Advanced Logic 2014–15

Assignment 1

(deadline: Friday, February 13)

1. Prove or disprove the universal validity of the following formulas:

(a) $\Box\bot \vee \Diamond\top$

(b) $\Diamond(p \vee q) \to \Diamond p \vee \Diamond q$

(c) $\Diamond p \wedge \Diamond q \to \Diamond(p \wedge q)$

(d) $\Box(p \to q) \to (\Diamond p \to \Box q)$

(e) $\Box(p \to q) \to (\Box p \to \Box q)$

2. If the formula $\Box\bot$ is valid in a frame $\mathcal{F} = (W, R)$, then what do you know of its transition relation R? Prove your answer.

Same question for $\Diamond\top$.

3. Let K be the class of frames defined by:

$$\mathsf{K} = \{\, (W, R) \mid \forall xyz \in W \,(\, Rxy \wedge Rxz \to Ryz \vee Rzy \,)\, \}$$

(a) Show that K is characterized by the formula κ:

$$\kappa = \Box(\Box p \to q) \vee \Box(\Box q \to p)$$

that is, prove that for all frames $\mathcal{F}$:

$$\mathcal{F} \in \mathsf{K} \quad \text{if and only if} \quad \mathcal{F} \vDash \kappa$$

(b) Why does it follow from the previous item that also the formula

$$\Box(p \to \Diamond q) \vee \Box(q \to \Diamond p)$$

characterizes the class K?

(see page 2)

4. Consider the frames $\mathcal{A} = (A, R_A)$, $\mathcal{B} = (B, R_B)$, $\mathcal{C} = (C, R_C)$:

$$A = \{a, b, c, d\} \qquad R_A = \{(a,b), (a,c), (b,a), (b,d)\}$$
$$B = \{1, 2, 3, 4\} \qquad R_B = \{(1,2), (1,3), (1,4), (2,1)\}$$
$$C = \{\circ, \bullet\} \qquad R_C = \{(\circ,\circ), (\circ,\bullet)\}$$

(a) Draw the transition graphs for $\mathcal{A}$, $\mathcal{B}$ en $\mathcal{C}$.

Let further $\mathcal{M}_\mathcal{A}$, $\mathcal{M}_\mathcal{B}$, $\mathcal{M}_\mathcal{C}$ be models based on $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$, respectively, such that all propositional variables are false in all states.

(b) Find a modal formula that *distinguishes* state a of model $\mathcal{M}_\mathcal{A}$ from state 1 of model $\mathcal{M}_\mathcal{B}$; that is, a formula φ such that $\mathcal{M}_\mathcal{A}, a \vDash \varphi$ while $\mathcal{M}_\mathcal{B}, 1 \nvDash \varphi$.

(c) Why can we conclude from the previous item that there cannot exist a bisimulation $Z \subseteq A \times B$ such that $(a, 1) \in Z$?

(d) Show that $\mathcal{M}_\mathcal{A}, a \leftrightarroweq \mathcal{M}_\mathcal{C}, \circ$.

(e) Can you find a modal formula which distinguishes state a in $\mathcal{M}_\mathcal{A}$ from state $\circ$ in $\mathcal{M}_\mathcal{C}$?