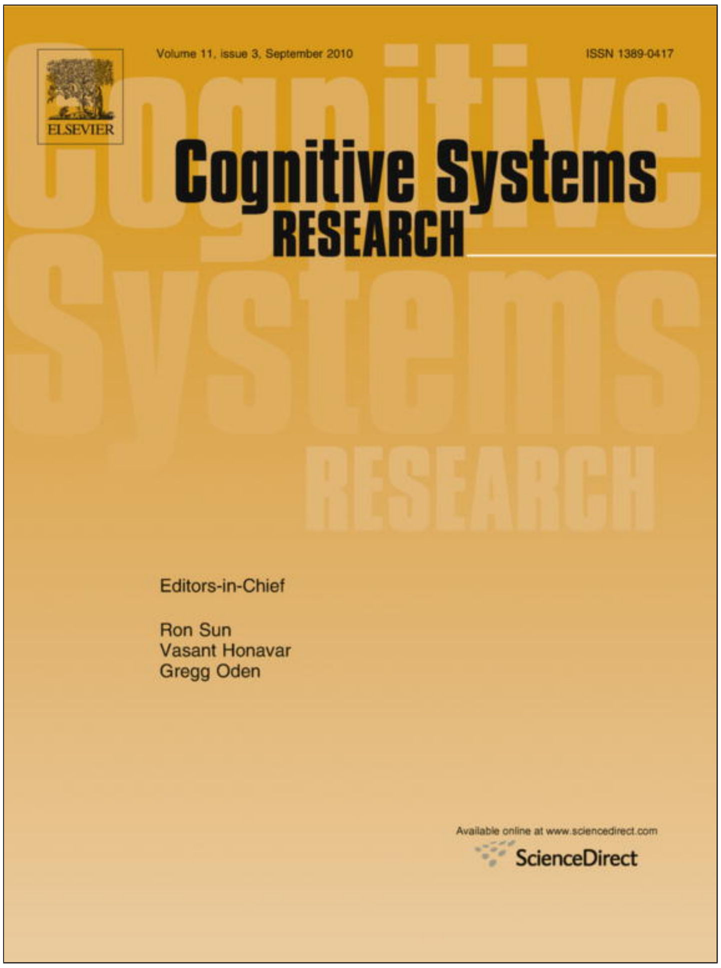
Volume 11, issue 3, September 2010
ISSN 1389-0417
ELSEVIER
Cognitive Systems
RESEARCH
Editors-in-Chief
Ron Sun
Vasant Honavar
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Available online at www.sciencedirect.com
ScienceDirect

Available online at www.sciencedirect.com
ScienceDirect
Cognitive Systems Research 11 (2010) 211–230
Cognitive Systems RESEARCH
www.elsevier.com/locate/cogsys

A computational model based on Gross' emotion regulation theory ☆

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Received 17 July 2009; received in revised form 6 October 2009; accepted 6 October 2009
Available online 20 October 2009

Abstract

Emotion regulation describes how a subject can use certain strategies to affect emotion response levels. Usually, models for emotion regulation assume mechanisms based on feedback loops that indicate how to change certain aspects of behavior or cognitive functioning in order to get a more satisfactory emotion response level. Adaptation of such feedback loops is usually left out of consideration. This paper introduces an adaptive computational model for emotion regulation by formalizing the model informally described by Gross (1998). The model has been constructed using a high-level modeling language, and integrates both quantitative aspects (such as levels of emotional response) and qualitative aspects (such as decisions to regulate one's emotion). This model includes mechanisms for adaptivity of the degree of flexibility of the emotion regulation process. Also, the effects of events like traumas or therapies on emotion regulation can be simulated. Based on this computational model, a number of simulation experiments have been performed and evaluated.

Keywords: Emotion regulation; Adaptivity; Cognitive modelling

1. Introduction

Historically, there has been much debate about the function of emotions. For example, Hebb saw emotions as neural activation states without a function (Hebb, 1949). However, recent research provides evidence that emotions are functional (e.g., Damasio, 2000). Emotions have a facilitating function in decision making (Oatley & Johnson-Laird, 1987), prepare a person for rapid motor responses (Frijda, 1986), and provide information regarding the ongoing match between organism and environment (Schwarz & Clore, 1983). Emotions also have a social function. They provide us information about others' behavioral intentions, and script our social behavior (Gross, 1998). In the past two decades, psychological research has started to focus more on *emotion regulation* (e.g., Gross, 1998, 2001; Ochsner & Gross, 2005; Thompson, 1994). In brief, emotion regulation is the process humans undertake in order to affect their emotional response. Recent neurological findings (such as bidirectional links between limbic centers, which generate emotion, and cortical centers, which regulate emotion) have changed the consensus that emotion regulation is a simple, top-down controlled process (Gross, 1998).

This article introduces a computational model to simulate emotion regulation, based on the process model described informally by Gross (1998, 2001). Note that Gross' definition of emotion is very much related to the well known notion of coping (see, e.g., Lazarus & Folkman, 1984; Scherer, 1984), but with some subtle differences. In particular, coping mainly focuses on decreasing negative emotion experience, whereas emotion regulation addresses increasing and decreasing both positive and negative emotions (Gross, 1998).

☆ Parts of this paper are based on work presented at the 2007 international conference on cognitive modeling (ICCM'07) (Bosse, Pontier, & Treur, 2007b), the 2007 IEEE/WIC/ACM international conference on intelligent agent technology (IAT'07) (Bosse, Pontier, & Treur, 2007a), and the 2008 European conference on modeling and simulation (ECMS'08) (Pontier, 2008).

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doi:10.1016/j.cogsys.2009.10.001

Computational models for emotion regulation, like the one described in this paper, can be used for different purposes (see Wehrle (1998) for an overview). In the first place, a model for emotion regulation can be used in the field of Artificial Intelligence; see e.g., (Bates, 1994; Hudlicka, 2008). For example, in the domain of virtual reality it can be used to let virtual agents show human-like behavior regarding emotion regulation. There are also various applications one could think of for which the model could be incorporated into robots, to make them show human-like emotion regulation behavior (Reilly, 1996). Similarly, in the gaming industry, there is much interest in manners to let game characters emotionally behave like humans. Finally, computational models for emotion regulation may play a role within the field of Ambient Intelligence (Aarts, Harwig, & Schuurmans, 2001). For instance, in settings where humans have to interact intensively with automated systems, it is useful if the system maintains a model of the emotional state of the user. This can enable the system to adapt the type of interaction to the user's needs; see e.g., (Klein, Moon, & Picard, 2002).

In addition, from a Cognitive Science perspective, such a model can provide insight in the process of emotion regulation; see e.g., (Sloman, 2003). An advanced model could also be helpful to make predictions about emotions, about behavior that is a consequence of emotions and about how to influence certain behaviors with, e.g., an anger management therapy. This may be useful for the purpose of developing therapies for persons that have difficulties in regulating their emotions (Burns, Bird, Leach, & Higgins, 2003; Towl & Crighton, 1996), for example, in work with forensic inpatients.

The perspective taken in this paper is mainly the Artificial Intelligence perspective. Thus, we aim to model the process of emotion regulation in such a way that it can be incorporated into intelligent entities (e.g., intelligent virtual agents, or Ambient Intelligence systems). Therefore, a main requirement of the model is that it is relatively lightweight (i.e., easy to plug into an existing applications), while nevertheless complying with the existing literature on emotion regulation.[1]

Often, models for emotion regulation are conceptualized as dynamical systems based on feedback loops that indicate how to change certain aspects of behavior or cognitive functioning to get a more satisfactory emotion response level. Such feedback loops have certain characteristics, for example, concerning sensitivity and flexibility of the adjustments made. Too sensitive feedback loops may result in stressful and energy-consuming behavior involving frequent adjustments, whereas feedback loops that are not sensitive enough may result in long periods of less desirable emotions. To obtain a balanced form of emotion regulation, either certain more or less ideal characteristics of the feedback loops in the emotion regulation system should be set at forehand, or an adaptation mechanism should be available that allows for tuning them on the fly to the required form of sensitivity. As it does not seem very plausible to have one set of 'ideal', innate characteristics applicable in various contexts (Wehrle, 1998, p. 5), this paper takes the latter assumption as a point of departure: adaptive emotion regulation. This adaptivity includes mechanisms to assess and adapt the degrees of flexibility of the emotion regulation process over longer periods; i.e., the subject changes its willingness to change behavior in favor of emotion regulation, regarding the success of its emotion regulation in the past.

This willingness to change behavior can be changed by certain events. For instance, if someone has a very low tendency to change his/her behavior in order to regulate his emotions, a therapy could help that person to change this tendency, and help him/her learn to adapt this regulation of emotions in a more flexible manner. Previous research suggests that therapies can help people to regulate their emotions. For instance, Beck and Fernandez describe, based on 50 studies, that people who were treated with a cognitive behavioral therapy as an approach for anger management were better off than 76% of untreated subjects, in terms of anger reduction (Beck & Fernandez, 1998). In addition, an article by Burns et al. (2003) suggests that a structured anger management training program is useful for forensic inpatients with learning disability. Finally, Deschner and McNeil (1986) describe an experiment, in which families that experienced violence, followed anger control training. After the training, 85% of the families were free of further violence and remained so, according to an independent survey completed 6–8 months later. On the other hand, an event like a trauma could decrease the tendency to change one's behavior in order to regulate his emotions significantly. Schore (2001) describes that an early trauma can cause impaired affect regulation.

An important choice to be made in modeling is the grain size by which the model represents reality. This choice is strongly related to the goals and may have a decisive effect on the feasibility and success of a modeling enterprise. On the one hand a model that has a too fine grain size may become unmanageable both conceptually and computationally. On the other hand a model with a too coarse grain size may miss patterns in reality that may be essential to fulfill the goals of the modeling enterprise. As indicated above, the goals of the work presented here are in the area of building relatively lightweight artificial, virtual agents which have a certain level of believability. Therefore a rather coarse-grained approach has been chosen, where a number of aspects have been addressed in an abstracted manner, such as the detailed processes underlying emotion elicitation and appraisal.

In Section 2, the process model of emotion regulation by Gross is explained. The model describes a number of strategies humans use to adapt their emotion response levels, varying from situation selection to cognitive change and response

[1] However, while developing our model, we always keep the Cognitive Science perspective 'in mind'. That is, if the modeling process itself provides us new insights into emotion regulation, we will not hesitate to explore these insights in more detail.

modulation. This model is used as a basis for the computational model of emotion regulation. In Section 3, a high-level overview of the model is provided, and the dynamical system style modeling approach is briefly introduced. In Section 4, the computational model is described in detail. Section 5 illustrates the model by a number of simulation experiments for different scenarios, both for ideal cases and for cases of over- and under-regulation. Also, simulation experiments have been performed to test whether events with a positive effect on the personal tendency to change behavior in favor of emotion regulation, like therapies, facilitate emotion regulation, and events with a negative effect on the tendency to change behavior, like traumas, impair emotion regulation. Section 6 addresses verification of global properties of the model, and Section 7 concludes the paper with a discussion.

2. Gross' model for emotion regulation

Gross (2001) describes a process model of emotion regulation using the following definition:

> 'Emotion regulation includes all of the conscious and nonconscious strategies we use to increase, maintain, or decrease one or more components of an emotional response'.
>
> (Gross, 2001).

Increasing components of an emotional response is called up-regulation of an emotion, and decreasing these components is called down-regulation of an emotion. The components he considers are (1) the *experiential* component, (the subjective feeling of the emotion), (2) the *behavioral* component (behavioral responses), and (3) the *physiological* component (responses such as heart rate and respiration). Humans use strategies to affect their level of emotional response for a given type of emotion, for example, to prevent a person from having a too high emotional or too low emotional response level. He differentiates between antecedent-focused strategies and response-focused strategies. *Antecedent-focused strategies* are applied to the process preparing for response tendencies before they are fully activated. *Response-focused strategies* are applied to the activation of the actual emotional response, when an emotion is already underway.

In his model, Gross distinguishes four different types of antecedent-focused emotion regulation strategies, which can be applied at different points in the process of emotion generation: *situation selection, situation modification, attentional deployment* and *cognitive change*. A fifth strategy, *response modulation*, is a response-focused strategy. Fig. 1 shows an overview of these strategies.

The first antecedent-focused emotion regulation strategy in the model is *situation selection*: a person chooses to be in a situation that matches the emotional response level the person wants to have for a certain emotion. For example, a person can stay home instead of going to a party, because he is in conflict with someone who is going to that party.

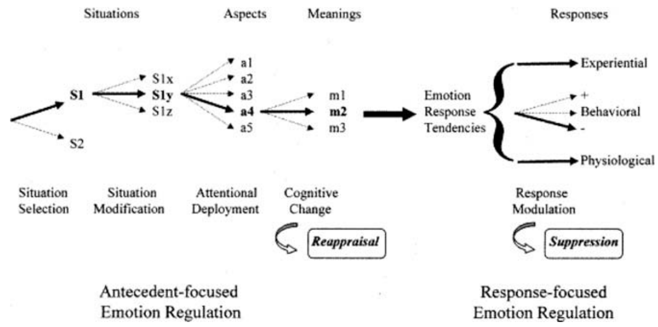

Fig. 1. Emotion regulation model by Gross (1998).

This is an example of down-regulating one's emotion (anger in this case). An example of situation selection to up-regulate one's emotion (excitement in this case) is taking a roller-coaster ride.

The second antecedent-focused emotion regulation strategy in the model is *situation modification*. When this strategy is applied, a person modifies an existing situation so as to obtain a different level of emotion. For instance, when watching an irritating television program, one may zap to another channel.

The third antecedent-focused emotion regulation strategy is *attentional deployment*. This strategy refers to shifting your attention to a certain aspect. For example, one may close his eyes when watching an exciting penalty shoot-out. The fourth antecedent-focused emotion regulation strategy is *cognitive change*: selecting a cognitive meaning to an event. A specific type of cognitive change, which is aimed at down-regulating emotion, is *reappraisal*:

> 'Reappraisal means that the individual reappraises or cognitively re-evaluates a potentially emotion-eliciting situation in terms that decrease its emotional impact'.
>
> (Gross, 2001).

An example of reappraisal is a case when a person loses a tennis match and blames the weather circumstances, instead of his own capacities. However, note that cognitive change could also be aimed at up-regulating emotion.

The fifth emotion regulation strategy, *response modulation*, a response-focused strategy, is applied after the emotion response tendencies have been generated: a person tries to affect the process of response tendencies becoming a behavioral response. A specific type of response modulation, again aimed at down-regulating, is *suppression*:

> 'Suppression means that an individual inhibits ongoing expressive behavior'.
>
> (Gross, 2001).

An example of suppression is a person that hides being nervous when giving a presentation.

In his article, Gross (2001) predicts that early emotion regulation strategies are more effective then strategies that are applied at a later time point in the process. He describes an experiment of which the results support this prediction. In this experiment, participants are shown a short film of a disgusting arm amputation in three different conditions: the reappraisal condition, the suppression condition, and the neutral condition. In the reappraisal condition, participants were asked to think about the film in such a way that they would not respond emotionally (for instance as if it were a medical teaching film). In the suppression condition, participants were asked to hide their emotional reactions to the film. In the natural condition, participants were given no specific instruction.

The results showed that reappraisal decreased emotion experience and expressive behavior, and did not have an effect on memory, or the physiological response. Suppression decreased expressive behavior, but had no effect on the emotion experience. Moreover, it impaired memory, and increased the physiological response. In this paper, our focus is on antecedent-focused strategies (i.e., *situation selection, situation modification, attentional deployment* and *cognitive change*). Thus, the consequent-focused strategy *response modulation* is not considered. The main reason for this is that, according to Gross (2001), p. 216, this strategy is not very effective: it does influence behavioral and physiological responses, but does not affect experiential responses. Nevertheless, it would not be difficult to incorporate this strategy in the computational model, by treating it in a similar manner as the antecedent-focused strategies.

Cognitive-behavioral therapies have the purpose to facilitate beneficial use of emotion regulation strategies. They focus on cognitive aspects, as well as behavioral aspects. The behavioral part focuses on replacing counterproductive emotional driven behaviors with alternatives. This has a facilitating effect on beneficial use of situation selection, situation modification, and attentional deployment. The cognitive part focuses on substituting irrational negative appraisals for evidence-based appraisals. This has a facilitating effect on beneficial use of cognitive change (Campbell-Sills & Barlow, 2006).

3. Global overview of the model

This section provides a global overview of our emotion regulation model. Section 3.1 introduces the basic concepts used, and the relations between them. Section 3.2 introduces the modeling environment that was used to formalize these concepts and relations.

3.1. Basic concepts and relations

Gross has described his process model for emotion regulation informally (i.e., in natural language, and not in a computational or mathematical notation). In order to convert this informal model to a computational model, a number of (iterative) steps have been performed, according to standard modeling and simulation methodologies (Banks & Carson, 1984; Shannon, 1975). First, Gross' theory was carefully analyzed in order to extract the relevant *concepts*. Second, the *relations* between these concepts were identified (i.e., how do the different concepts depend on each other?). Third, the concepts and their relationships were *formalized* (in this case, in terms of mathematical concepts: variables with real numbered values and differential equations). Fourth, the resulting model was used to perform *simulation*. And fifth, the results of the simulation were *verified*. Below the results of the different steps are described in more detail.

Table 1
Strategies and elements addressed in the model.

Strategy	Corresponding element
Situation selection	Situation
Situation modification	Sub_situation
Attentional deployment	Aspect
Cognitive change	Meaning

As a first step, for any given type of *emotion* a number of variables have been introduced. For convenience, the model concentrates on one specific type of emotion. In principle, this can be any emotion, e.g., sadness, happiness, or anger.

In order to describe the regulation of such an emotion, the model takes into account a number of emotion regulation *strategies* that can be chosen. In the variant of the model as described in this paper, the four antecedent-focused emotion regulation strategies discussed by Gross are used (i.e., situation selection, situation modification, attentional deployment, and cognitive change). For the moment, response modulation is not considered. However, the model is generic in the sense that this set of strategies considered can easily be adapted. Based on the four strategies mentioned, in the formalization four corresponding *elements* k are introduced, denoting the objects that are influenced by the particular strategies (see Table 1).

In the model it is assumed that at each point in time, for each element k a certain choice is in effect, and this choice has a certain *emotional value* v_k attached. This emotional value contributes to the *emotion response level ERL* via an element-specific weight factor w_k, thereby taking into account a *persistency factor* β indicating the degree of persistence or slowness of adjusting of the emotion response level when new emotional values are obtained. Someone whose emotions can change rapidly (e.g., who stops being angry in a few minutes after a fight) will have a low β.

Humans are always searching for a certain level of emotion.[2] The location of this optimum varies per person and through time. For instance, there are people who love excitement and enjoy extreme sports or roller coasters, while others prefer a more quiet kind of recreation. However, the people who *do* like roller coasters generally do

[2] Although we use words like ‘searching for’ and ‘choose’ to describe this process, it is not claimed that this process is always a conscious, deliberate activity. The mechanism by which this ‘choosing’ is performed is described in detail in Section 4.2.

not want to sit in a roller coaster all the time, but after a period of excitement they usually switch to a calmer activity. This point of view is similar to theories of emotion that are based on the idea of *homeostasis*, i.e., the perspective that the human body continuously tries to keep certain (physiological) variables between a certain range (e.g., Cañamero, 1997; Velasquez, 1997).

The level of emotion aimed at depends also on the type of emotion. Most humans aim at a relatively high level of emotion for happiness, while they aim at a lower level of emotion for fear. The regulation process starts by comparing the actual emotion response level *ERL* to the *emotion response level aimed at* ERL_{norm}. The *difference d* between the two is the basis for adjustment of the choices made for each of the elements *k*; based on these adjusted choices, each element *k* will have an adjusted emotional value v_n. The strength of such an adjustment is expressed by a modification factor α_n, which can be seen as a flexibility or willingness (conscious or unconscious) to change one's emotional value for a certain element. For instance, the α for the element 'situation selection' can be seen as the flexibility to change one's situation.

In order to obtain a model that can adapt itself to various circumstances, the modification factors α_n have been made adaptable. The flexibility to 'choose'[3] different emotional values v_n can be adapted to an assessment of the emotion regulation process: a sort of reflection or meta-cognition about the emotion regulation process based on the history of differences *d*. The *adaptation factor* γ_n mediating this process represents the personal flexibility to adjust the emotion regulation behavior based on such an assessment. It takes some effort to change behavior in favor of emotion regulation. This effort, or the costs of adjusting the modification factor for element *n*, is represented by c_n. Table 2 shows a summary of all the treated variables.

Some of these variables were chosen to be set at forehand and remain constant during the process (in particular ERL_{norm}, β, w_n, c_n, γ_n). The other variables depend on each other and on the fixed variables, as shown in a qualitative manner in the graph depicted in Fig. 2. Note that the model contains two cycles. One is the basic emotion regulation cycle from the v_n to ERL via d back to the v_n. The other one is the adaptation cycle from the α_n to the basic regulation cycle and back (via v_n, *ERL* and *d* back to α_n). Note that the basic regulation cycle is described literally by Gross (see, e.g., (Gross, 1998), Fig. 4). He does not explicitly describe the adaptation cycle, but he refers several times to 'individual differences' in terms of people's preferred regulation strategies (p. 278–279, 281), which was our motivation to introduce parameter α.

Fig. 2 shows that the emotion response level *ERL* is affected by the emotional values v_n for the different elements, the weights w_n attached to these elements, and the persistency factor β that indicates to what extent the previous response level affects the current one. The difference *d* between response level and norm obviously depends on both of these factors. Finally, the emotional values v_n for the different elements are affected by this difference *d* and the modification factor α_n.

In the model described so far, the modification factors α_n could be taken fixed. In order to obtain a model that can adapt itself based on past experiences (with respect to the successfulness of the chosen strategies), however, the flexibility in adjusting emotional values v_n as expressed by the modification factors α_n need to be adaptable. For example, when a subject is adjusting its behavior all the time in order to obtain certain emotion levels aimed at, this may result in a stressful and energy-consuming life. In such a case it is useful if the emotion regulation process can adapt itself to obtain a more peaceful mode of functioning (Wehrle, 1998, p. 5). To obtain such adaptive capabilities, the flexibility to 'choose'[2] situations, sub-situations, attention focuses and cognitive meanings with different emotional values v_n, as expressed by the modification factors α_n can be adapted to an assessment of the emotion regulation process: a sort of reflection or meta-cognition about the emotion regulation process based on the history of differences d. The *adaptation factor* γ_n mediating in this adaptation

Table 2
Summary of All Variables.

Variable	Meaning
ERL	Level of emotion
ERL_{norm}	Optimal level of emotion
d	Difference between *ERL* and ERL_{norm}
β	Slowness of adjustment *ERL*
w_n	Weight of element n in adjusting the *ERL*
v_n	Chosen emotional value for element *n*
α_n	Modification factor that represents the 'willingness' to change the emotional value of element *n*
γ_n	Personal tendency to adjust the emotional value of element *n* much or little
$\gamma_{basic\ n}$	Basic personal tendency to adjust the emotional value of element *n* much or little
c_n	Costs of adjusting emotional value v_n
Event	Value of an event that reflects the impact it has on personal tendency γ
ζ_n	Variable that determines the speed with which events influence personal tendencies
Δt	Time step

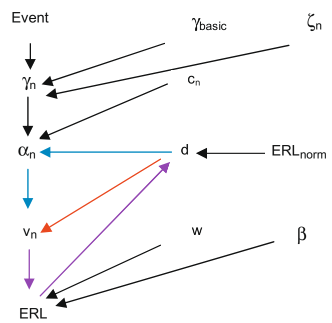

Fig. 2. Dependencies between the variables. The basic emotion regulation cycle has been indicated by the red arrows, and the adaptation cycle by the blue arrows. Arrows that are part of both cycles are colored purple. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

process represents the personal flexibility to adjust the emotion regulation behavior based on such an assessment. This adaptation factor is not described literally by Gross, but several other authors emphasize the importance of such a flexibility (e.g., Schore, 2001; Wehrle, 1998). With changing one's behavior, there are always costs or effort involved. These costs of adjusting the modification factor for element n are represented by c_n.

3.2. Modeling approach

Modeling the various aspects introduced above in an integrated manner poses some challenges. On the one hand, qualitative aspects have to be addressed, such as decisions to regulate one's emotion (e.g., by selecting a different situation). On the other hand, quantitative aspects have to be addressed, such as levels of emotional response.

The modeling approach based on the modeling language LEADSTO (Bosse, Jonker, van der Meij, & Treur, 2007c) fulfils these desiderata. It integrates qualitative, logical aspects such as used in approaches based on temporal logic (e.g., Barringer, Fisher, Gabbay, Owens, & Reynolds, 1996) with quantitative, numerical aspects such as used in Dynamical Systems Theory (e.g., Ashby, 1960; Port & van Gelder, 1995). Direct temporal dependencies between two state properties in successive states are modeled by *executable dynamic properties* defined as follows. Let a and b be state properties of the form 'conjunction of ground atoms or negations of ground atoms', then the notation $a \twoheadrightarrow_{e,f,g,h} b$ means:

If state property a holds for a certain time interval with duration g,

then after some delay (between e and f) state property b will hold for a certain time interval of length h.

Atomic state properties can have a qualitative, logical format (e.g., *desire*(d), expressing that desire d occurs), or a quantitative, numerical format (e.g., *has_value*(x, v) expressing that variable x has value v).

For a precise definition of the LEADSTO format in terms of the language TTL, see (Bosse, Jonker, van der Meij, Sharpanskykh, and Treur, 2009). A specification of dynamic properties in LEADSTO format has as advantages that it is executable and that it can often easily be depicted graphically. The LEADSTO format has also shown its value especially when temporal or causal relations in the (continuous) physical world are modeled and simulated in an abstract, non-discrete manner. Another advantage is that it is compatible with (and is a sublanguage of) the Temporal Trace Language (TTL) verification environment (Bosse et al., 2009). As a result, simulation traces generated in the LEADSTO environment can be directly taken as input by the TTL environment, in order to perform formal analysis of these traces. However, in the current paper, LEASDTO is mainly used as a vehicle. We do not claim that this is the only possible way to formalize our model.

4. The detailed model

The emotion response level and the emotional values for the different elements for a given type of emotion are represented in the model by real numbers in the interval [0, 2] (where 0 is the lowest possible emotion response level and 2 the highest). Although, obviously, a large simplification, this point of departure is quite common in affective computing, especially in the area of Artificial Intelligence (e.g., Gmytrasiewicz & Lisetti, 2002; Hudlicka, 2002; Velasquez, 1997). Moreover, a fixed level of emotion to aim at is assumed (the *ERL norm*), also expressed in a real number in the domain [0, 2]. This assumption has similarities with the approach taken by Cañamero (1997), who assumes a certain range within which agents should keep certain variables. As a simple illustration, suppose one wants to influence its state of anger by selecting an appropriate situation, and one deliberates whether to go to a party or not. This can be represented by introducing two different situations *sit1* and *sit2*, for example with $v_{sit1} = 1.5$ (since going to the party will increase the state of anger) and $v_{sit2} = 0.5$ (staying home will decrease the state of anger). Moreover, the *ERL norm* can for instance be 0.7 (i.e., one aims at being a bit angry, but not too angry). In that case, if one's current *ERL* is already high, one will be likely to stay home (i.e., choose *sit2*), and vice versa.

The process of emotion regulation has a continuous, interactive and cyclic nature. At any point in time, the characteristics of the current situation affect a person's emotional response level. Meanwhile, this emotional response level affects the person's choice for the emotional values v_n, which in turn influence the current situation (see also the cycle in Fig. 2). An approach to model such a process is the Dynamical Systems Theory (DST) based on differential equations; e.g., (Port & van Gelder, 1995). To use differential equations for simulation, some form of *discretization* is needed. Therefore, instead of differential equations, a set of difference equations is used, with a fixed step size s, that can be taken any size as desired (as long as $s \leqslant 1$).

4.1. Updating the emotion response level

Based on the above ideas, the emotion response level is recalculated each step by the following difference equation formula:

$$new\_ERL = (1 - \beta) * \Sigma n(w_n * v_n) + \beta * ERL$$

In this formula[3], *new_ERL* is the new emotion response level, and *ERL* is the old emotion response level. The persistency factor β is the proportion of the old emotion response level that is taken into account to determine the

[3] Note that the formula can also be rewritten into the following difference equation format. This format shows more explicitly how β determines the speed of adaptation of *ERL* to the new contribution $\sum_k w_k * v_k$; here Δt is taken 1. $\Delta ERL = (1 - \beta) * \sum_k (w_k * v_k) - ERL)\ \Delta t$ with $\Delta ERL = new\_ERL - ERL$.

new emotion response level. The new contribution to the emotion response level is determined by the emotional impact of the 'chosen'[3] situation, subsituation, attention focus and cognitive meaning. This is calculated by the weighted sum of the emotional values: $\Sigma_n\, w_n * v_n$. By normalization, the sum of all the weights w_n is taken to be 1. According to the indication of Gross (2001), elements that are affected at an earlier point in the emotion regulation process have higher weights. Within the simulation model, the update of the emotional response level is expressed by the following dynamic property in LEADSTO format (where s is the step size):

LP1 (Update Emotion Response Level)

emotion_response_level(erl)
and has_weight(situation, w1)
and has_weight(sub_situation, w2)
and has_weight(aspect, w3)
and has_weight(meaning, w4)
and has_emotional_value(situation, v1)
and has_emotional_value(sub_situation, v2)
and has_emotional_value(aspect, v3)
and has_emotional_value(meaning, v4)
$\twoheadrightarrow_{0,0,s,s}$ emotion_response_level((1 − beta) ∗ (w1 ∗ v1 + w2 ∗ v2 + w3 ∗ v3 + w4 ∗ v4) + beta ∗ erl)

The remaining formulas will only be shown in mathematical format. The dynamic properties in LEADSTO will be shown in Appendix A.

4.2. Updating the emotional values

The chosen emotional values v_n, which affect the emotion response level, are on their turn recalculated each step by the following set of difference equations:

$$d = ERL - ERL\_norm$$
$$\Delta v_n = -\alpha_n * d/d_{\max}\Delta t$$
$$new\_v_n = v_n + \Delta v_n$$

These formulas manage that a situation, subsituation, attention focus and cognitive meaning are chosen that better fit the desired level of emotion. The speed with which the emotional values are changed is determined by the willingness to change behavior in favor of emotion regulation. In these formulas, new_v_n is the new emotional value, and v_n is the old emotional value, while Δv_n is the change of the emotional value v_n (either positive or negative), and Δt the time step, which is taken 1 in this paper. The change in the emotional value v_n is calculated by the formula $-\alpha_n * d/d_{max}$. In this formula, α_n is the modification factor, and d is the difference between the actual emotion response level and the desired emotion response level (represented by ERL_{norm}). Here d_{max} is an estimation of the maximum difference that can be reached. So d/d_{max} is the proportion of the maximal reachable level of emotion above the level of emotion aimed at (or below this level, if d is negative).

When the actual emotion response level equals the desired emotion response level, then $d = 0$; this means that $\Delta v_n = 0$, so the emotion response level will not change. Moreover, when $d \neq 0$, a person will 'choose' an element with a more extreme emotional value v_n (i.e., Δv_n will be bigger), when (s)he is more flexible in this emotional value v_n (this is the case when α_n is high), or when (s)he experiences an emotion response level that is further away from the desired emotion response level (this is the case when d deviates more from 0).

4.3. Adaptation of the modification factors

In order to be able to simulate adaptive emotion regulation in the detailed computational model, the success of emotion regulation over a period of time is evaluated, and based on this evaluation, the willingness to change behavior in favor of emotion regulation can be adjusted. The following evaluation function is used:

$$Eval(d_{t-t} + p) = mean(abs(d))_{t-t} + p$$

To evaluate the emotion regulation process over the time points t until $t + p$ (where currently $p = 5$), the absolute difference of the actual level of emotion and the level of emotion aimed at is taken for all time points. The (arithmetic) mean value of these absolute differences gives the value of the evaluation function.

Until the model has done enough steps to perform this evaluation function for two different periods of time, the $\alpha_k^{'}$s are kept constant. After that, the evaluation function is used to adjust the modification factors α_k using the following difference equations:

$$\Delta\alpha_n = \gamma_n * (\alpha_n/1 + \alpha_n) * ((Eval(new\_d)/Eval(old\_d)) - c_n)\Delta t$$
$$new\_\alpha_n = \alpha_n + \Delta\alpha_n$$

In these formulas, new_α_n is the new modification factor α_n and γ_n represents in a numerical manner the personal flexibility to adjust the emotion regulation behavior. When γ_n increases, in a proportional manner $\Delta\alpha_n$ will increase, and α_n will change more. The part $\alpha_n/1 + \alpha_n$ assures that $\Delta\alpha_n$ is more or less proportional to α_n. The denominator $1 + \alpha_n$ prevents α_n from under- or over-adaptation when it gets very high. Furthermore, new_d is the mean value of d in the last time interval, and old_d is the mean value of d in an older time interval. The ratio $Eval(new\_d)/Eval(old\_d)$ will be smaller, if the actual level of emotion response deviated relatively more from the level of emotion aimed at in the older interval than in the newer interval. Currently, for the new interval the interval from $t - 5$ to t is taken, with t the current time point, and for the old interval the interval from $t - 10$ to $t - 5$. If $Eval(new\_d)/Eval(old\_d)$ is smaller, $\Delta\alpha_n$ will be lower. Finally, c_n represents the costs of adjusting the modification factor for element n. When there are higher costs to adjust α_n, the value c_n is higher, and $\Delta\alpha_n$ will be lower. However, due to the prevention from under- or over-adaptation, α_n will never reach a value under 0, even with high costs.

4.4. Adding the possibility to simulate events like trauma's and therapies

In order to simulate events that can change the personal tendency to adjust behavior in favor of emotion regulation (γ_n), we have chosen to express these events in real numbers in the domain $[-1, 1]$. If an event has a high value, for instance a successful therapy, it will lead to a higher tendency to adjust behavior in favor of emotion regulation. If the value gets closer to 0, it will have a smaller effect, and when it reaches 0 it will have no effect at all. An event with a negative value, for instance a trauma, will result in a lower tendency to adjust emotion regulation behavior. The following formula is used to let events influence the tendency to adjust behavior in favor of emotion regulation:

$$\Delta\gamma_n = \xi_n * Event/(1 + (\gamma_n - \gamma_{basic\,n}) * Event) * \Delta t$$
$$new\_\gamma_n = \gamma_n + \Delta\gamma_n$$

In these formulas, new_γ_n is the new personal tendency, and γ_n is the old personal tendency. $\Delta\gamma_n$ is the change of γ_n. The new γ_n is derived by adding $\Delta\gamma_n$ to the old γ_n. The variable 'Δt' is the time step, which is taken 1 in this paper. ζ_n is a variable that determines the speed with which the personal tendencies are adjusted by events. We performed simulation experiments which showed that a ζ_n in the range 0.10–0.20 produced the most realistic simulations. *Event* is the value that is attributed to a particular event that is simulated in the model. $\gamma_{basic\ n}$ is a person's basic personal tendency to change its behavior in favor of emotion regulation. Assumed is that a person is born with a basic personal tendency to change behavior in favor of emotion regulation, and this personal tendency can be changed by events. However, when the γ_n deviates more from $\gamma_{basic\ n}$, and an event influences γ_n to deviate even more from $\gamma_{basic\ n}$, γ_n will be changed less than when it is influenced by an event with the same strength in the different direction. So for instance, when a person has a very low $\gamma_{basic\ n}$, but a series of events made the γ_n rise to a much higher level, an *Event* with the value 0.5 will make the γ_n raise only a little bit more, while an *Event* with the value -0.5 will make the γ_n decrease significantly. In other words, events can change a person's personal tendency, but it gets harder when the personal tendency has already changed much.

5. Simulation results

In order to test whether the model produces realistic behavior for different circumstances, a number of experiments (under different parameter settings) have been performed. Each subsection below addresses a specific type of scenario. Various types of cases are addressed: those where the agent (or person) shows an optimal form (compared to the emotion response level aimed at) of regulation (Section 5.1), where it performs over- or under-regulation (Section 5.2), where it adapts its emotion regulation strategies (Section 5.3), where it experiences a successful anger management therapy, and where it changes its personal tendency to change behavior in favor of emotion regulation (Section 5.4). The different scenarios are established by taking different settings for some of the parameters involved (in particular, in the first two subsections, for the modification factors α_n). The values of the other variables are the same for all experiments described in this section, see Table 3.

As shown in the table, the person considered has an optimal level of emotion of 0.5 in the domain [0, 2]. The factor β is set to 0.7, which means that in each step, 70% of the old emotional response level persists, and the remaining 30% is determined by the new emotional values. The weight attached to situation selection is 0.35, which means that the selected situation determines 35% of the 30% of the new emotion response level that is determined by the emotional values. Similarly, the weights for situation modification, attentional deployment, and cognitive change are set to 0.30, 0.20, and 0.15, respectively. The results of the experiments are shown and explained below. By comparing these traces with predictions made by Gross, we perform an initial check on the validation of the model (see also Section 6). If such a check is successful, in a next step the model may be used for different applications (both from an Artificial Intelligence and a Cognitive Science perspective), as mentioned in the introduction.

5.1. Optimal forms of emotion regulation

5.1.1. Experiment 1

For the first experiment, we experimented with various values of the α_n. With all modification factors α_n set to 0.15 the emotion regulation seemed optimal. The results of this simulation experiment are shown in Fig. 3. In such figures, time is on the horizontal axis; the values of the different variables are shown on the vertical axis.

As shown in the upper graph, the emotional response level decreases monotonically without decreasing below the level aimed at. So, the subject gradually reaches his level of emotion aimed at. The emotional values show similar behavior.

5.1.2. Experiment 2

In the second experiment, we experimented with setting the α_n at different levels for each strategy. This should result

Table 3
Values of variables used in the simulations.

Variable	Fixed value	Variable	Initial value
ERL_{norm}	0.5	ERL	1.85
β	0.7	v_1	1.90
w_1	0.35	v_2	1.85
w_2	0.30	v_3	1.80
w_3	0.20	v_4	1.75
w_4	0.15		
s	1		

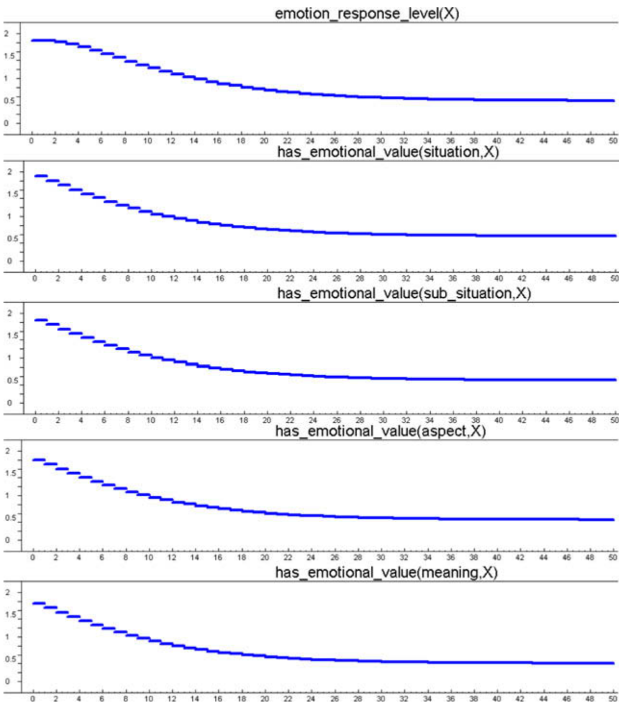

Fig. 3. Results for an optimal case (equal α_n).

in a preference for certain emotion regulation strategies. In this experiment, the subject has the following flexibilities α_n in emotion regulation:

$$\alpha_1 = 0.20, \quad \alpha_2 = 0.15, \quad \alpha_3 = 0.10, \quad \alpha_4 = 0.05$$

The results of this experiment are shown in Fig. 4. Here, the emotion response level reaches the emotion response level of 0.5 aimed for in a reasonable amount of time, just like in the optimal case. However, the way the emotional values change in order to achieve this differs from the first experiment. Here, it is important to note that the scale on the vertical axis is not the same for the different graphs in Fig. 4. The graphs show that the emotion response levels of the elements with a higher α descend much quicker and further than the elements with a lower α. For example, situation selection ($\alpha = 0.20$) has reached an emotional value of 0 at the end of the simulation, whereas cognitive change ($\alpha = 0.01$) changes only a little bit, and reaches an emotional value of about 1.3. This means that the subject finds a way to reach his/her level of emotion aimed for, and does this by changing his/her behavior more for the elements for which (s)he has a higher flexibility. So indeed, these settings resulted in the subject having a preference for certain emotion regulation strategies.

5.2. Over- and under-regulation

5.2.1. Experiment 3

In the third experiment, we set the modification factors for all elements α_n to a very high-level: 0.4. This means that the subject has a relatively high flexibility in emotion regulation, for all elements. This should result in a too high

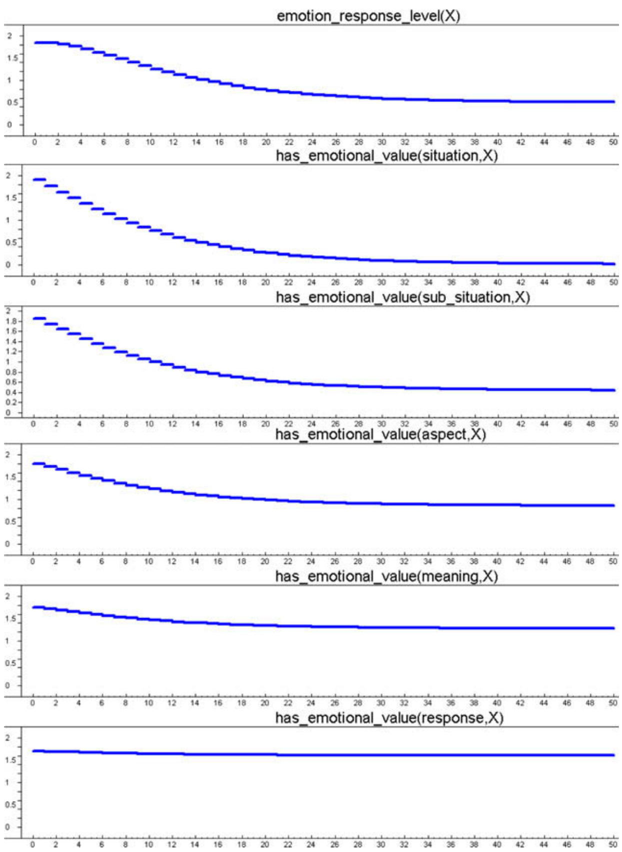

Fig. 4. Results for an optimal case (different α_n).

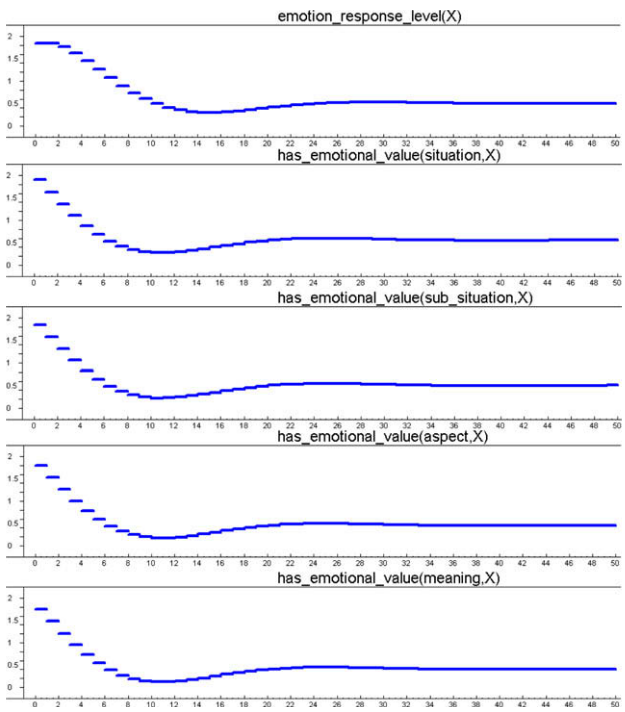

Fig. 5. Results for the over-regulation case.

adjustment of behavior: over-regulation. The behavior of the emotion response level in this experiment is shown in Fig. 5.

In this case, the emotion response level starts to decrease rapidly, immediate after the experiment has started. However, it decreases below the level of 0.5 aimed at. It reaches its minimum after 15 steps in the simulation, at about 0.3: the subject over-regulates his/her emotion. After this, the emotion response level starts to rise until it is just above the optimal level of 0.5, and stays more or less at this value aimed at for the rest of this simulation. This confirms that setting the flexibility in emotion regulation to a very high-level results in over-adaptation.

The lower part of Fig. 5 shows how the subject changed his/her emotional values in order to achieve this. These emotional values all show similar behavior, since the α_n's, which represent the flexibility and willingness to change behavior, were set to the same value. Also, the graphs of the emotional values are comparable to the graph of the emotion response level. The emotional values make a somewhat steeper curve, especially at the start of the graph. This makes sense, because the emotion response level is only for 30% determined by the emotional values, and for 70% by its own old value.

5.2.2. Experiment 4

In the fourth experiment presented, we set the flexibility in emotion regulation to a very low level, with the α_n set to a static level of 0.01 for all elements. A very low flexibility in emotion regulation should result in under-regulation. The results of this experiment are shown in Fig. 6.

In this experiment, the emotion response level decreases extremely slowly, so this indeed leads to under-regulation. After 50 steps, it has only decreased by 0.3 until 1.55, as can be seen in the graph. After 50 steps, the emotion response level continued decreasing gradually slower over time.

5.3. Adaptive case

5.3.1. Experiment 5

To test the model for adaptivity of the emotion regulation process, some more experiments have been performed. In this experiment, the modification factors $\alpha_1 - \alpha_4$ are dynamic. Initially, they have a value of 0.01, the same as in the under-regulation experiment (Experiment 4). However, in this experiment the α_n can be adjusted by evaluating the success of the emotion regulation in the past. This adaptivity should prevent the subject from under-regulation, and enable the subject to successfully regulate its emotions. In this experiment, all adaptation factors γ_n were set to 0.09 and the costs c_n were set to 0.7, 0.4, 0.4, and 0.6, respectively. The subject starts with a very high *ERL* of 1.8, and very high emotional values, all set to the same level of 1.8. The weights attached to the various elements are the same as earlier. The result for the emotion response value is shown in Fig. 7.

In this experiment, the ERL starts to decrease faster after a period of time. While with static α_n (i.e., without adaptivity in emotion regulation), this lead to under-regulation, with adaptive α_n, the optimal level of emotion, 0.5, is reached after 100 steps (see Fig. 7). After these 100 steps the *ERL* reached an equilibrium and did not increase or decrease significantly anymore. This confirms the hypothesis that adaptivity in emotion regulation can prevent the subject from under-regulation.

The emotional values v_n show a pattern similar to the *ERL*s over time. Some emotional values obviously decrease faster than others. The emotional value for situation selection decreases only until 1.1, and the emotional value for cognitive meaning until 0.8, while the emotional values for situation modification, attentional deployment, and response modification decrease until they are almost 0. The costs c_n for situation selection are 0.7, and for cognitive meaning 0.6, while the costs for the other elements are 0.4. These results show that the emotional values for elements with higher costs are changed less. Fig. 8 shows how the modification factors α_n change over time.

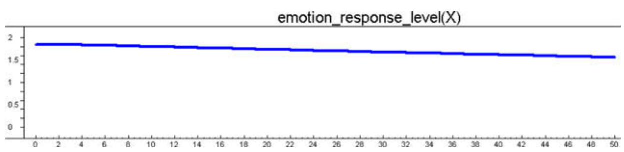

Fig. 6. Results for the under-regulation case.

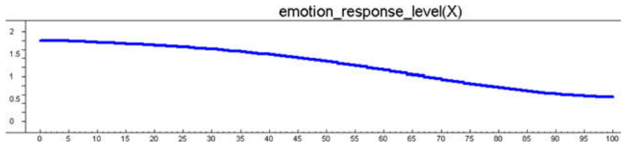

Fig. 7. Results for an adaptive regulation case ($\gamma_k = 0.09$).

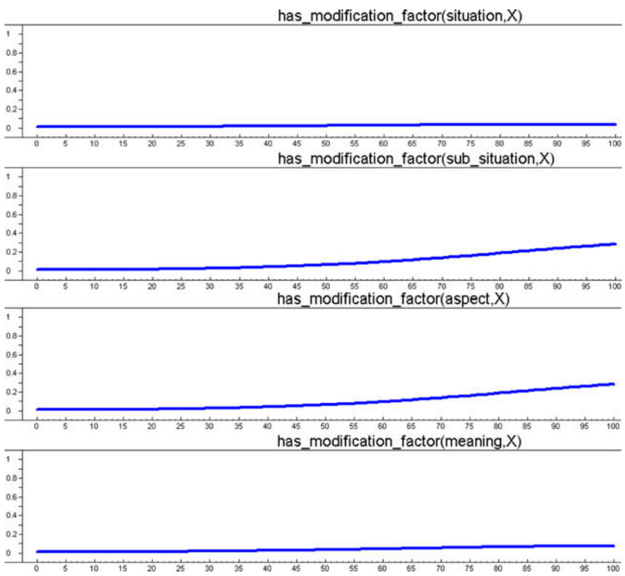

Fig. 8. Modification factors for the adaptive case.

All modification factors show similar behavior, the main difference is that the modification factors with higher costs, situation selection and cognitive meaning, rise much less than the other modification factors. While the other three modification factors rise to a value of 0.3, the modification factor for meaning rises only until around 0.08, and the modification factor for situation selection only until around 0.04. Also, the modification factor for situation selection starts to decrease again, before the simulation stops. This is the effect of the higher costs to change behavior in situation selection or cognitive change.

5.4. The effect of events

5.4.1. Experiment 6

A number of experiments have been performed, to test whether the model can simulate the effects of events like traumas or therapies on emotion regulation. The variables that were used in the experiments are summarized in Table 4. The values of the fixed variables and the initial values of the remaining variables are shown (note that this table contains some overlap with Table 3).

The simulated emotion in the experiments in this subsection is anger, and the agent's optimal level of anger is 0.5. The agent starts with a very high emotion response level of 1.8, and very high emotional values, all set to the same level of 1.8. So at the start of the simulation, our agent is very angry, and is in a situation that keeps him angry. The weights, and the costs attached to the various elements are set to the same values as in the previous experiments. The γ_n's, which represent the personal tendency to change behavior in favor of emotion regulation, are initially set to 0.01, which is a very low value. Another simulation experiment showed that without introducing any events during the simulation, this would lead to under-regulation. Therefore, in this case events were introduced. The value for all $\gamma_{basic\ n}'$s is set to 0.05. This is somewhat lower than average, which means that by nature the agent has a relatively low personal tendency to change behavior in favor of emotion regulation. The initial γ_n is even lower, which means that before the start of the simulation, the agent has had some experiences, for example a trauma, which decreased its personal tendency to change behavior in favor of emotion regulation. The ζ_n are all set to 0.15. Simulation experiments showed that this is a normal value. In the experiments in this chapter, the manipulated variables are the events that influence the personal tendency to change behavior in favor of emotion regulation.

In this simulation, we let our agent experience an event that will increase its personal tendency to change behavior in favor of emotion regulation very much: a successful cognitive-behavioral anger management therapy. This event takes place at time point 40, and has the value of 0.9 in the domain $[-1, 1]$. This should stop the subject from under-regulating its emotions, and enable it to reach its optimal level of emotion at the end of the simulation.

The results of this simulation can be seen in Fig. 9. Because all emotional values and modification factors show similar behavior, and the only difference is made by the costs, only the graphs of the element with the highest costs, situation, and of one of the elements with the lowest costs, subsituation, are shown.

As can be seen in Fig. 9, the emotion response level first decreases very slowly. Later in the simulation, after the therapy has taken place, the emotion response level starts

Table 4
Values of variables used for simulations in Sections 5.4.

Variable	Value	Fixed/initial
ERL_{norm}	0.5	Fixed
β	0.7	Fixed
ERL	1.8	Initial
$v_1 - v_4$	1.8	Initial
w_1	0.35	Fixed
w_2	0.30	Fixed
w_3	0.20	Fixed
$\alpha_1 - \alpha_4$	0.01	Initial
c_1	0.7	Fixed
c_2	0.4	Fixed
c_3	0.4	Fixed
c_4	0.6	Fixed
$\gamma_1 - \gamma_4$	0.01	Initial
$\gamma_{basic\ 1} - \gamma_{basic\ 4}$	0.05	Fixed
$\zeta_1 - \zeta_4$	0.15	Fixed

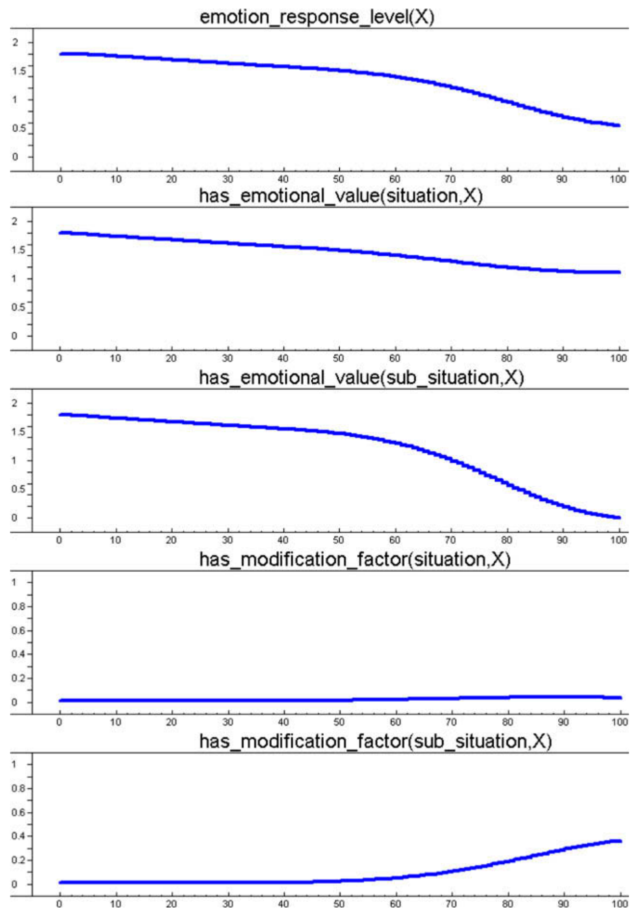

Fig. 9. Simulation of the ERL, the emotional values and the modification factors in Experiment 6.

to descend more quickly, and at the end of the simulation the optimal level of emotion is reached. So at the beginning of the simulation, the agent is not able to let its level of anger decrease to its optimal level, but at the end of the simulation it is. This confirms the hypothesis that introducing a positive event, such as a successful cognitive-behavioral anger management therapy, can enable a person to successfully regulate its emotions.

This can also be seen in the simulations of the emotional values. First these values decrease very slowly, and later in the simulation they decrease much quicker. This is the clearest in the emotional values of elements with lower costs, such as subsituation. These emotional values decrease much more than the emotional values of elements with higher costs. At the end of the simulation, the emotional values of elements with lower costs have almost reached 0, while the emotional value of situation selection, with higher costs, has decreased only until 1.1.

The modification factors α_n increase very slowly at the beginning of the simulation, and start to increase more quickly after the therapy has taken place. The modification factors of elements with lower costs increase much quicker than the elements with higher costs. At the end of the simulation, the modification factors of elements with lower costs have increased until 0.37, while the modification factor of situation selection, with higher costs, has increased only until 0.042, and has started to decrease again.

It makes sense that the modification factors increase very slowly at first, and start to increase quicker at a later time point in the simulation. At time point 40, an event takes place, which makes the personal tendency to change behavior in favor of emotion regulation, represented by the γ_n, rise from 0.01 to 0.15. These γ_n have a direct effect on the modification factors, as can be seen in Fig. 8. Especially in the simulation of the modification factor of situation selection, the impact this has on the modification factors can be seen. A few steps after time point 40, the modification factor starts to increase much quicker. So after the anger management therapy, it immediately starts to increase its willingness to change its behavior in favor of emotion regulation.

The impact this has on the emotional values can be seen very clearly. After time point 40, the emotional values start to decrease much quicker. After a while, the emotion response value has decreased enough to make the emotional value for situation selection decrease more slowly again. So our agent is not able to reach its optimal level of emotion by choosing different situations, cognitive meanings, etc. in the first part of the simulation, but after the anger management therapy at time point 40, it starts to change its behavior, and at the end of the simulation it has reached its optimal level of emotion, and is able to keep it stable.

5.4.2. Experiment 7

In this simulation, we let our agent experience various events that change its personal tendency to change behavior in favor of emotion regulation, which are shown in Table 5.

Table 5
A summary of the events in Experiment 7.

Event at time point 20	0.9
Event at time point 40	−0.9
Event at time point 60	0.4
Event at time point 80	−0.3

First, at time point 20, the agent experiences an event that is just as ‘strong’ as the event at time point 40 in Experiment 6. So the same agent follows the same cognitive-behavioral anger management therapy with the same amount of success as in Experiment 6, only now it already takes place at time point 20. At time point 40, the agent experiences an event that is just as strong as the event at time point 20, but now in the opposite direction, so that it will decrease its personal tendency to change behavior

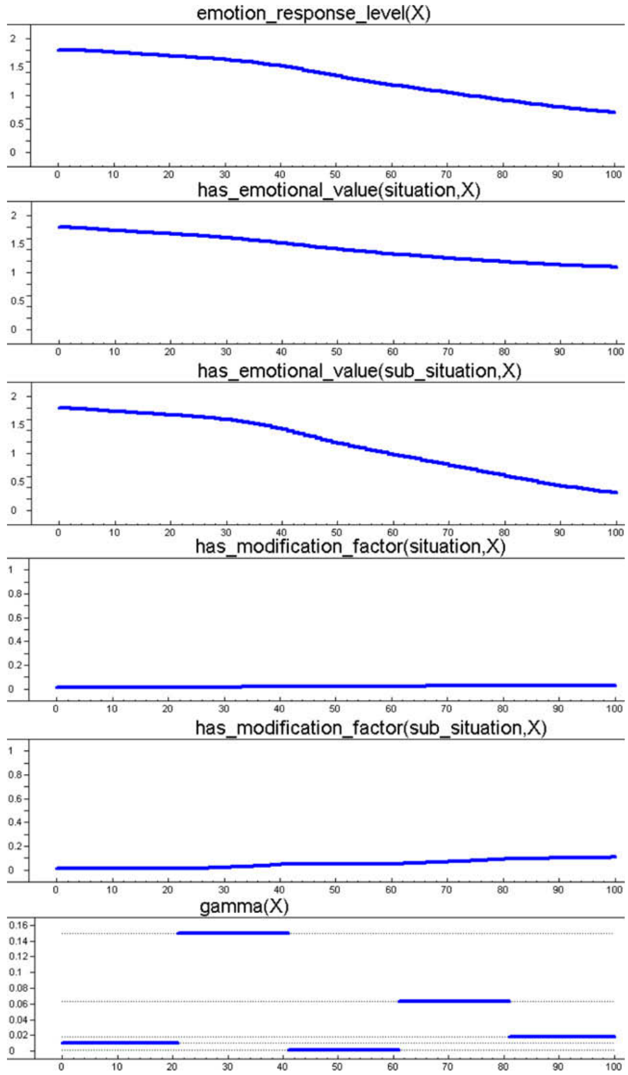

Fig. 10. Simulation of the *ERL*, the emotional values, the modification factors, and the γ_n in Experiment 7.

in favor of emotion regulation. In real life, this event could be for instance a severe traumatic experience in which an applied emotion regulation strategy has the opposite effect, which severely decreases the belief of the subject that trying to regulate its emotions will help it to reach its optimal level of emotion. At time point 60, the agent experiences a positive event with the strength 0.4, for instance a therapy that helps it deal with the traumatic event it experienced at time point 40. Finally, at time point 80, the agent experiences a negative event with the strength of 0.3.

The results of this simulation can be seen in Fig. 10. Again, because all emotional values and modification factors show similar behavior, and the only difference is made by the costs, only the graphs of the element with the highest costs, situation, and of one of the elements with the lowest costs, subsituation, are shown.

As can be seen in Fig. 10, the emotion response level decreases very slowly at the start of the simulation. After the anger management at time point 20, which makes the γ_n rise to 0.15, the emotion response level starts to decrease somewhat quicker. Because the traumatic event at time point 40 makes the γ_n decrease to an even lower level than it was at the beginning of the simulation, this decreasing trend does not proceed the way it did in Experiment 6. The effects of the less powerful events at time points 60 and 80 can clearly be seen by the kinks in the graphs of the modification factors, but the effects on the emotional values and the emotion response level are less clear. It can clearly be seen that the less powerful events have a smaller impact on the γ_n.

6. Verification of global properties

In the previous section, it was shown that the presented simulation model is capable of producing various simulation traces for different circumstances. Although intuitively these traces seem to show realistic patterns, no proof has been provided that they indeed match the predictions made by Gross' theory. However, obtaining such a 'proof' is not trivial (Parkinson, 2001; Reilly, 1996). In order to do this, different perspectives can be chosen. Ideally, the results of the simulation are compared in detail with empirical data obtained from experiments. Such experiments could, for example, involve a setup where participants are confronted with an undesirable situation (i.e., their emotional response value for happiness is manipulated in such a way that it is far below the *ERL norm*). Next, by means of modern techniques (e.g., 'face reading' devices, or EEG scans) their *ERL* could be measured, while keeping track of which emotion regulation strategy they use. However, these types of experiments have various drawbacks. For instance, current measurements of emotional states are still rather unreliable (Busso et al., 2004; Cowie et al., 2001; Gunes & Piccardi, 2007; Larsen & Fredricksen, 1999, chap. 3) and difficult to perform. A second type of validation would be to develop virtual characters, to endow them with the emotion regulation model presented here, and to ask human observers to judge whether their behavior is believable (Reilly, 1996). A drawback of this second type of validation is that building the virtual characters would be a waste of effort if one does not have a clue that the developed model is correct. Moreover, to build such agents, in addition to the presented emotion regulation model, also an emotion generation model would be needed.

For these reasons, as a first step, in the current paper a more modest form of validation is performed. The idea of this type of validation is that the literature by Gross (1998, 1999, 2001, 2002) is taken as a basis, and is carefully inspected in order to extract qualitative statements about the types of patterns that are expected to occur in emotion regulation processes. Next, these statements are interpreted and translated into expressions in a formal language in a step-wise manner (cf. Bosse, 2005, chap. 1). The resulting formulae (which are in fact 'formalized interpretations of Gross' theory') are then verified (using automated checking software) against the simulation traces, to check whether these traces show the expected behavior. Since Gross' theory itself is based on empirical evidence, this type of checking can be seen as some kind of 'indirect validation'.

The formal language that is used for this verification process is the Temporal Trace Language (TTL) (Bosse et al., 2009). This predicate logical language supports formal specification and analysis of dynamic properties, covering both qualitative and quantitative aspects. TTL is built on atoms referring to *states* of the world, *time points* and *traces*, i.e. trajectories of states over time. In addition, *dynamic properties* are temporal statements that can be formulated with respect to traces based on the state ontology *Ont* in the following manner. Given a trace γ over state ontology Ont, the state in γ at time point t is denoted by $state(\gamma, t)$. These states can be related to state properties via the formally defined satisfaction relation denoted by the infix predicate $|=$, comparable to the Holds-predicate in the Situation Calculus: $state(\gamma, t)| = p$ denotes that state property p holds in trace γ at time t. Based on these statements, dynamic properties can be formulated in a formal manner in a sorted first-order predicate logic, using quantifiers over time and traces and the usual first-order logical connectives such as , $\wedge$, $\vee$, $\Rightarrow$, $\forall$, $\exists$. A special software environment has been developed for TTL, featuring both a Property Editor for building and editing TTL properties and a Checking Tool that enables formal verification of such properties against a set of (simulated or empirical) traces.

Based on this TTL language, a number of Gross' informal statements about emotion regulation have been formalized in terms of dynamic properties. Below, a number of them are introduced, both in semi-formal and in informal notation (note that they are all defined for a particular trace and time interval between t_b and t_e):

P1a – Emotion approaches norm monotonically
For all time points t1 and t2 between tb and te in trace γ
if the value of the ERL norm is n
and at t1 the value of v is x1
and at t2 the value of v is x2
and t1 < t2
then $|n - x1| \geqslant |n - x2|$.

P1a(γ:TRACE, tb, te:TIME) $\equiv$
$\forall$ t1,t2:TIME $\forall$ x1,x2:REAL
state(γ, t1)| = erl_norm(n) &
state(γ, t1)| = emotion_response_level(x1) &
state(γ, t2)| = emotion_response_level(x2) &
tb $\leqslant$ t1 $\leqslant$ te & tb $\leqslant$ t2 $\leqslant$ te & t1 < t2
|n – x1| $\geqslant$ |n – x2|

By checking property *P1a*, one can check for a given trace whether the emotion response level eventually approaches the emotion norm (in a monotonic manner). Note that this property subsumes both upward and downward regulation of emotions, similar to Gross' interpretation: "I focus on five aspects of this definition of emotion regulation. First, individuals increase, maintain, and decrease negative and positive emotions..." (Gross, 1998, p. 275). For example, this property could be confirmed (for a down-regulation case) for the trace that resulted from Experiment 1: property *P1a(trace1, 0, 100)* succeeded. Overall, this property turned out to be satisfied for all generated simulation traces, except for the one produced in Experiment 3. This is conform expectations, since Experiment 3 addressed a case of over-regulation, which means that the emotion is regulated in such a way that the discrepancy between *ERL* and *ERL norm* may become larger.

P1b – Emotion approaches norm with speed s
For all time points t1 and t2 between tb and te in trace γ
if the value of the ERL norm is n
and at t1 the value of v is x1
and at t2 the value of v is x2
and t2 = t1 + 1
then $s * |n - x1| \geqslant |n - x2|$.

P1b(γ:TRACE, tb, te:TIME, s:REAL) $\equiv$
$\forall$ t1,t2:TIME $\forall$ x1,x2:REAL
state(γ, t1)| = erl_norm(n) &
state(γ, t1)| = emotion_response_level(x1) &
state(γ, t2)| = emotion_response_level(x2) &
tb $\leqslant$ t1 $\leqslant$ te & tb $\leqslant$ t2 $\leqslant$ te & t2 = t1 + 1
|n – x1| * s $\geqslant$ |n – x2|

Property *P1b* is a refinement of *P1a*. It can be used to check not only whether the *ERL* approaches the norm, but also to determine the speed *s* with which this happens (where $0 < s < 1$, and a high *s* denotes a slow speed). For the case of Experiment 1, this s turned out to be 0.87. The resulting values of *s* for the different simulation traces, as determined by the checks, are shown in Table 6. For Experiment 3, no regulation speed was found, since the *ERL* does not approach the norm for this case (see *P1a* above).

P2 – Early strategies are more effective
For all traces $\gamma1$, $\gamma2$, $\gamma3$, $\gamma4$,
if in $\gamma1$ only situation selection takes place
and in $\gamma2$ only situation modification takes place
and in $\gamma3$ only attentional deployment takes place
and in $\gamma4$ only cognitive change takes place
then the change in emotion response level over the first 10 time points is highest for $\gamma1$, followed by $\gamma2$, $\gamma3$, and $\gamma4$.

P2 $\equiv$
$\forall$ γ1,γ2,γ3,γ4:TRACE $\forall$t:TIME
$\forall$ x1b,x2b,x3b,x4b,x1e,x2e,x3e,x4e:REAL
state(γ1, t) | = has_modification_factor(situation, 1) &
state(γ1, t) | = has_modification_factor(sub_situation, 0) &
state(γ1, t) | = has_modification_factor(aspect, 0) &
state(γ1, t) | = has_modification_factor(meaning, 0) &
state(γ2, t) | = has_modification_factor(situation, 0) &
state(γ2, t) | = has_modification_factor(sub_situation, 1) &
state(γ2, t) | = has_modification_factor(aspect, 0) &
state(γ2, t) | = has_modification_factor(meaning, 0) &
state(γ3, t) | = has_modification_factor(situation, 0) &
state(γ3, t) | = has_modification_factor(sub_situation, 0) &
state(γ3, t) | = has_modification_factor(aspect, 1) &
state(γ3, t) | = has_modification_factor(meaning, 0) &
state(γ4, t) | = has_modification_factor(situation, 0) &
state(γ4, t) | = has_modification_factor(sub_situation, 0) &
state(γ4, t) | = has_modification_factor(aspect, 0) &
state(γ4, t) | = has_modification_factor(meaning, 1) &
state(γ1, 0) | = emotion_response_level(x1b) &
state(γ1, 10) | = emotion_response_level(x1e) &
state(γ2, 0) | = emotion_response_level(x2b) &
state(γ2, 10) | = emotion_response_level(x2e) &
state(γ3, 0) | = emotion_response_level(x3b) &
state(γ3, 10) | = emotion_response_level(x3e) &
state(γ4, 0) | = emotion_response_level(x4b) &
state(γ4, 10) | = emotion_response_level(x4e)
$\Rightarrow$ |x1b – x1e| > |x2b – x2e| & |x2b – x2e| > |x3b – x3e| &
|x3b – x3e| > |x4b – x4e|

Property *P2* was designed to verify Gross (2001), p. 218 prediction that "adjustments made early in the emotion trajectory are more effective than adjustments made later

Table 6
Speed by which *ERL* approaches the *ERL norm* (for different experiments).

Experiment	Speed *s*
1	0.87
2	0.88
3	–
4	0.99
5	1.00
6	1.00
7	1.00

on". In other words, situation selection results in a larger (and faster) change in emotional response level than situation modification, attentional deployment, and cognitive change. However, it makes no sense to check this property against the traces introduced earlier, since in these experiments multiple strategies were used simultaneously. Therefore, a number of simulation traces have been generated in addition to the ones shown before, such that in each of these traces only one strategy was performed at a time. This has been done by always setting the modification factor α to 1 for one of the strategies, and setting them to 0 for the other strategies. Given the new simulation traces, property *P2* turned out to be satisfied, which confirms Gross' prediction for these traces.

P3 – High strategy flexibility leads to large adjustments

For all traces γ, time points t, and strategies s1 and s2,
if at t the modification factor of s1 is higher than
the modification factor of s2
then at t + 1 the emotional value of s1 will have changed more
than the emotional value of s2.

P3 ≡
∀ γ:TRACE, ∀ t:TIME, ∀ s1,s2:STRATEGY
∀ α1,α2,v1a,v1b,v2a,v2b:REAL
state(γ, t) |= has_modification_factor(s1, α1) &
state(γ, t) |= has_modification_factor(s2, α2) &
state(γ, t) |= has_emotional_value(s1, v1a) &
state(γ, t + 1) |= has_emotional_value(s1, v1b) &
state(γ, t) |= has_emotional_value(s2, v2a) &
state(γ, t + 1) |= has_emotional_value(s2, v2b) &
α1 > α2
⇒ |v1a – v1b| > |v2a – v2b|

Since different individuals may differ in their preferred emotion regulation strategies (Gross, 1998, p. 279), it is useful to check whether more preferred strategies (i.e., in our model, strategies with a higher flexibility, expressed by a higher modification factor α) result in larger adjustments in the emotional values. This can be done by means of property *P3*. For example, is it the case that people who prefer situation selection over attentional deployment indeed perform situation selection more often? *P3* checks this for all combinations of strategies. It turned out to be satisfied for all generated simulation traces.

All in all, the above checks have pointed out that the presented model satisfies a number of relevant dynamic properties that were formulated on the basis of Gross' theory. Of course, this is by no means an exhaustive proof, but it is a first indication that the global behavior of the model is satisfactory. In addition, these kinds of checks allow the modeler to distinguish different groups of simulation traces from each other. For example, property *P1* enables one to separate situations of over-regulation from other cases. Note that this feature is useful not only for simulation traces, but also for empirical traces. Since the TTL checker also takes empirical traces as input (in case these are available), in principle the approach can be used to classify different traces obtained from real world experiments.

7. Discussion

Below, Section 7.1 provides a brief summary and conclusion about the presented work. Section 7.2 compares the approach with similar approaches in the literature, and Section 7.3 describes some possible directions of future research.

7.1. Conclusion

In this paper, a formal model for Gross' (informally described) model of emotion regulation has been introduced. The emotion regulation model has been constructed using the high-level simulation language LEADSTO as a modeling vehicle, and integrates both quantitative, dynamical system aspects (such as levels of emotional response) and qualitative aspects (such as decisions to regulate one's emotion). The model is adaptive, and gives the possibility to simulate the effects of events like trauma's and therapies on emotion regulation.

It is important to evaluate the relation of this model to the real world. Of course, a model is always a simplification of the real world. Emotion regulation is a complex process, and it would be overconfident to try to represent every aspect of emotion regulation in a computational model at this time point. The model is, however, able to simulate a simplification of an emotion regulation process, as illustrated in the simulation experiments described in this paper.

Simulation experiments have been performed for different situations. The first experiments were constructed by using different settings for the modification factors α_n: for ideal cases (all α_n are medium, or the α_n have different values), for cases of over-regulation (all α_n are high), and for cases of under-regulation (all α_n are low). The experiments show that different values for the modification factors α_k indeed result in different patterns. In these experiments, the subject uses the difference between its actual level of emotion, and its optimal level of emotion to 'choose' different situations, sub-situations, attentional aspects and cognitive meanings. In Experiment 1, the subject successfully regulates its emotions. In Experiment 2, the subject has a specific preference for adjusting certain types of behavior above others. These choices have an influence on its level of emotion, and by doing this repeatedly, the subject regulates its emotions. Also over-regulation (Experiment 3) in which the subject adjusts its behavior too much, and under-regulation (Experiment 4), in which the subject adjust its behavior too little, could be simulated.

By making use of a variable that represents the willingness to change behavior in favor of emotion regulation, adaptive emotion regulation can be simulated, as can be seen in Experiment 5. In this simulation, the subject evaluates its emotion regulation over a past period of time, and uses this to change its willingness to change behavior in favor of emotion regulation.

Finally, some experiments were performed to demonstrate that the model can simulate the effects of events that influence the personal tendency to change behavior in favor of emotion regulation, like trauma's, or therapies. In Experiment 6, the agent has at first a low tendency to change its behavior, and is because of this low tendency not able to reach its optimal level of emotion. After an anger management therapy, its personal tendency to change behavior in favor of emotion regulation has increased, and the agent is able to regulate its emotions, and reach its optimal level of emotion. In Experiment 7, a series of events influence the agent's emotion regulation. In this experiment, the relatively 'stronger' events have a bigger impact on the emotion regulation process. These results are consistent with the literature (e.g., Beck & Fernandez, 1998; Deschner & McNeil, 1986). Validation involving extensive comparison with detailed empirical data is left for future work.

As a preliminary validation of the model, the simulation results have been compared with the predicted behaviors for different situations as described by Gross (1998, 2001), which are (partly) based on empirical evidence. The patterns produced by the model were found consistent with Gross' descriptions of examples of human regulation processes. Validation involving extensive comparison with detailed empirical data is left for future work. When doing this, two types of validation will be performed, namely (1) aligning the results of the simulations with empirical data of human emotion regulation processes (see Section 6) and (2) incorporating the emotion regulation model with virtual characters, and asking human observers to judge their believability (see Reilly, 1996).

7.2. Related work

Although the process of emotion regulation is widely investigated in the literature (e.g., Eisenberg, 2000; Goldsmith & Davidson, 2004; Gross, 1998, 1999, 2001, 2002; Ochsner & Gross, 2005; Thompson, 1994), not so many contributions address the possibility of developing a computational model of this process. The computational models that have been developed so far either address some very specific aspects of the process at a more detailed (neurological) level, see e.g. (Thayer & Lane, 2000), or they aim at incorporating emotions into software agents, in which case they focus more on emotion elicitation (appraisal) in general (e.g., Armony, Servan-Schreiber, Cohen, & Ledoux, 1997; Bates, 1994; Burkitt & Romano, 2008; Dias & Paiva, 2005; Reilly, 1996; Reisenzein, 2009; Velasquez, 1997) than on emotion regulation, which can be seen as a sub-process of emotion elicitation. The current paper can be seen as an attempt to build a bridge between both directions. It provides a relatively coarse-grained model for emotion regulation using a high-level modeling language, but still in enough detail to be able to generate useful simulation traces, and to provide believable behavior of virtual agents.

As such, it has some similarities with the work by Gratch and Marsella (2004), Gratch, Marsella and Mao (2006), Marsella and Gratch (2002, 2003), who propose an approach to incorporate both appraisal and coping behavior into virtual humans. However, an important difference in that their approach takes appraisal theory as point of departure, which emphasizes that emotions are rooted in cognitions. Thus, in these models, emotions arise when discrepancies between beliefs and desires (or other beliefs) are detected by automatic appraisal processes. Based on that perspective, they propose a "content model", in which appraisal and regulation operate on rich representations of the emotion-evoking situation. Moreover, their approach (which has been evaluated against clinical data) makes use of plan-based causal representations, augmented with decision-theoretic planning techniques, whereas our approach uses dynamical systems representations.

The model presented here abstracts from a number of the details addressed in the work by Gratch, Marsella and Mao, such as the processes underlying emotion elicitation and appraisal. Due to this, the presented model is more abstract and makes fewer commitments. From this higher abstraction level it can be related to more fine-grained models based on different perspectives.

Another existing model that may be worth aligning with the presented approach is the model by Reisenzein (2009). Similar to Gratch, Marsella and Mao, he presents a computational model of affect based on the belief-desire theory of emotions (*BDTE*), which views emotions as nonconceptual metarepresentations. Due to the difference in grain size, these types of models seem to complement the model presented here. Therefore, an interesting possibility for future work would be to explore whether (parts of) our model may be useful in extending other computational models, in particular of emotion elicitation. For example, the parameters used in our model could be used to incorporate regulatory mechanisms in Reisenzein's model. For example, as a first step, our parameters γ and ζ could be incorporated, respectively, into the belief/belief and belief/desire discrepancy detection engines.

Furthermore, the model shows similarities with existing cognitive architectures, such as ACT-R (Anderson & Lebiere, 1998) or CLARION (Sun, 2006; Wilson, Sun, & Mathews, 2009). For example, CLARION is composed of a number of functional systems, such as an action-centered subsystem, a non-action-centered subsystem, a motivational subsystem, and a meta-cognitive subsystem. Within CLARION, emotion regulation could be modeled by adjustment of behavior (within the action-centered subsystem) in response to motivational drives (from the motivational subsystem), and/or meta-cognitive regulation (within the meta-cognitive subsystem). The presented model does not attempt to connect the regulation process to these detailed cognitive factors (again, due to a difference in grain size). Instead, it focuses on the patterns in emotional response that are produced as a result of differences in regulation strategies. Nevertheless, for future work it may be worthwhile to compare the generated simulation traces with the emotion regulation behavior shown by CLARION or ACT-R.

Finally, our model may be useful in computational studies of social norms. For example, Staller and Petta (2001) exploit computational modeling to investigate the interplay between emotions and social norms. They explicitly distinguish emotion generation from emotion regulation, and claim that both processes play an important role when studying people's tendency to adhere to social norms. For example, if a person desires to perform a certain action that is disapproved by other people, she may decide not to perform the action and to regulate her emotions instead. It may be worth exploring whether frameworks such as in Staller and Petta (2001) can be extended with our formalized emotion regulation strategies.

7.3. Future work

Many improvements could still be made to the model. For example, emotion response level ERL_{norm} is currently fixed. This could be made dynamic, so that it can depend on specific circumstances, as humans are usually searching for emotion variation. Another improvement could be introducing decay for the γ_n, which represent the personal tendency to change behavior in favor of emotion regulation. This way, the γ_n would slowly return to the value of $\gamma_{basic\ n}$ if no events occur that influence the γ_n. Also, in the formulas that are used to calculate the new emotional values, the emotion response level could be modified by a random factor. The size of this random factor could be changed in order to simulate emotion regulation in people that are not able to recognize their level of emotion very well. Moreover, response modification is not considered in this model. In the real world, people modify their emotional response, regarding for instance social desirability. Additions could be made to the model, in order to be able to simulate this. Also, response modulation has no effect on the emotion regulation at all. It could be the case that response modulation at the long term, for instance the suppression of a traumatic experience, could have an effect on emotion regulation. The model could be changed in order to make it able to simulate this. Additionally, in the current model, a trauma cannot cause a relapse in the emotion regulation process. It can only slow down the process. The model could be changed so that a trauma would not only influence the personal tendency to change behavior in favor of emotion regulation, but also the willingness to change behavior in favor of emotion regulation, and the chosen emotional values. A final extension would be to represent the different elements k using more complex knowledge structures, and to enable the model to dynamically derive the different emotional values from these structures, as is done, for example, in Marinier and Laird (2004). Future work will explore such possibilities.

When the model has been sufficiently refined, we will combine the model presented in this paper with an existing computational model of perception and affective decision making: I-PEFiC^{ADM} (Hoorn, Pontier, & Siddiqui, 2008). Whereas the current model focuses on the regulation of emotions and affective decision making, I-PEFiC^{ADM} addresses the elicitation of emotion, without explicit regulatory mechanisms. We expect that both models will smoothly fit together, since the affective decision making process of I-PEFiC^{ADM} could also be applied to emotion regulation strategies such as situation selection, situation modification, and attentional deployment. Initial steps for this approach have been taken in Pontier and Siddiqui (2009). Finally, in a later stage of the project, the formalization will be validated against empirical data of human affective trade-off processes. As soon as the model has been validated and adapted, we will start exploring the possibilities to apply it to real humans instead of agents; i.e., to develop a robot that can communicate affectively with humans in a more natural way, that is, with a mind of its own, in pursuit of its own goals.

Acknowledgements

We would like to thank the anonymous reviewers for their useful comments.

Appendix A:. Dynamic properties in LEADSTO format

LP1 (Update Emotion Response Level)

emotion_response_level(erl)
and has_weight(situation, w1)
and has_weight(sub_situation, w2)
and has_weight(aspect, w3)
and has_weight(meaning, w4)
and has_emotional_value(situation, v1)
and has_emotional_value(sub_situation, v2)
and has_emotional_value(aspect, v3)
and has_emotional_value(meaning, v4)
$\twoheadrightarrow_{0,0,s,s}$ emotion_response_level((1 − beta) ∗
(w1 ∗ v1 + w2 ∗ v2 + w3 ∗ v3 + w4 ∗ v4) + beta ∗ erl)

LP2 (Update Emotional Values Vn)

emotion_response_level(erl)
and erl_norm(erl_norm)
and has_emotional_value(element, v)
and has_modification_factor(element, a)
$\twoheadrightarrow_{0,0,s,s}$ has_emotional_value(element, v − a ∗ (erl-erl_norm)/
dmax ∗ s)

LP3 (Remember Emotion Response Level step 1)

emotion_response_level(erl)
$\twoheadrightarrow_{0,0,s,s}$ erl_steps_back(1, erl)

LP3 makes the model remember the current emotion response level as the emotion response level of 1 step back.

LP4 (Remember Emotion Response Level step 2–10)

>erl_steps_back(steps, erl)
>$\twoheadrightarrow_{0,0,s,s}$ erl_steps_back(steps + 1, erl)

LP4 makes the model remember the emotion response level of n steps back as the emotion response level of $n + 1$ steps back. So for instance, the emotion response level of 5 steps back, is now remembered as the emotion response level of 6 steps back.

LP5 (Keep modification factors alpha_n)

>has_modification_factor(element, a)
>and not alphas_not_persistent_anymore
>$\twoheadrightarrow_{0,0,s,s}$ has_modification_factor(element, a)

LP5 manages that the α_n's can not be changed when there are not enough steps done to be able to apply the formula for updating the α_n's.

LP6 (Alpha's not persistent anymore)

erl_steps_back(10, X)
$\twoheadrightarrow_{0,0,s,s}$ alphas_not_persistent_anymore

LP6 manages that the α_n's can be changed when there are enough steps done to be able to apply the formula for updating the α_n's.

LP7 (Update modification factors alpha_n)

alphas_not_persistent_anymore
and has_modification_factor (element, a)
and has_cost(element, costs)
and gamma(gamma)
and erl_steps_back(1, erl1)
and erl_steps_back (2, erl2)
and erl_steps_back(3, erl3)
and erl_steps_back(4, erl4)
and erl_steps_back(5, erl5)
and erl_steps_back(6, erl6)
and erl_steps_back(7, erl7)
and erl_steps_back(8, erl8)
and erl_steps_back(9, erl9)
and erl_steps_back(10, erl10)
$\twoheadrightarrow_{0,0,s,s}$ has_modification_factor(element, a + gamma * a/
(1 + a) * ((abs(erl1 – erl_norm) + (abs(erl2 –
erl_norm) + (abs(erl3 – erl_norm) +
(abs(erl4 – erl_norm) + (abs(erl5 – erl_norm))/5/
(((abs(erl6 – erl_norm) + (abs(erl7 – erl_norm) +
(abs(erl8 – erl_norm) + (abs(erl9 – erl_norm)
+ (abs(erl10 – erl_norm))/5) – costs))

LP7 manages that the modification factors α_n are updated every step using the formula's described in Section 4.

LP8 (Keep old gamma)

gamma(gamma)
and not change_gamma
$\twoheadrightarrow_{0,0,s,s}$ gamma(gamma)

LP9 (Change gamma)

experience(exp)
and gamma(gamma)
$\twoheadrightarrow_{0,0,s,s}$ gamma(gamma + zeta * exp/
(1 + (gamma – gamma_basic) * exp))

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