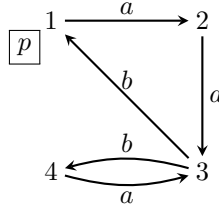# Advanced Logic 2014–15

Assignment 3

(deadline: Monday, March 16)

1. Let $\widehat{\mathcal{M}}$ be the PDL-extension of the $\{a, b\}$-model $\mathcal{M}$ given by:

(a) Compute the transition relations $\widehat{R}_\alpha$, $\widehat{R}_\beta$, $\widehat{R}_\gamma$, $\widehat{R}_\delta$, $\widehat{R}_\varepsilon$ of $\widehat{\mathcal{M}}$ corresponding to the programs:

$$\alpha = aa \qquad \beta = ba \qquad \gamma = \beta^* \qquad \delta = \alpha\gamma b \qquad \varepsilon = \delta^*$$

(25 pt)

(b) Show that $\widehat{\mathcal{M}} \vDash p \leftrightarrow \langle \varepsilon \rangle p$. *(10 pt)*

(c) Give a PDL-formula ψ such that $\widehat{\mathcal{M}} \nvDash \psi \leftrightarrow \langle \varepsilon \rangle \psi$. *(10 pt)*

2. The Hilbert system T is the extension of the system K with the truth axiom $A1$. System $S4$ is the extension of T with $A2$, the axiom of positive introspection.

(a) Formulate the completeness theorem for the system $S4$. *(5 pt)*

(b) Show that $A3$, the axiom of negative introspection, is not deducible in $S4$. *(10 pt)*

(c) Prove that a reflexive and euclidean relation is an equivalence relation. *(10 pt)*

(d) Give a derivation or construct a countermodel:

    (i) $\vdash_{K^2} K_1 K_2 p \to K_2 K_1 p$ *(10 pt)*

    (ii) $\vdash_T (K(p \to q) \wedge p) \to q$ *(10 pt)*