Cognitive and Social Simulation of Criminal Behaviour: 
the Intermittent Explosive Disorder Case

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ABSTRACT
Criminal behaviour often involves a combination of physical, mental, social and environmental (multi-agent) aspects, such as neurological deviations, hormones, arousal, (non)empathy, targets and social control. This paper contributes a dynamical agent-based approach for analysis and simulation of criminal behaviour, covering the above aspects, illustrated for the case of an Intermittent Explosive Disorder. It involves dynamically generated desires and beliefs in opportunities within the social environment, both based on literature on criminal behaviour.

Categories and Subject Descriptors
I.6.3 [Simulation and Modeling]: Applications.

General Terms
Experimentation, Human Factors, Verification.

Keywords
Criminal, Cognitive Simulation, Social Simulation, Analysis.

1. INTRODUCTION
Within Criminology the analysis of criminal behaviour addresses physical, mental, environmental and social aspects; e.g., [4, 10, 17, 19, 21]. Only few contributions to the literature address formalisation and computational modelling of criminal behaviour, usually focussing only on some of the factors involved; e.g., [2, 15, 16]. This paper presents a modelling approach for criminal behaviour illustrated for Intermittent Explosive Disorder, IED, extending a BDI-model for action preparation based on motivations [20] by specific models for generation of desires and for generations of beliefs in opportunities. These extensions are based on available literature on criminal behaviour and the underlying aspects. For the generation of desires, dynamical models were incorporated involving internal states, for example, for neurological, hormonal, and emotional aspects and their interaction; e.g., [17, 19]. For the generation of beliefs in opportunities, a model was incorporated formalising the well-known Routine Activity Theory within Criminology; e.g., [10]. This (informal) theory assumes motivation of the criminal and covers environmental and social aspects such as the presence of targets and social control.

To formalise and analyse such criminal behaviour, an artificial society has been modelled, where on a map (represented by a labeled graph) agents move around and meet each other. Agents may be of four types: IED criminal, agent with negative appearance, potential victim, and guardian. The models for the agents and their environment have been formally specified in dynamical systems style [1, 18] by executable temporal/causal logical relationships, extended by probabilities. To obtain these, knowledge from the literature in Criminology, and the different disciplines underlying it, was exploited; e.g., [12, 17, 19, 21].

The challenge is to model the variety of physical, mental and social aspects as mentioned above in an integrated manner. On the one hand, qualitative aspects have to be addressed, such as beliefs, desires and intentions, certain brain deviations, and some aspects of the environment such as the presence of certain agents. On the other hand, quantitative aspects have to be addressed, such as testosterone and serotonin levels, and (in the environment) distances and time durations. Furthermore, it should be possible to model on a higher level of aggregation or abstraction, as it would not be feasible, for example, to model the brain anatomy at the level of neurons. The modelling language LEADSTO [6] fulfils these desiderata. It allows to model at higher levels of aggregation, and it integrates qualitative, logical aspects and quantitative, numerical aspects; cf. [8]. In LEADSTO direct temporal dependencies between two state properties in successive states are modelled by executable dynamic properties. The format is briefly defined as follows. Let \( \alpha \) and \( \beta \) be state properties of the form ‘conjunction of ground atoms or negations of ground atoms’. In the LEADSTO language the notation \( \alpha \rightarrow_{d, t, g, h} \beta \), means:

\[
\text{If state property } \alpha \text{ holds for a certain time interval with duration } g, \text{ then after some delay (between } e \text{ and } f) \text{ state property } \beta \text{ will hold for a certain time interval of length } h.
\]

Here atomic state properties can have a qualitative, logical format, such as an expression \( \text{has_value}(x, v) \), expressing that desire \( d \) occurs, or a quantitative, numerical format such as an expression \( \text{has_value}(x, v) \) which expresses that variable \( x \) has value \( v \). For more details of the language LEADSTO, see [6].

Section 2 discusses a summary from the literature on criminals with Intermittent Explosive Disorder. In Section 3 the simulation model is presented, and Section 4 discusses a resulting simulation trace. Section 5 presents a number of global dynamic properties of the society and their logical formalisation and discusses automated verification of the simulation results against them. Section 6 discusses a probability-based analysis of similar properties, also automatically verified on a set of generated traces. Finally, Section 7 is a concluding discussion about the approach.
Positive and negative emotional attitudes towards how those concepts play a role in his or her behaviour. A person's understanding that others (also) have minds, described by different and separate mental concepts, such as the person's own beliefs, desires, and intentions, and being able to form theories as a result of these concepts. This can lead to changes in hormonal (physical) and emotional (mental) states, which lead to a certain type of desire, providing the motivation for some criminal action (mental aspect). As soon as an opportunity of a suitable potential victim with not much social control (social aspect) is perceived (mental aspect), the desire leads to the criminal action. To create a simulation model of this type of behaviour, the following characteristics are used:

The **anxiety threshold** has to be passed by stimuli, in order to make a person anxious. When a person's anxiety threshold is high, it is very difficult for this person to become anxious (and as a result, (s)he hardly knows any fear). An IED person normally has a medium anxiety threshold, but within an episode it decreases and anxiety may occur easily. The **excitement threshold** has to be passed by stimuli, in order to make a person excited. When a person's excitement threshold is high, it is very difficult for this person to become excited (and as a result, (s)he is often bored). Someone with IED normally has a medium threshold, but within an episode it increases (which can be related to a low serotonin level): a desire occurs for actions that provide strong stimuli, which might be criminal actions; e.g. [19].

The notion of **theory of mind** (e.g., [3, 13, 14]) covers the understanding that others (also) have minds, described by different and separate mental concepts, such as the person’s own beliefs, desires, and intentions, and being able to form theories as to how those concepts play a role in his or her behaviour. A person with IED has a limited theory of mind, especially during episodes. Positive and negative emotional attitudes towards others express the extent to which a person may have positive or negative feelings with respect to other persons. For the IED criminal, these attitudes usually are normal, but within an episode the positive emotional attitude towards others decreases and the negative attitude towards others increases.

Behaviour of a person with IED is usually characterised by **impulsiveness** (e.g., [17, p.176; 22]) and **aggressiveness**, which can be related to a high level of testosterone (e.g., [12, pp. 65, 68, 83-85, 112]). Impulsiveness means that actions are not planned. During an episode, impulsiveness and aggressiveness are more apparent. Moreover, persons with IED usually have high sensitivity to alcohol and easily turn to drinking and taking drugs. In addition, for them only a small amount of drugs or alcohol can become a compulsion and lead to an increased aggressiveness and impulsiveness, eventually resulting in violent behaviour [17, pp.187, 201, 258].

### 2. CASE STUDY: A CRIMINAL WITH IED

An Intermittent Explosive Disorder (IED) is a disorder of impulse control, characterised by several episodes (usually of 10 to 20 minutes each) in which aggressive impulses are released and expressed in serious assault or destruction of property, although no such impulsiveness or aggressiveness is shown in the periods (usually weeks or months) between episodes. To evoke such episodes, often only a minor stimulus is sufficient, such as an encounter with someone that has a negative, provoking, appearance or behaviour. It is estimated that about 7% of the adult population in the US can be diagnosed as having IED. Offences by persons with IED concern a disproportionate reaction, usually to an acquaintance or family member. After the episode the offender has no recollection of his actions and, when interviewed, has feelings of remorse [17, p.184]. The following sketch illustrates the interplay of the physical, mental and social aspects involved. Suppose the criminal meets somebody with negative, provoking behaviour (social aspect). This is interpreted by the criminal (mental aspect), provokes stress, and leads to an episode with an epileptic state of the brain (neurological, physical aspect). This state leads to changes in hormonal (physical) and emotional (mental) states, which lead to a certain type of desire, providing the motivation for some criminal action (mental aspect). As soon as an opportunity of a suitable potential victim with not much social control (social aspect) is perceived (mental aspect), the desire leads to the criminal action. To create a simulation model of this type of behaviour, the following characteristics are used:

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3. THE SIMULATION MODEL

In this section the simulation model as developed is described in more detail. The combination of physical, mental and social aspects involved requires integration of models for internal physical and mental functioning of an agent with a model at the social level. To this end, the simulation model has been composed from submodels, integrating different aspects, including (but not limited to) a cognitive model based on beliefs, desires and intentions (BDI) and a model for the social environment. The BDI-submodel describes how actions relate to desires and intentions, when appropriate opportunities are there. It needs as input desires and beliefs in opportunities. For these elements additional models have been developed. Thus the simulation model is composed of five submodels:

1. a submodel for reasoning about beliefs, desires and intentions
2. a submodel to determine desires needed as input for the BDI-model
3. a submodel to determine how observations lead to beliefs in an opportunity as needed as input for the BDI-model; this model is based on the Routine Activity Theory
4. a **geographical model** of the world; this is represented by a labeled graph of locations and connections
5. a submodel for the multi-agent society; this lets agents move in the world and determines the effects of actions performed.

Note that submodels 1. and 2. address physical and mental aspects, submodels 4. and 5. address social and environmental aspects, and submodel 3. relates society aspects to mental aspects.

### The BDI-submodel

The BDI-submodel bases preparation and performance of actions on beliefs, desires and intentions, e.g. [20]; an action is performed when the subject has the intention to do this action and it has the belief that the opportunity to do the action is there. Beliefs are created on the basis of stimuli that are observed. The intention to do a specific type of action is created if there is a certain desire, and there is the belief that in the given world state, performing this action will fulfill this desire. The generic rule to generate the action performance from the intention and the belief in the opportunity is specified within the BDI-submodel as:

\[
\text{LP32} \quad \text{belief(opportunity(a))} \land \text{intention(a)} \rightarrow \text{performed(a)}
\]

This intention is generated by a desire and a belief in a good reason, according to the following rule:

\[
\text{LP31} \quad \text{Desire(d(x1, x2, x3, x4, x5, x6, x7, x8))} \land \text{belief(satisfies(a, d(x1, x2, x3, x4, x5, x6, x7, x8)))} \rightarrow \text{intention(a)}
\]

1 For a complete overview of the model see URL: http://www.cs.vu.nl/~rhosse/crime/AAMAS07.pdf
Within the BDI-submodel, for reasons of simplicity, per desire only one action that can satisfy the desire is included. What remains to be generated are the desires and the beliefs in opportunities. For desires, there is no generic way (known) in which they are to be generated. Generation of desires often depends on domain-specific knowledge, which also seems to be the case for criminal behaviour. In particular, a number of physical aspects play a role here, such as certain brain deviations and serotonin levels, as discussed below in some further detail. For beliefs in opportunities, they are strongly dependent on the (social) environment, which is another theme discussed below.

The Submodel to Determine Desires
To determine desires a rather complex submodel was built, incorporating, for example, testosterone, serotonin, adrenalin, blood sugar levels and brain configuration aspects. These physical aspects relate to mental aspects such as arousal, aggressiveness, impulsiveness, risk-taking, thrill-seeking, understanding others, and feeling for others. The aspects involved contain both qualitative and quantitative aspects. To model these, both causal and logical relations (as in qualitative modelling) and numerical relations (as in differential equations) have to be integrated in one modelling framework, using the LEADSTO language.

The variety of physical and mental aspects that were found relevant in the literature such as \[4, 12, 17, 19\] and are taken into account in this model, covers: (a) the extent to which a theory of mind was developed (to understand others), (b) testosterone levels and aggressiveness, (c) dealing with anxiety, (d) social-emotional attitudes with respect to others (e.g., to feel pity for someone), (e) stimuli assessment; excitement arousal and thrill seeking, (f) serotonin levels, (g) interactions of blood sugar and impulsiveness. Stimuli from the world are labeled with two aspects, indicating the strength with respect to anxiety (risk), and with respect to excitement (thrill), respectively. For both aspects, certain thresholds represent characteristics of the person considered. The excitement threshold depends on other aspects in the model, such as sensitivity for and use of drugs and alcohol. A stimulus with excitement strength below the excitement threshold leads to being bored, and being bored leads to a desire for an action with stronger stimuli. Similarly, a stimulus with anxiety strength above the anxiety threshold leads to internal alarm bells and (eventually) to an increased adrenalin level, which (depending on another characteristic, the level of oxytocine) leads to the desire to take into account anxiety. Different combinations of such elements lead to different types of (composed) desires, for example, the desire to perform an exciting planned nonaggressive nonrisky action that harms somebody else (e.g., a pick pocket action in a large crowd). Some example specifications are:

\[
\begin{align*}
\text{LP41} & \quad \text{When agent } a1, \text{ who is a criminal, is at location } l \text{ and observes a } \text{negative} \text{ agent at location } l, \text{ then agent } a1 \text{ will have an episode.} \\
& \quad \forall a1,a2: \text{AGENT} \ (l: \text{LOCATION}) \\
& \quad \text{observes}(a1, \text{agent_of_type_at_location}(a1, \text{criminal}))(l) \land \\
& \quad \text{observes}(a1, \text{agent_of_type_at_location}(a2, \text{neg_agent}))(l) \\
& \quad \implies \text{has_episode}
\end{align*}
\]

\[
\begin{align*}
\text{LP17} & \quad \text{An episode leads to a negative emotional attitude towards others.} \\
& \quad \forall a1: \text{AGENT} \ (l: \text{LOCATION}) \\
& \quad \text{has_episode} \implies \text{emotional_attitude_towards_others}(\text{neg}, \text{high})
\end{align*}
\]

\[
\begin{align*}
\text{LP15c} & \quad \text{A current adrenalin level higher than 5 combined with a low oxytocine level leads to the desire to cope with anxiety.} \\
& \quad \forall a1: \text{INTEGER} \ (x: \text{REAL}) \\
& \quad \text{chemical_state(adrenalin, current, } x < 5) \land \text{chemical_state(oxytocine,low)} \\
& \quad \implies \text{desire_to_cope_with_anxiety(\text{high})}
\end{align*}
\]

\[
\begin{align*}
\text{LP30} & \quad \text{A combination of values and qualifications for theory of mind, aggressiveness, the desire to cope with anxiety, the desire to ignore anxiety, the desire for actions with strong stimuli, impulsiveness, emotional attitude towards others(pos) and emotional attitude towards others(neg) leads to a specific composed desire, represented as } \text{"d(x1, x2, x3, x4, x5, x6, x7, x8)".} \\
& \quad \text{Note that these desires may be unconscious.} \\
& \quad \forall x1,x2,x3,x4,x5,x6,x7,x8: \text{SCALE} \\
& \quad \text{theory_of_mind}(x1) \land \text{aggressiveness}(x2) \land \text{desire_to_cope_with_anxiety}(x3) \land \\
& \quad \text{desire_to_ignore_anxiety}(x4) \land \text{desire_for_actions_with_strong_stimuli}(x5) \land \\
& \quad \text{impulsiveness}(x6) \land \text{emotional_attitude_towards_others}(\text{pos},x7) \land \\
& \quad \text{emotional_attitude_towards_others}(\text{neg},x8) \\
& \quad \implies \text{desire}(\text{d(x1, x2, x3, x4, x5, x6, x7, x8)})
\end{align*}
\]

The Submodel to Determine Opportunities
As another input for the BDI-model, the notion of opportunity is based on criteria indicated in the Routine Activity Theory [10]: a suitable target and absence of a guardian. This was specified by:

\[
\begin{align*}
\text{LP40} & \quad \text{When agent } a1, \text{ who is a criminal, is at location } l \text{ and observes a passer-by at location } l \text{ and does not observe a guardian at location } l, \text{ then agent } a1 \text{ believes that there is an opportunity to assault someone.} \\
& \quad \forall a1,a2: \text{AGENT} \ (l: \text{LOCATION}) \\
& \quad \text{observes}(a1, \text{agent_of_type_at_location}(a1, \text{criminal}))(l) \land \\
& \quad \text{observes}(a1, \text{agent_of_type_at_location}(a2, \text{passer_by}))(l) \land \\
& \quad \text{not}\text{observes}(a1, \text{agent_of_type_at_location}(a3, \text{guardian}))(l) \\
& \quad \implies \text{belief(opportunity(assault))}
\end{align*}
\]

In dynamic property LP32 shown earlier, the third criterion of the Routine Activity Theory, the motivated offender, is represented by the intention to perform some action.

The Geographical Environment Model
The social, multi-agent aspect is modelled by an environment, in which a number of agents move around and sometimes meet at a location. One of the agents is the criminal agent with IED, the others are guardian agents, potential victims (passers-by) and agents with provoking behaviour (so that they may trigger an episode in the criminal when (s)he encounters them), from now on referred to as negative agents. The passers-by are assumed to be suitable targets, for example, because they appear rich and/or weak. However, as also the guardians are moving around, such targets may be protected, whenever at the same location a guardian is observed by the criminal: social control.

Figure 1. Example World Geography (with an initial distribution of agents over locations, agent 1 is the criminal with IED, agent 2 and 3 are guardians, agent 4, 5 and 6 are passers-by (potential victims), and agent 7 and 8 are ‘negative’ agents).

The interaction between a specific agent and the environment is modelled by (1) observation, which takes information on the environment as input for the agent (e.g., at which location it is, where suitable targets are, and whether social control is present), and (2) performing actions, which is an output of the agent affecting the state of the world (e.g., going to a different location, or committing a crime). The geographical information of the
world is described by a labeled graph as depicted in Figure 1. Relevant locations are indicated by nodes A, B, …, and routes connecting locations by edges E1, E2, … The agents move from location to location via these edges. Edges have lengths; travelling takes time, depending on these lengths.

The Submodel for the Society
To model the dynamics of an agent moving in the environment, the following cycle is used: observe, determine next action, determine effects of this action. In some more detail, the model is based on (1) properties expressing what is observed, for example, stimuli or other agents; if another agent is present at the agent’s location, then the agent will observe this, (2) properties expressing which next action is to be undertaken; for example, if the agent has stayed at its location for duration s, and the next location to reach is I, then it will move to this next location (probabilities are used to make random choices between options), and (3) properties expressing the results of actions undertaken; for example, if the agent starts to move to a next location over edge e and edge e has length d, then it will arrive at the next location after duration d.

Settings for the Model
The model was set with characteristics of the criminal with IED; the model has to show as output the behaviour as known for this type of criminal. More specifically, characteristics set at forehand include: basic adrenalin level, basic serotonin level, basic oxytocine level, brain configuration for sensitivity for alcohol, anxiety threshold, excitement threshold, emotional attitude towards others, and beliefs about which action satisfies which desire. Inputs for the model over time are: stimuli (i.e., external events that have a certain impact), taking alcohol, taking Prozac, taking Ritalin. Finally, some characteristics of the environment were set at forehand: the initial presence of certain types of agents (criminals, guardians, passers-by, negative agents) at certain locations. These inputs are included in scenarios for simulation.

4. AN EXAMPLE SIMULATION TRACE
A large number of simulation traces (200 in total) have been generated for the behaviour of the IED criminal under different circumstances. In this section, one specific simulation trace is described in detail. In Figure 2, the results of the chosen example simulation trace (run) for the behaviour of the IED criminal are shown. In this picture, time is on the horizontal axis; state properties are on the vertical axis. A dark box on top of the line indicates that the property is true during that time period, and a lighter box below the line indicates that the property is false. In the lower part of the picture, some quantitative information is shown, such as the adrenalin level and the strength of the stimuli from the world. Due to space limitations, this picture only shows a selection of relevant state properties.2

As shown by Figure 2, the IED criminal (agent 1) initially has a desire (represented as desire(low, high, low, low, high, high, medium, medium)) for actions that are characterised by the following elements: a low theory of mind, high aggressiveness, a low desire to cope with anxiety, a low desire to ignore anxiety, a high desire for actions with strong stimuli, high impulsiveness and both a high desire (represented as desire(low, high, high, high, high, medium, high, low)).

At time point 75 the criminal is at location G, where he meets a negative agent (agent 8). This causes an episode, which leads (via an increased adrenalin level) to a strong desire to cope with anxiety, a highly negative emotional attitude towards others, and a new composed desire: (low theory of mind, high aggressiveness, high desire to cope with anxiety, low desire to ignore anxiety, high desire for actions with strong stimuli, high impulsiveness, low positive emotional attitude towards others and a high negative emotional attitude towards others). Notice that aggressiveness and impulsiveness already occurred in the previous desire, but now the brakes are no longer there: no concern for other persons or anxiety plays a role anymore. This desire, combined with the belief that performing an assault leads to the satisfaction of this desire, leads to the intention to assault someone. At time point 83, the IED criminal is again at location G, but now together with a passer by (agent 6) without a guardian present (agents 2 and 3 are respectively on location H and E).

Figure 2. Example Simulation Trace
This leads to the belief that there is an opportunity to assault someone. This belief combined with the intention leads to the performance of the assault. Because of the assault, the stimuli of the world increase, which satisfies the desires of the criminal. Although the simulation example as presented here involves only 8 agents, it has been found that it easily scales to a society of several hundreds of agents (processing time staying within one hour). However, the overall pattern for such larger numbers is not essentially different from the trace shown here.

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2 Larger pictures of (other) simulation traces are shown at URL: http://www.cs.vu.nl/~tbosse/crm/AAMAS07.pdf
5. LOGICAL ANALYSIS

A number of properties of the multi-agent system have been identified and formalised. Some of these properties have a logical character and some have a probabilistic character. Both types of properties have been automatically verified for the simulation traces. In this section the logical properties are discussed, in the next section the probabilistic ones. For a multi-agent system, dynamic properties can be identified at different aggregation levels, roughly spoken (1) the level of the behaviour of a single agent (external perspective), (2) the level of the internal functioning of an agent (internal perspective), and (3) the level of the multi-agent system as a whole (society behaviour). For each of these levels, relevant properties are identified and formalised.

Behavioural Properties of Agents

The properties that have been identified and formalised in the logical language TTL [5] to characterise the behaviour of the criminal agent (from an external perspective) are as follows (where f is the duration of the reaction time from observations to internal states or from internal states to actions):

BP1 From Circumstances to Criminal Action If a criminal agent meets a negative agent, and within duration e an opportunity occurs, then an assault will be performed.

\[ \forall t \forall a_1, a_2, t' \text{agent} \forall \text{location} \]
\[ \text{[state}(t), t \Rightarrow \text{observes}(a_1, \text{agent_of_type_at_location}(a_1, \text{criminal, } l_1)) \land \text{observes}(a_1, \text{agent_of_type_at_location}(a_2, \text{passer_by, } l_1)) \land \text{v3:agent} \]
\[ \text{state}(t), t \Rightarrow \text{observes}(a_1, \text{agent_of_type_at_location}(a_3, \text{criminal, } l_1)) \land \text{v3:agent} \]
\[ \text{state}(t), t' \Rightarrow \text{performed}(assault) \]

Here state(t), t denotes that within the state state(t), t at time point t in trace γ state property X holds (and with # that it does not hold), with the infix predicate |= within the language denoting the formalised satisfaction relation. See [5] for more details of TTL.

BP2 From Criminal Action to Circumstances If an assault is performed, then the opportunity was there and earlier (at most e back in time) the criminal agent encountered a negative agent.

\[ \forall t \exists t_0 \text{agent} \exists \text{location} \]
\[ \text{[case}(\text{negative_encounter}(a_1, a_3, l_1), t_0) \land \text{case}(\text{negative_encounter}(a_2, a_3, l_1), t_0) \land \text{v3:agent} \]
\[ \text{state}(t, t_0) \Rightarrow \text{observes}(a_1, \text{agent_of_type_at_location}(a_1, \text{criminal, } l_1)) \land \text{observes}(a_2, \text{agent_of_type_at_location}(a_2, \text{passer_by, } l_1)) \land \text{v3:agent} \]
\[ \text{state}(t, t_0) \Rightarrow \text{performed}(assault) \]

Notice that these properties summarise how the agent functions in the context of society, abstracting from the internal mechanisms underlying this behaviour. Logical consequences of these external agent behaviour properties include the following external behavioural property:

BP3 No Opportunity No Crime If no opportunities are offered, then no criminal action occurs.

\[ \forall t \forall a_1, a_2, \text{agent} \forall \text{location} \]
\[ \text{[case}(\text{negative_encounter}(a_1, a_3, l_1), t_0) \land \text{case}(\text{negative_encounter}(a_2, a_3, l_1), t_0) \land \text{v3:agent} \]
\[ \text{state}(t, t_0) \Rightarrow \text{observes}(a_1, \text{agent_of_type_at_location}(a_1, \text{criminal, } l_1)) \land \text{observes}(a_2, \text{agent_of_type_at_location}(a_2, \text{passer_by, } l_1)) \land \text{v3:agent} \]
\[ \text{state}(t, t_0) \Rightarrow \text{observes}(a_1, \text{agent_of_type_at_location}(a_3, \text{guardian, } l_1)) \]
\[ \Rightarrow \text{[case}(\text{negative_encounter}(a_1, a_3, l_1), t_0) \land \text{case}(\text{negative_encounter}(a_2, a_3, l_1), t_0) \land \text{v3:agent} \]
\[ \text{state}(t, t_0) \Rightarrow \text{observes}(a_1, \text{agent_of_type_at_location}(a_3, \text{guardian, } l_1)) \]
\[ \Rightarrow \text{false} \]

Internal Properties of Agents

Although for an analysis at the level of the society as a whole, details of internal mechanisms and processes are not needed, from the perspective of justifying, understanding and explaining whether, how and when such behaviour can occur, still the internal agent dynamics are interesting to formalise. Knowledge of these mechanisms may also be useful as a basis for therapy and/or medication. The following internal behavioural properties of the criminal agent were identified and formally specified. Due to space limitations, the formalisations have been left out:

IP1a (Episode Provoking) If the criminal agent C observes an agent A which has a negative appearance, then from t to t+ε the criminal agent C will have an episode.

IP1b (Episode Grounding) If the criminal agent C has an episode, then at some time point between t-ε and t the criminal agent C observed an agent A which has a negative appearance.

IP2a (Crime Committing) If the criminal agent C has an episode, and it believes there is an opportunity to commit a crime, then it will perform criminal action a.

IP2b (Crime Grounding) If the criminal agent C performs criminal action a, then it believes there is an opportunity to commit a crime, and it has an episode.

IP3a (Belief Generation) If the criminal agent C observes X, then it will believe X.

IP3b (Belief Grounding) If the criminal agent C believes X, then before it has observed X.

In fact, the properties IP1a and IP1b express that having an episode has a backward representation relation (within duration ε) to meeting a negative agent, and IP2a and IP2b that it has a forward representation relation to conditionally performing a criminal action as soon as (within duration ε) an opportunity occurs; cf. [7]. The properties IP1ab and IP2ab can be refined further into more local properties describing the criminal agent’s internal mechanisms, as modelled in the simulation model.

Society Behaviour Properties

At the level of the society as a whole, one (especially as a policy maker) may typically be interested in under which circumstances how many crimes are committed, according to a certain measure. In the case considered here, two main factors to be taken into account are the negative encounters and the opportunities. In this section, some properties have been identified and logically specified. In Section 6, from a probability analysis perspective, other types of society properties have been formalised. The logical properties identified require counting of the number of certain types of observations or actions over time. For this, a very useful summation feature is available in TTL, denoted by \( \sum_{\gamma} \). For example, if \( \phi \) indicates the value \( v1 \) if \( \phi \) is true, and \( v2 \) otherwise. So, for the kth term, this summation adds \( v1 \) if \( \phi(0) \) is true and \( v2 \) if \( \phi(0) \) is not true. In particular, if \( v2=0 \) is taken and \( v1=1 \), then it counts the number of time points t at which \( \phi(t) \) is true.

SP1 More Negative Agents More Crime The more often negative agents are encountered, the more often criminal actions will occur.

\[ \forall\gamma; \gamma(t) = 1, \gamma(t, 0) = 1 \]
\[ \sum_{\gamma} \text{case}(\text{negative_encounter}(\gamma, k), 1, 0) = v1 \]
\[ \sum_{\gamma} \text{case}(\text{negative_encounter}(\gamma, k), 1, 0) = v2 \]

SP2 More Opportunities More Crime The more often opportunities are present, the more often criminal actions will occur.

\[ \forall\gamma; \gamma(t) = 1, \gamma(t, 0) = 1 \]
\[ \sum_{\gamma} \text{case}(\text{opportunities}(\gamma, k), 1, 0) = v1 \]
\[ \sum_{\gamma} \text{case}(\text{opportunities}(\gamma, k), 1, 0) = v2 \]
Here, the following abbreviations are used:

- negative_encounter(x): \( \exists \gamma, l \) state(\( \gamma, l \)) \& observes(a1, agent_of_type_at_location(a2, neg_agent, l))
- opportunity(x): \( \exists \gamma, l \) state(\( \gamma, l \)) \& observes(a1, agent_of_type_at_location(a2, passer_by, l)) \& va3:agent
- state(y, l): \( \exists \gamma, t \) state(\( \gamma, t \)) \& observes(a1, agent_of_type_at_location(a3, guardian, l))

Notice that the above properties compare two traces with each other. In the language TTL, it is possible to express such properties, in contrast to, for example, modal temporal logics.

 Verification of the Logical Properties

Verification of properties at the three aggregation levels can be done in different ways. One way is to check whether the properties hold in the different simulation traces that have been generated. All of the properties as discussed have been checked automatically for all 200 simulation traces using the TTL Checker tool [5]. Using these checks, the behavioural and internal agent properties were all found satisfied. However, the society properties turned out not to hold for all combinations of traces. The reason for this is that, by chance, there are some traces in which there is not much crime although many negative agents are encountered (for example, because there are no opportunities). Likewise, there are some traces where there is not much crime although many opportunities arise (e.g., because the criminals have no episodes). These individual traces cannot be distinguished by checking properties such as SP1 and SP2. For this reason, a probabilistic approach is sometimes more useful. Such a probabilistic approach is worked out in the next section.

Another way of verification is by establishing interlevel relations between dynamic properties. For example, the properties IP1a, IP2a and IP3a together (logically) imply behaviour property BP1, and IP1b, IP2b and IP3b together imply BP2, by the following interlevel relations:

- IP1a & IP2a & IP3a \( \Rightarrow \) BP1
- IP1b & IP2b & IP3b \( \Rightarrow \) BP2

These interlevel relations have been verified as well.

6. PROBABILISTIC ANALYSIS

In this section properties are analysed from a probability perspective. At the society level, a main property is the parameterised global property below, addressing the expected number of crimes occurring within a certain time interval.

\[ \text{GP1}(t, d, EC) \text{ Crime Occurrence Expectation} \]

The expected number of crimes that take place from \( t \) within duration \( d \) is \( EC \).

Later on an expression will be shown for the expected number of crimes \( EC \) in this property with the following parameters:

- \( M \) total number of locations that can be visited
- \( N \) total number of agents with negative appearance
- \( V \) total number of agents offering an opportunity (potential victims)
- \( G \) total number of guardian agents

To analyse property GP1 in more detail, it is related to two more refined properties:

- the probability that within a certain duration \( d1 \) (for the first time) a negative agent is met
- the probability that (after meeting a negative agent) within a duration \( d1 \) (for the first time) an opportunity for crime is met

Here \( e \) is the assumed duration of the episode.

GP2(1, d1, p1) Provocation Occurrence Probability

The probability that from \( t1 \) after duration \( d1 \) a negative agent is met is \( p1 \).

GP3(2, e1, p2) Opportunity Occurrence Probability

The probability that from \( t2 \) after duration \( e1 \) a first opportunity is met and in the meantime no negative agent is met is \( p2 \).

A first step is to assume invariance over time, so that these probabilities do not depend on the time parameters. Then these parameters will be left out. As a next step it is assumed that meeting a negative agent before \( t1 \) and an opportunity after \( t1 \) are independent events. Moreover, the behavioural properties IP1 and IP2 of the criminal agent are used.

Relating the probabilities and expected crimes

As a first step the probability \( p \) in GP1 will be related to the probabilities \( p1 \) and \( p2 \) in GP2 and GP3. This is done by the following logical relation.

\[ p1 \& p2 \& EC = \sum_{d1 \leq d} p1 \text{ with } \sum_{e1 \leq e} p2 \text{ with } p1 \cdot p2 \Rightarrow \text{GP1}(d1+e, EC) \]

This relation collects all paths that can lead to a crime, indicated by the time that a negative agent was met and the (first) time that an opportunity was met. A next step is to find out what reasonable estimations are for the probabilities in GP2 and GP3. After this step the relation above will be used to find an estimation for \( EC \). First GP2 is addressed. For convenience the following short notations are used: \( a = (1 - 1/M), b = 1 - (1 - aV)aG \).

Estimating the probability to meet a negative agent

A next assumption is that, by their moving, the agents will be present at locations according to a uniform probability distribution, so for any agent \( A \) and location \( L \), at any point in time, the probability that agent \( A \) is at location \( L \) is \( 1/M \). The probability that it is not at \( L \) is \( 1 - 1/M = a \). A further assumption is that agents move independently, and hence their locations are independent. Therefore for a given location \( L \) at time point \( t \), the probability that there is no agent with negative appearance at \( L \) is given by \( p(\text{no_negative_agent_at_L}) = a^N \), and the probability that there is at least one agent with negative appearance at \( L \) is:

\[ p(\text{at_least_one_negative_agent_at_L}) = 1 - a^N \]

This gives an estimation of how the probability \( p1 \) in property GP2(1, d1, p1) depends on \( d1 \), or, expressed differently, it has been found that by estimation it holds: \( GP2(d1, (1 - a^N)) \).

Estimating the probability to meet an opportunity

The next step addresses the probability to meet an opportunity within duration \( e \), as indicated by property GP3. Here, the additional condition is that at \( e1 \), it is the first time in that the interval \( e \) an opportunity is met, and that no further negative agents were met in the meantime. Then the probabilities that at that location no victims and no guardians are met are as follows (with \( a = 1 - 1/M \)):

\[ p(\text{no_victim_at_L}) = a^V \]
\[ p(\text{no_guardian_at_L}) = a^G \]

Therefore the probabilities of the presence of a victim and of a guardian are:

\[ p(\text{victim_at_L}) = 1 - a^V \]
\[ p(\text{guardian_at_L}) = 1 - a^G \]

The probability that an opportunity is met (i.e., a victim and no guardian) is (with \( b = 1 - (1 - a^V)a^G \)):

\[ p(\text{opportunity_at_L}) = (1 - a^V)a^G = (1 - b) \]
The probability that no opportunities and no negative agents are met is:
\[ p_{\text{no opportunity}_t \text{, no neg}_t} = \delta(1 - (1 - b)^t) = \delta e^t \]

The probability that at e1 locations \([0, \ldots, e1-1]\) of a sequence of no opportunities and no negative agents are met is:
\[ p_{\text{no opportunity}_t \text{, met}_t \text{, no neg}_t} = (\delta e^t)^{e1} \]

Based on this, the probability that in a sequence of e1 locations at the e1-th element a first opportunity is met, whereas at all locations before e1 no opportunity and no negative agent was met is given by:
\[ p_{\text{first opportunity}_t \text{, met}_t \text{, no neg}_t} = (\delta e^t)^{e1}(1 - b) \]

This gives an estimation of how the probability p2 in property GP3(e1, p2) depends on e1, or, expressed differently, it has been found that by estimation it holds: GP3(e1, b^a)^{e1}(1 - b).

**Estimating the Expected Number of Crimes**

Now that estimations for the probabilities in GP2 and GP3 have been found, it is possible to estimate the expected number of crimes in GP1, on the basis of the following calculation for EC:
\[ \sum_{d=0}^{t=0} p_{\text{no opportunity}_t \text{, met}_t \text{, no neg}_t} = \sum_{d=0}^{t=0} (\delta e^t)^{e1}(1 - b) \]

Substituting here the probabilities as specified by GP2 and GP3 in the form GP2(d1, (1 - a^b)) and GP3(e1, (a^b)^{e1}, (1 - b)) obtains the following for the probability in GP1 that a crime is committed within duration \(d + e\):

\[
EC = \sum_{d=0}^{t=0} p_{\text{no opportunity}_t \text{, met}_t \text{, no neg}_t} = \sum_{d=0}^{t=0} (\delta e^t)^{e1}(1 - b) \\
= \sum_{d=0}^{t=0} (\delta e^t)^{e1} \cdot (\delta e^t)^{e1}(1 - (a^b)) \cdot (1 - b) \\
= \sum_{d=0}^{t=0} d \cdot (1 - a^b) \cdot ((1 - a^b)(e+1))/ (1 - (aN b)) \cdot (1 - b) \\
= d \cdot (1 - a^b) \cdot ((1 - a^b)(e+1))/ (1 - (aN b)) \cdot (1 - b) \\
\]

This implies that
\[ GP1(d,e, d \cdot (1 - a^b) \cdot ((1 - a^b)(e+1))/ (1 - (aN b))) \]

holds. Substituting \(b = 1 - (1 - a^b)^{e1}\) and \(a = (1 - 1/M)^{d1}\) provides for EC the following expression in the basic parameters:
\[
d \cdot (1 - a^b) \cdot ((1 - a^b)(e+1))/ (1 - (a^b)) \cdot (1 - b) = d \cdot (1 - a^b) \cdot ((1 - a^b)(e+1))/ (1 - (a^b)) \cdot (1 - b) \\
\]

To evaluate the behaviour of this expression for the expected number of crimes, depending on different parameter settings for the 6 basic parameters M, V, G, N, d, e, the expression has been implemented in a spreadsheet\(^3\) (in Microsoft Excel). Some observations that are plausible from the context are shown by the implementation and were also analysed from the form of the formula. e.g.:

- EC is monotonically increasing in its dependence on each of N, V, d, e
- EC is monotonically decreasing in G
- for N = 0 or V = 0 or M very large, EC becomes 0

Furthermore, the expected number of crimes has been automatically verified against a set of simulation traces. To this end, another set of n=200 simulation traces has been generated. These traces were similar to the ones mentioned in Section 4, but used a fully connected graph for the geographical model (because of the assumption that the location of agents is independent of their previous location). For these traces, the following TTL formula:

\[ \exists w (w = \sum_{k=1}^{n} \sum_{t=0}^{\delta} \text{case(state}(w(k), t) = \text{performed}(assault)), \text{1, 0}) / n \]

was checked with suitable values for parameters EC (given above) and \(\delta\). For the expected number of crimes EC, the value of 4.14 was chosen, as predicted by the above probabilistic analysis (with \(M=8, N=2, V=3, G=2, d=40, e=10\)). For \(\delta\), the value of 0.1 was chosen (i.e., about 2.5% of EC). Based on these parameter values, the TTL formula mentioned above indeed succeeded (in a few minutes), since in the 200 traces under investigation 809 crimes were performed. This is an average of 4.04 crimes per trace, which lies just within \(\delta\) from the number of 4.14 expected crimes.

### 7. DISCUSSION

In this paper, an agent-based modelling approach is presented to analyse criminal behaviour in its social context. Agent-based modelling approaches often either address the internal functioning of an agent in an extensive manner but leave the social context limited, or address the social interactions at the level of the multi-agent system as a whole, thereby taking the internal models of the agents of limited complexity. As in the case considered here the interaction of physical, mental and social aspects is a central issue, a model covering both levels is required. The model as presented extends the general BDI-agent-model [20] by specific models to generate desires and beliefs in opportunities, exploiting literature on criminal behaviour, in particular [12, 17, 19]. It involves both qualitative aspects (such as the anatomy of brain deviations, and presence or absence of agents at a specific location in the world), and quantitative aspects (such as distances and time durations in the world and hormone and neurotransmitter levels). The model has been illustrated for the case of a person with Intermittent Explosive Disorder (IED). It has been found that the model indeed shows the behaviour as known for this type of criminal within the given social context. Moreover, to analyse the model in more detail, a number of dynamic properties have been formalised in the TTL language, and (using an automated checker tool) have been (successfully) verified against a large set of simulated traces. These dynamic properties, both of logical and probabilistic type, comprise not only behavioural and internal properties of the agents involved, but also properties that address the society as a whole. Especially the latter type of properties may have a complex structure, e.g., because they compare multiple traces with each other, or because of the probabilistic aspects involved. The language TTL and its software environment turned out useful for these purposes.

In literature such as [20], within standard BDI-models no general model for generation of desires is included. For the case considered, various aspects as found in the literature have been taken into account in the dynamic generation of desires, varying from specific types of brain deviations, and serotonin and testosterone levels, to the extent to which a theory of mind was developed. The generation of beliefs in opportunities has been based on environmental and social aspects involving two specific criteria (suitable target, presence of guardian) as indicated by the Routine Activity Theory in [10]. Within the BDI-submodel, for reasons of simplicity, per desire only one action that can satisfy the desire is included (and one intention for that action). When a number of intentions are possible for one desire, then the model can be extended by a more specific decision making approach, such as utility-based multi-objective decision making; cf. [11].
Although it is recognised that computer support in the area of crime investigation is an interesting challenge, only few papers on simulation and formal analysis of criminal behaviour can be found in the literature; they usually address a more limited number of aspects than the approach presented in this paper. For example, Brantingham and Brantingham [9] discuss the possible use of agent modelling approaches to criminal behaviour in general, but do not report a specific model or case study. Moreover, in [2] a model is presented with emphasis on the social network and the perceived sanctions. However, this model leaves the mental and physical aspects largely unaddressed. The same applies to the work reported in [16], where an emphasis is on the environment, and police organisation. The contribution put forward in the current paper shows that an agent-based modelling approach is possible where both a complex internal agent model is involved (addressing physical and mental aspects) and a model for the multi-agent society.

Dynamical models for criminal behaviour as presented in this paper can be used in a number of ways. In the first place, they can be used to simulate behaviour for given scenarios of circumstances occurring over time. This can be used to find out for such a given scenario of circumstances, whether a criminal of a certain type may show certain behaviour under these given circumstances. Second, the models can be used in the opposite direction, i.e., given a certain behaviour, to determine what kind of scenario of circumstances could lead to this behaviour. Third, the models can be used in the situation that the circumstances and/or the behaviour are only partially given. In that case, the models can be used to complete this partial information, i.e., to find out which (completed) behaviour could be consistent (or inconsistent) with which (completed) circumstances, and to find out which additional information should be investigated to determine one or more completions of the partial information that are realistic. Fourth, the models can be used for therapeutic reasoning. For example, they may be used to predict which behaviour certain types of criminals will show if circumstances are avoided or slightly changed (what-if reasoning). Using this approach, the behaviour of the subject can be modified by selecting or avoiding the appropriate circumstances. Another possible use of such models in therapeutic reasoning is to determine a (cognitive) training program for the criminal to adapt the relationship between circumstances and behaviour.

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9. REFERENCES