Methods for a Smart Thermostat to Estimate the Characteristics of a House Based on Sensor Data

Wim van der Ham\textsuperscript{a}, Michel Klein\textsuperscript{b}, Seyed Amin Tabatabaei\textsuperscript{b}, Dilhan J. Thilakaratne\textsuperscript{b}, Jan Treur\textsuperscript{b}\textsuperscript{*}

\textsuperscript{a}Quby, Joan Mayskenweg 22, 1096 CJ Amsterdam, The Netherlands
\textsuperscript{b}Agent Systems Research Group, Dep. of Computer Science, VU University Amsterdam

Abstract

Smart thermostats can play an important role in achieving more economic energy usage in domestic situations. This paper focuses on the energy used for natural gas-based heating, and monitoring of gas usages versus indoor and outdoor temperatures over time. Two methods are presented that enable the smart thermostat to learn, over time, characteristics of the house such as heat loss rate and heat capacity. Through this, the thermostat can make some homeowners aware, for example, that there is room for improvement in insulation of the house. The presented methods are able to deal with sensor data with varying extents of imperfection concerning their completeness.

© 2015 The Authors. Published by Elsevier Ltd.
Peer-review under responsibility of Riga Technical University, Institute of Energy Systems and Environment.

Keywords: heat loss rate; heat capacity; cooling down rate; heating degree days, smart thermostat

1. Introduction

In northern parts of Europe much energy is used for domestic heating. Smart thermostats are devices used more and more to get insight in domestic energy usage; e.g., [1]–[3]. They can contribute to more economic energy use in two ways. On the one hand, they can control the heating system in an economic manner, for example by not heating the house if they detect that nobody is at home. But, on the other hand, they play an important role in making the

\textsuperscript{*} Corresponding author. Tel.: +31 20 598 7763; fax: +31 20 598 7653.
E-mail address: j.treur@vu.nl

1876-6102 © 2015 The Authors. Published by Elsevier Ltd.
Peer-review under responsibility of Riga Technical University, Institute of Energy Systems and Environment.
persons in the house aware of their energy usage; so, that they are encouraged to change their behaviour or their	house into a more sustainable situation.

A challenge here is to give the smart thermostat the intelligence needed to analyze what are the energetic
characteristics of the house. This could provide the basis for comparisons with other users and tailored advices about
measures to reduce energy usage. To achieve this, both monitoring devices and analysis methods for interpreting the
data are used. The work described in this paper uses the data of gas consumption used for heating and the indoor
and outdoor temperatures over time to derive the energetic characteristics of the house. A particular additional challenge
is that often sensor data do not provide complete time series. From time to time data may be missing. To analyse the
heating characteristics of a house, often so called heating degree day methods are used; e.g., [4]–[9]. These methods
allow to estimate how the heating demand of a house relate to differences between indoor and outdoor temperatures.

Taking such heating degree day and other information, in this paper two methods are discussed that enable the
smart thermostat to estimate characteristics of the house such as heat loss rate and heat capacity; so, that home
owners can possibly be made aware that there is room for improvement in insulation of the house. The proposed
methods are also able to handle sensor data with certain extents of imperfection concerning their completeness.

In the paper, in Section 2 some background theory about gas-based heating is discussed. In Section 3, the
available dataset is described. Section 4 describes how the adaptive methods have been applied to the dataset and
which outcomes were achieved. Finally, Section 5 is a discussion.

2. Theoretical basis

This section discusses the theoretical background concerning gas-based heating and the relation to outdoor
temperature, and central concepts characterizing a house such as heat loss rate $\varepsilon$ (depending on insulation level of the
house) and heat capacity $C$ of the house (depending on volume of the house).

2.1. Central concepts

The energy loss of a house per time unit with indoor temperature $T_{id}$ and lower outdoor temperature $T_{od}$ is
proportional to the temperature difference between indoor and outdoor temperatures. The proportion factor $\varepsilon$ is the
loss rate:

\[ \text{energy loss per time unit} = \varepsilon(T_{id} - T_{od}) \]  \hfill (1)

This loss rate depends on insulation level of the area of the house in contact with the outside: the walls, windows,
floor and roof.

Another central concept is the heat capacity of a house. The amount of energy needed to increase the indoor
temperature is proportional with the difference $\Delta T_{id}$ in indoor temperature. The proportion factor $C$ is the heat
capacity:

\[ \text{energy needed for increase} = C \Delta T_{id} \]  \hfill (2)

The heating capacity of a house, $C$, depends on the volume (content) of the house. Note that, during a time
interval of temperature increase, still energy loss takes place as well. More specifically, there are three types of
situations: A) Indoor temperature increase: In periods of increase of indoor temperature both types of energy
described by (1) and (2) have to be added to each other to get the total amount of energy spent. B) Maintaining
constant indoor temperature: When heating takes place just to maintain a given indoor temperature, only the
energy loss is compensated by the heating. The amount of energy needed for this is described by (1). C) Natural
cooling down without heating: When no heating takes place the house follows a natural cooling down process.
During such a time interval the energy loss (per time unit) described by (1) leads to a temperature decrease (per time
unit) described by (2):
So in a cooling down process, the speed of indoor temperature decrease is proportional to the difference between indoor and outdoor temperature. This proportion factor $\varepsilon/C$ is called the cooling down rate, indicated by $\mu$.

The concepts discussed above can be applied for specific time instants or very short time durations, but they can also be used for longer time periods, such as hours, days, months, seasons or years. In the latter case, the formulae (1), (2) and (3) can still be applied but some form of summation or integration over time is needed. This has been done in the two approaches that have been developed. The two proposed approaches put different requirements on the data to which they can be applied.

2.2. First Approach

The first approach is based on a calculus of what is called (heating) degree days. Degree days based energy consumption analysis is a well-known approach to quantify the relation between energy usage and outdoor and indoor temperatures; e.g., [4]–[9]. Through this, it is possible to approximate energy consumption and energy performance of a building based on historical data and that can be used to estimate the energy loss per degree per day (kWh/°C day). In this approach, only the energy which needed to compensate for the energy loss of the house is counted. Therefore (1) above applies, but still an integration process over a longer time period has to be applied to it. Since both indoor and outdoor temperatures are varying, a specific adapted definition of a number of degree days (dd) during a 24-hour period can be expressed mathematically in the following manner:

$$dd = \int_0^{24 \text{hrs}} (T_{\text{in}}(t) - T_{\text{od}}(t)) \, dt,$$

where $T_{\text{in}}(t) > T_{\text{od}}(t)$ for all $t$ (4)

Here, $T_{\text{in}}(t)$ and $T_{\text{od}}(t)$ are indoor and outdoor temperatures as function of time $t$. In this approach, the quality of calculating degree-day value depends on the quality of indoor and outdoor air temperature values. In practical applications, this equation can be transformed into a more discrete version as:

$$dd = \sum_{\varepsilon \in 24 \text{hrs and } \Delta t \text{ intervals}} (T_{\text{in}}(t) - T_{\text{od}}(t))$$

(5)

The smaller the $\Delta t$ values, the higher the accuracy (in our experiments, $\Delta t$ was taken as one hour). When energy usage data for heating are available, based on (1) above the relation between heating energy usage and the number of degree days can be mathematically expressed as:

$$\text{energy usage} = \varepsilon dd$$

(6)

Here, $\varepsilon$ is the energy loss rate that gives a good indication about quality of insulation of a house. When $\varepsilon$ has a low value, that house has good insulation, but high values indicate bad insulation. This can be used as an indication to take initiative to enhance the awareness of householders to take necessary actions.

2.3. Second Approach

In the second approach, the focus is not (only) on the energy loss over 24 hour periods, but also more in particular on analysing the shorter time periods in which the indoor temperature has a downward or upward trend. For example, in the morning many houses are heated from a lower night indoor temperature to the higher day indoor temperature. This approach enables to estimate not only the loss rate $\varepsilon$ but also the capacity $C$, and the cooling down
rate $\mu$. For cooling down periods, when no heating energy is given to the house, by integration of (3) the equation (7) and (8) can be obtained. Furthermore, by combining (7) and (8) we get (9).

\[
\text{energy loss} = C \Delta T_{in} 
\]  
\[
\text{energy loss} = \varepsilon \int (T_{in} - T_{od}) dt 
\]  
\[
\frac{\varepsilon}{C} = \frac{\Delta T_{in}}{\sum (T_{in} - T_{od}) \Delta t} 
\]

As mentioned, $\varepsilon/C$ is the cooling ratio $\mu$; and shows the rate or speed of decreasing the indoor temperature of a house when no heating energy is provided. On the other hand, for the periods of time in which heating energy is provided to increase the indoor temperature, by integration from (1) and (2) the following is found:

\[
\text{energy demand} = \text{energy for maintaining a temperature} + \text{energy for increasing temperature} = \varepsilon \int (T_{in} - T_{od}) dt + C \Delta T_{in} 
\]

In the second approach, equations (9) and (10) are used to calculate the capacity $C$, cooling down rate $\mu$ and loss rate $\varepsilon$ for a house. To do this, four main steps are used: 1) Data extraction to find the time intervals in which upward or downward trends in indoor temperature occur. 2) Determining cooling down rate $\mu$ for the time intervals in which the house is cooling down (based on equation (9)). This also provides one relation $\varepsilon = \mu C$ between the capacity $C$ and loss rate $\varepsilon$. 3) Determining another relation between the capacity $C$ and loss rate $\varepsilon$ using the time intervals in which the indoor temperature (monotonically) increases (based on equation (10)). 4) Determining the values for the capacity $C$ and loss rate $\varepsilon$ by combining the two relations obtained in 2 and 3.

Details of each step are described below:

**Data Extraction and Cleaning:** Since this article is on the space heating usage, the focus was on the colder season, which for the available dataset was limited to most of the days in March and some days in April. As a first step, data of hours which have all required information were extracted. The required information for an hour are: Gas usage in that hour, Indoor temperature per minute, Outdoor temperature for three time points (at minutes 5, 25 and 55). For the outdoor temperature in other minutes of an hour, a linear interpolation was used.

**Calculating the cooling down rate, $\mu$:** To do this, for each house the set of continuous hours with no gas usage and a downward trend for the indoor temperature are extracted. As mentioned, to calculate the cooling down rate, it is necessary to analyse time intervals in which no heating energy is provided by heating system. To get rid of delays in the heating and metering system (it takes some time after gas usage to deliver the energy to the house environment through the hot water inside the radiators and the meter only provides data per hour), the first hour of each set of continuous hours is removed, and the cooling down rate $\mu$ is determined according to equation (9). Given the number of time intervals in which cooling down takes place, for each house several estimated values for the cooling down rate are calculated, some of which are outliers. When, for example, windows or doors are open, the energy loss will increase significantly. In this condition, (9) does not apply anymore, and the value of the calculated rate will be much higher than the real one. However, it can still be used as a technique in smart thermostats, to alert the residence that probably a window or door is remained open for hours. So, to remove the effect of such outliers on the result, the median (not average) of these values as the cooling down rate of the house is used.

**Calculating loss rate $\varepsilon$ and capacity $C$:** To do this, the time intervals are addressed in which the heating system is activated (gas usage > small threshold) and the indoor temperature has an upward trend. As the capacity has an effect only when the temperature is changing (not when it is constant), the focus is on the time intervals in which the indoor temperature is increasing (from when indoor temperature reaches its local minimum, till 2 hours after that).
The amount of energy usage is calculated from (10). So, by combining the outcomes of (10) and the cooling down rate \( \mu \), the following is obtained:

\[
C = \frac{\text{energy usage}}{\mu \int (T_{in} - T_{od}) dt + \Delta T_{in}}
\]  
(11)

On average, the efficiency of natural gas based heating systems in North Western Europe is taken as 7 kWh/m³; for example, see [http://nl.wikipedia.org/wiki/Aardgas](http://nl.wikipedia.org/wiki/Aardgas). Using this, the energy usage based on the amount of gas used is calculated by (12). Again, the median of the obtained capacities and of \( \varepsilon \) is chosen.

\[
C = \frac{7 \times \text{gas usage}}{\mu \sum (T_{in} - T_{od}) \Delta t + \Delta T_{in}}
\]  
(12)

3. Dataset

The data that was used to validate the above methods was collected by a smart thermostat company in the period starting from the 26th of March 2014 until the 23th of July 2014. During this period, data was collected from 67 households that have a smart thermostat (Toon) installed at home and agreed to be part of this data collection. Depending on the boiler type and the software installed, the number of collected variables is between 10 and 35. For this study, only the following variables are used that were collected from the thermostat: Set point:- value of the goal temperature set in the thermostat, Indoor temperature:- actual indoor temperature as measured by the thermostat; Gas use:- total gas used. The set point and indoor temperature have a value for every minute in degree Celsius. Gas use is in liters (0.001 m³) and has a value for every hour. The outdoor temperature is missing from this data because the thermostat does not directly measure this value. These temperatures were acquired using the Weather Underground API and the postal code of the user to retrieve the location. The dataset was used in an anonymized way. The data that was collected from the thermostat was combined with general information that is known about the characteristics of the household. These include the construction period of the house divided in the following six groups: before 1946, between 1946 and 1964, between 1965 and 1974, between 1975 and 1987, between 1988 and 1999, after 1999. The total floor area in square meters of the house is represented in the following eight groups: between 50 and 75, between 75 and 100, between 100 and 125, between 125 and 150, between 150 and 200, between 200 and 300, between 300 and 500, more than 500, all in square meters. The type of household is in the dataset used for this research one of the following: apartment, terraced, semidetached or detached. And, the last characteristic of the household is the number of people that live in the house, which can be: one to four, or more than four.

4. Experimental analysis

In this section, we describe how the methods introduced in Section 2.2 and 2.3 have been applied to the data set. Approach 1, based on the degree-days, depends on the integration of consecutive data points for the indoor and outdoor temperature. Approach 2 requires the identification of specific intervals in which heating takes place and intervals in which no heating takes place.

The main research question in this paper is whether the heating characteristics of a house can be estimated based on temperature and gas usage data, given an imperfect and incomplete data set. A number of hypotheses were defined:

H1. The calculated capacity \( C \) of a house is positively correlated with the size of a house (a large house requires more energy to be heated).

H2. The calculated loss rate \( \varepsilon \) is correlated with insulation level of the house. As the data set does not contain specific information about this, we operationalize this hypothesis in the following ways:
H2a. The calculated loss rate $\varepsilon$ is correlated with the building year of the house: the older the house, the higher the loss rate.

H2b. The calculated loss rate $\varepsilon$ is correlated with the type of the house: the more detached a house, the higher the loss rate.

H3. The calculations of the loss rate $\varepsilon$ via each of the approaches will result in similar values.

H4. Approach 2 will be better applicable to incomplete data, as it requires smaller intervals of consecutive data.

It turned out that the dataset of indoor temperature and gas usage of 67 households had a lot of missing values. For approach 1, only 40 days were identified in which there were sufficient consecutive data points to calculate a degree day value. These 40 days belonged to 12 households, resulting in 2 till 4 usable days for each of the 12 households. Table 1 shows the resulting loss rate calculations for Approach 1.

<table>
<thead>
<tr>
<th>House#</th>
<th>loss rate $\varepsilon$</th>
<th>Building year</th>
<th>Type</th>
<th>Size category (m2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.83</td>
<td>1988_2000</td>
<td>SEMI_DET</td>
<td>125_150</td>
</tr>
<tr>
<td>2</td>
<td>2.92</td>
<td>1975_1988</td>
<td>SEMI_DET</td>
<td>125_150</td>
</tr>
<tr>
<td>3</td>
<td>4.68</td>
<td>1946_1965</td>
<td>DETACHED</td>
<td>200_300</td>
</tr>
<tr>
<td>4</td>
<td>3.39</td>
<td>After 2010</td>
<td>TERRACED</td>
<td>150_200</td>
</tr>
<tr>
<td>5</td>
<td>3.58</td>
<td>1946_1965</td>
<td>TERRACED</td>
<td>125_150</td>
</tr>
<tr>
<td>6</td>
<td>4.98</td>
<td>Before 1946</td>
<td>TERRACED</td>
<td>200_300</td>
</tr>
<tr>
<td>7</td>
<td>10.47</td>
<td>After 2010</td>
<td>TERRACED</td>
<td>150_200</td>
</tr>
<tr>
<td>8</td>
<td>3.36</td>
<td>Before 1946</td>
<td>APARTM</td>
<td>075_100</td>
</tr>
<tr>
<td>9</td>
<td>2.92</td>
<td>1965_1975</td>
<td>TERRACED</td>
<td>125_150</td>
</tr>
<tr>
<td>10</td>
<td>5.02</td>
<td>1946_1965</td>
<td>TERRACED</td>
<td>150_200</td>
</tr>
<tr>
<td>11</td>
<td>3.00</td>
<td>1975_1988</td>
<td>SEMI_DET</td>
<td>125_150</td>
</tr>
<tr>
<td>12</td>
<td>3.63</td>
<td>1988_2000</td>
<td>TERRACED</td>
<td>150_200</td>
</tr>
</tbody>
</table>

Applying approach 2 resulted in 166 usable intervals belonging to 11 different households. The number of respective cooling and heating intervals are shown in Table 2. The outcomes of approach 2 are listed in Table 3. Table 1. Results Approach 1

Table 2. Number of intervals used in approach 2.

<table>
<thead>
<tr>
<th>House#</th>
<th>cooling intervals</th>
<th>heating intervals</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
<td>15</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>11</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>7</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>9</td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>10</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>11</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>12</td>
<td>11</td>
<td>10</td>
</tr>
<tr>
<td>Total</td>
<td>71</td>
<td>95</td>
</tr>
<tr>
<td>Average</td>
<td>5.92</td>
<td>7.92</td>
</tr>
</tbody>
</table>

Table 3. Results Approach 2.

<table>
<thead>
<tr>
<th>House#</th>
<th>C loss rate $\varepsilon$</th>
<th>Build. year</th>
<th>Type</th>
<th>Size category (m2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.28</td>
<td>1988_2000</td>
<td>SD</td>
<td>A125_150</td>
</tr>
<tr>
<td>2</td>
<td>3.93</td>
<td>1975_1988</td>
<td>SD</td>
<td>A125_150</td>
</tr>
<tr>
<td>3</td>
<td>9.78</td>
<td>1946_1965</td>
<td>D</td>
<td>A200_300</td>
</tr>
<tr>
<td>4</td>
<td>5.18</td>
<td>After 2010</td>
<td>T</td>
<td>A150_200</td>
</tr>
<tr>
<td>5</td>
<td>4.67</td>
<td>1946_1965</td>
<td>T</td>
<td>A125_150</td>
</tr>
<tr>
<td>9</td>
<td>4.20</td>
<td>1965_1975</td>
<td>T</td>
<td>A125_150</td>
</tr>
<tr>
<td>11</td>
<td>10.08</td>
<td>1975_1988</td>
<td>SD</td>
<td>A125_150</td>
</tr>
<tr>
<td>7</td>
<td>3.71</td>
<td>After 2010</td>
<td>T</td>
<td>A150_200</td>
</tr>
<tr>
<td>8</td>
<td>11.76</td>
<td>Before 1946</td>
<td>A</td>
<td>A075_100</td>
</tr>
<tr>
<td>12</td>
<td>7.39</td>
<td>1988_2000</td>
<td>T</td>
<td>A150_200</td>
</tr>
<tr>
<td>6</td>
<td>4.54</td>
<td>Before 1946</td>
<td>T</td>
<td>A200_300</td>
</tr>
<tr>
<td>10</td>
<td>6.79</td>
<td>1946_1965</td>
<td>T</td>
<td>A150_200</td>
</tr>
</tbody>
</table>

Table 4. Average loss rate for approach 2 per type of apartment.

<table>
<thead>
<tr>
<th>Type</th>
<th>Average $\varepsilon$</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>APARTMENT</td>
<td>3.6</td>
<td>1</td>
</tr>
<tr>
<td>TERRACED</td>
<td>1.8</td>
<td>3</td>
</tr>
<tr>
<td>SEMI_DETACHED</td>
<td>2.0</td>
<td>7</td>
</tr>
<tr>
<td>DETACHED</td>
<td>4.0</td>
<td>1</td>
</tr>
</tbody>
</table>

Relation between capacity and size of the house
The size of the houses in the dataset is represented by four categories. Only approach 2 results in estimate of the capacity. Fig. 1 shows the correlation between the capacity and the size of the households.

There is only one household in the smallest category. This one house has a very high capacity. The mean value of the capacity for the category A100_150 (5 items) is 4.3, while for A150_200 (4 items) the value is 5.6. The average value of the capacity of the houses in the largest category A200_300 is 7.2.

**Relation between loss rate and type of the house**

Both approach 1 and approach 2 can be used to determine the relation between the loss rate $\varepsilon$ and the type of house. Fig. 2 shows the derived loss rates for the different types of houses. It can be seen that the average loss rate calculated via approach 1 is higher than the one calculated by approach 2. Also, it is visible that the ranking is different. Calculating the averages of the loss rates per house type leads to the following results (Table 4). The only instance of the type “apartment” scores relatively high. The average of the “terraced” is slightly lower than the average of “semi_detached”. This is in line with the fact that a terraced house has a higher percentage of walls with contact with the outside.

Another aspect of the type of house is the building year. The same analysis has been performed as for the building type. Fig. 3 depicts relation between calculated loss rates and building year. There is no clear correlation visible for approach 1. For approach 2, the newer buildings seem to have lower loss rates. When the average loss rates per category are calculated, this becomes even clearer. Fig. 4 shows the average of loss rate for different categories. Except for one category (based on one house), it holds that newer buildings have lower loss rates.

With respect to the hypotheses, the following conclusions can be drawn. It appears that the outcomes of approach 1 are less consistent with the expected results than the outcomes of approach 2. Also, the number of useful segments of data is higher for approach 2 than approach 1. Both facts support H4: approach 2 is better for handling incomplete data. Related to this, it has been shown that the outcomes of the calculations of the loss rates were quite different for
approach 1 and 2. H3 therefore has to be rejected: the outcomes are not similar. Hypothesis H1 and H2 are largely validated for approach 2. The average values of the capacity per size category on the one hand and the loss rate per type of house and per building year on the other hand increase for each higher category, except for one single measurements in each analysis. It has to be noted, however, that the data set is too small to draw strong conclusions.

5. DISCUSSION

In this paper, it has been addressed how a smart thermostat (cf. [1]–[3]) can be equipped with the means to estimate in an adaptive manner some of the characteristics of a house over time: heat loss rate, heat capacity and cooling down rate. Two approaches have been presented one of which is based on heating degree day analysis methods; cf. [4]–[9]. The approaches have been tested using a data set including data of a variety of houses of different types. This dataset shows a certain extent of incompleteness. The approaches both turn out to be able to handle data with some extent of incompleteness, where the second approach seems to do this a bit better than the first one. The proposed methods actually can be incorporated in a smart thermostat to make it adaptive over time, and smarter due to that.

There are few commercial products (e.g., Joulo and Nest) available mainly to provide personalised home heating advice to households. Joulo [10] has used a model-based approach for this purpose, which does not use gas or electricity consumption of the heating system in the calculations but only indoor and outdoor temperatures. However, this technique is based on some unrealistic assumptions. The first assumption is that "the thermostat has a single set-point through a day", which in most of the real houses is not the case. Another (and more important) unrealistic assumption is that “energy produced by heating system in all intervals is always the same”. Even if the heating system has just two modes (ON/Off) this cannot be a true assumption, since the system may be ON just for part of an interval.

In the proposed techniques, focus is to calculate the characteristics of a house based on used heating energy and to improve the awareness of users about impact of the current state of their house on heating energy cost. From an application perspective, these may be useful features on specific thermostats and maybe it is useful to integrate features of most of these commercial products to improve the sustainability of energy usage. Another interesting feature is to also incorporate dynamic pricing (especially with smart grid based systems) in the calculations; e.g. [11].

References