter, thus are exchanged directly between two parties, without a “middleman”, such as an organized securities exchange.

The value of a 4:3 spark spread [SS] is given as follows:  $[SS] = 4[E] - 3[NG]$ . This spread consists of 4 forward contracts of power [E] (E for energy) and 3 contracts of natural gas [NG] at time t.

#### 1.4.1.1. Heat Rate

To be able to calculate the spark spread and work with it in practice, the term heat rate [HR] should be well-understood and will thus be discussed to some extent in this section.

The heat rate is the number of British thermal units (Btu) that is needed to make 1 kWh of electricity. In ideal situations, no inefficiency, one needs 3412 Btu to produce 1 kWh of electricity. Constant heat rates are approximations and simplify things because in reality the rate varies with a number of parameters such as the temperature of the environment and the generation level. Different levels and different temperatures, in particular, cause the heat rate to vary. The heat rate can also be used to determine the cost of fuel needed to generate one unit of power.

So let us say that the price for one Btu of fuel is:  $P_{fuel} \left( \frac{\text{dollars/euros}}{\text{Btu}} \right)$ .

Then the costs of fuel to produce 1 kWh of power is equal to

$$Fuel\_Cost_{power} \left( \frac{\text{dollars/euros}}{\text{kWh}} \right) = HR \left( \frac{\text{Btu}}{\text{kWh}} \right) \cdot P_{fuel} \left( \frac{\text{dollars/euros}}{\text{Btu}} \right),$$

or

$$\begin{aligned} Fuel_{cost\_power} \left( \frac{\text{dollars}}{\text{MWh}} \right) &= \frac{1000 \left( \frac{\text{kWh}}{\text{MWh}} \right)}{1.000.000 \left( \frac{\text{Btu}}{\text{MMBtu}} \right)} \cdot HR \left( \frac{\text{Btu}}{\text{kWh}} \right) \cdot P_{fuel} \left( \frac{\text{dollars/euros}}{\text{MMBtu}} \right) \\ &= \frac{1}{1000} \cdot HR \left( \frac{\text{Btu}}{\text{kWh}} \right) \cdot P_{fuel} \left( \frac{\text{dollars/euros}}{\text{MMBtu}} \right). \end{aligned}$$

So now that we have defined the Heat Rate, the spark spread can be defined as

$$Spark\ Spread = Power\ Price - HR \cdot Fuel\ Price.$$

As you have probably already noticed, the different parts in this formula can have a different unit. But, as a rule, electricity is quoted in dollars/euros per MWh, gas is quoted in dollars/euros per MMBtu, and the heat rate is quoted in Btu per kWh which leaves us with the following spark spread quotes:

$$\begin{aligned} Spread \left( \frac{\text{dollars/euros}}{\text{MWh}} \right) &= P_{power} \left( \frac{\text{dollars/euros}}{\text{MWh}} \right) \\ &\quad - \frac{1000 \left( \frac{\text{kWh}}{\text{MWh}} \right)}{1.000.000 \left( \frac{\text{Btu}}{\text{MMBtu}} \right)} \cdot HR \left( \frac{\text{Btu}}{\text{kWh}} \right) \cdot P_{fuel} \left( \frac{\text{dollars/euros}}{\text{MMBtu}} \right), \end{aligned}$$

or

$$Spread \left( \frac{\text{dollars/euros}}{\text{MWh}} \right) = P_{power} \left( \frac{\text{dollars/euros}}{\text{MWh}} \right) - \frac{1}{1000} \cdot HR \left( \frac{\text{Btu}}{\text{kWh}} \right) \cdot P_{fuel} \left( \frac{\text{dollars/euros}}{\text{MMBtu}} \right),$$

where  $P_{power} \left( \frac{\text{dollars/euros}}{\text{MWh}} \right)$  is the price of one MWh of power.

Now what if the spread should be quoted in dollars or euros per MMBtu, then we would get the following quote:

$$Spread \left( \frac{\text{dollars/euros}}{\text{MMBtu}} \right) = \frac{P_{power} \left( \frac{\text{dollars/euros}}{\text{MWh}} \right)}{\frac{1}{1000} \cdot HR \left( \frac{\text{Btu}}{\text{kWh}} \right)} - P_{gas} \left( \frac{\text{dollars/euros}}{\text{MMBtu}} \right),$$

where  $P_{gas} \left( \frac{\text{dollars/euros}}{\text{MMBtu}} \right)$  is the price of one MMBtu of gas

### 1.4.2. Crack spread

The crack spread [CS] is the price differential between refined products and the crude oil [CO], output and input, respectively. It is used to replicate a short position in crude oil and a long position in the refined products. The 3:2:1 crack spread is the most commonly used in the market (see the section on Oil for a description). They are traded over the counter and on NYMEX.

The value of the 3:2:1 crack spread is given by:  $[CS] = \frac{2}{3}[UG] + \frac{1}{3}[HO] - [CO]$  at time t.

Here [UG] stands for unleaded gasoline and [HO] stands for heating oil

In practice beware that crude oil prices are quoted in dollars/barrel while heating oil and unleaded gasoline are quoted in dollars/gallon. So a conversion has to be applied that takes into account that a barrel contains 42 gallons.

### 1.4.3. Spread options

In the energy market one cannot talk about spreads without mentioning something about spread options. The market itself forces this concept upon its key-players because all most market constructions can be seen as complex spread options. A spread option is an option on a spread, the holder has the right, but not the obligation, to enter into a spot or forward spread contract. These options, being a put or a call, consist of two underlying commodities instead of one and are thus more complex than plain vanilla options. The most common spread option is the one with the underlying spread  $S = Fe - H \cdot Fg$ , where  $Fe$  and  $Fg$  are the forward contracts on electricity and natural gas, respectively, and  $H$  is the heat rate.

(More on this important concept in the last section of this paper)



# Chapter 2. Characteristics of energy prices

The most important step in understanding a certain market is analyzing the available data. Then we will be able to understand and quantify the most important features of a particular market, in our case the energy market. If we want to be able to adequately select the most appropriate model, we need to know the important and significant elements that influence the market and take care that these elements are to some extent part of the model.

It is well known that commodity prices display a different behavior than other prices in the financial market. This is due to the difference between supply and demand on short term, which means that there is a difference between spot prices and forward prices of commodities. So when one wants to sell a certain product now, the prices will be different then when that same product is delivered in the near future. In this chapter we will list important market characteristics. This will be a good foundation for upcoming chapters.

## 2.1. Distribution

The first characteristic that we are going to look at is the price distribution of the underlying products. In common models it is mostly assumed that prices are log-normally distributed. This means that the logarithm of the prices is normally distributed. From knowledge of the normal distribution it is known that this distribution does not capture outliers in data. The absence of fat tails displays this fact well. Also skewness and kurtosis in the data indicate non-normality. So when we want to model, this fact needs to be taken into account. There is thus a possibility that the generated scenarios may be biased and not capture the extremely high values (fat tails) as well as we would want to(Figure 1).

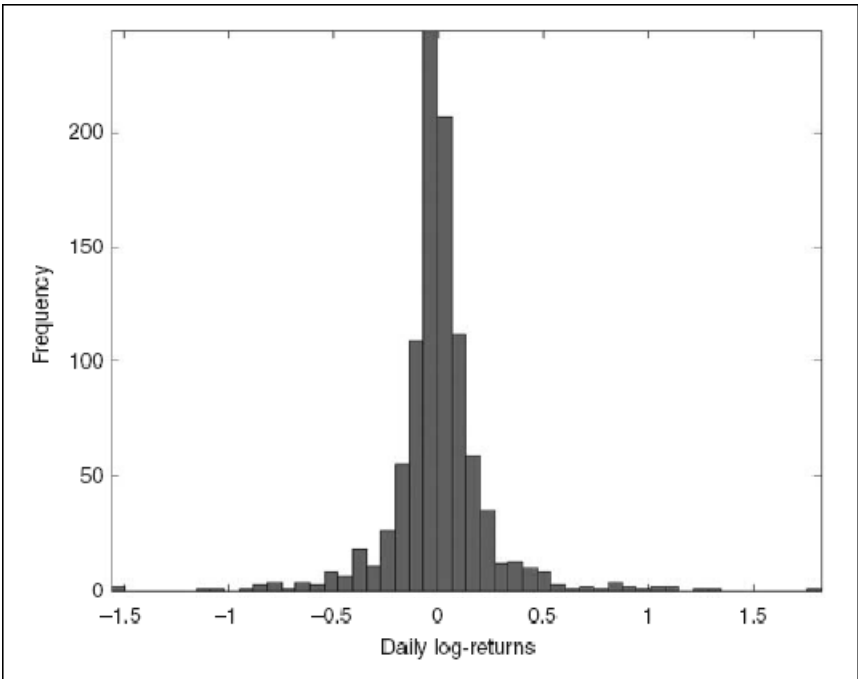


Figure 1: Histogram of daily Log-returns for PJM prices ([A1])

## 2.2. Spikes

Electricity spot prices have the tendency to jump to a new level at certain time points. When these prices return back to their normal level we call them spikes. So spikes are jumps that return back to their normal level as quickly as possible(Figure 2).

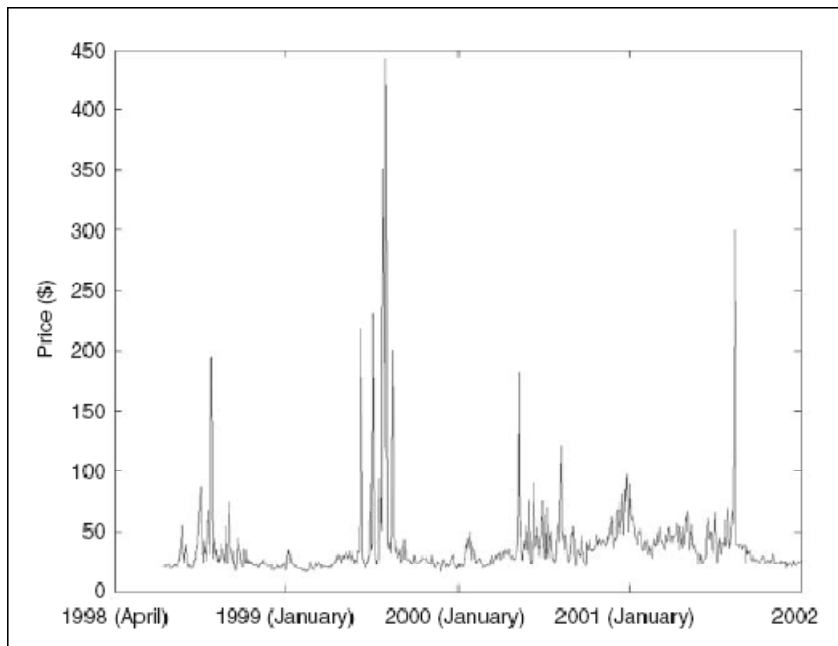


Figure 2: PJM daily power prices from April 1998 to December 2001 ([A1])

## 2.3. Mean reversion

Evidently, see the previous characteristic, the energy spot prices display mean reversion. Moving from a new level to the normal level again is just the characteristic that we are referring to. Mean reversion means that the higher the jump to a new level the bigger the probability that the price moves back to its normal level in the near future(Figure 4).

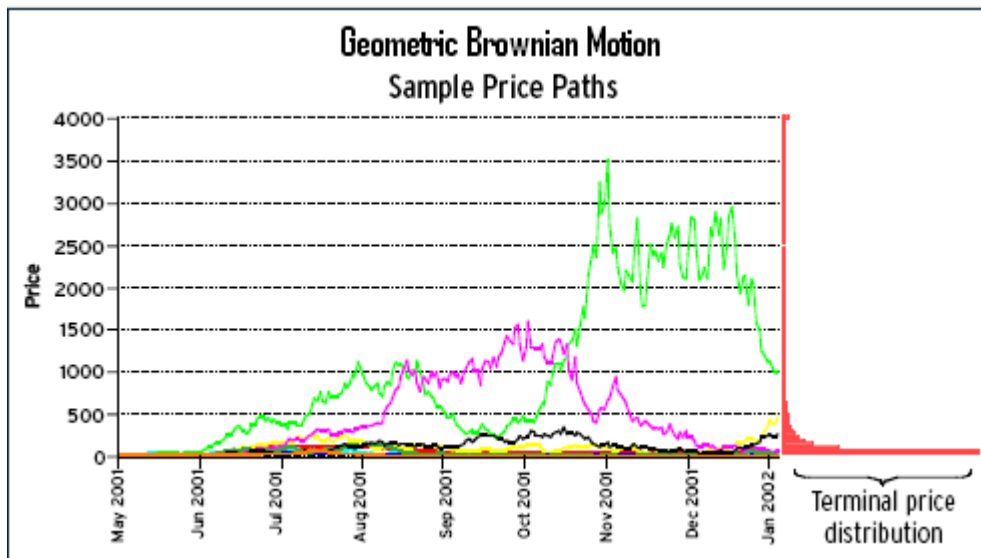


Figure 3: Geometric Brownian motion [A3]

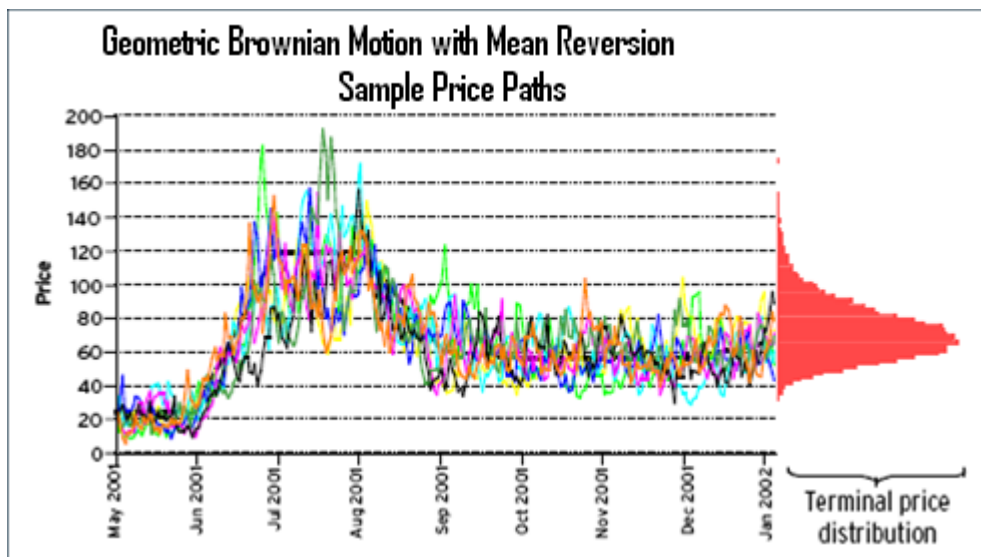


Figure 4: Geometric Brownian Motion with Mean reversion [A3]

## 2.4. Volatility

Energy spot prices display high and stochastic volatility. We can talk about historical or implied volatility. The historical volatility is a measure of how the prices of contracts, have actually been changing over a given period. And the implied volatility is implied by the option prices on a given futures contract. The historical volatility and the implied volatility can be quite different at some times but on the long run they tend to be similar. The implied volatility is the market's best prediction of how the price of the underlying commodity, the traded product, will move during the life time of the option, although it is not a stable measure. It can change multiple times intra-day. The volatility of energy products especially electricity is significantly higher than other commodities. This is due to the above characteristics which allow for discontinuities in energy prices that can take on great magnitudes (Figure 5).

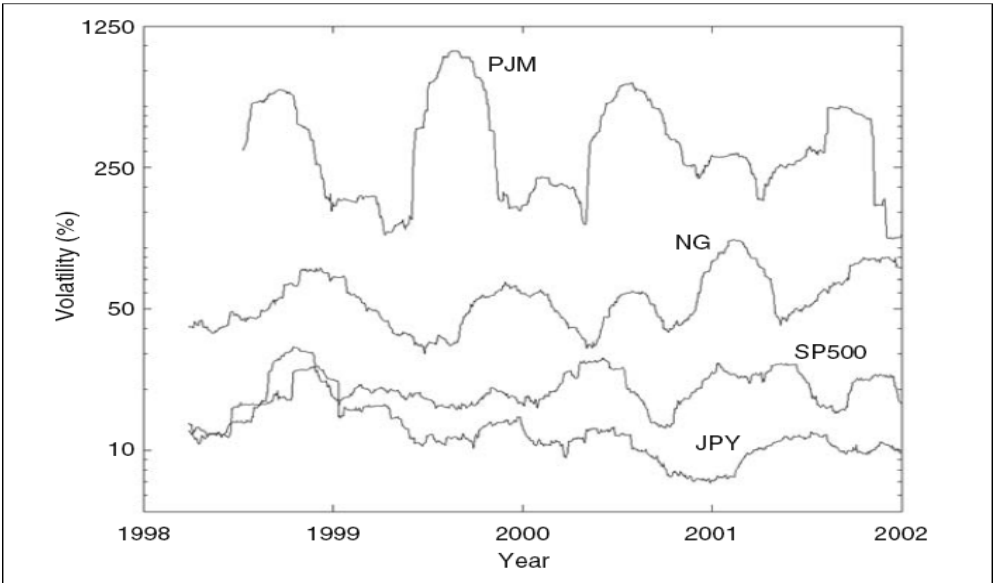


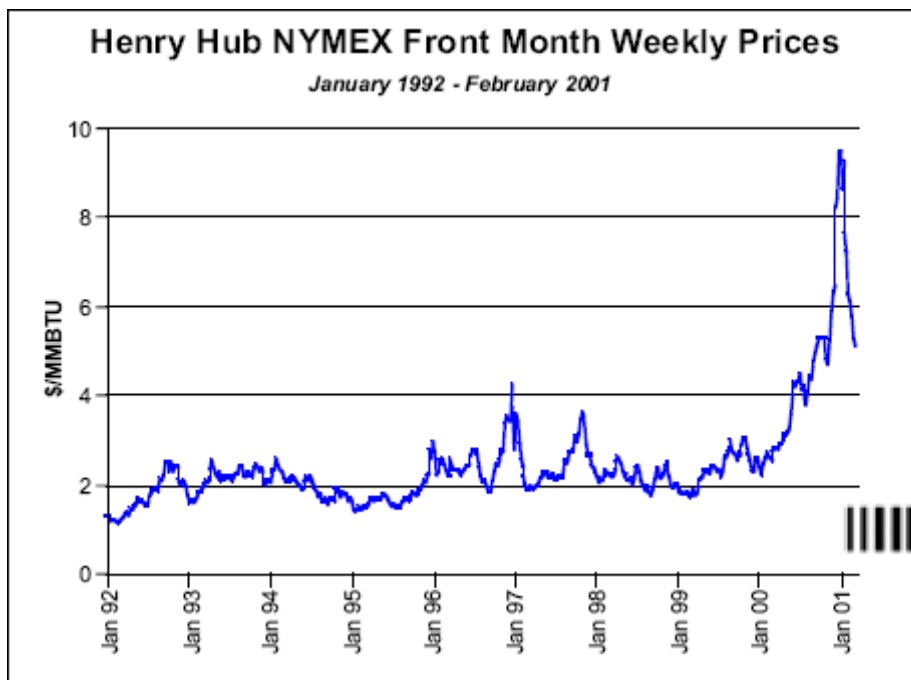
Figure 5: Annualized volatility of PJM electricity spot prices compares with SP500, NG and JPY

### Chapter 3. Modeling spot price processes

The aim of accurately modeling price processes is to lay a good foundation for derivatives pricing and risk management. As the models in this field, derivatives pricing and risk management, have a certain assumption with respect to the underlying commodity's price process it is a must to capture the underlying process as good as possible.

When choosing a model we need to understand the model's strengths and weaknesses as well as its limits of application with respect to a certain part of the market. It is great to take a very complex model that captures all the given characteristics of the commodity at hand, but you will probably agree that it is much better to take a simple and easy to interpret model and adjust it to fit the process that you are looking at. Important to note is that though the simple models are easy to interpret, they might also give a simplistic view of reality. On the other hand sophisticated models let us capture more information but are also harder to interpret as more parameters need to be estimated.

If we want to capture the processes very well no matter the process that we use, we need to take into account the information on expected prices for delivery at any given point in time and also the variability, volatility, of these prices at any point in time. Now that is just what we will try to do in this chapter. Some common and intuitively clear models will be reviewed.



*Energy prices typically display seasonal variations in volatility, occasional price spikes, and a tendency to quickly revert to the average cost of production. Stochastic Processes used to model Commodity prices should capture the specific characteristics of the commodity.*

Figure 6: Energy price processes [A2]

### 3.1. Brownian motion

The Brownian motion or the Wiener process is the most well known price process and is also called the Random Walk. There is numerous literature devoted to this process and the famous Black & Scholes derivatives pricing model is based on it. The main properties of the Random Walk are that price processes are independent of each other and the processes have a constant mean and volatility. We say that a stochastic process  $W$  is a Brownian motion if it has the following properties:

- The increment  $W_t - W_s$  is normally distributed with mean 0 and variance  $t-s$  for any  $s$  between 0 and  $t$  (zero included);
- The increment  $W_t - W_s$  is stochastically independent of  $(W_d : d < s)$  for any  $s$  between 0 and  $t$  (zero included);
- With probability one,  $t \rightarrow W_t$  is continuous;
- $W_0 = 0$ .

Whenever we are talking about modeling spot price processes this process is worth mentioning. It makes intuitively clear how price processes behave randomly and do not follow a predictable path.

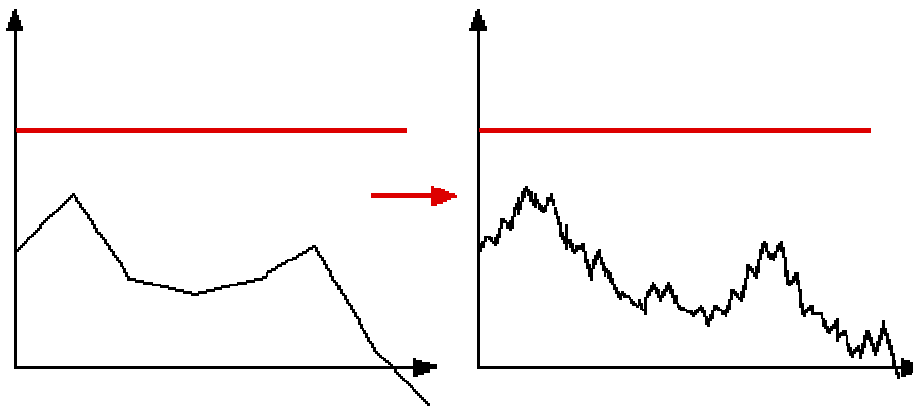


Figure 7: Random walk (left), Brownian motion (right)  
[<http://plus.maths.org/issue15/features/doubling/index.html>]

## 3.2. Geometric Brownian motion

The most frequently used process to model a price evolution as follow up to the Brownian motion is the Geometric Brownian Motion (*GBM*), Figure 3. It is as common and as well understood as the standard Brownian motion with the additional benefit of generating only positive random numbers. This is an important property in finance; prices cannot be negative. We say that a price process follows a *GBM* when the logarithms of the randomly generated random numbers follow a *Brownian motion* or *Wiener process*. The most standard form of a *GBM* is given as follows:

$$S_t = S_0 e^{(\mu - \sigma^2/2)t + \sigma W_t},$$

or by Ito's lemma

$$dS_t = \mu S_t dt + \sigma S_t dW_t,$$

where:  $S_t$  = spot price at  $t$ ;

$dS_t$  = random movement of spot price in  $[t, t+dt]$ ;

$\mu$  = drift parameter;

$\sigma$  = volatility parameter;

$dW_t$  = movement of standard Brownian motion in  $[t, t+dt]$ .

The parameters can be estimated, for instance, using empirical data, historical measures, or through option prices, implied measures. Thereafter we can determine the spot price  $S_t$  by *Ito's lemma*. So if we can determine the parameter values,  $\mu$  and  $\sigma$ , we can also determine the spot prices resulting in a model for a given series of prices (Figure 3).

### 3.2.1. Parameter estimation

To make the model complete it would now be appropriate to take a look at how we can estimate the two significant parameters in the model, namely  $\mu$  and  $\sigma$ . This will help us construct the model for ourselves and understand the model much better if we want to apply it to a given price process.

#### 3.2.1.1. Volatility

We can distinguish two types of volatility, the implied volatility and the historical volatility. In the broadest sense, the price volatility is a randomness of price changes, so the higher the degree of randomness, the higher the volatility. In the financial world the volatility is mostly associated with the standard deviation of the distribution of the logarithm of prices. The assumption is that prices are log-normally distributed or that price logarithms are normally distributed with a certain mean and volatility.

### Historical volatility lognormal case

Let us assume that  $\{P_i\}$  is a series of observed historical prices at times  $t_i$ ,  $i=0, \dots, n$  and the increments  $\log P_t - \log P_{t+1}$  are independent and normally distributed. Also the standard deviation of the normal distribution is proportional to the square root of time between observations so that it can be represented as  $\sigma\sqrt{t_i - t_{i-1}}$ . We can estimate the return volatility,  $\sigma$ , with the following sample variance  $s_x^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2$  where  $X_i = (\log P_i - \log P_{i-1}) / \sqrt{t_i - t_{i-1}}$  and thus

$$\sigma = \sqrt{\frac{1}{n-1} \sum_{i=1}^n \left( \frac{\log P_i - \log P_{i-1}}{\sqrt{t_i - t_{i-1}}} - \frac{1}{n} \sum_{i=1}^n \frac{\log P_i - \log P_{i-1}}{\sqrt{t_i - t_{i-1}}} \right)^2}.$$

Helpful to note is that time increments  $t_i - t_{i-1}$  are usually represented as year fractions. The volatility is then called the annualized price volatility. Now in this log-normal case it is assumed that the normalized increments on price logarithms have the same standard deviation,  $\sigma$ , but actually this does not always hold in cases where the standard deviation is time dependent. In this latter case we can use the common assumption that the volatility will be constant over at least a short period of time (typically 20 to 30 days). The moving window method is then used, where for any time  $t$  the estimate of the volatility depends on the number of observations that fall into this time-frame of constant volatility. Now the estimate of the volatility at any time  $t_k$  is equal to

$$\sigma(t_k) = \sqrt{\frac{1}{m-1} \sum_{i=k-m+1}^k \left( \frac{\log P_i - \log P_{i-1}}{\sqrt{t_i - t_{i-1}}} - \frac{1}{m} \sum_{i=k-m+1}^k \frac{\log P_i - \log P_{i-1}}{\sqrt{t_i - t_{i-1}}} \right)^2}.$$

Here  $m$  is the width of the moving window, e.g., 20 or 30 for the number of observations preceding time  $t_k$ .

In statistics it is common to give the accuracy of estimates when one has worked with observations. This is just to give a view on how good the estimate really is. So to give the estimates more weight, we can calculate the confidence interval of the estimate to accompany and represent with it [B1].

If we look at the historical volatility of energy products that we are discussing, we can see that it is significantly higher than that of other commodities. From the existing literature we can see that the implied volatilities display spikes of great magnitude [B1]. The volatility of power prices is even exceptionally high for energy market standards. So this means that energy products are much riskier than other commodities and one should take this into consideration when one is entering into the energy market as a trader for instance [A6].

## Implied volatility

The historical volatility is only measured from past data, making it a bit impossible to take along future price behavior. This is why another volatility measure has been developed that can be seen as forward looking and is based on option prices. This volatility measure is called, implied volatility. So in short, this is an anticipated volatility of prices in the future such that option prices can match the market quotes. So if under the normality condition an option price is given as  $V_{\text{option}} = f(F, X, t, T, r, \sigma)$ , then the implied volatility is the volatility that causes  $V_{\text{option}(\text{market})} = f(F, X, t, T, r, \sigma_{\text{implied}})$ . This means that the option value is a function of the forward price  $F$ , strike  $X$ , time  $t$  at which the option is evaluated, exercise time  $T$ , interest rate  $r$ , and of the volatility  $\sigma$ . To find the implied volatility, standard techniques such as the Newton method, a root finding technique, can be used. So now  $\sigma_{\text{implied}} = \sigma(F, X, t, T, r, V_{\text{option}(\text{market})})$ . In the Black Scholes world of option pricing, the implied volatility is constant,  $\sigma_{\text{implied}(\text{BlackScholes})}(t, T, X) = \text{constant}$ . In the energy market this constant volatility assumption does not hold for option pricing.

The Newton method is proposed because at some point the inverse of the function will have to be computed and in general the inverse of the pricing formula does not have a closed-form formula solution. Therefore root finding techniques have to be applied and Newton's method is then worth considering.

## Convenience yield

The spot prices and forward prices have a certain relationship that is derived through consideration of exercising strategies of forward contracts. In the financial world this relationship is as follows

$$F_{t,T} = (S_t)e^{(r-q)(T-t)},$$

where  $r$  is the risk free rate and  $q$  is the dividend yield or foreign interest rate in case of equity products and currencies, respectively. By no-arbitrage arguments, the price of a forward contract was developed where the crucial assumption was to borrow a commodity without cost.

Now for gas and oil this crucial assumption is not very realistic. There is a cost for storing them that is denoted as  $U$ , and can make a big difference. Also the owners of the commodity may require a premium for selling the commodity when they anticipate shortages and might also be reluctant to sell in that case. So in this case the relationship is only proven to be an inequality

$$F_{t,T} \leq (S_t + U)e^{r(T-t)}.$$

So one would exercise the contract and enter into different hedging constructions, buying back commodities, selling them, or borrowing money, the above is proven and the following relationship is derived [B1]:

$$Y \geq (S_t + U)e^{r(T-t)} - F_{t,T} > 0.$$

The most that can be proven is that the relationship is an inequality as can be seen. This inequality is represented as equality by introducing the convenience yield,  $y$ , as the measurement of the benefits of physical ownership of the commodity. The relationship is then given by:

$$F_{t,T} = (S_t + U)e^{(r-y)(T-t)}.$$

The storage costs are proportional to time and by substituting  $U = S_t e^{u(T-t)} - S_t$ , the following most commonly used representation is derived

$$F_{t,T} = S_t e^{(r+u-y)(T-t)}.$$

This relationship is attractive when  $u$  and  $y$  are constants, and if that is the case it is the same representation as in the financial world.

### *Drift*

The drift parameter is the risk free rate of return and in case of gas and oil adjusted for storage and convenience yield. In some cases it is not possible to fully hedge away risk associated with derivative trading. Therefore the use of a risk free rate does not always hold up. In these cases, the need for introducing another drift measure arises. One adjustment is to introduce a price for the risk that cannot be hedged away and adjust the risk free rate by a compensation for taking the risk. So instead of just using a measure such as  $\mu_p$  in the risk-neutral world, the adjusted return could be

$$\mu_p - \sum \lambda_i s_i,$$

where  $\lambda_i$  is the price of risk that is defined by a random factor with volatility  $s_i$ . Now the main challenge is to separate the hedgeable from the unhedgeable risk factors. The modification of the drift parameter can be done by finding the appropriate convenience yield or by using the forward curve. For more info on these approaches see [B1].

### *Modifications of the Geometric Brownian Motion*

We have seen the standard GBM and have given some ideas on estimating the parameters and some challenges that go along with that task. Now in relation to the energy market, this model seems to have some setbacks due to the fact that it does not capture the special characteristics concerning some energy commodities.

Various modifications of this process have been proposed with the goal of a more realistic representation of energy prices; the majority emphasizes two essential attributes of spot prices in energy markets, namely, mean reversion and seasonality. Mean reversion means that the prices are not completely independent of each other as assumed in the Random Walk. The Random Walk assumes that historical price paths are not relevant for future predictions and thus prices are independent of each other.

The most common models that consider mean reversion in the GBM mainframe are the following two:

- 1. Mean reversion to the long term price;**
- 2. Mean reversion to the long term price logarithm.**

**Ad 1.** In this modification of the GBM it is assumed that the spot prices on average revert to the long term mean and the process is modeled as follows:

$$\frac{dS_t}{S_t} = \kappa(S_\infty - S_t)dt + \sigma dW_t,$$

where  $S_\infty$  is the long term mean of spot prices and  $\kappa$  is the magnitude of mean reversion. All other variables have the same meaning as with the standard GBM. The left hand side of the equation is the exact spot price change.

**Ad 2.** Now the spot prices on average revert to the long term mean of logarithms of spot prices:

$$\frac{dS_t}{S_t} = \kappa(\theta - \log S_t)dt + \sigma dW_t,$$

where  $\theta - \frac{1}{2\kappa}\sigma^2$  is the long term mean of logarithms of spot prices and  $\theta$  is the long-term price logarithm.

We can rewrite this model by Ito's Lemma and substitute  $Z_t = \log S_t$  as

$$dZ_t = \kappa\left(\theta - \frac{1}{2\kappa}\sigma^2 - Z_t\right)dt + \sigma dW_t.$$

Finding the parameters can be done by use of historical data.

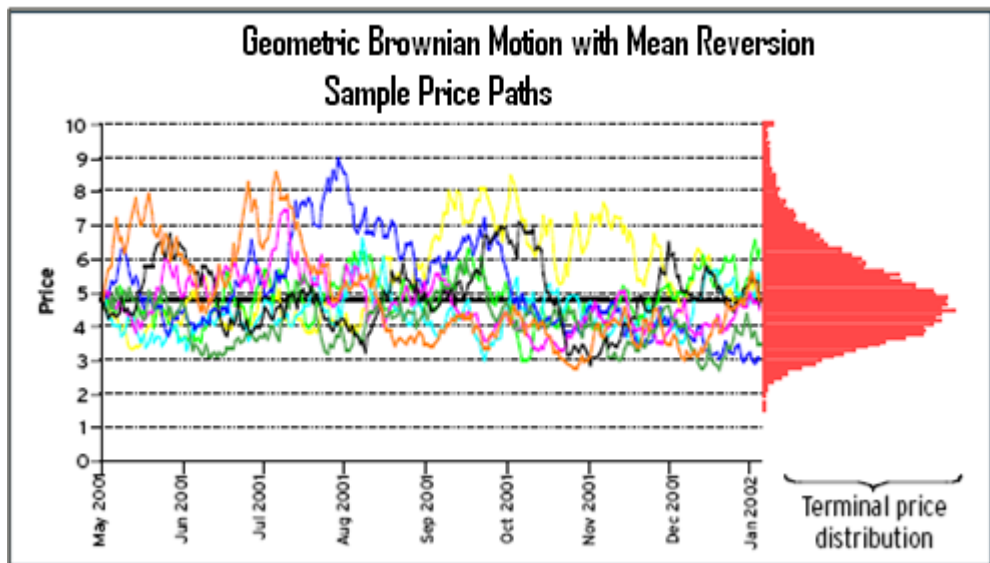


Figure 8: Geometric Brownian motion with mean reversion [A3]

### 3.3. Jump Diffusion Processes

Jump diffusion models have the characteristic of adding random jumps to the *GBM*. The assumption is that these jumps follow a certain probability law. These models have been appearing more and more in favor of the *GBM* as price spikes are common in energy markets and thus have to be well modeled. This then leads to Jump diffusion processes. These processes also capture the fat tails of the price distribution. A typical jump diffusion process is a combination of the diffusion process such as the standard *GBM* and a jump process. For the discontinuous jumps the Poisson process is most frequently used. This means that in a given interval of a certain length, for instance  $dt$ , either no jumps occur or a jump of magnitude one occurs with a certain probability that is proportional to the length of the interval. It is unlikely for two or more jumps to occur in a small interval.

Now by definition all jumps in the Poisson process are of magnitude one. In pricing models, we assume jumps are of random magnitude. The most frequently used jump diffusion process [B1] is the following:

$$\frac{dS_t}{S_t} = (\mu - \lambda k)dt + \sigma dW_t + (Y_t - 1)dq_t,$$

where:  $S_t$  = spot price at  $t$ ;

$W_t$  = standard Brownian motion;

$\mu$  = expected instantaneous rate of relative changes in the spot prices;

$\sigma$  = volatility of relative changes of spot prices on any interval not containing jumps;

$q_t$  = the Poisson process;

$\lambda$  = the intensity of the Poisson process;

$Y_t - 1$  = a random variable presenting the magnitude of jumps in price returns,  $Y_t \geq 0$ ;

$k$  = the expected jump magnitude,  $E(Y_t - 1)$ .

### 3.3.1. Jump diffusion processes with mean reversion

We still have not modeled the important characteristic of electricity prices, spikes, with the jump diffusion process. These processes are well capable of modeling sudden jumps in price processes but they assume that the process will stay at a certain level until another jump occurs. We can imagine that price processes do not behave like that for all energy products, namely electricity. As the processes of these commodities display discontinuous behavior and move up to a certain level they quickly tend to return to their normal level, thus not portraying jumps but spikes. This calls for a jump diffusion process that considers mean reversion. As in the GBM case we now add a mean reverting component to the standard process and assume that the prices move to the long-term mean of logarithm of spot prices. So the process becomes

$$\frac{dS_t}{S_t} = \kappa(\theta - \lambda k - \log S_t)dt + \sigma dW_t + (Y_t - 1)dq_t.$$

The parameters  $\kappa$  and  $\theta$  have the same meaning as in the GBM modification for mean reversion. All other parameters have the same meaning as mentioned in the common jump diffusion process. As we can see in the model the number of parameters has increased to six. This is probably not in favor of the model's popularity. So even though the model is realistic, and captures the spiky characteristic of electricity prices, one might still not use such a model for modeling purposes.

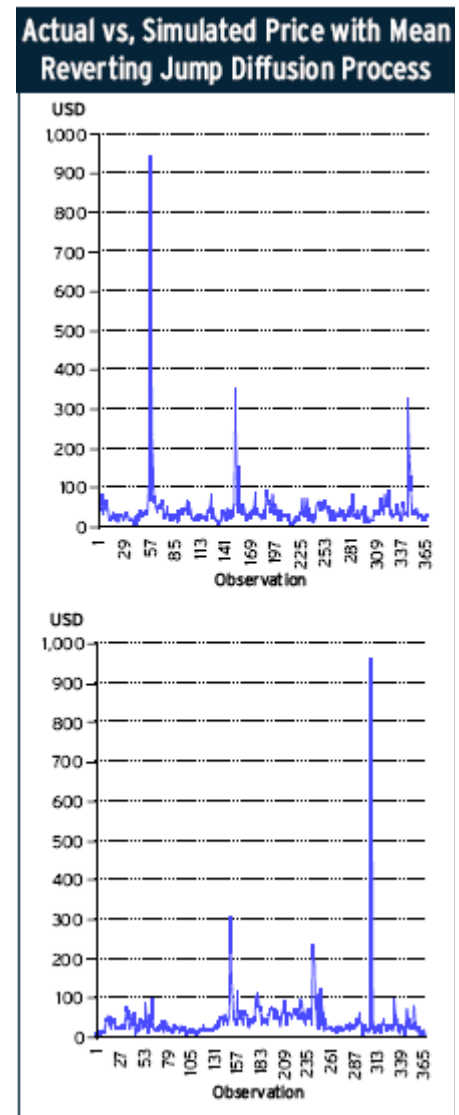


Figure 9: Jump diffusion process with mean reversion [A4]

### 3.3.2. Completeness of the market

A market is said to be complete when all risks can be hedged away. So if for each random shock affecting the price process, a product exists that can hedge this uncertainty away, the market is complete. So with jump-diffusion processes it is much harder to say that the market is still complete due to the random shocks whereas this is not the case with the GBM. So some approaches have to be considered to get the market complete or incompleteness has to be accepted. In the case of incompleteness we cannot use a risk-free rate for derivative pricing.

The risk-adjusted process for Jump diffusion processes is equal to the following:

$$\frac{dS_t}{S_t} = (\mu - \gamma - \tau\kappa)dt + \sigma dW_t + (Y_t - 1)dq_t,$$

where,  $\gamma$  is the price of risk. Now we can use the mean-variance strategy to hedge [B3].

Another jump diffusion process that can be considered with much fatter tails than the process with log-normally distributed jump-sizes is the one introduced by Kou and Wang, where  $\log(Y_t)$  has the following distribution

$$f(y) = \begin{cases} p\eta_1 e^{\eta_1 y}, & y \geq 0, \\ (1-p)\eta_2 e^{\eta_2 y}, & y < 0, \end{cases}$$

where  $p, \eta_1, \eta_2$  are positive constants.

Jump diffusion processes have the benefit of capturing the fat tails of energy prices and the ability to model spikes. They are gaining popularity in the world of energy derivatives as an important modeling tool. The disadvantage is that it adds another parameter to the list of parameters that needs to be estimated. There is not enough data to estimate this number of parameters and the parameters seem to change significantly when more data is added. So although the process has benefits to uphold it, it is still not a very good and favorable model.

### 3.4. Regime switching models

Let us assume we have different states that prices can take on and we want to be able to model these changes. We could say that the price takes on different regimes in a given period and at certain time spans in that period. Now to model these regime switches the literature introduces regime switching models. Here some different states are specified among which the process moves. So we can say that with a certain probability prices will move to a certain regime and back to their normal state or the other way around. The probabilities do not have to be the same when going back and forth between given regimes, and in fact they are not.

The way to model this mathematically is by using Markov processes. We introduce a transition matrix with the probabilities of going from one state to another dependent on the number of states that can be reached. Let us assume we can take on 2 different states and split the price process in two geometric Brownian motions with different return volatilities, drifts, and starting points. We will particularly make use of a continuous time Markov process and characterize the model with the probability rates per unit of time. In the case of constant rates we are dealing with a Poisson-distributed Markov chain.

The process is given by:

$$\frac{dS}{S} = \begin{cases} \mu_L dt + \sigma_L dW_L \rightarrow P_L = 1 - \lambda_{LU} dt \\ \mu_U dt + \sigma_U dW_U \rightarrow P_U = 1 - \lambda_{UL} dt \end{cases}$$

And the probability transition matrix is given by:

$$P = \begin{pmatrix} 1 - \lambda_{LU} dt & \lambda_{LU} dt \\ \lambda_{UL} dt & 1 - \lambda_{UL} dt \end{pmatrix}$$

On the diagonal we see the probabilities of remaining in a given state and the off-diagonals give the probability of switching states.

We can extend this model for more transition states and make the matrix as big as we want but within reasonable boundaries. A big matrix will not be efficient and will probably give interpretation difficulties in practice

In the papers [A8] and [A11] more regime switching models are proposed and also applied to real data. Paper [A11] of Ronald Mahieu and Ronald Huisman proposes transition matrices with 3 states. They assume the three states are defined as follows: a normal regime (0), where prices just behave normally, an initial jump regime where the price either increases or decreases (+1), and a reversion regime where the price goes back to its normal level (-1).

So they assume that after a normal price progress, a jump occurs and is immediately followed by reversion to the normal level. Now we need to find the transition probabilities and supply the probability transition matrix. So the authors give the following explanation: Assume  $\pi(i, j)$  is the probability of moving from state  $i$  to  $j$  in a given time interval  $[t, t+1]$ . A reachable state is  $\pi(0,0)$ , the probability of staying in the normal situation without jumps and reversions. The probability that a jump occurs is 1 - the probability of no jump occurring, so  $\pi(0, +1) = 1 - \pi(0,0)$ . Of course we cannot move from a normal situation to a reversion state immediately and therefore this probability is zero,  $\pi(0, -1) = 0$ . Important to note is that the time interval is equal to 1 day, meaning that if  $t$ =today, then  $t+1$  is tomorrow. So far all probabilities are intuitively clear. Now we will take into consideration the one day time interval. If we are in a jump state today, we assume the reversion will certainly take place tomorrow and then  $\pi(+1, -1) = 1$ , and if we are in a reversion state today, the jump state cannot be reached directly and also from a jump state we cannot go directly to the normal state so  $\pi(+1, +1) = \pi(+1,0) = 0$ . Further, if we are in the state of reversion today, the normal state will be reached the next day with probability 1, and the other states cannot be reached and are zero,  $\pi(-1,0) = 1$ ,  $\pi(-1, +1) = \pi(-1, -1) = 0$ .

The authors also propose different values for the time interval. Now the only thing missing is the probability of staying in the normal state  $\pi(0,0)$  and that is defined as  $\pi(0,0) = \frac{e^p}{1+e^p}$  to ensure that the values stay between 0 and 1.

In contrast to [A11], paper [A8] by Michael Bierbrauer, Stefan Trück and Rafal Weron proposes a different price process for the jump reversion and normal part. Where [A11] proposes mean

reversion to model these price parts and a GBM for the initial jump, [A8] suggests a Gaussian or log-normal random variable in these cases. They do not specify the direction of the initial jump but do restrict the model by saying that the reversion is always opposite to the initial jump on average.

# Chapter 4. Energy derivatives

The financial world knows some commonly used risk management tools. Such tools are forward contracts, future contracts, swaps and options. They are standardized and well understood products and thus easy to use. In the energy market these products are also used but with underlying products in the energy market. We can have an option on natural gas or on electricity as well as futures contracts and forward contracts on these types of commodities. In addition, the energy market also knows some market specific contracts such as spread options and volumetric options such as swings, recalls and nominations. In this chapter we will take a brief look at some examples of common energy derivatives as well as the additional contracts, the very important spread option, and the market specific volumetric option, called swing.

## 4.1. Basic products & Hedge examples

The forward and futures contract are the easiest known derivatives, easy to implement and use as are the plain vanilla options. For more information on these derivatives see the books [B1] and [B3]. Other very popular derivatives are the swaps which will be mentioned and discussed in this section.

Physical			Financial		
Standard	Exotic		Assets	Standard	Exotic
<i>Futures</i>	<b>Price-based</b> American options	<b>Volumetric</b> <i>Swing options (recall, nominational etc.)</i>	Storage	<i>Futures</i>	American options
<i>Forwards</i>	Asian options		Power plants	<i>Forwards</i>	Asian options
<i>Swaps</i>	Swaptions	Load following	Transmission	<i>Swaps</i>	Swaptions
<i>European options</i>	<i>Spread Options</i> Tolling, etc.			<i>European options</i>	<i>Spread options, etc.</i>

Table 1. Energy Derivatives ([B1])

### 4.1.1. Swaps

Swaps in the energy market are just the same as swaps in the financial market. They are an exchange between two parties of different wanted, by both parties, financial “goods”. In the financial world this is an interest rate exchange, floating for fixed and vice versa. Their popularity mainly stems from the following three reasons:

1. Flexibility, OTC, easily customizable transactions;
2. Typically financially settled, no physical delivery, off balance sheet and non-regulated;
3. Uniquely suited for hedging applications.

The most frequently encountered swap is the fixed price swap: The counterparty pays a fixed amount for a time period to the other party while he receives a floating amount for that same period.

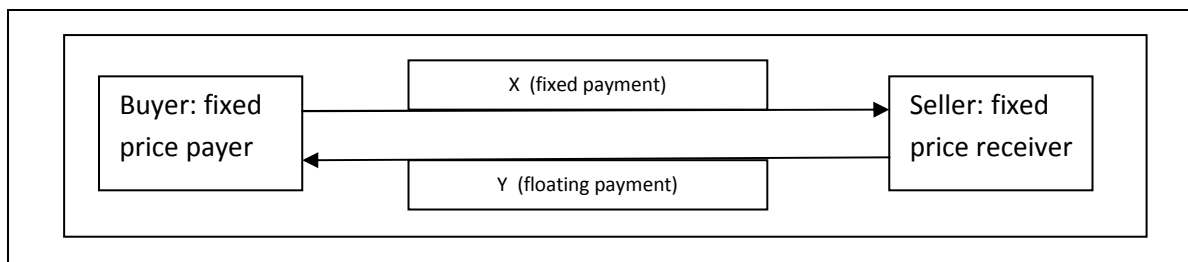


Table 2: Fixed-price Swap

The fixed price can change or stay the same for the whole period while the floating price is linked to a spot price index.

Let us take a look at an example of how this derivative can be used for hedging purposes. This example is taken from [B1].

Assume corporation ABC monthly pays a certain spot index for each MMBtu of its delivered natural gas. Now at some point ABC finds that the volatility of this index is too high and wants to eliminate the price risk from this contract. So ABC is looking for another company that would like to take on this volatility and enter into a financial swap with this particular company. Let us say that company XYZ is willing to take on this risk and enters a swap contract with company ABC. Of course XYZ should then want to receive a fixed payment from ABC in order for this swap to work, which XYZ also wants. So now ABC is not exposed to the floating rate risk which is now carried by XYZ. In this case, XYZ probably can better carry this risk for him to be willing to enter this swap settlement.

The situation first looked something like this:

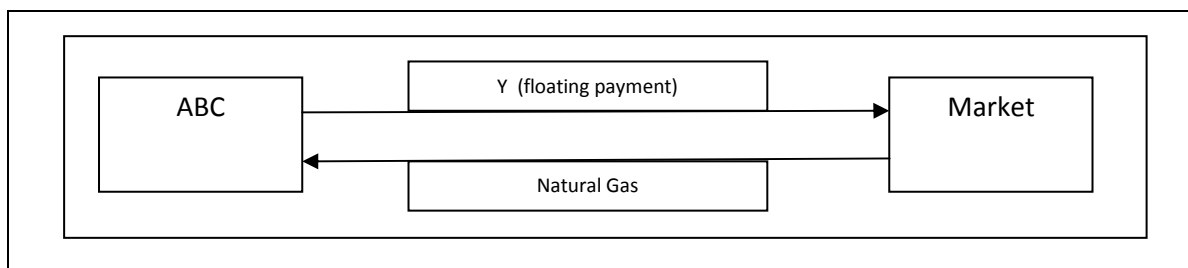


Table 3: No hedge yet

Then by entering the swap with company XYZ, the situation changed to the following:

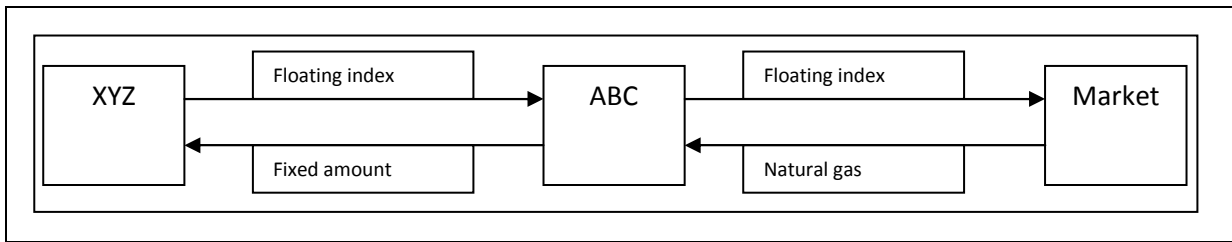


Table 4: ABC's hedge

Now we can value this swap as a strip of monthly fixed price forward contracts and the price will look as follows:

$$PV_{swap}(t) = \sum_i N_i [f_i \cdot DF(t, T_{i,pay}^{float}) - X \cdot DF(t, T_{i,pay}^{fixed})],$$

where

$N_i$  is the volume;

$f_i = F_{i,t}$  is the forward price at the valuation time  $t$  of the commodity for delivery in the  $i$ -th month;

$DF(t, T_{i,pay}^{float}) = e^{-r_i(t, T_i)(T_i - t)}$  is the discount factor in period  $(t, T_{i,pay}^{float})$ , the time of payment for the commodity delivered during the  $i$ -th month;

$DF(t, T_{i,pay}^{fixed})$  is the discount factor in period  $(t, T_{i,pay}^{fixed})$ ;

$PV_{swap}(t)$  is the present value of the swap at time  $t$ .

To make this a zero sum game the fixed payment needs to have a fair value which can be calculated as follows:

$$SwapPrice(t) = \frac{\sum_i N_i \cdot f_i \cdot DF(t, T_{i,pay}^{float})}{\sum_i N_i \cdot DF(t, T_{i,pay}^{fixed})}.$$

In this case of energy swaps the frequency of both payments is the same, as is the assumption in this formula. In the financial world this assumption does not always hold and the floating and fixed parts have to be specified explicitly.

Month	Volume of MMBtu	Cash payment dates	Discount factors	Forward prices/MMBtu
Nov	600000	20-dec	0,96	1,1
Dec	620000	20-jan	0,97	1,2
Jan	620000	20-feb	0,98	1,3
Feb	560000	20-mrt	0,99	2,1
Mar	620000	20-apr	0,95	2

Table 5: Market & Contractual Conditions

Now let us look at an example where the fair value needs to be calculated. The demand for gas in the winter is 20000 MMBtu/day. A company wants to secure the baseload supply of gas for this period at the beginning of the summer and wants to pay a fixed price. The information needed is given in Table 5. So the fair price can be calculated with the given formula and is equal to

$$\begin{aligned}
 & \text{SwapPrice} \\
 &= \frac{600000 * 0,96 * 1,1 + 620000 * 1,2 * 0,97 + 620000 * 1,3 * 0,98 + 560000 * 2,1 * 0,99 + 620000 * 2 * 0,95}{600000 * 0,96 + 620000 * 0,97 + 620000 * 0,98 + 560000 * 0,99 + 620000 * 2 * 0,95} \\
 &= \frac{633600 + 721680 + 789880 + 1164240 + 1178000}{576000 + 601400 + 607600 + 554400 + 589000} = \frac{4487400}{2928400} = 1,53.
 \end{aligned}$$

So on June 1<sup>st</sup> the company should agree to a price of 1,53/MMBtu.

This is just a simple example of working with swaps. This market actually knows many more swap variants and for more information on these types of swaps one can take a look at Chapter 8 of [B1].

#### 4.1.2. Swaptions

One can imagine that there can probably also be options on swaps, these are called swaptions. They are calls or puts on a swap contract giving the owner the right, but not the obligation, to enter into a swap contract. This class of options belongs to a class of derivative products called compound options ([B1], Chapter 8). Their valuation is quite challenging and mostly calls for numerical methods to estimate the value.

In the financial world, swaptions are options to enter into a swap on an interest rate in the future. To ensure that future interest rate payments will not exceed a certain level ([B3]).

For a single period these options boil down to plain vanilla calls and puts, spread options (see next section) or compound options.

For multi-period swaps the payoff is given as follows

$$\text{Swaption} = \max\left(\frac{\sum_i df_i \text{Volume}_i F_i}{\sum_i df_i \text{Volume}_i} - K, 0\right).$$

## 4.2. Spread options

As mentioned before, spread options are very important to the energy market in contrast to the financial markets. We will take a look at spread options in the Black Scholes world and thereafter in a world without Black Scholes. Although we can argue that the Black Scholes model, based on the Brownian motion is not a very good model in the energy market we can still value spread options under this assumption to make the idea intuitively clear. It is also proven that the behavior, with respect to quality, of the spread options and their risk measures is robust given the price distribution.

A spread has the following payoff,

$$\text{SpreadOption} = \max(F_1 - F_2 - K, 0).$$

First, we calculate the difference between the two asset prices and the strike price and then we take the maximum of this difference and zero.

Let us take as example that the forward price of electricity and natural gas are X and Y dollars/MWh, respectively. And the price of natural gas is already taken times the heat rate. Then the payoff of the call option on the spread between these two assets is equal to

$$SpreadCall = \max(X - Y - K, 0).$$

And the payoff of the put option is equal to

$$SpreadPut = \max(K - X - Y, 0).$$

Now this is intuitively clear that it works analogously as the put and call options in the financial world. So let us now look at the value of such a European call on a spread in the Black Scholes world, where the Brownian motion is the process that is being used to model the underlying price process. This model is also called the Margrabe formula.

#### 4.2.1. Margrabe formula

The value of a call option with zero as strike price according to the Margrabe formula is given by:

$$Call_s = e^{-rt}(X\Phi(d1) - Y\Phi(d2)),$$

where

$$d1 = \frac{\ln\left(\frac{X}{Y}\right) + \frac{\sigma^2 t}{2}}{\sigma\sqrt{t}},$$

and

$$d2 = d1 - \sigma\sqrt{t}.$$

Also

$$\sigma = \sqrt{(\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2)},$$

where

$$\rho = \text{the correlation coefficient}$$

So now apart from the volatility of this formula, this looks almost like the Black Scholes formula but instead of the strike price, the forward price of the asset is inserted. This is a closed-form formula and can thus easily be calculated.

Now, we can look at what happens when we assume a non-zero strike price under the Black Scholes assumption. For this valuation no closed-form formula exists yet, but one approach is to adjust the Margrabe formula and replace the price for asset 2, Y, by Y+K and proceed as in the Margrabe formula and also adjust the volatility by multiplying it with the factor X/(Y+K). It is known that this approximation performs quite well (see [B1]).

### 4.2.2. Adjusted Margrabe formula

The value of a call option with a non-zero strike price is according to the Margrabe formula:

$$Call_s = e^{-rt}(X\Phi(d1) - (Y + K)\Phi(d2)),$$

where

$$d1 = \frac{\ln\left(\frac{X}{Y + K}\right) + \frac{\sigma^2 t}{2}}{\sigma\sqrt{t}},$$

and

$$d2 = d1 - \sigma\sqrt{t}.$$

Also

$$\sigma = \sqrt{(\sigma_1^2 + (\sigma_2 [X/(Y + K)])^2 - 2\rho\sigma_1\sigma_2 [X/(Y + K)])}.$$

### 4.2.3. The Bachelier model

The limitation of the Margrabe model is that it only allows positive numbers as it uses the GBM which leads to the log-normal distribution. As spreads can also take on negative numbers this assumption does not really apply to the spreads themselves. So now the arithmetic Brownian motion was proposed as underlying price process to ensure positive as well as negative random numbers. This model is called the Bachelier model and causes simple closed-form formulae to calculate the option price through Gaussian integrals. The difference boils down to the specification of the volatility. The model allows time dependent volatility  $\sigma_t$  instead of  $\sigma$ . See paper [A11] and references therein for more information on this model and comparison with other models.

### 4.2.4. Kirk's formula

Another proposed model is Kirk's formula which is given as follows:

$$p^K = X\Phi\left(\frac{\ln\left(\frac{Y}{X + Ke^{-rT}}\right) + \frac{\sigma^K}{2}}{\sigma^K}\right) - (X + Ke^{-rT})\Phi\left(\frac{\ln\left(\frac{Y}{X + Ke^{-rT}}\right) - \frac{\sigma^K}{2}}{\sigma^K}\right),$$

where

$$\sigma^K = \sqrt{\left(\sigma_2^2 - 2\rho\sigma_1\sigma_2 \frac{X}{X + Ke^{-rT}} + \sigma_1^2 \left(\frac{X}{Y + Ke^{-rT}}\right)^2\right)}.$$

For more information on this model see paper [A11] and references therein.

### 4.3. Swings, Recalls, and Nominations

Swings, Recalls, and Nominations are volumetric options that give the holder the right, but not the obligation, to adjust the volume of the order at any time before the delivery date. One can imagine why these options are very important in the energy market. If not, this is the reason: The energy market has limited storage capacity which means that it is hard to store energy, especially electricity, and traders would only profit from such options which make it possible for them to adjust their volumes as the market fluctuates. For instance, the natural gas market has great need for these options as well, as supply and demand can fluctuate daily.

#### 4.3.1 Swings

The swing option consists of two parts: a baseload and a swing. The baseload specifies the nominated amount that has to be delivered for a given period, whereas the swing allows for adjustments in this amount. There are different types of swing options but they all have some common characteristics [A9]. There is a certain period  $[T1, T2]$  in which the option can be exercised up to  $N$  times (mostly  $N$ ), if  $0$  is the time point at which the contract is written. So the period  $[0, T2]$  is the baseload period and within that period the swing allows for  $N$  adjustment rights. A right can be exercised only at discrete dates between  $T1$  and  $T2$  with at most 1 right exercised on a certain date.

When the number of swing rights is equal to the number of days in the period, the value of the swing option is given by the value of a strip of daily European call options with the nomination price as the strike price. Now if the number of swing rights is less than the number of days, the valuation is more challenging and pricing the swing options has the same difficulties as pricing American options. The hard task is finding the optimal exercise boundary which is the critical price that separates the continuation region and exercise region ([B1]). So if  $S > \text{boundary}(t)$  we should exercise and otherwise wait. This is also the case with swing options as time gradually progresses the outlook on the option changes. As long as the option is in the money, we would want to exercise depending on the outstanding option rights. If the expiration date is day 30, and we are at day 25 with all rights still intact, we should probably start exercising some rights, hence the time dependence. So the critical surface we are looking for is  $S=S(t,K)$  where  $K$  is the number of remaining option rights.

The algorithm for valuation of swing options involves building a two-dimensional tree. One dimension is the standard tree, while the other is the discrete tree of the number of available swing rights.

This can be described in the following steps which are a direct implementation of the backward induction method of solving the relevant stochastic dynamic program ([B1] and [B9]):

1. Start at last time node  $N$  in all the trees. The payoffs of the swing options are evaluated in all trees;
2. Move one step back to time node  $N-1$ . In every tree evaluate two expectations of the value of the option at expiration for the feasible pairs of swing rights. So assume we have  $k$  swing rights left in the node that we are looking at. Calculate:  $E[V(S_N, k) | S_{N-1}, k]$  and  $E[V(S_N, k-1) | S_{N-1}, k]$ ;

3. Compare the sum of the second expectation and the current exercise value against the first expectation. Compare the strategy of exercising at N-1 with the k-1 available exercises with the strategy of not exercising at N-1. So the value is given by the maximum of these two values;
4. Go back until t=0 is reached.

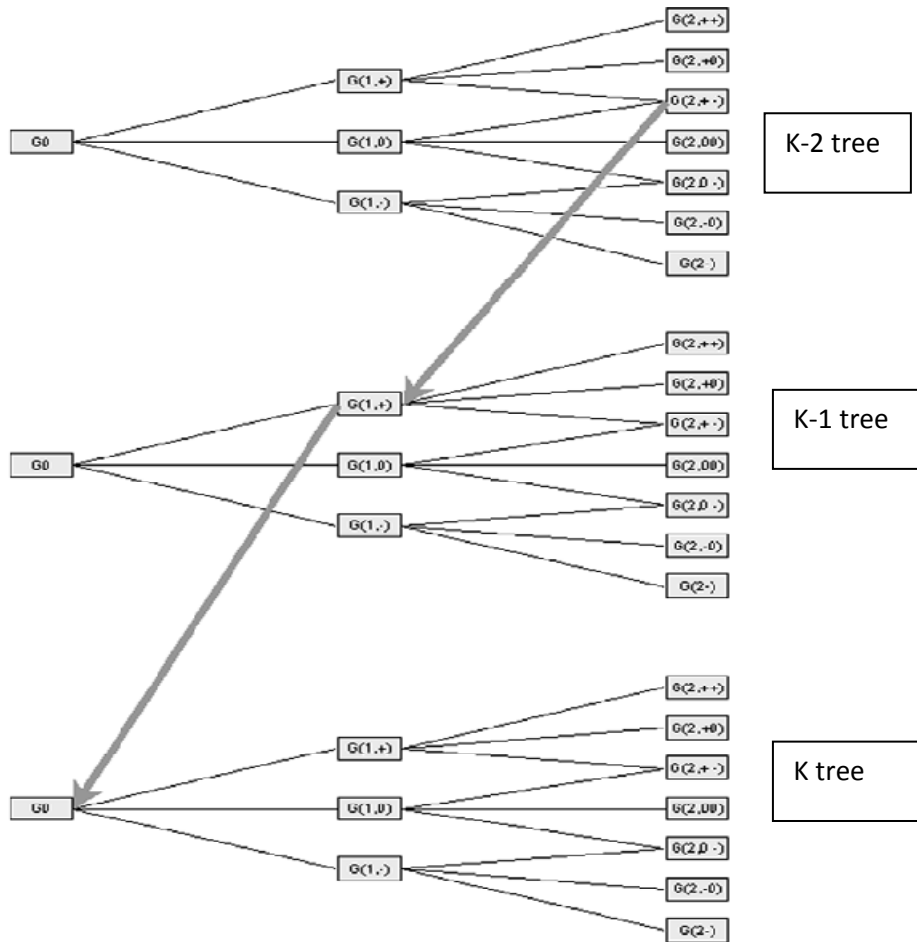


Figure 5: Backward induction method [B1]

### 4.3.2 Recalls

Recalls are just swing options but they are used differently. They are used to interrupt delivery under stressful circumstances, while swings are used to manage demand. The number of swing rights is always less than N periods. Where the customer can use the swing option to adjust the volumes, a supplier on the other hand has the right not to deliver, or recall, the nominated volume a couple of times during the period. The recall provision gives the supplier a way out of delivering due to extreme conditions such as earthquakes or pipeline failures. These rights are still very small because these unforeseen situations do not occur that often and under exceptional circumstances ([B1]).

### 4.3.3 Nominations

Nomination options are just like swings and offer the holder the right to adjust the volume received  $K$  ( $<N$ ) times during the contract period. The level of the volume though, is adjusted upwards or downwards until the next nomination is exercised. Let us take a look at an example to make this clear.

Assume the original nomination is specified for 5000 MMBtu of natural gas every day. Now if the holder doubles the volume to 10000 MMBtu, he will receive this amount for the remaining time of the option every day unless this amount is changed again at a following nomination right.

There is a limited number of nominations to be exercised. The timing of these exercises is usually prespecified as a contract that may give the right to nominate the volume at the beginning of each month up to the option expiration month. There might even not be a limit to the minimum and maximum amount to be nominated ([B1]).



## Conclusion

We can conclude that the energy market is a very dynamic market. Dynamic in the sense that it changes constantly which can make it difficult, and in practice it actually is, to model the price processes correctly. The price processes that we need to model can be very complex and so can the models. The modeling process is also very important as it flows through the pricing mechanism of derivatives in this market. The prices of the different derivatives indirectly derive their values from the underlying price processes. Also a good understanding of the market itself and the traded products and their characteristics is an important part in the modeling process. As this market keeps on changing, more models are being developed as well as others are improved to better fit the data. All in all, the energy market is a very complex market to deal with. A lot of features have to be taken into consideration when one wants to operate in this market; either to trade or model price processes.

The goal of this paper has been to state some simple interesting market characteristics that have to be taken into consideration when trying to model price processes. Also to take some intuitively clear models and see how they can be adjusted to fit energy data. Lastly, some market specific derivatives have been “introduced”, due to the fact that trading to hedge daily risk is the ultimate goal in any commodity market, as with the energy market.

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**Note:** This paper is mainly based on the first book listed.

All other books, papers and websites are used to verify and better understand the book as well as to add and to give a different perspective on the book.