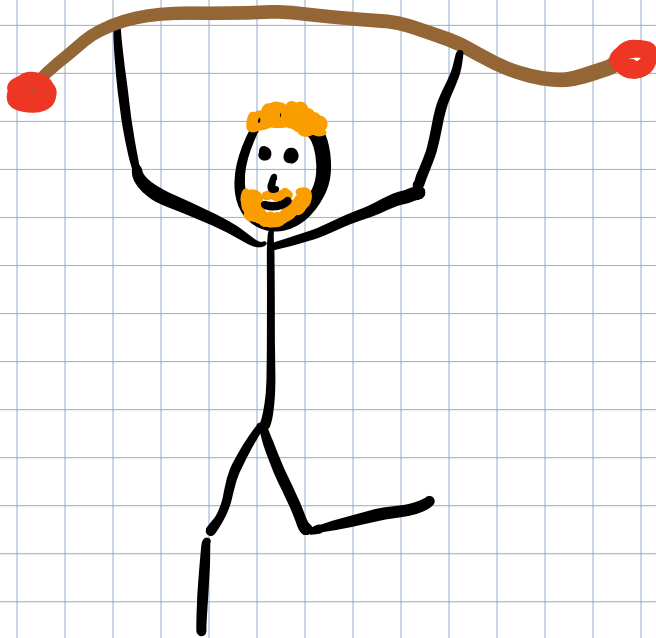


Een touw heeft een even aantal uiteinden

(een introductie tot cobordisme theorie)

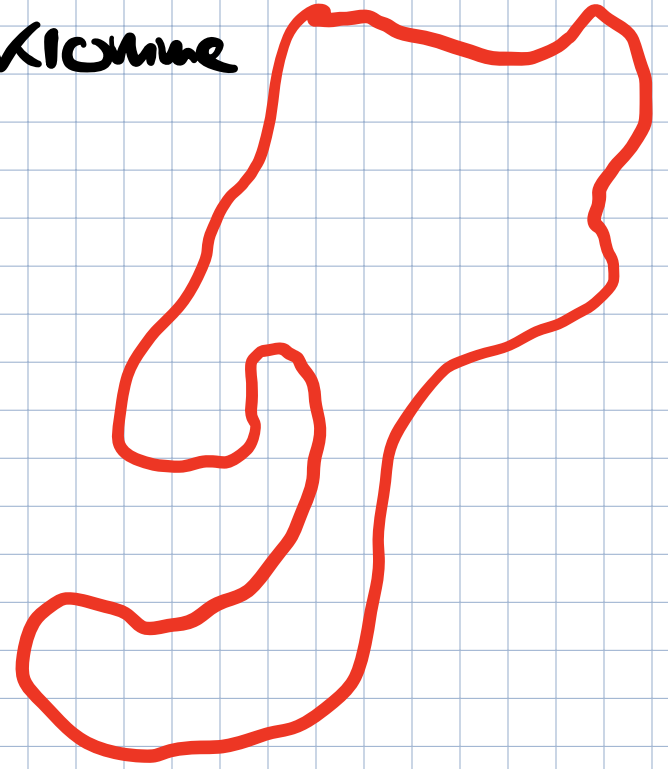
Thomas Rot
VU Amsterdam



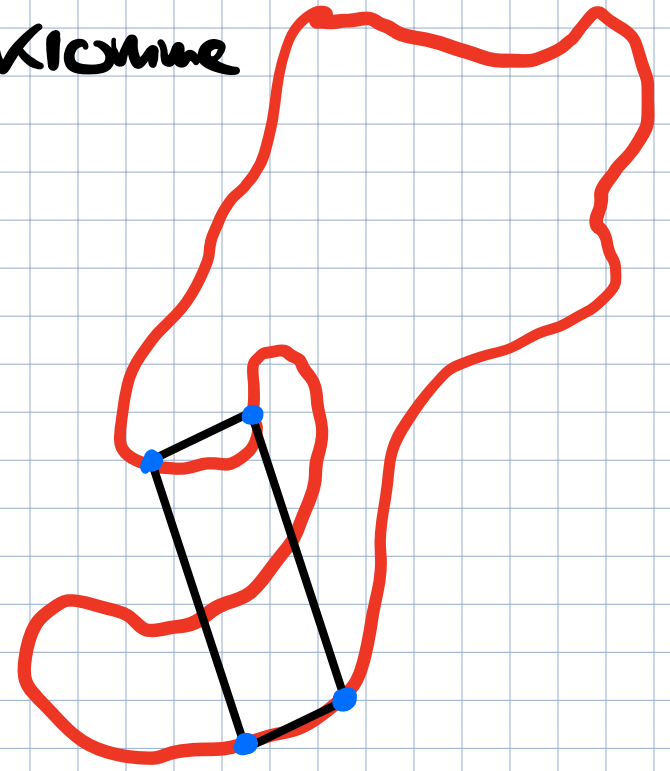
KWG

Wintersymposium 2024

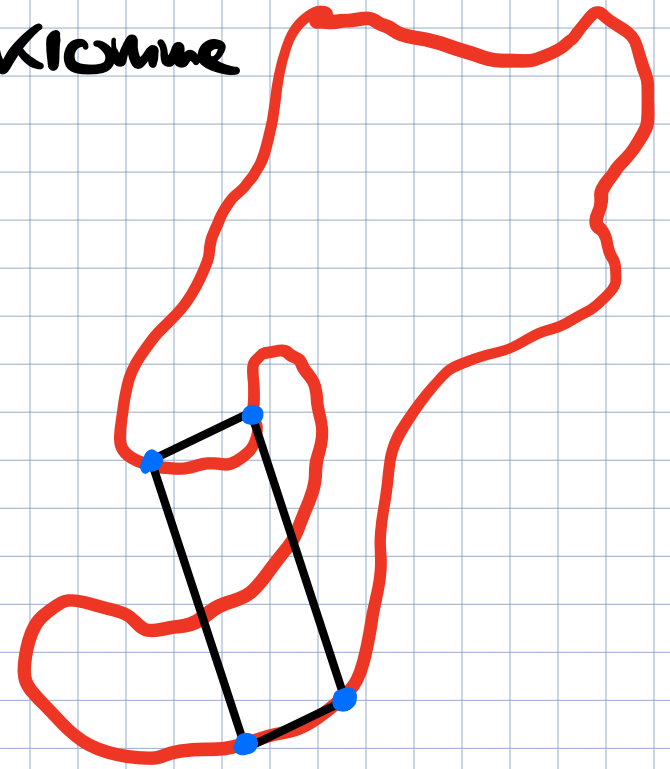
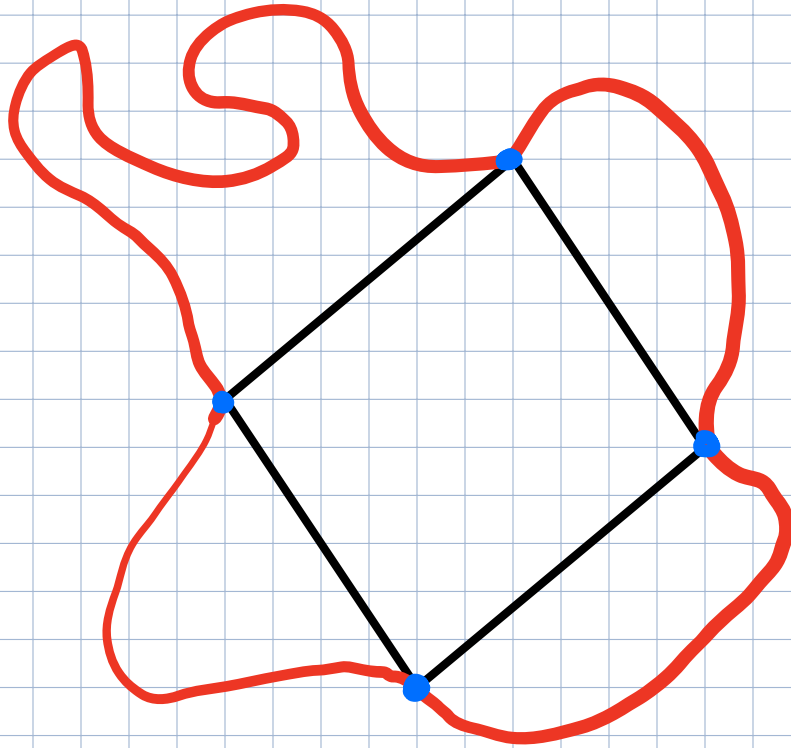
Doel: Rechthoeken op een kromme



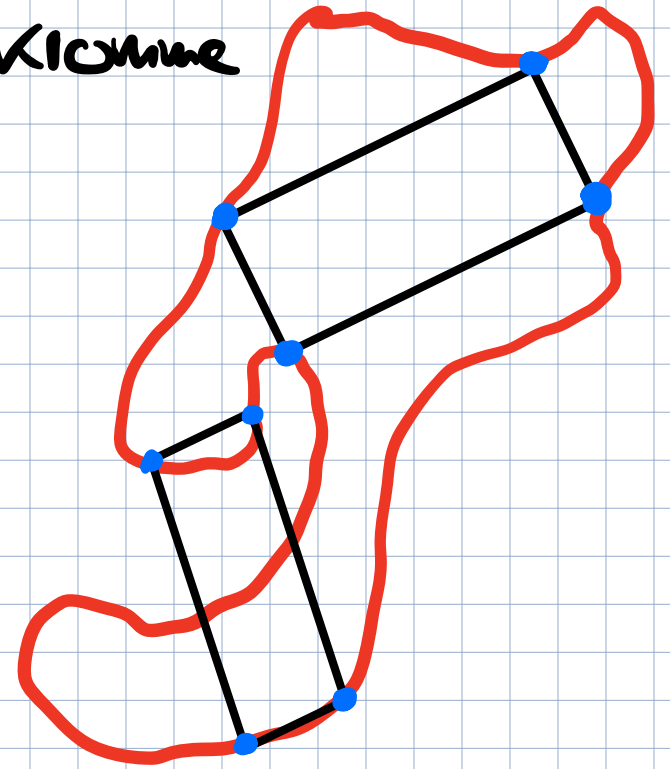
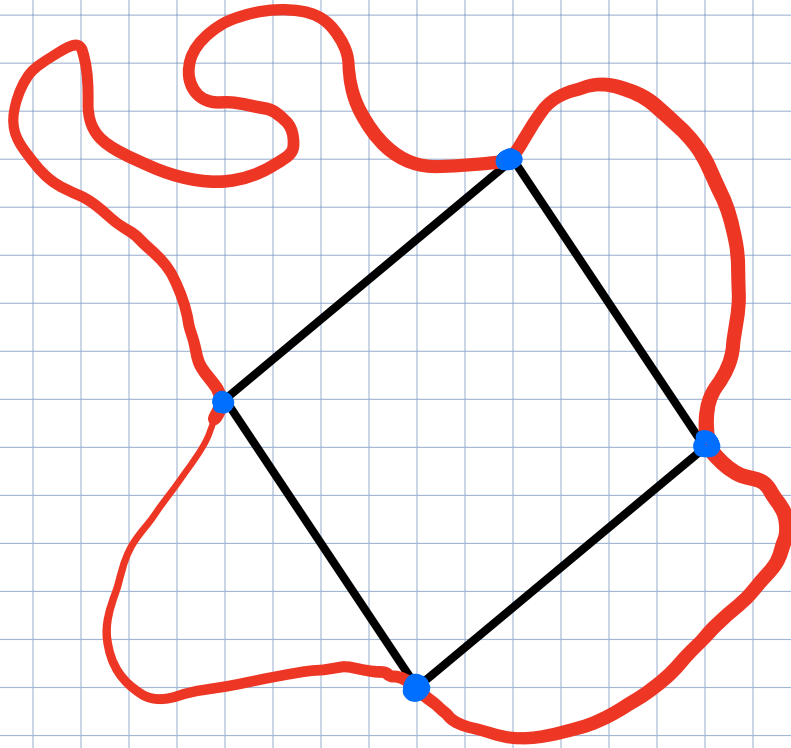
Doel: Rechthoeken op een kromme



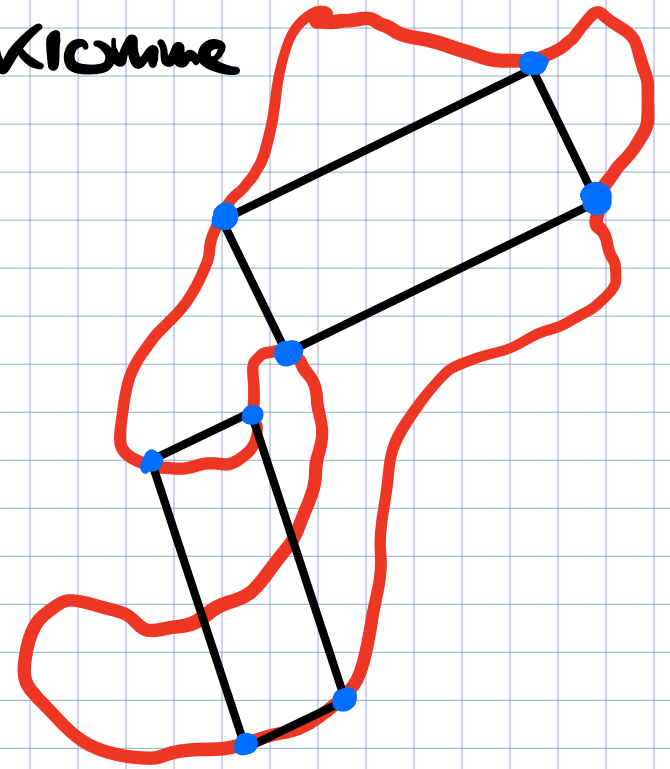
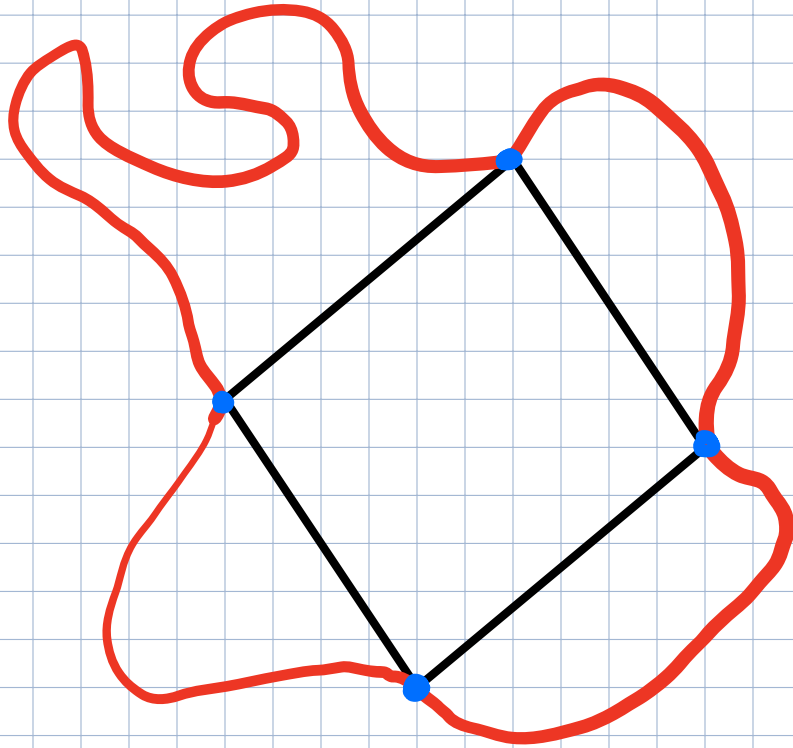
Doel: Rechthoeken op een Kromme



Doel: Rechthoeken op een Kromme



Doel: Rechthoeken op een kromme



Stelling op elke gesloten kromme is een rechthoek
(vandaag) te vinden met hoekpunten op de kromme

Korte historie

(1913)

- Probleem van Toeplitz

(1913 -)

- Bewijzen voor speciale klassen van Krommen Emch, Schnirelman,

(1981)

- Op Elke Kromme is een rechthoek (Vaughn). - Dit bewijs zien we vandaag -

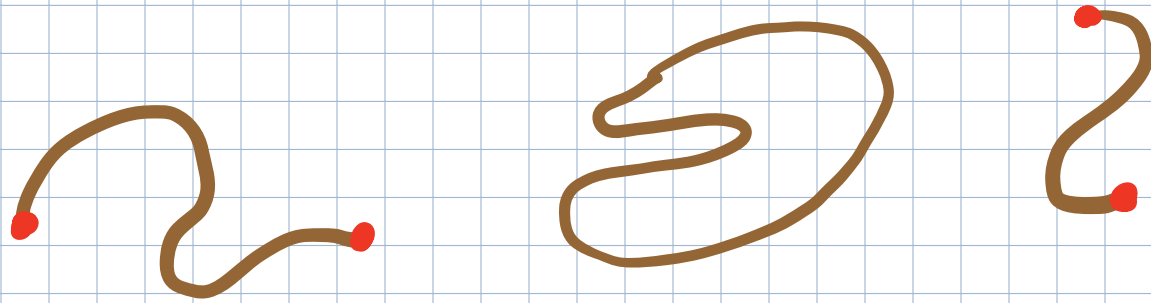
(2020)

- Op elke Kromme is er een rechthoek van elke verhouding (Green - Lobb)

Mijn favoriete stelling:

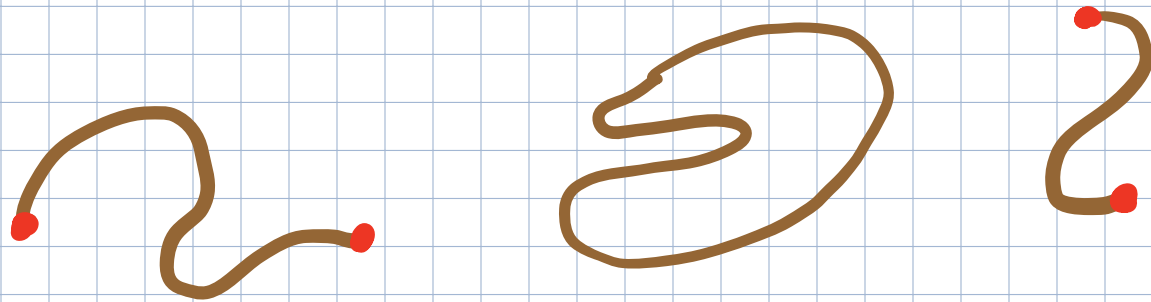
Mijn favoriete stelling:

Stelling Een touw heeft een even aantal uiteinden



Mijn favoriete stelling:

Stelling Een touw heeft een even aantal uiteinden



Opmerking

Een touw mag voor mij
uit meerdere stukken bestaan

Gevolg:

Een aantal punten is het uiteinde van een touw, dan en slechts dan, als het aantal punten even is.



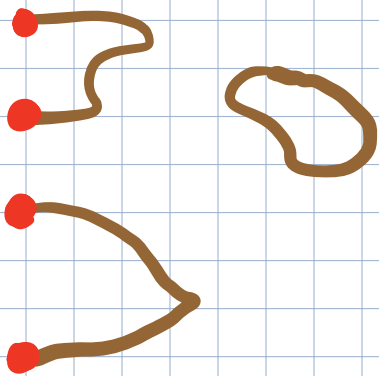
Even aantal
Punten



Oneven aantal
Punten

Gevolg:

Een aantal punten is het uiteinde van een touw, dan en slechts dan, als het aantal punten even is.



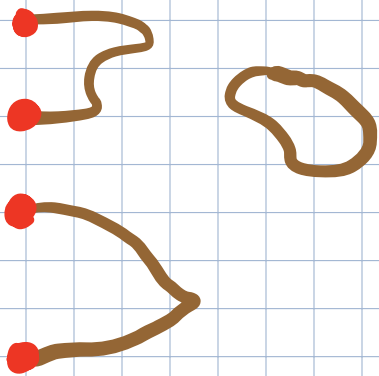
Even aantal
Punten
is **wel** het uiteinde



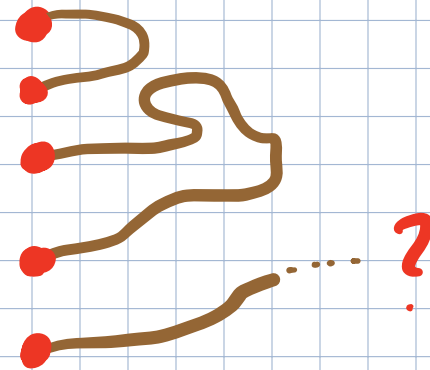
Oneven aantal
Punten

Gevolg:

Een aantal punten is het uiteinde van een touw, dan en slechts dan, als het aantal punten even is.

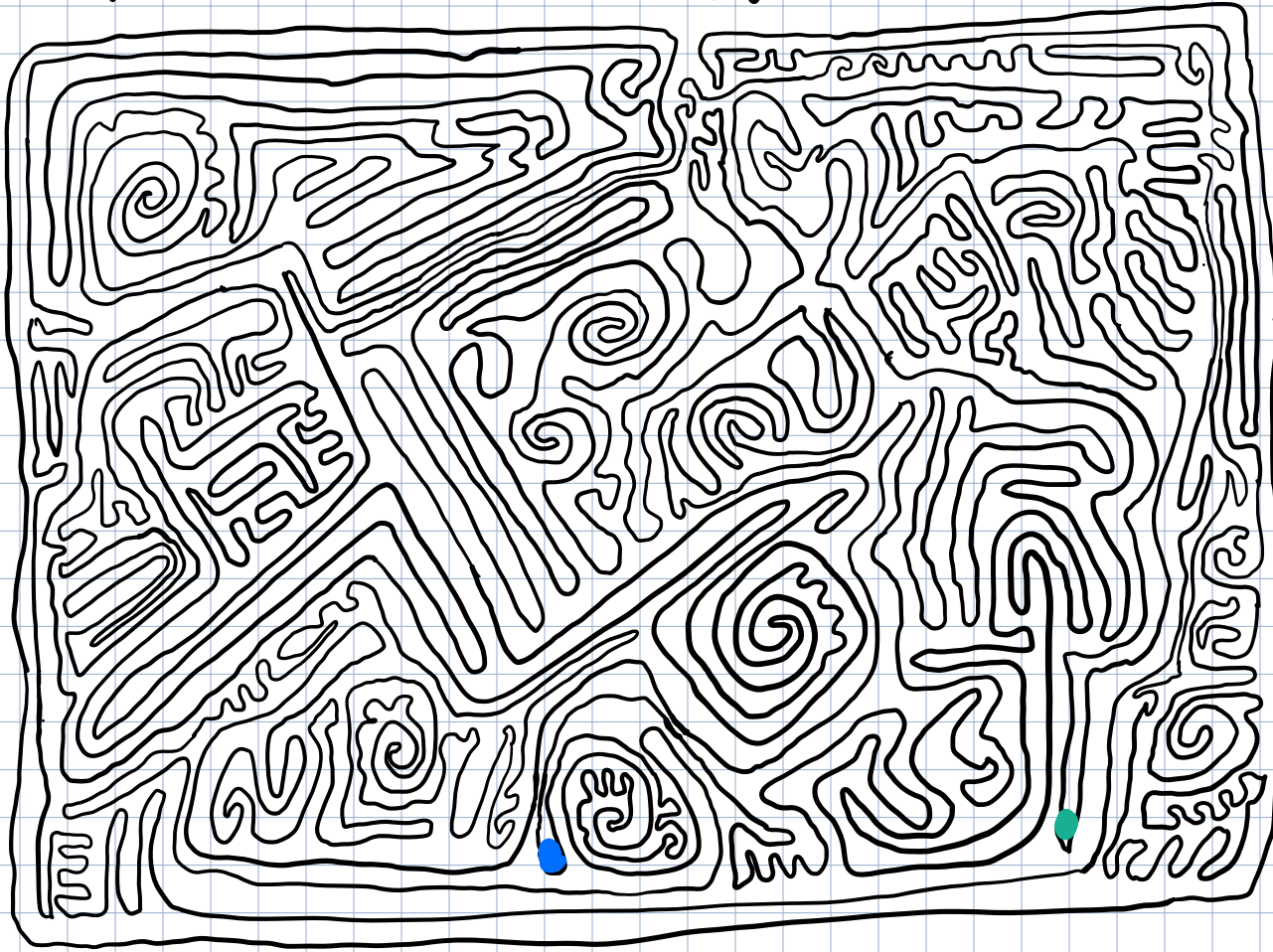


Even aantal
Punten
is **wel** het uiteinde



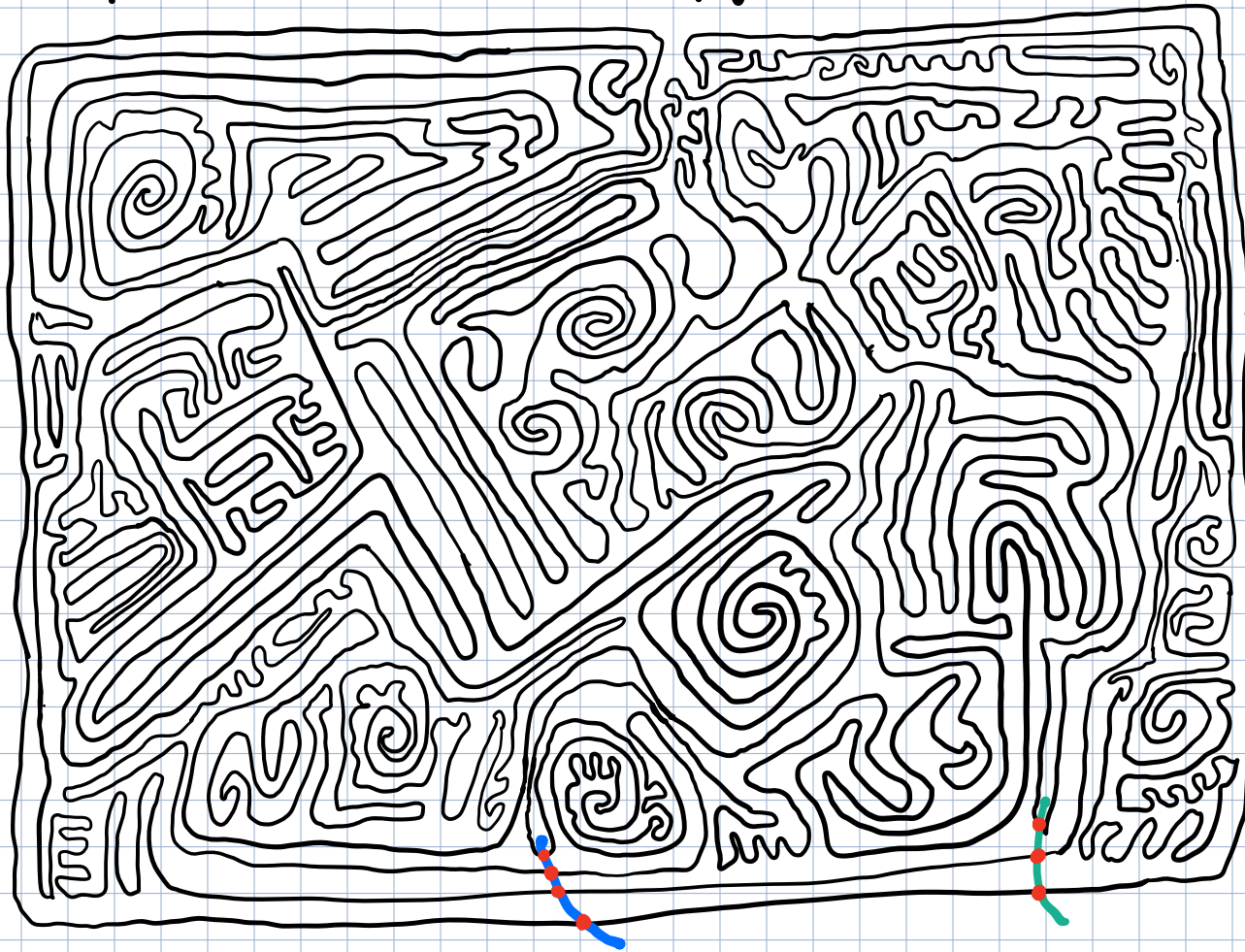
Oneven aantal
Punten
is **niet** het uiteinde

Toepassing: Ontsnappen uit een doolhof.



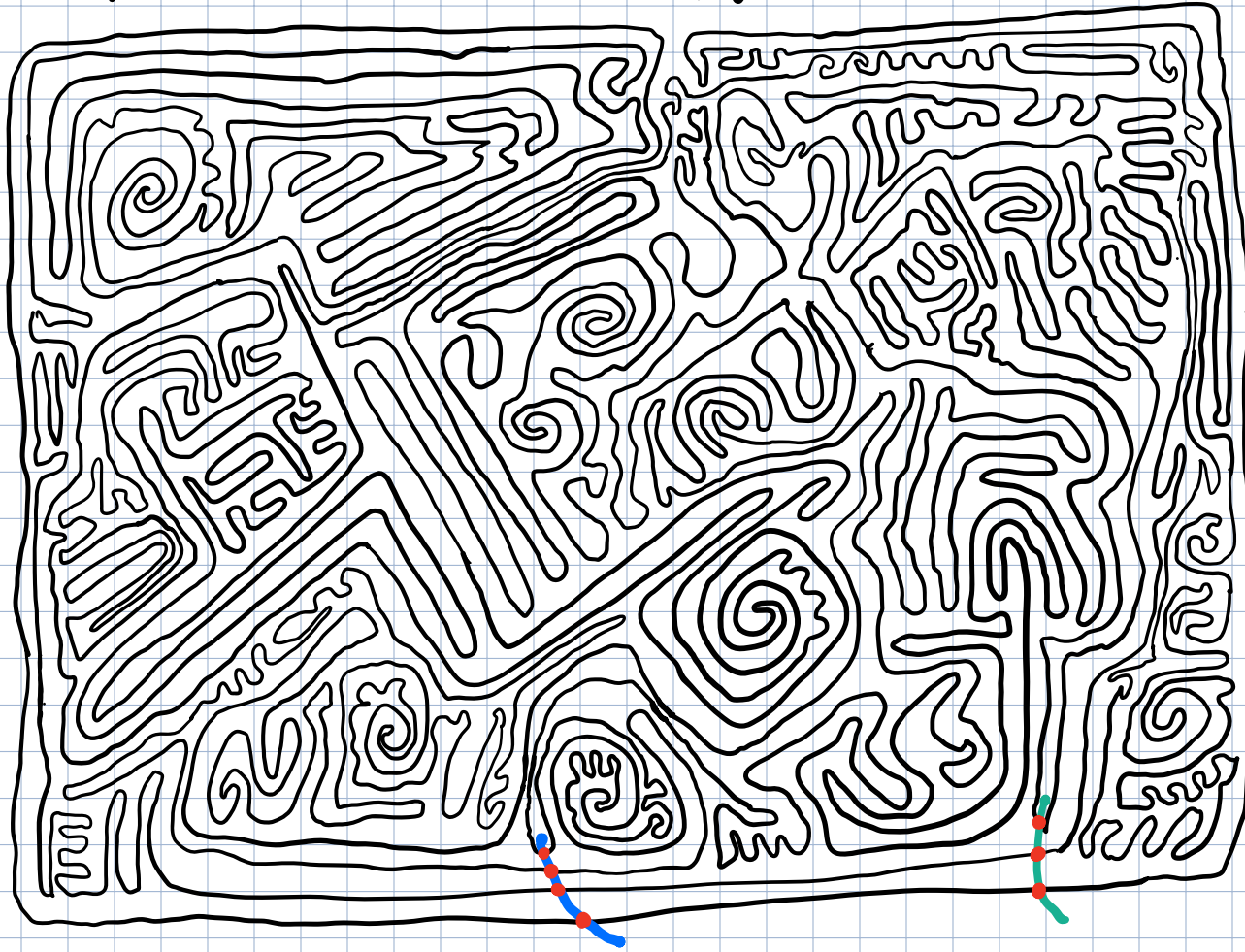
Vragen: Kan je ontsnappen van • of •?

Toepassing: Ontsnappen uit een doolhof.



Vragen: Kan je ontsnappen van • of •?

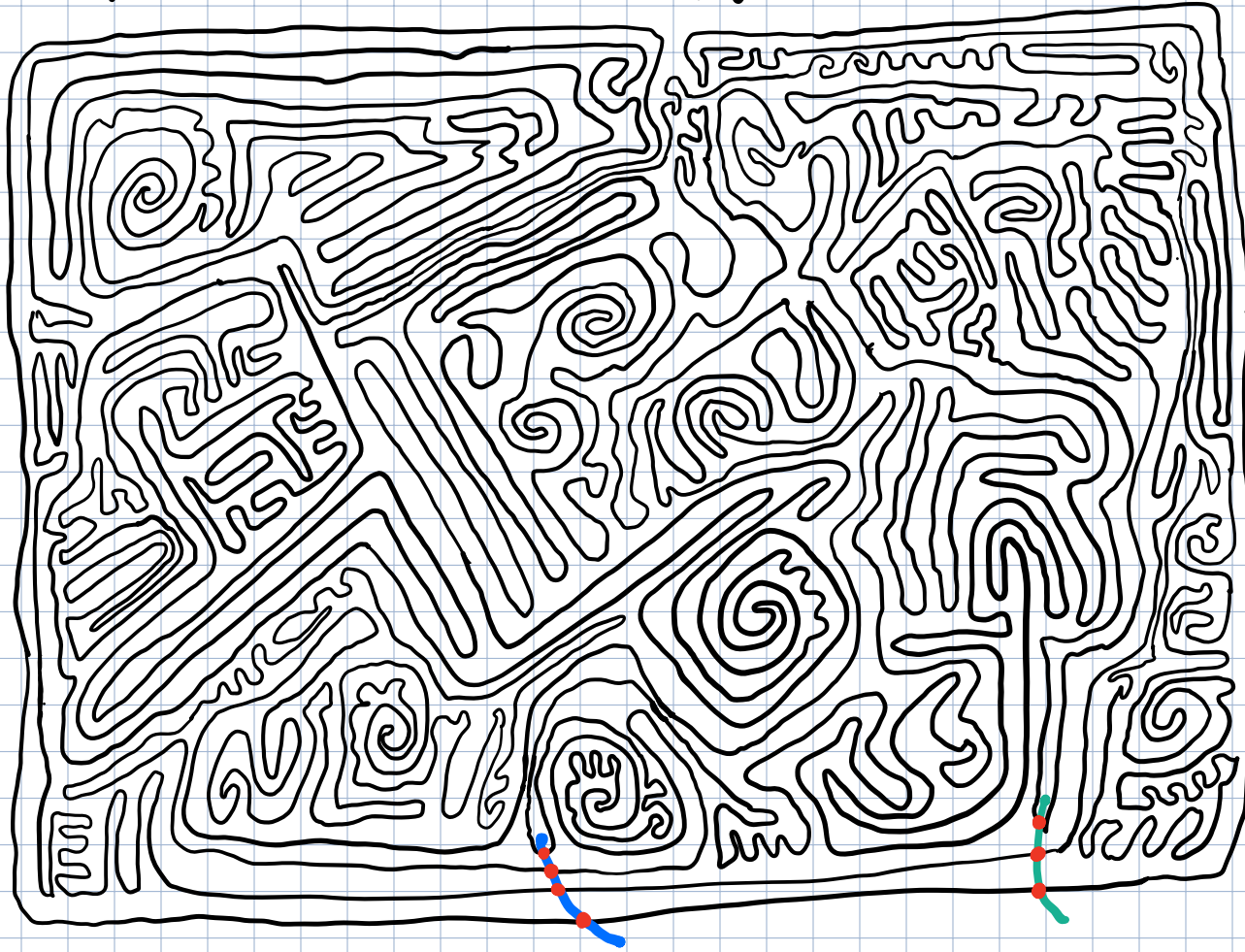
Toepassing: Ontsnappen uit een doolhof.



Trek een pad naar buiten en tel of het aantal snijpunten met doolhof **even** of **oneven** is.

Vragen: Kan je ontsnappen van • of •?

Toepassing: Ontsnappen uit een doolhof.

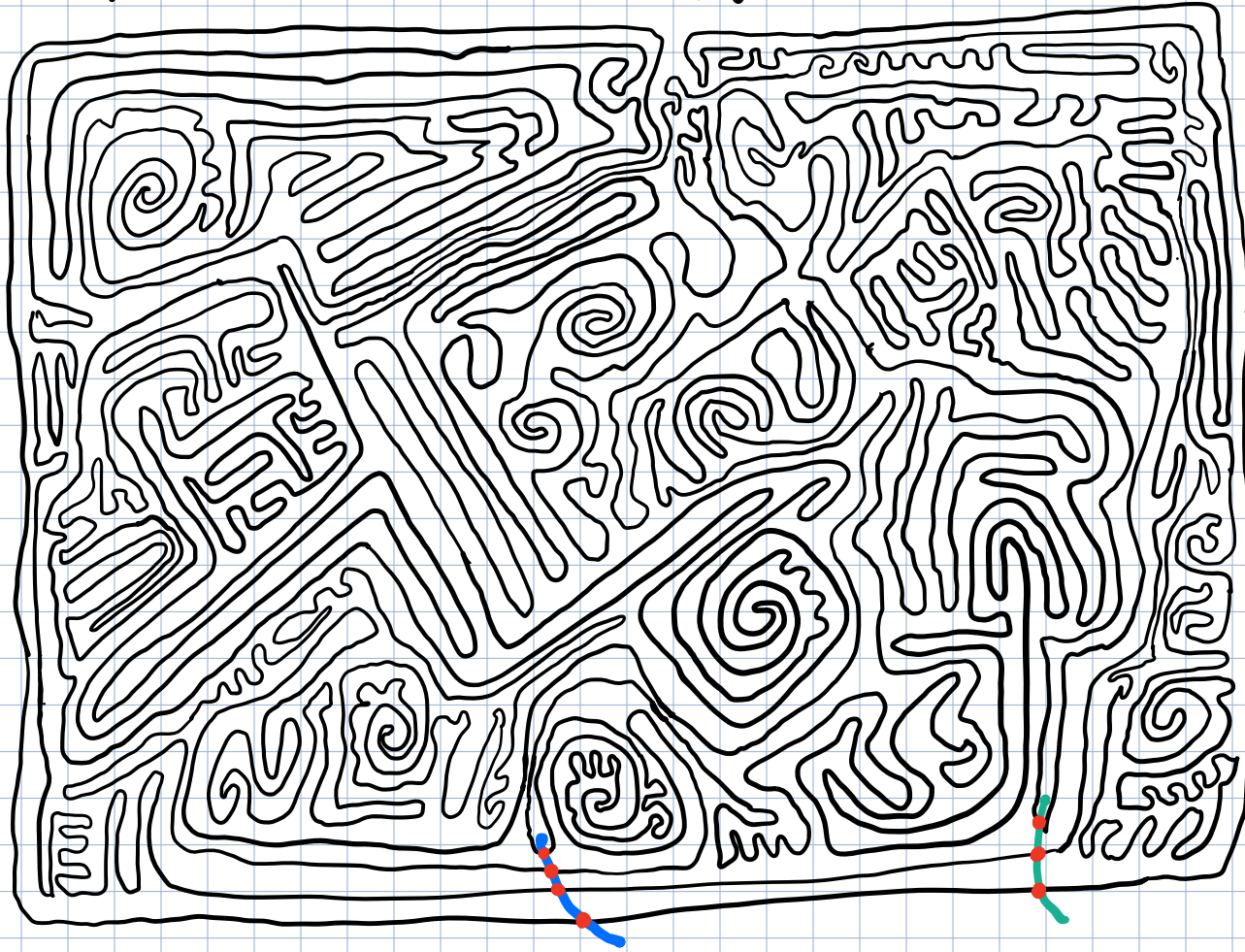


Trek een pad naar buiten en tel of het aantal snijpunten met doolhof **even** of **oneven** is.

- Als **even**: Kan **wel** ontsnappen
- Als **oneven**: Kan **niet** ontsnappen.

Vragen: Kan je ontsnappen van • of •?

Toepassing: Ontsnappen uit een doolhof.



Trek een pad naar buiten en tel of het aantal snijpunten met doolhof **even** of **oneven** is.

- Als **even**: Kan **wel** ontsnappen
- Als **oneven**: Kan **niet** ontsnappen.

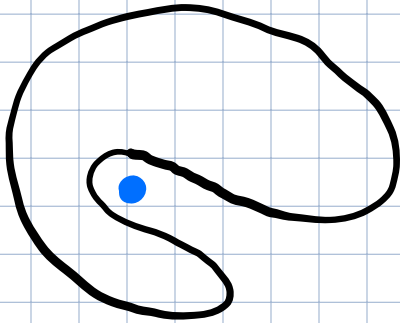
Vragen: Kan je ontsnappen van • of •?

Opmerking: Het maakt **niet** uit hoe je het pad trekt!

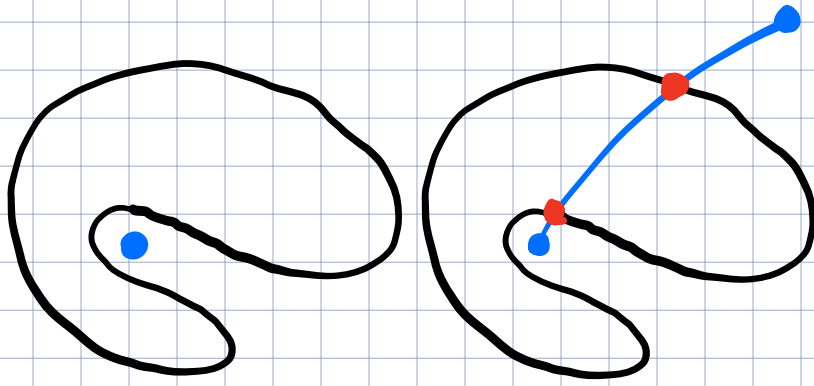
De oplossing



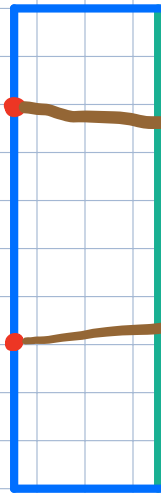
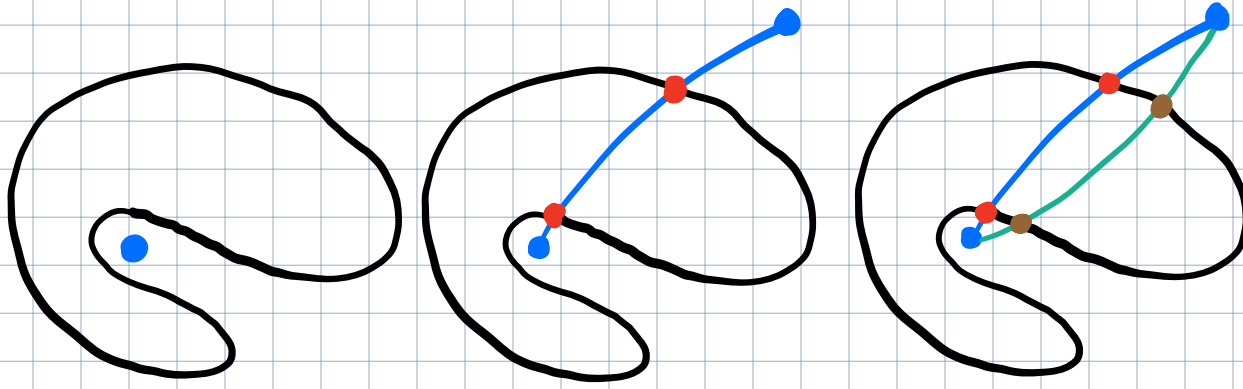
Waarom werkt dit?



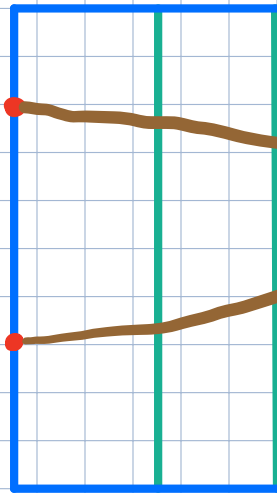
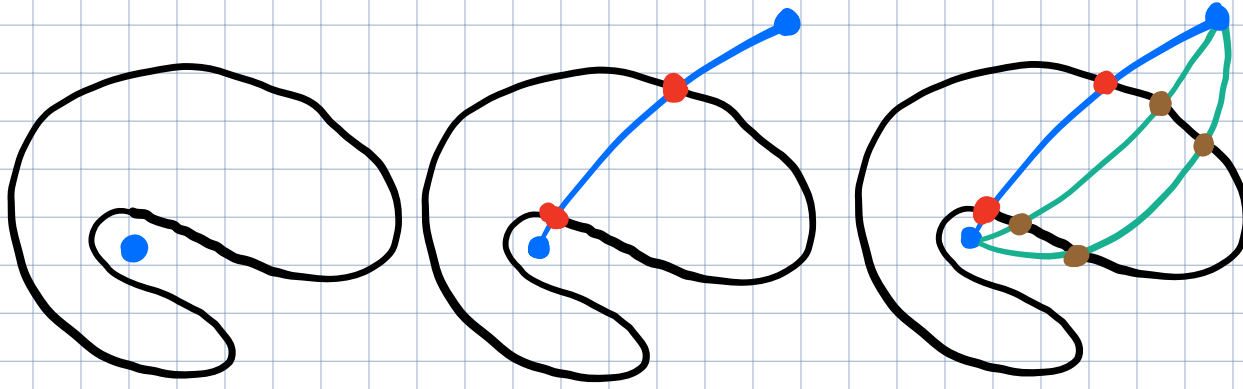
Waarom werkt dit?



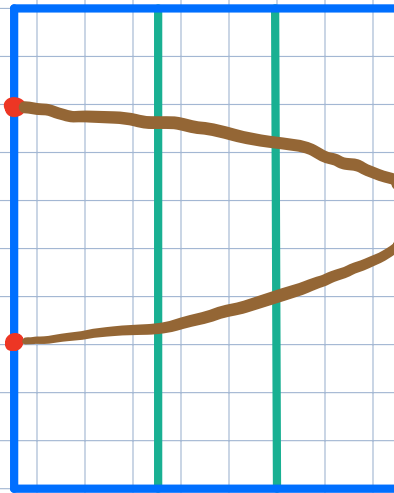
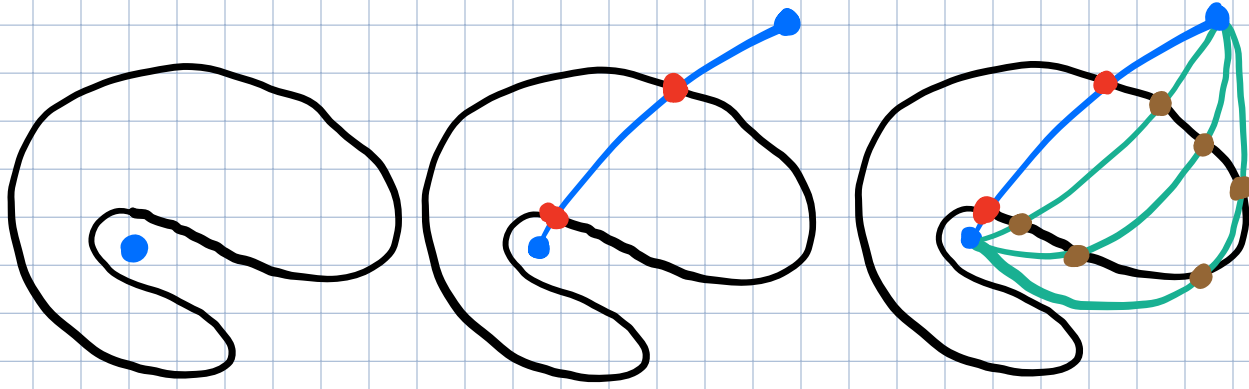
Waarom werkt dit?



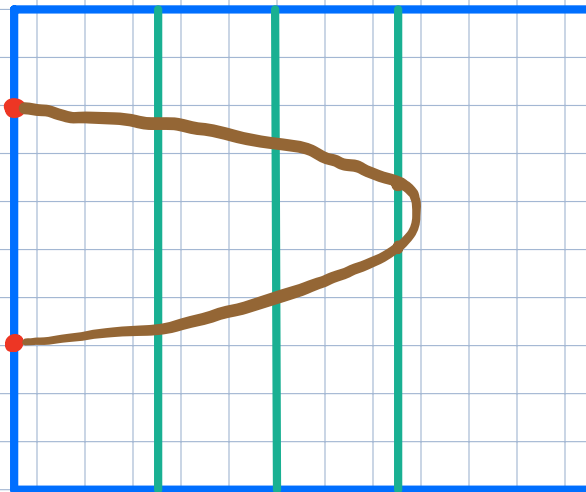
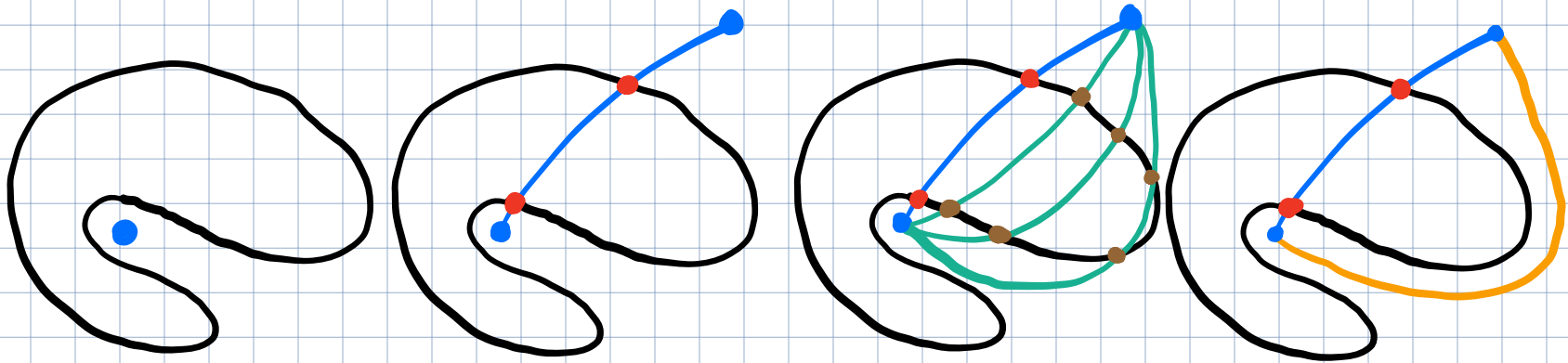
Waarom werkt dit?



Waarom werkt dit?

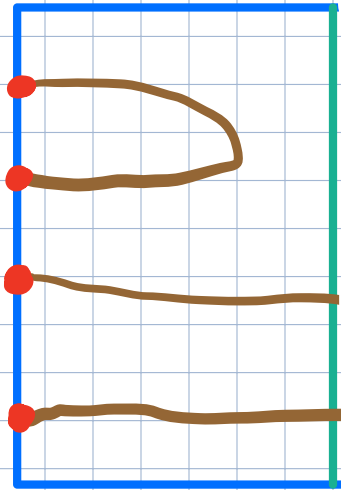
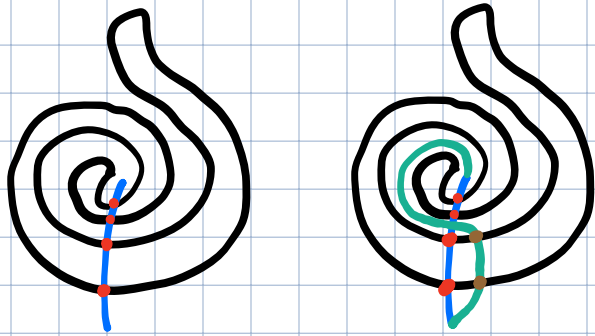


Waarom werkt dit?

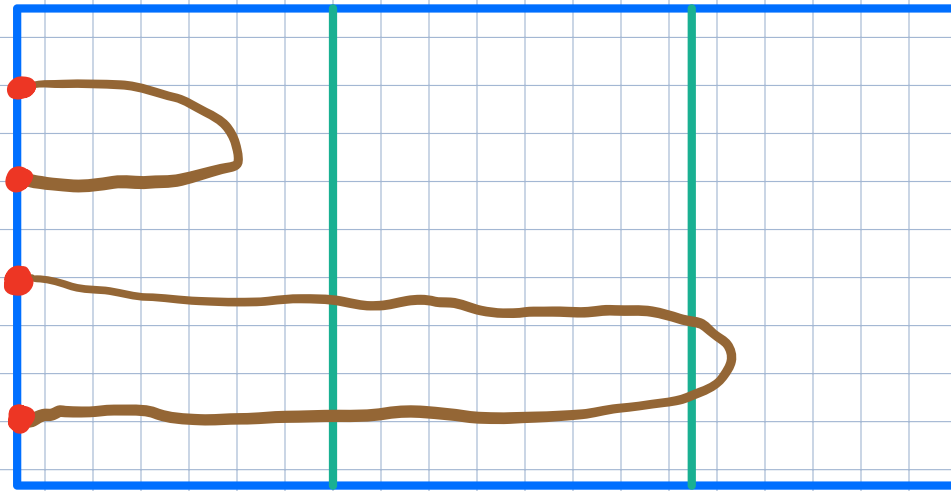
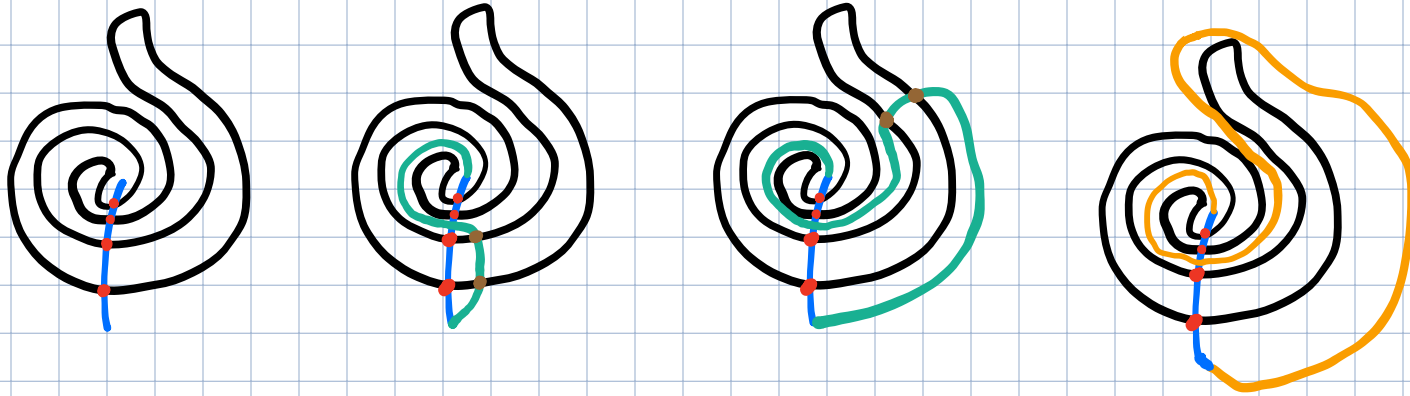


De **snijpunten** zijn de uiteindes van een **touw**
dus zijn er een **even** aantal

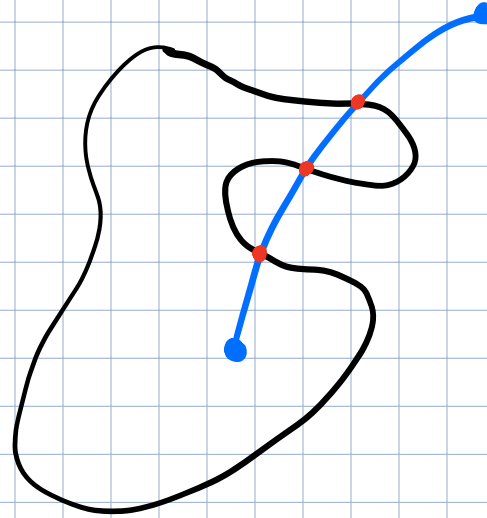
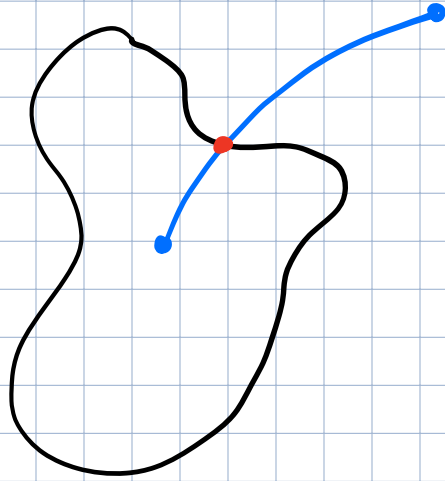
Waarom werkt dit?



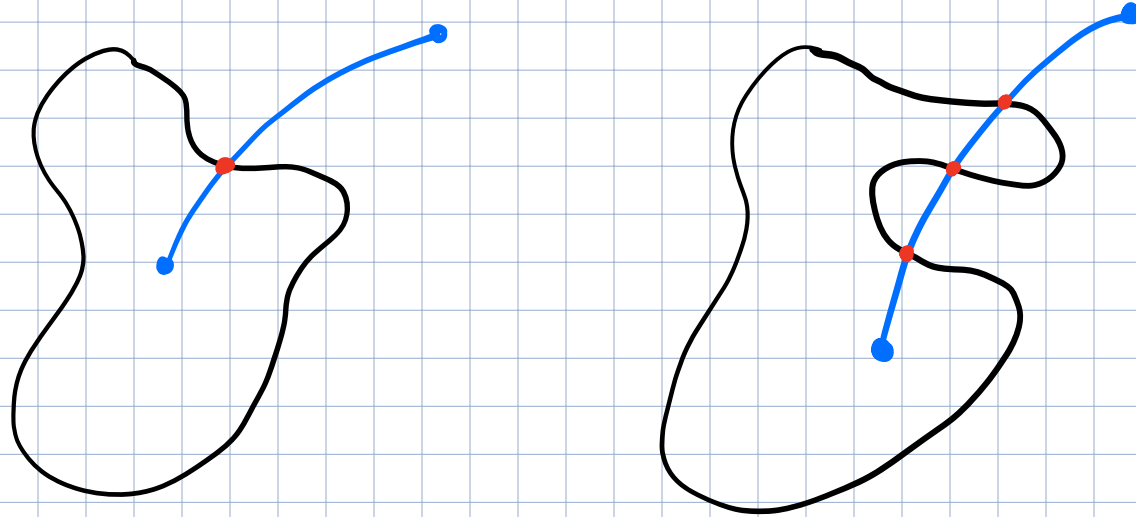
Waarom werkt dit?



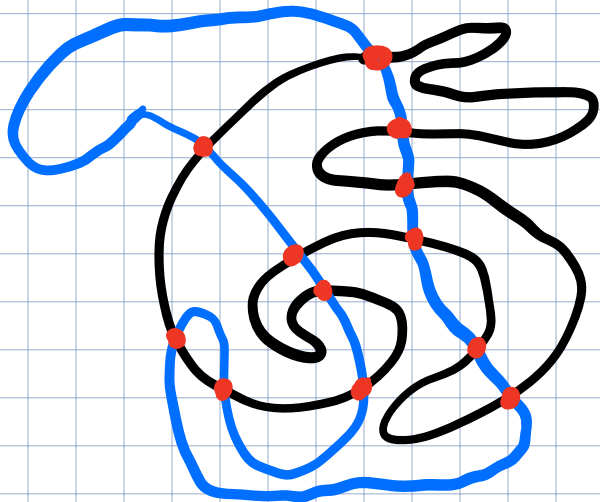
Bij **oneven** aantal snijpunten kan je **niet** ontsnappen



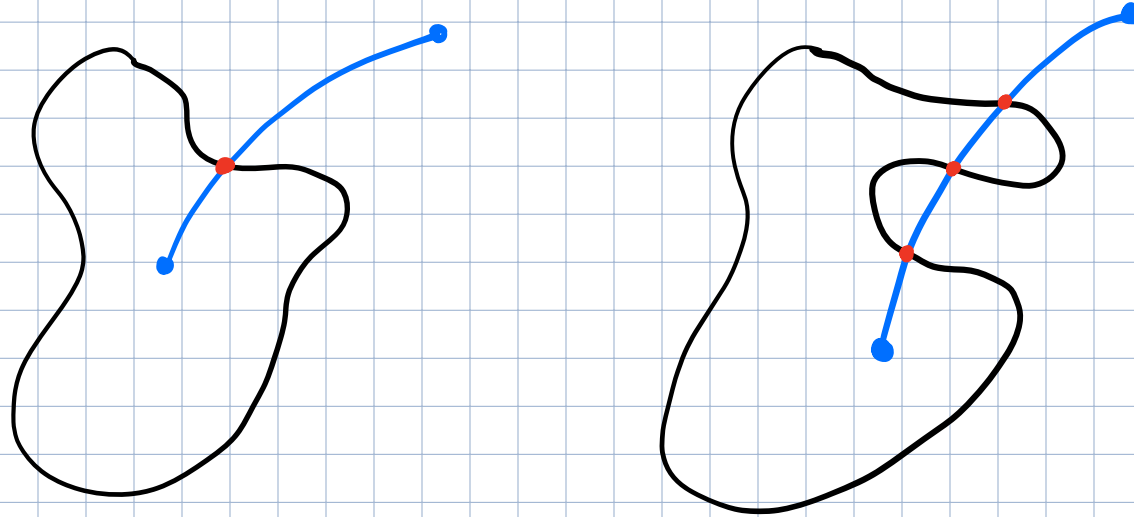
Bij **oneven** aantal snijpunten kan je **niet** ontsnappen



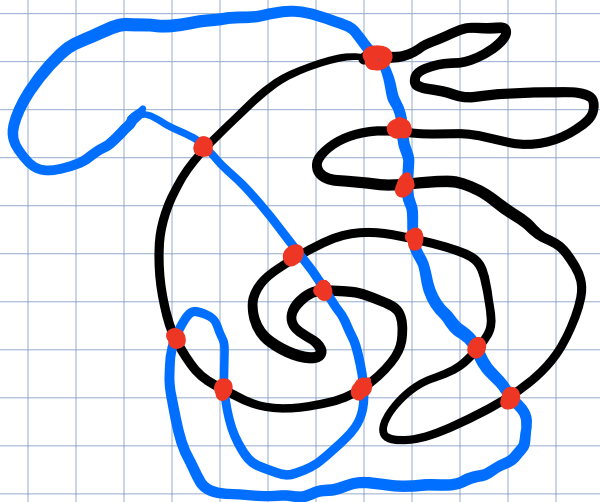
Stelling twee gesloten krommen doorsnijden
elkaar een **even** aantal punten



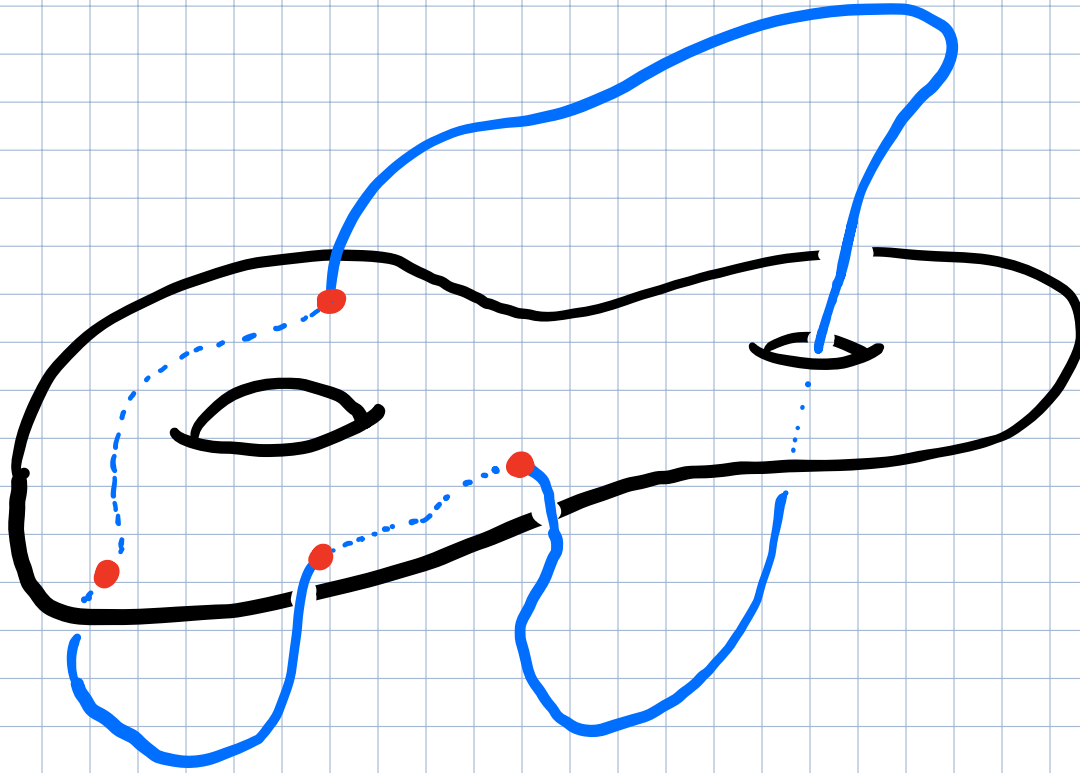
Bij oneven aantal snijpunten kan je niet ontsnappen



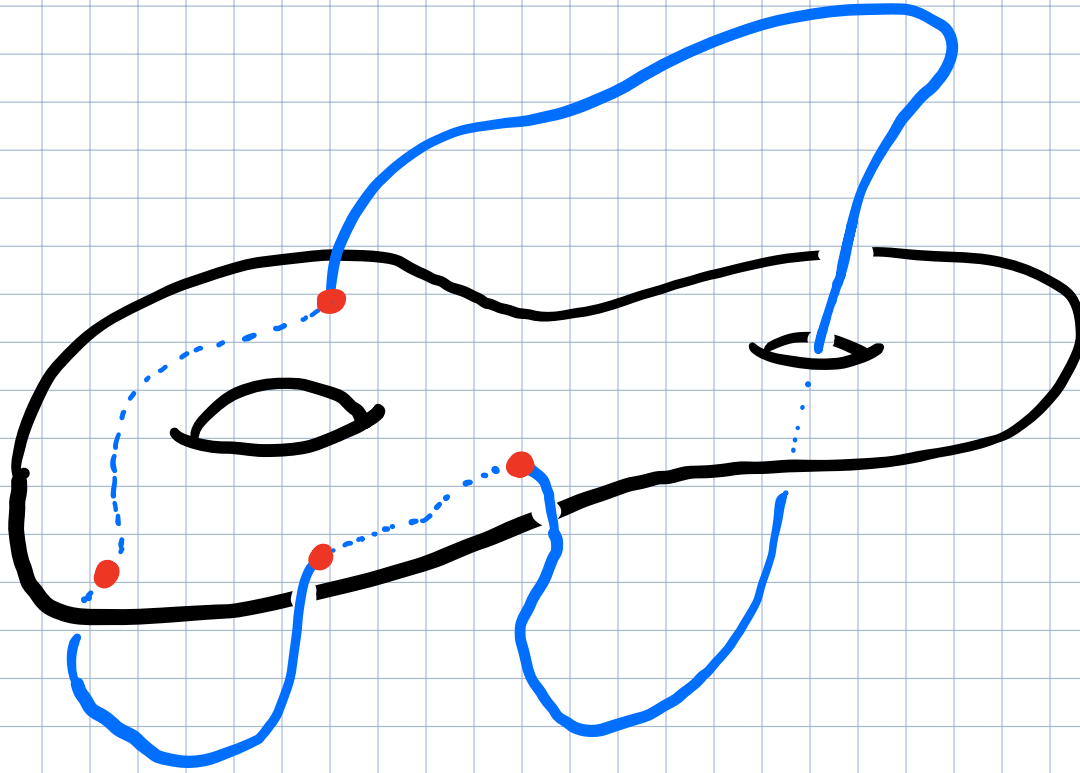
Stelling twee gesloten krommen doorsnijden elkaar een even aantal punten



Hogere dimensies

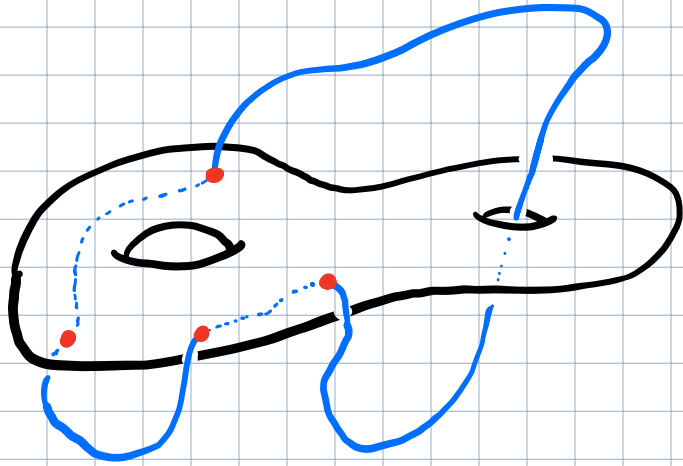


Hogere dimensies

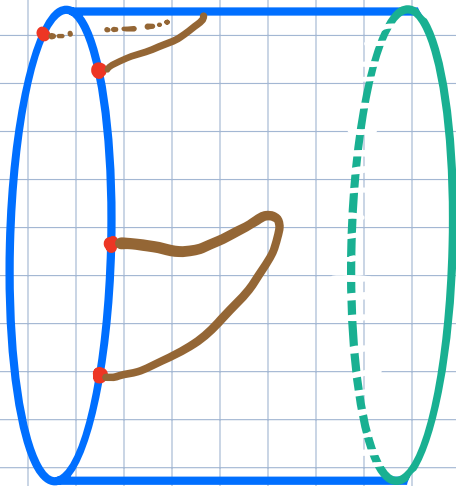
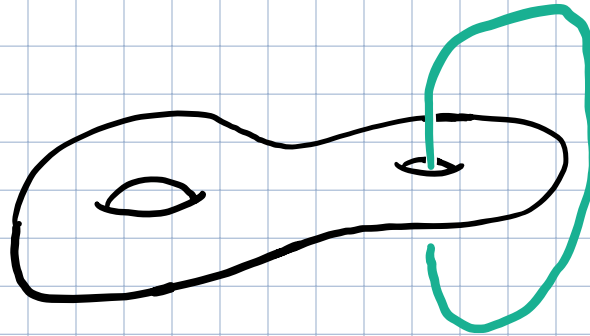
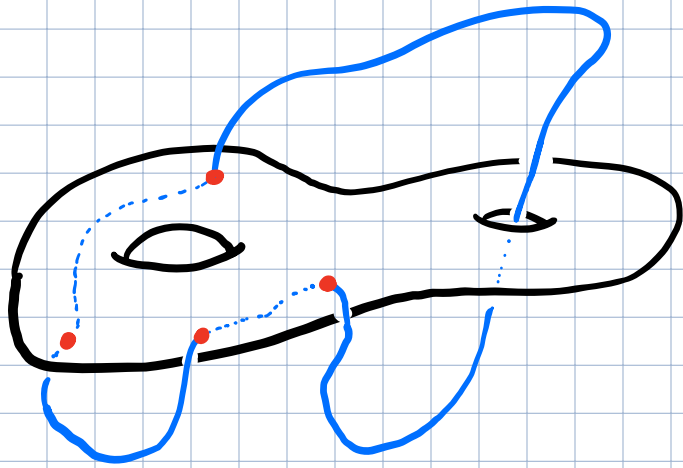


Stelling: Een gesloten oppervlak en een gesloten kromme snijden elkaar in een **even** aantal punten in drie dimensies.

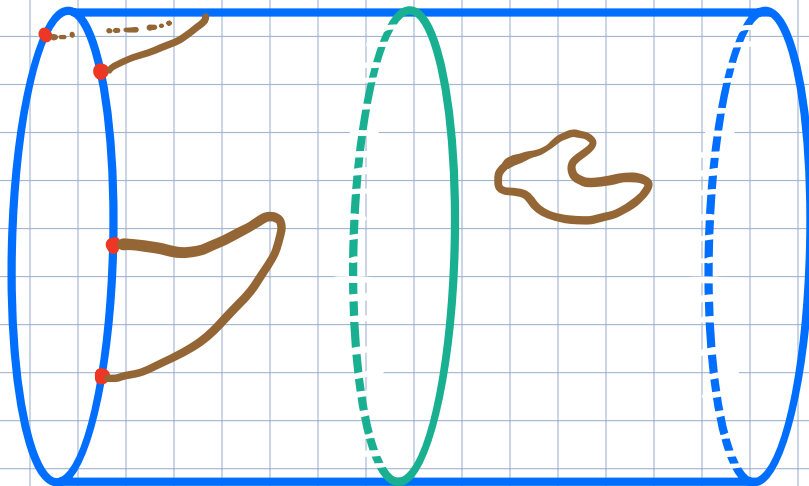
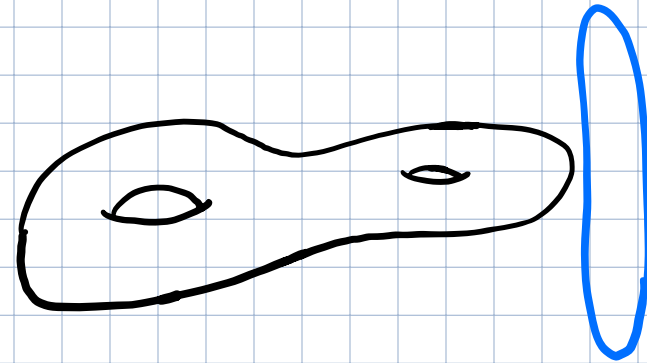
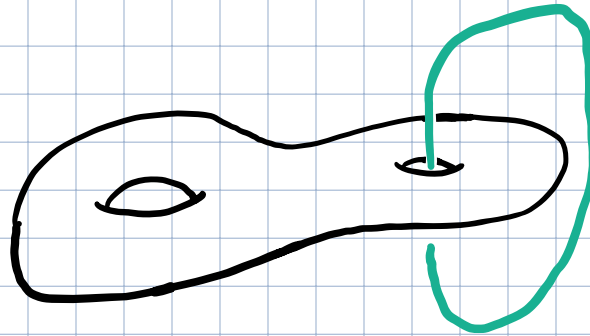
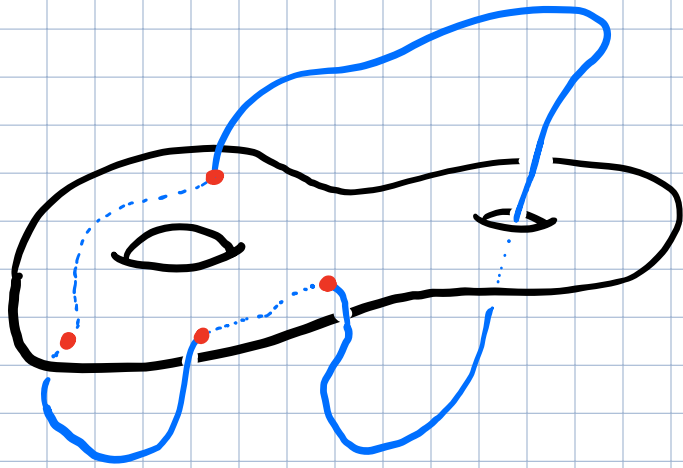
He-zelfde argument



Hetzelfde argument



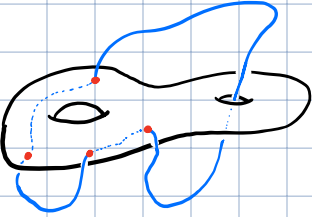
He-zelfde argument



Een touw
heeft
even
aantal uiteinden



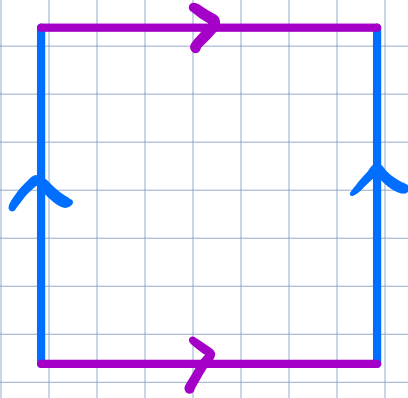
even



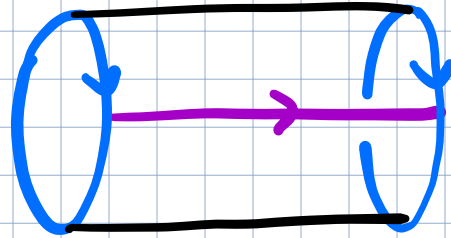
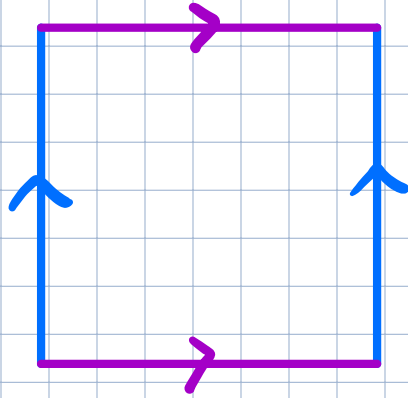
doorsnijdingen

De logica

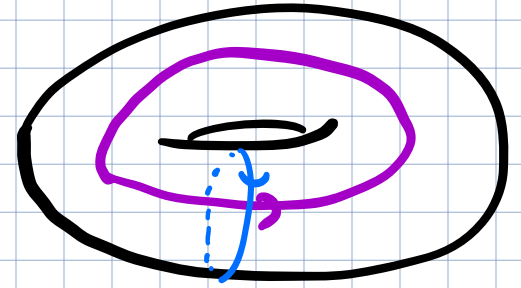
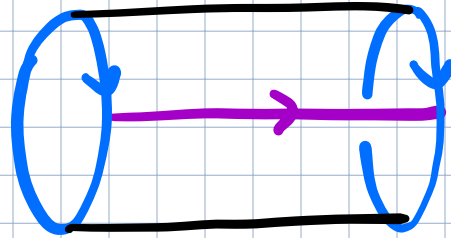
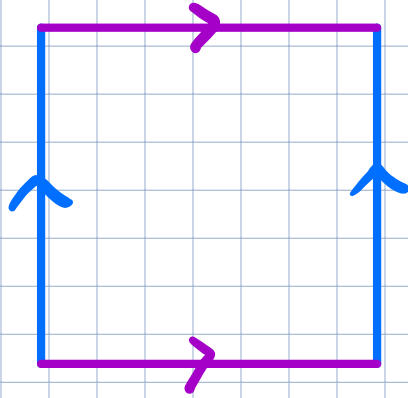
Maken van oppervlakken



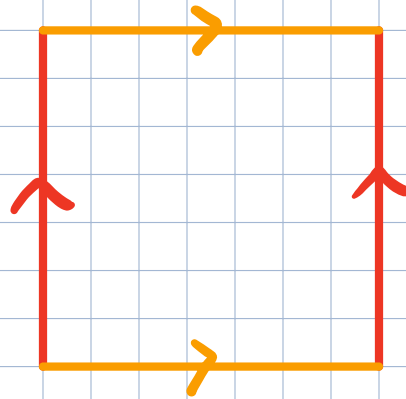
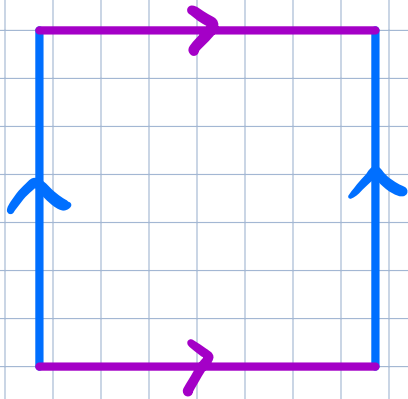
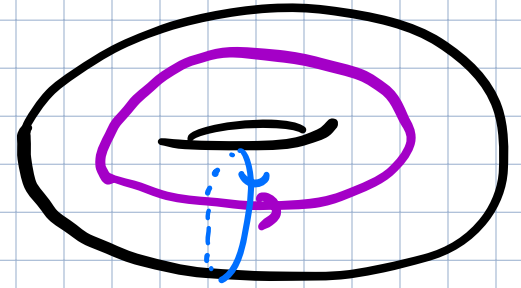
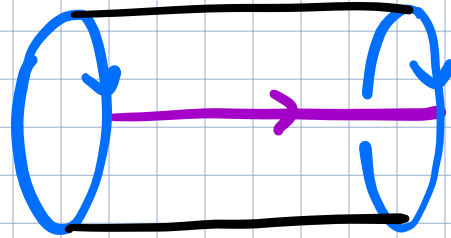
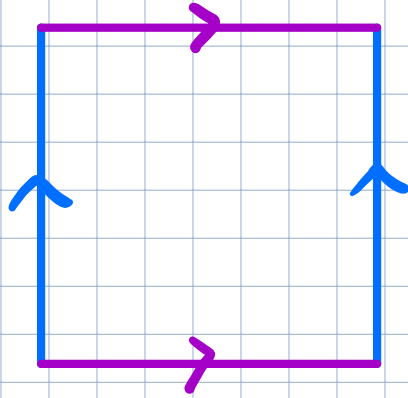
Maken van oppervlakken



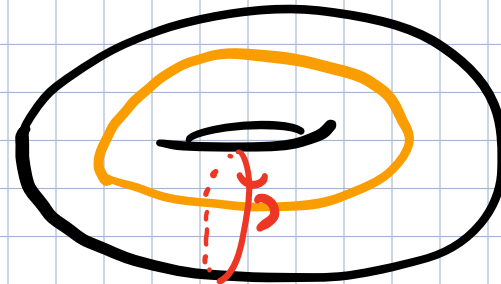
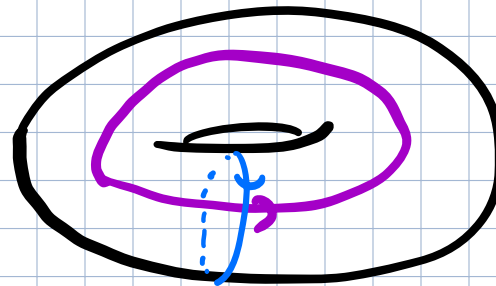
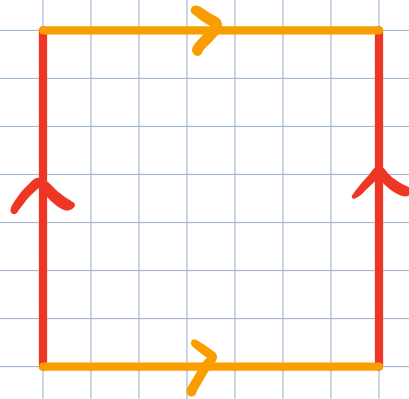
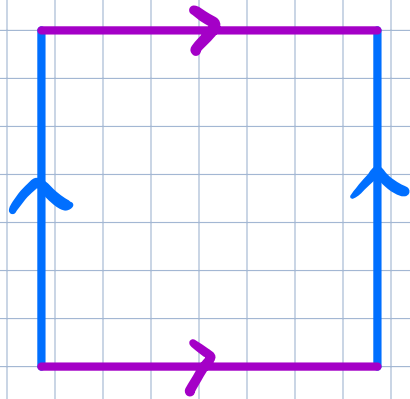
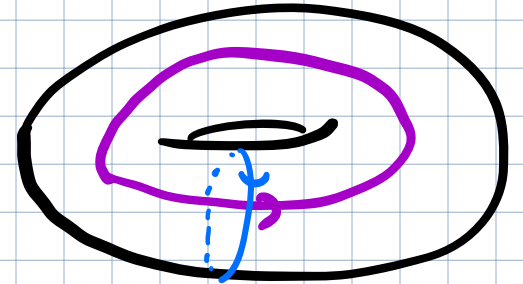
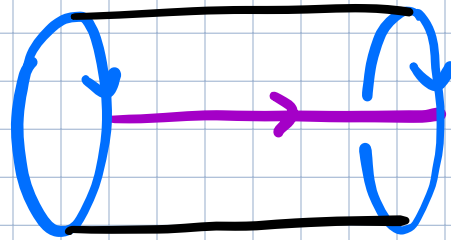
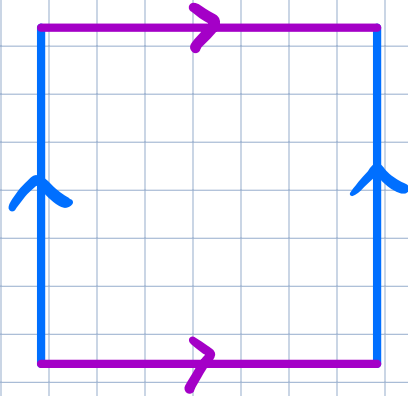
Maken van oppervlakken



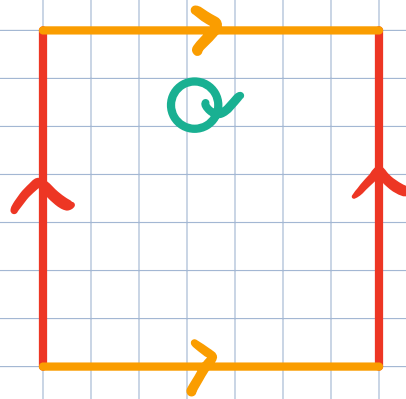
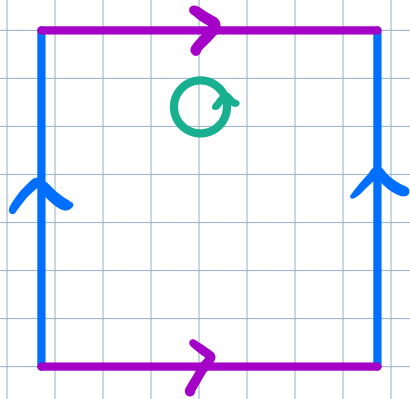
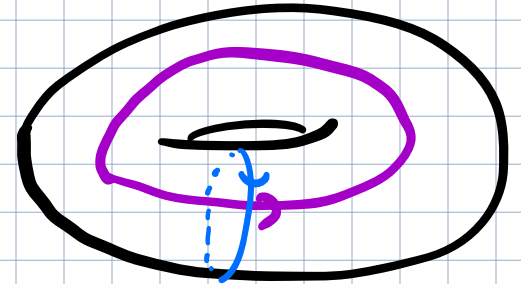
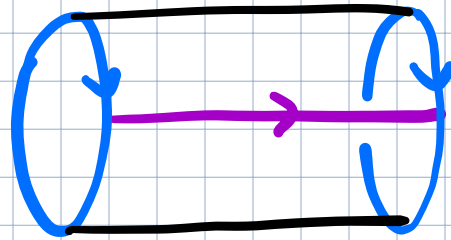
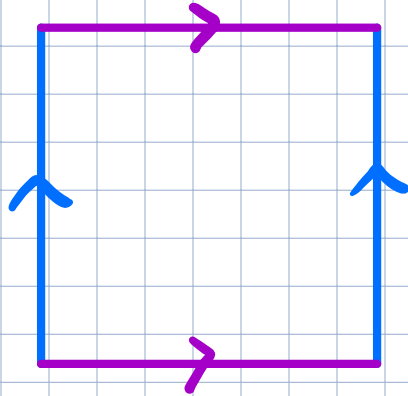
Maken van oppervlakken



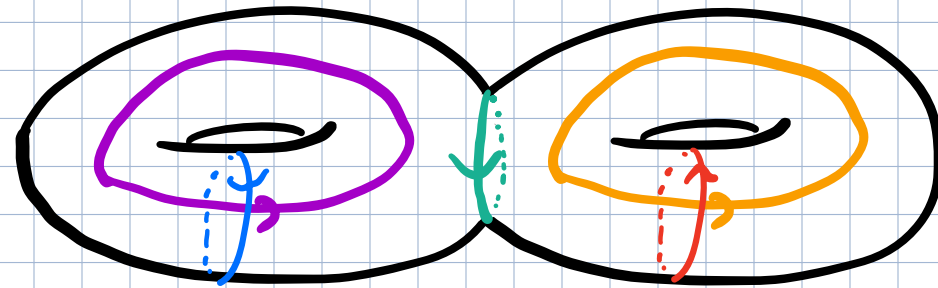
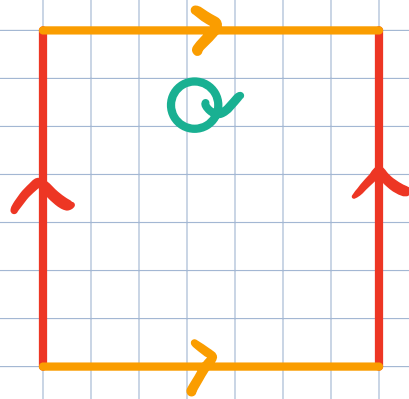
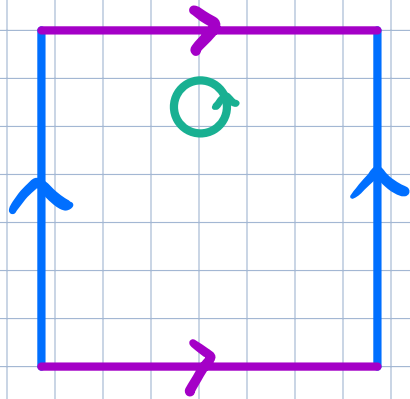
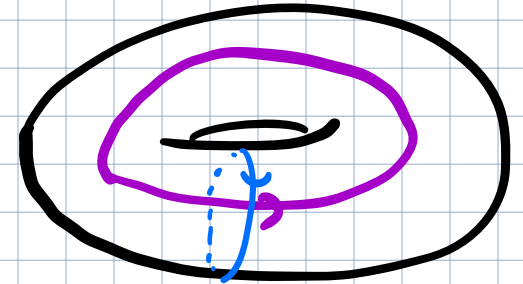
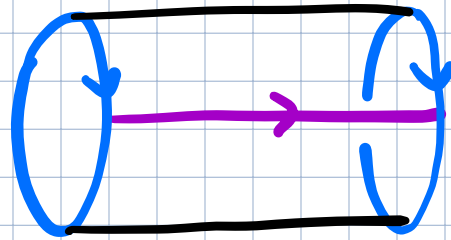
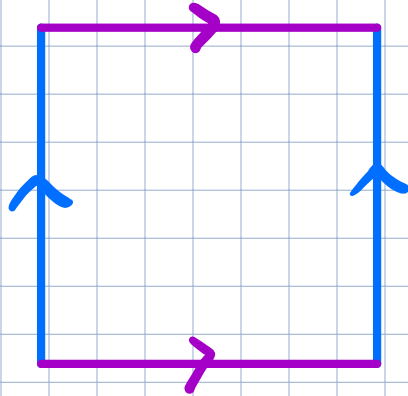
Maken van oppervlakken



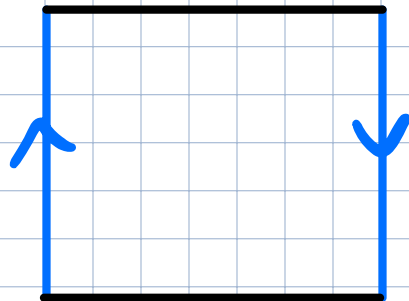
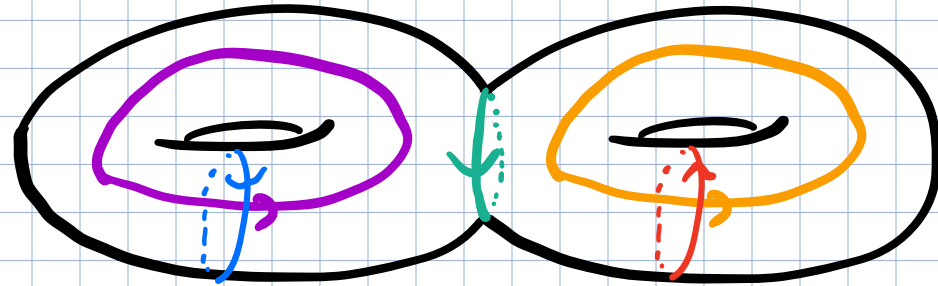
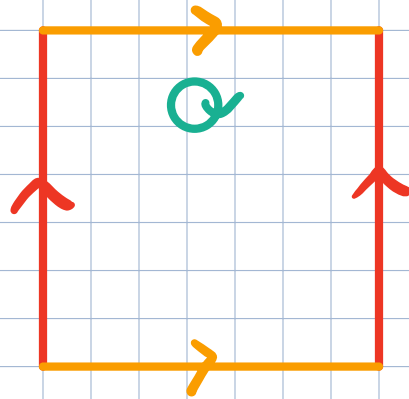
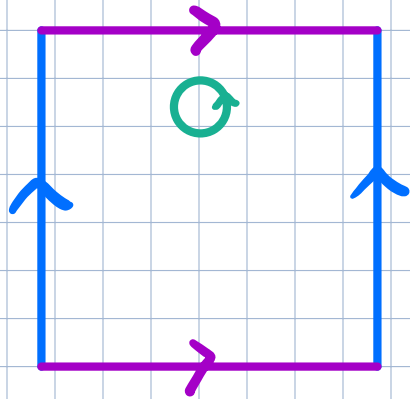
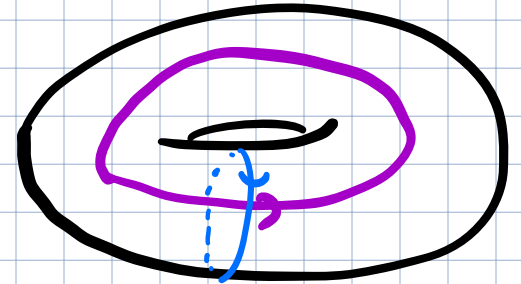
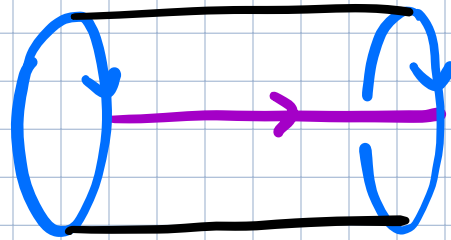
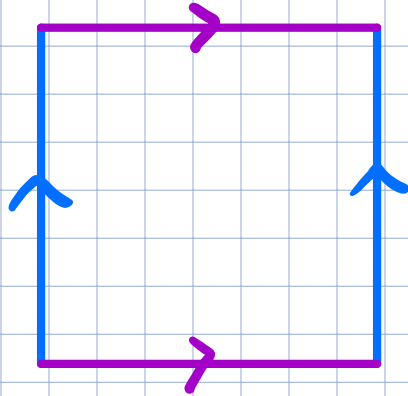
Maken van oppervlakken



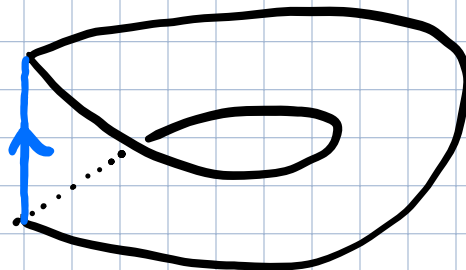
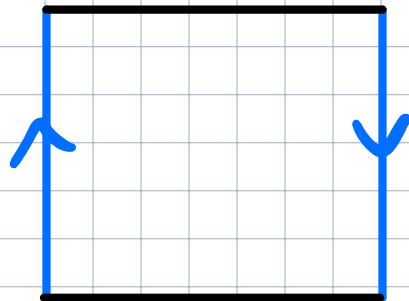
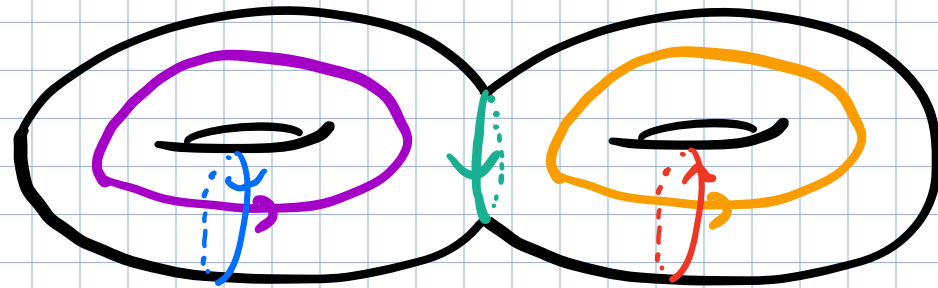
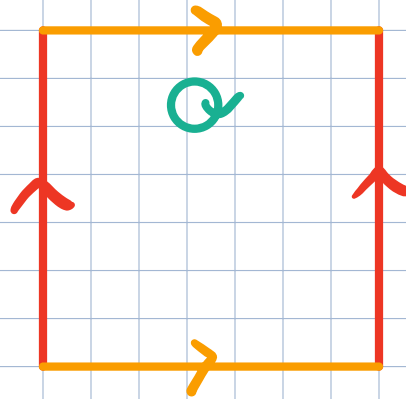
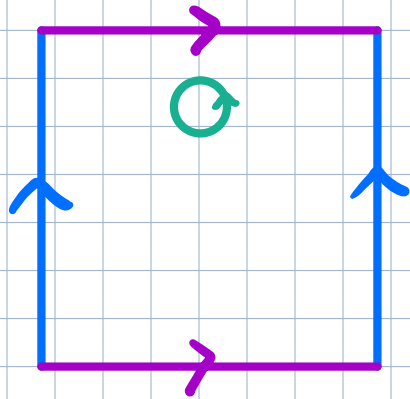
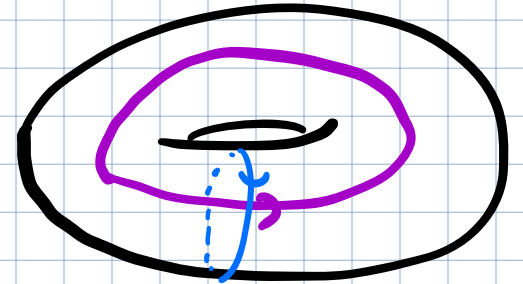
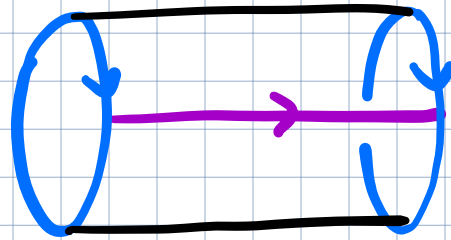
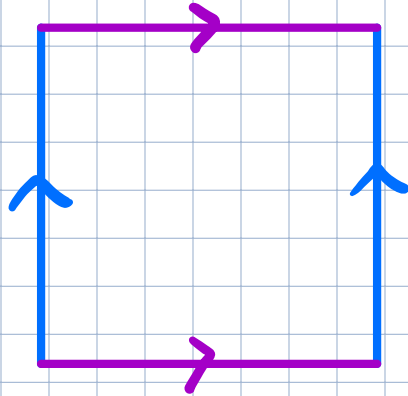
Maken van oppervlakken



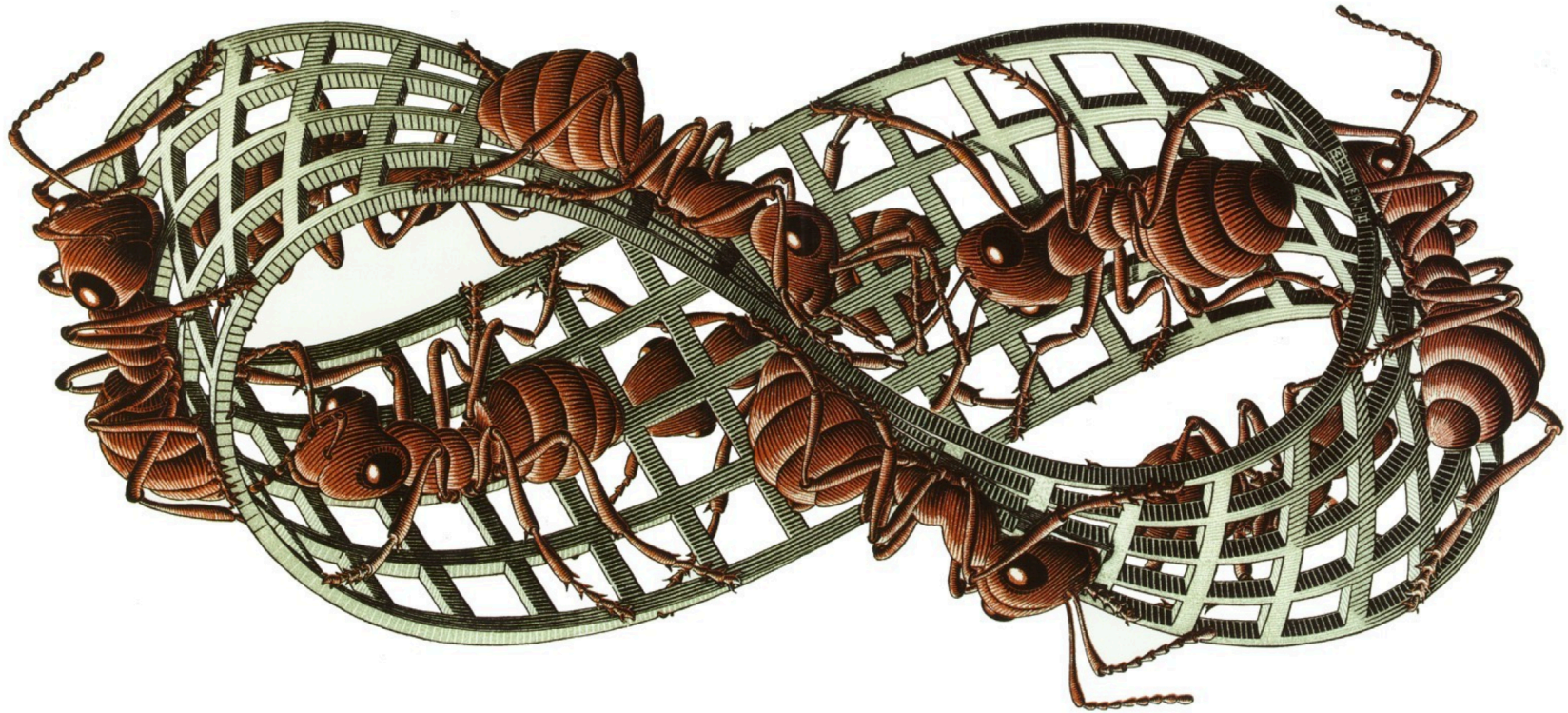
Maken van oppervlakken



Maken van oppervlakken

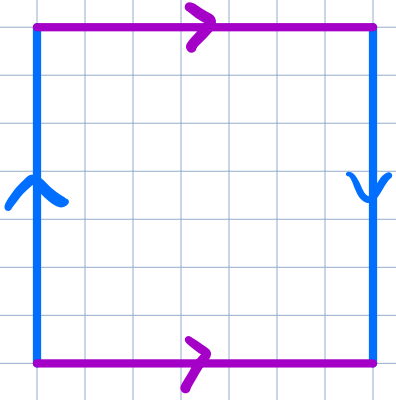


De Möbius strip heeft maar één zijde

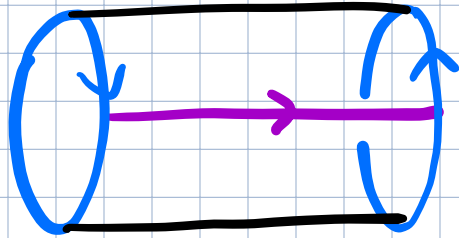
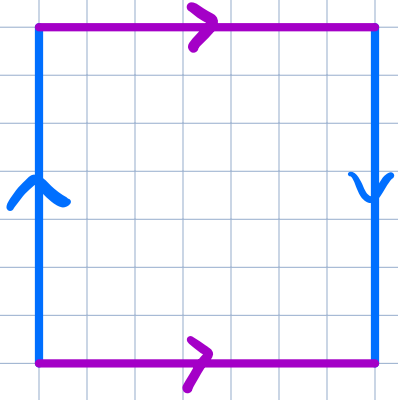


(Escher)

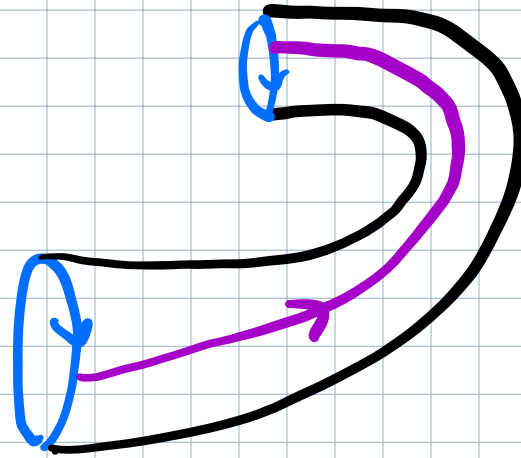
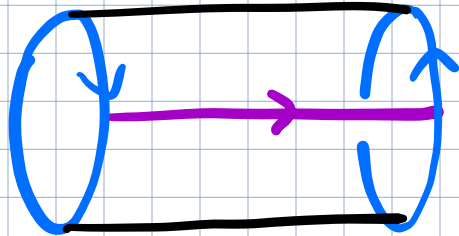
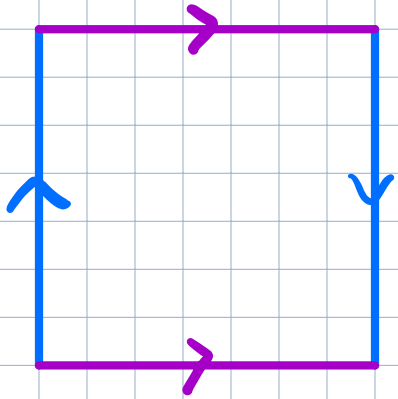
De Kleinse fies :



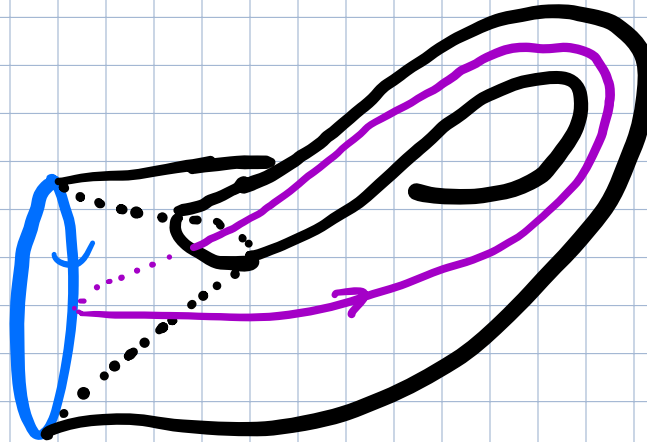
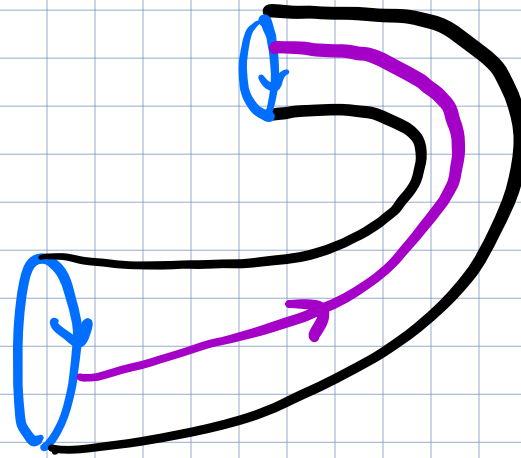
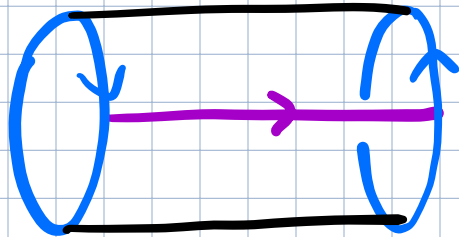
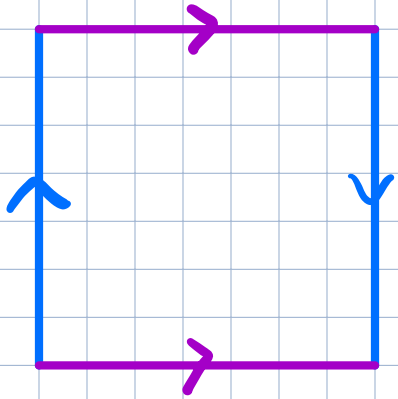
De Kleinse fies :



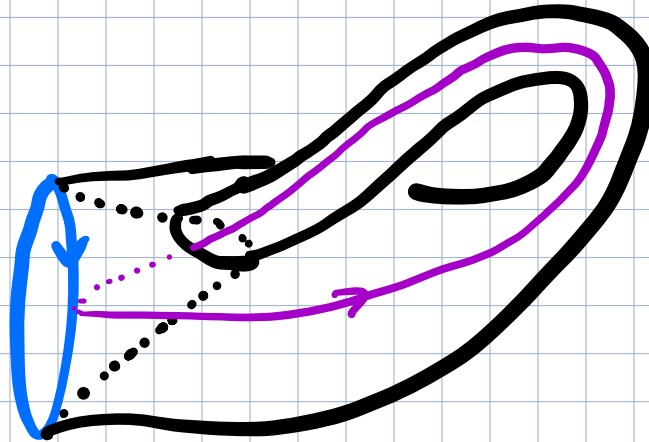
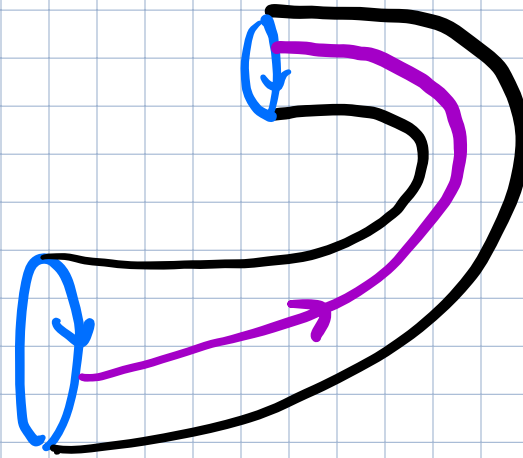
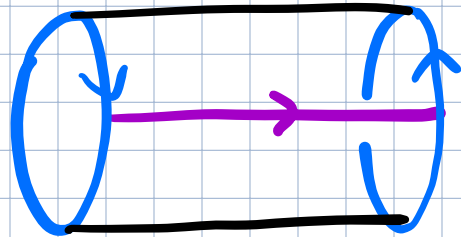
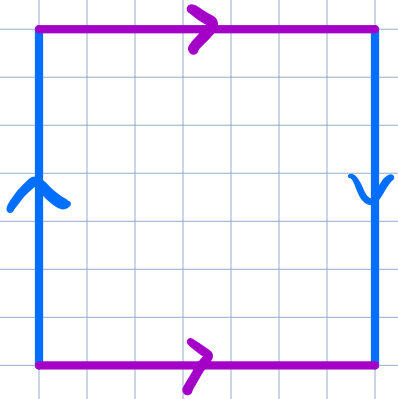
De Kleinse fles :



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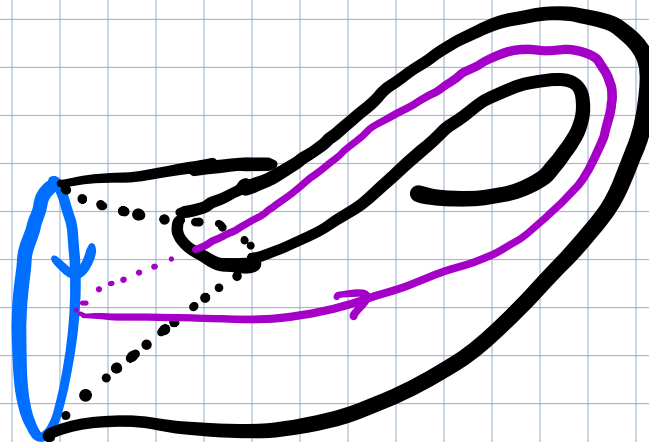
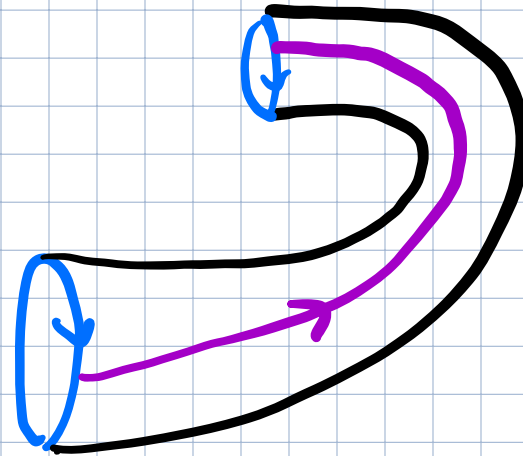
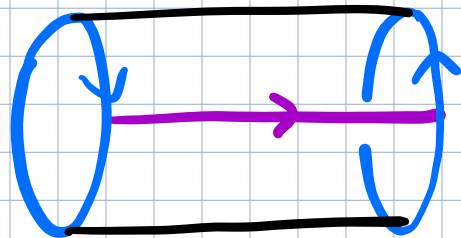
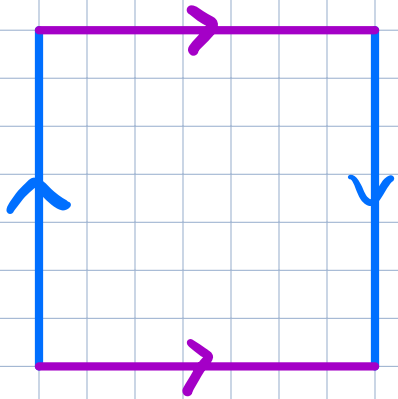


De Kleinse fles:



Vraag: Past de Kleinse fles in $\mathbb{R}^3$ zonder zelf doorsnijdingen?

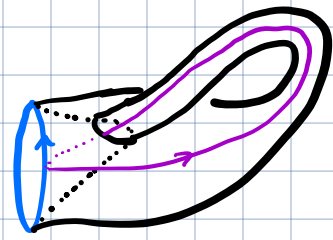
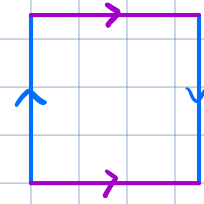
De Kleinse fles:



Vraag: Past de Kleinse fles in $\mathbb{R}^3$ zonder zelf doorsnijdingen?

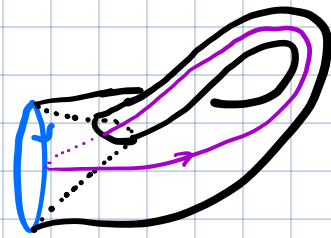
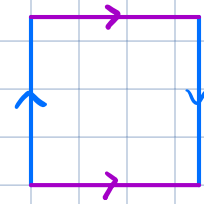
Nee! Dit gaan we nu bewijzen.

De tegenspraak: Stel het past

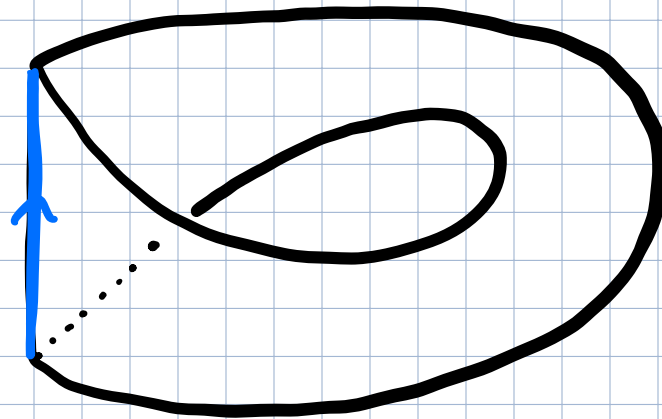
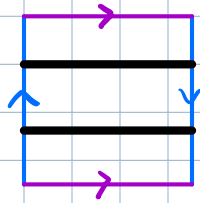


Maar dan **zonder** zelf doorsnijdingen

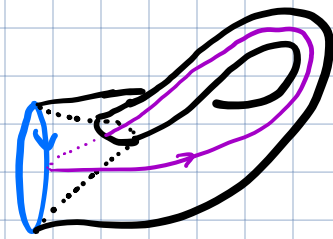
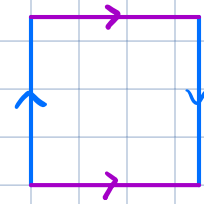
De tegenspraak: Stel het past



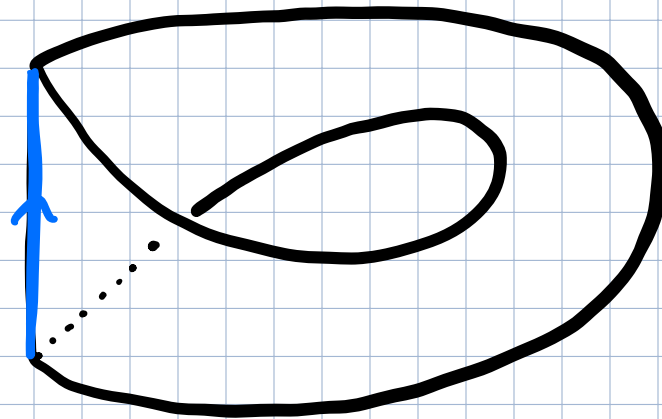
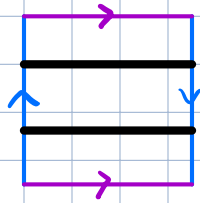
Stap 1: In de kleinse fles zit een Möbius strip



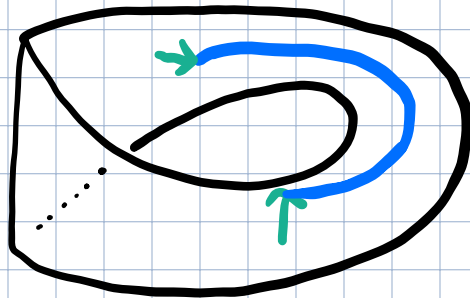
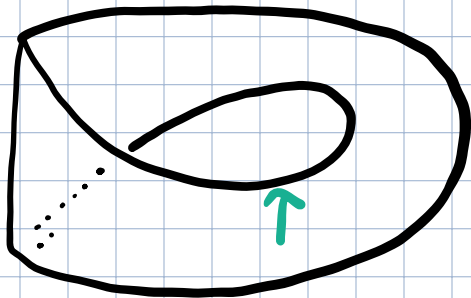
De tegenspraak: Stel het past



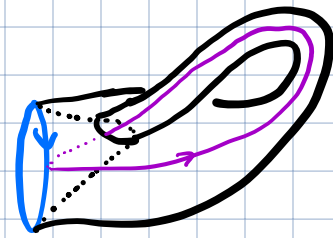
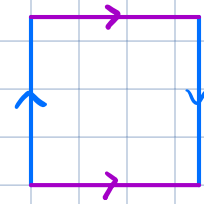
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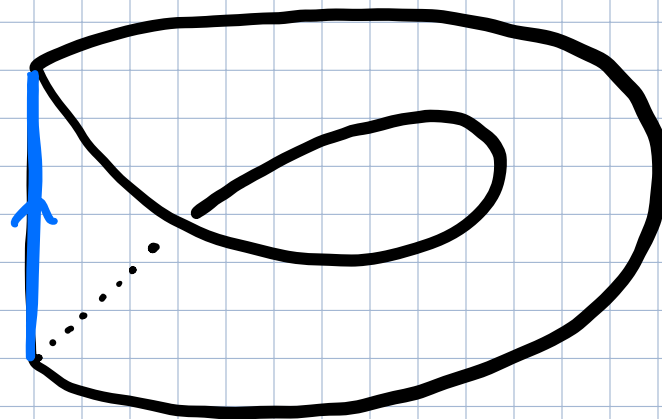
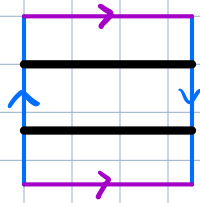
Stap 2: laat een miez over de Möbius strip lopen



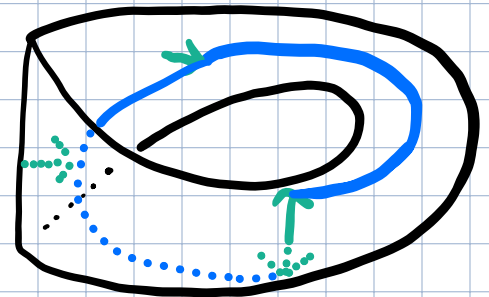
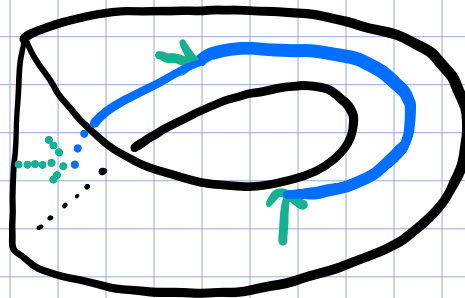
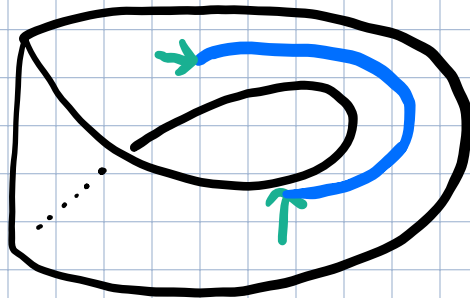
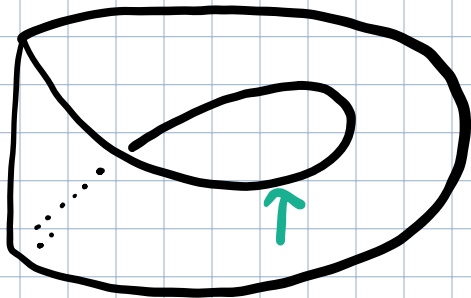
De tegenspraak: Stel het past



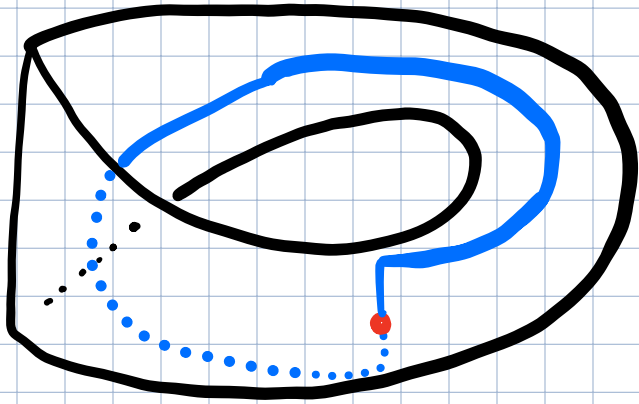
Stap 1: In de kleinse fles zit een Möbius strip



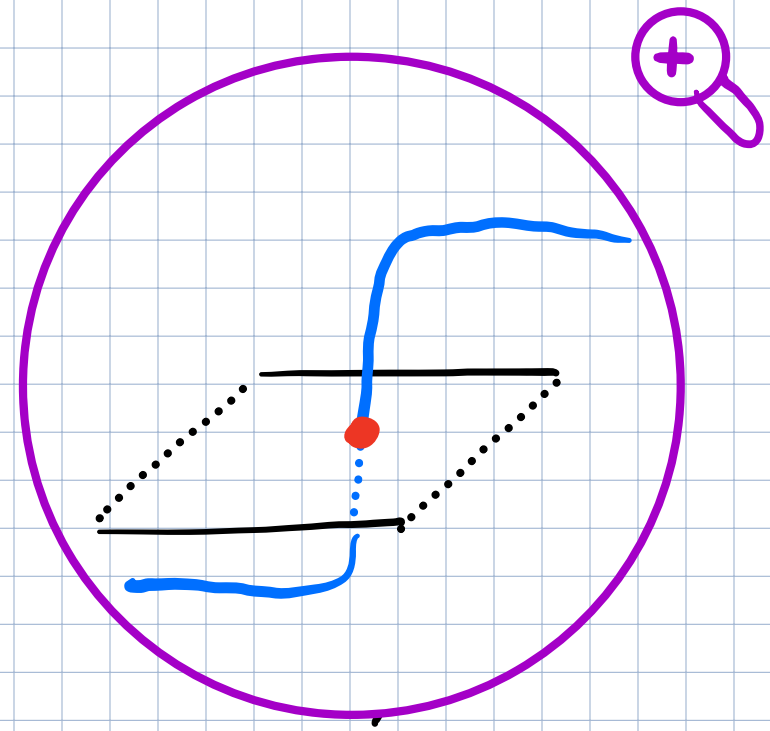
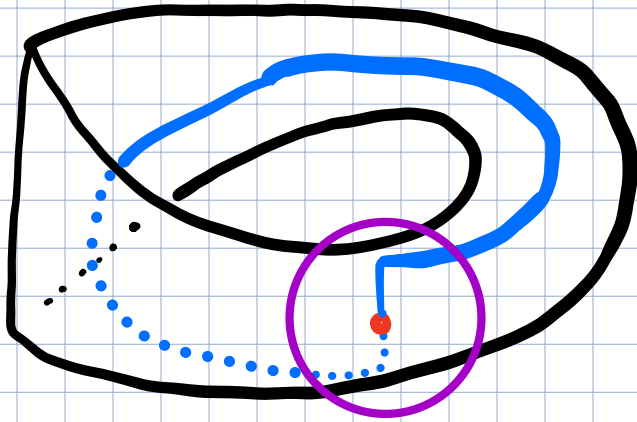
Stap 2: laat een miez over de Möbius strip lopen



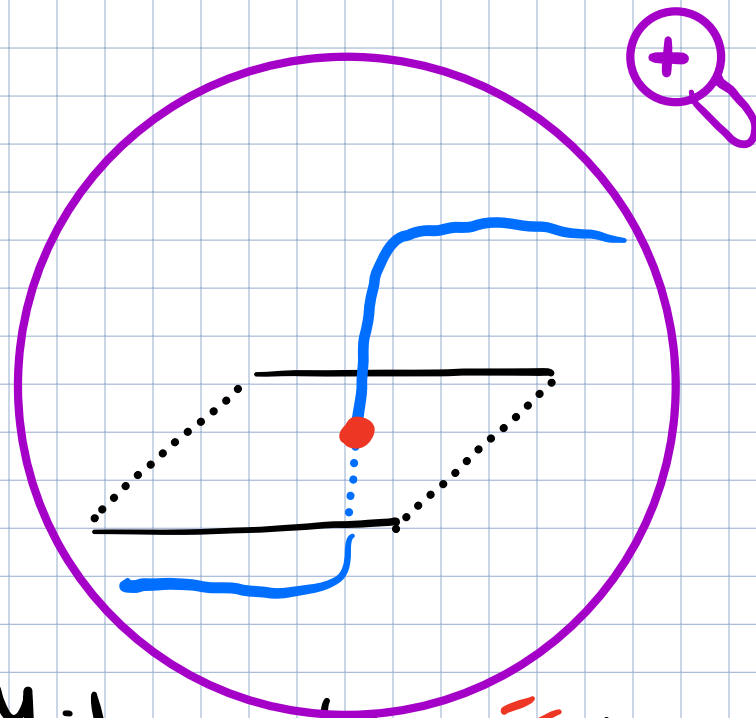
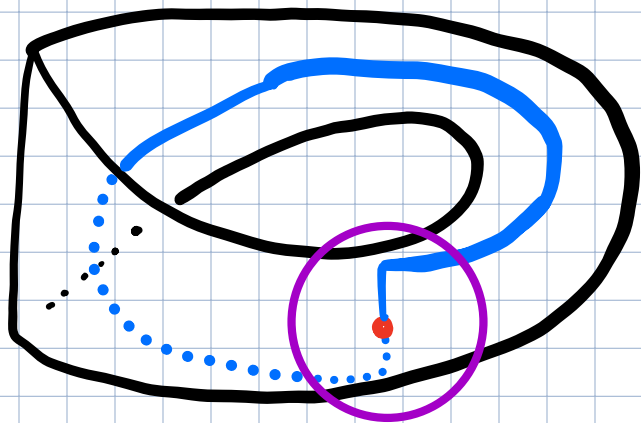
Step 3: Verbindt het pad.



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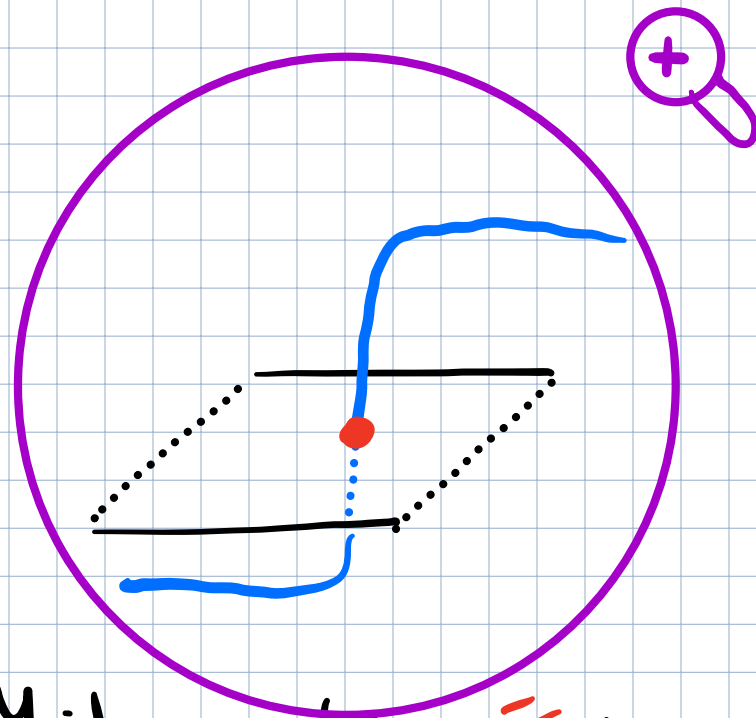
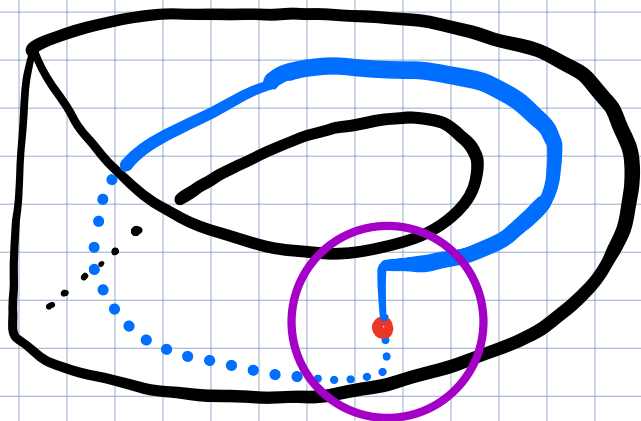


Dit pad snijdt de Möbius strip **één** keer

Step 4: Conclusie:

Als de Kleinse fles in $\mathbb{R}^3$ zou passen
Dan vinden we een kromme die de fles
een **oneven** keer snijdt. Dat kan niet!

Step 3: Verbindt het pad.



Dit pad snijdt de Möbius strip **één** keer

Step 4: Conclusie:

Als de Kleinse fles in $\mathbb{R}^3$ zou passen
Dan vinden we een kromme die de fles
een **oneven** keer snijdt. Dat kan niet!

(De Möbius strip en de kromme kunnen elkaar
wel oneven keer doorsnijden. De Möbius strip heeft een **rand**)

We hebben laten zien:

- De Kleinse fles past niet in $\mathbb{R}^3$

Via hetzelfde argument

- De Kleinsche fles past niet in $\mathbb{R}^3$
- Geen enkel gesloten oppervlak dat niet orienteerbaar is past in $\mathbb{R}^3$

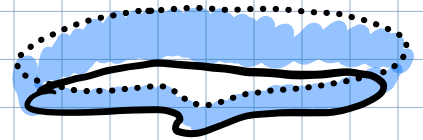
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Stelling Het is niet mogelijk de Möbius strip in $\mathbb{R}^3$ te passen zodat

Via hetzelfde argument

- De Kleinsse fles past niet in $\mathbb{R}^3$
- Geen enkel gesloten oppervlak dat niet orienteerbaar is past in $\mathbb{R}^3$



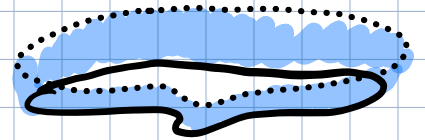
Stelling

Het is niet mogelijk de Möbius strip in $\mathbb{R}^3$ te passen zodat

- 1) de rand in het x,y vlak ligt

Via hetzelfde argument

- De Kleinsse fles past niet in $\mathbb{R}^3$
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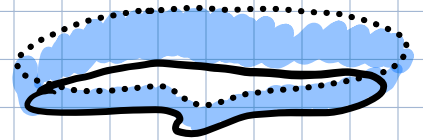
Stelling

Het is niet mogelijk de Möbius strip in $\mathbb{R}^3$ te passen zodat

- 1) de rand in het x,y vlak ligt
- 2) de rest van de strip in (x,y,z) met $z > 0$ ligt

Via hetzelfde argument

- De Kleinsse fles past niet in $\mathbb{R}^3$
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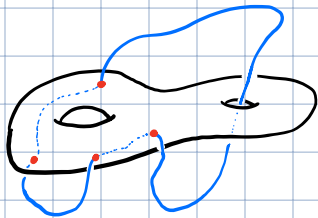
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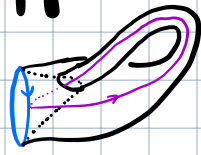
Laten we dit een **onmogelijke** Möbius strip noemen

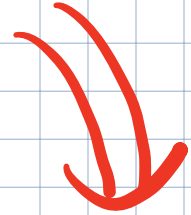
Een touw
heeft
even
aantal uiteinden

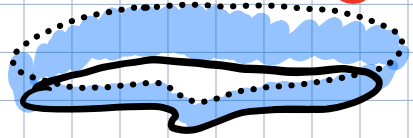


even

doorsnijdingen

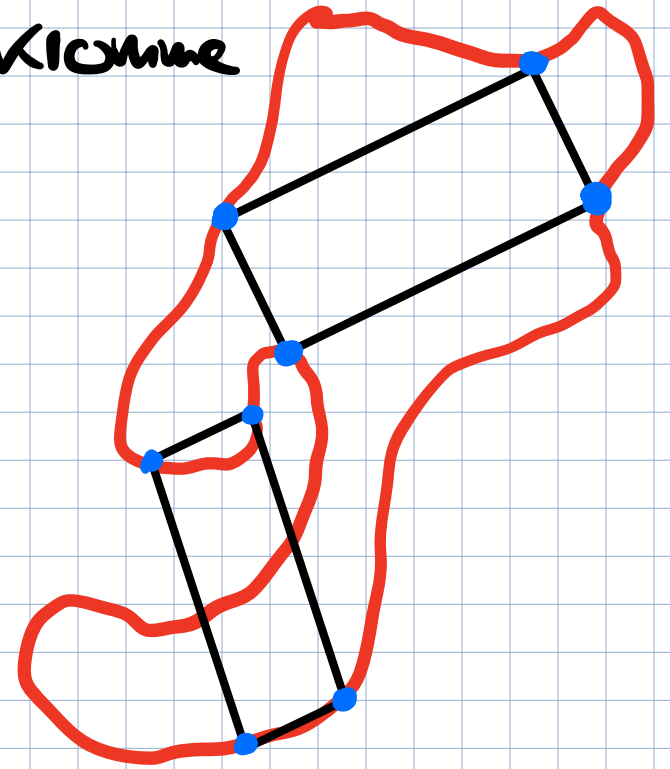
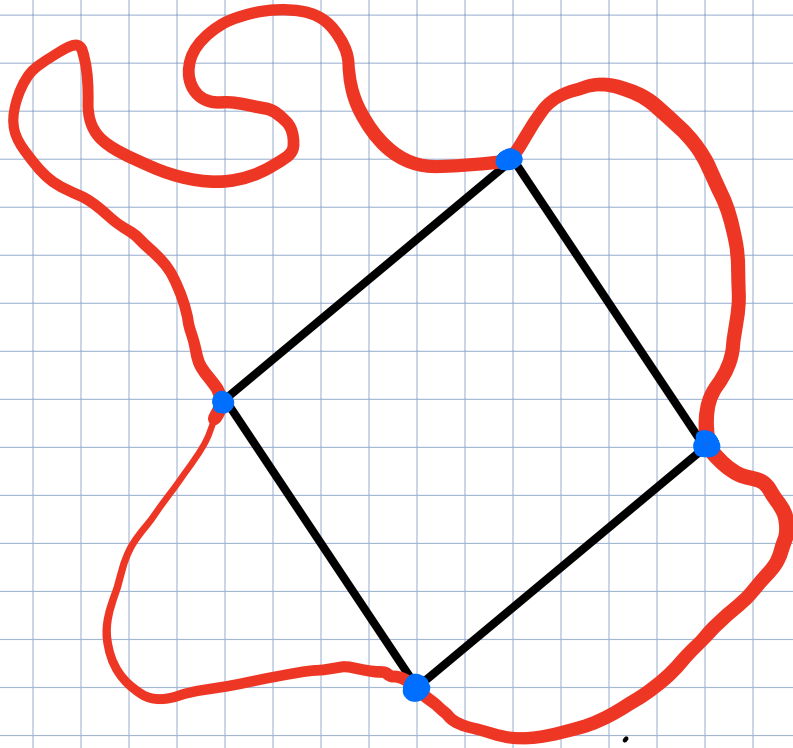


onoriënteerbare
oppervlakken

passen niet in $\mathbb{R}^3$



Geen **onmogelijke**

Möbius strip

Doel: Rechthoeken op een kromme

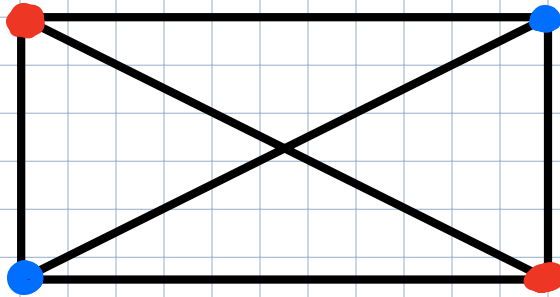


Stelling op elke gesloten kromme is een **rechthoek** te vinden met hoekpunten op de kromme

Strategie: tegenspraak

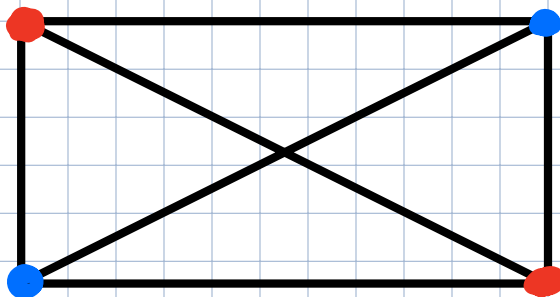
- laten zien dat **als** er een kromme bestaat zonder rechtehoek, dat er dan een onmogelijke Möbius strip bestaat. Deze bestaat niet, dus heeft elke kromme een rechtehoek.

Euclidische meetkunde:

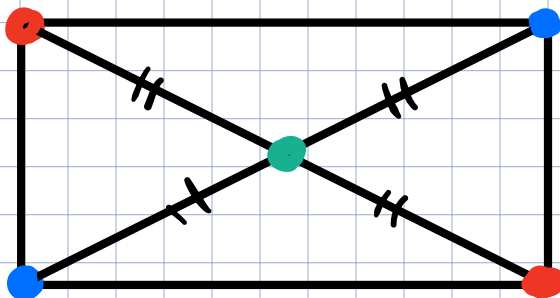


- twee paren punten vormen een rechtehoek dan en slechts dan als

Euclidische meetkunde:

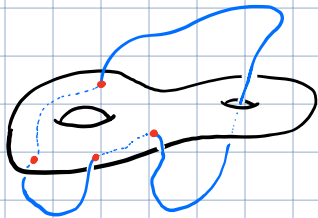


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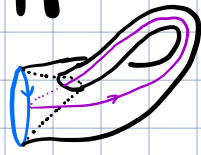


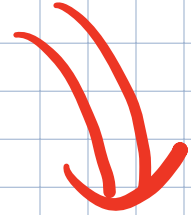
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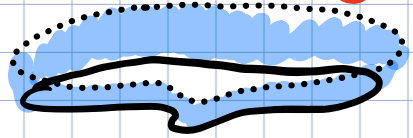


even

doorsnijdingen

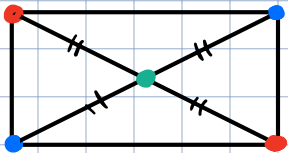


onoriënteerbare
oppervlakken

passen niet in $\mathbb{R}^3$



Geen **onmogelijke**

Möbius strip

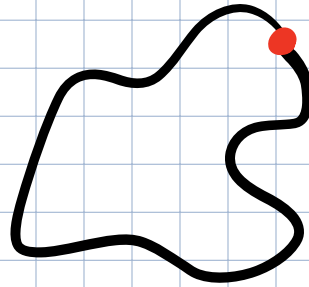
Euclidische
Meetkunde



De logica

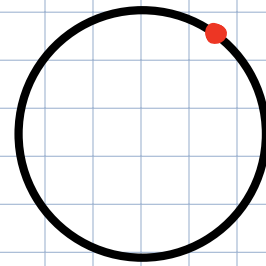
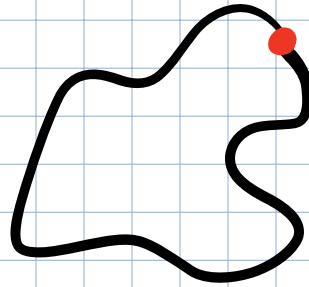
De ruimte van geozdende paren punten op de kromme

G. ruimte van één
punt op de kromme
is een cirkel



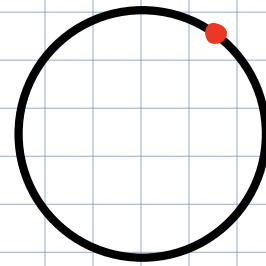
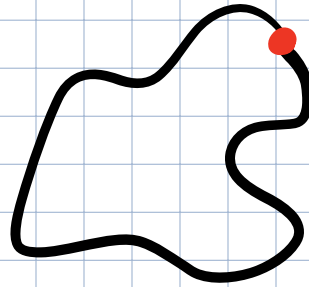
De ruimte van geozdende paren punten op de kromme

G. ruimte van één
punt op de kromme
is een cirkel



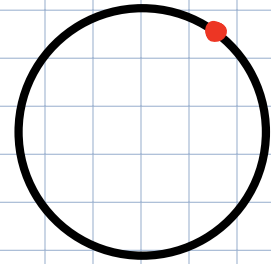
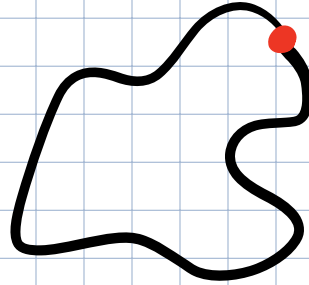
De ruimte van geordende paren punten op de kromme

G. ruimte van één
punt op de kromme
is een cirkel

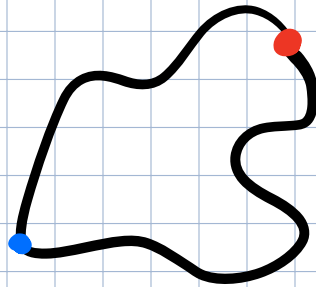


De ruimte van geordende paren punten op de kromme

G_1 ruimte van één
punt op de kromme
is een cirkel

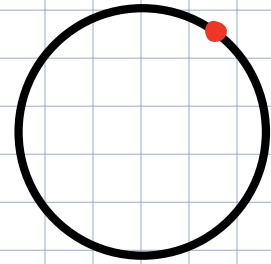
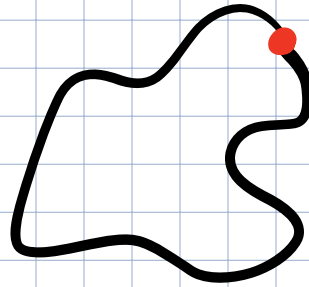


G_2 ruimte van twee
punten op de kromme
is een torus

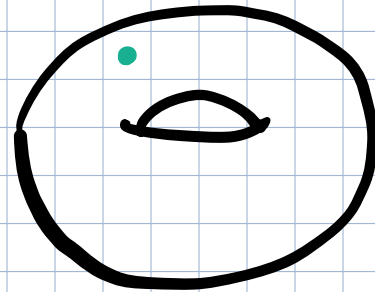
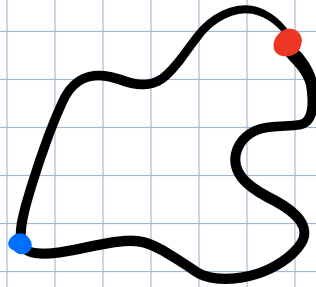


De ruimte van **geordende** paren punten op de kromme

G_1 ruimte van één
punt op de kromme
is een cirkel

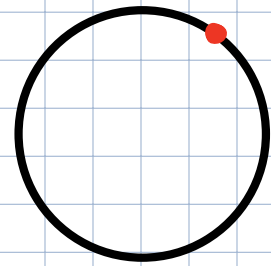
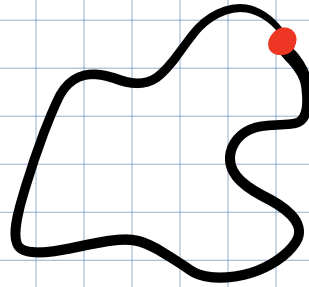


G_2 ruimte van twee
punten op de kromme
is een torus

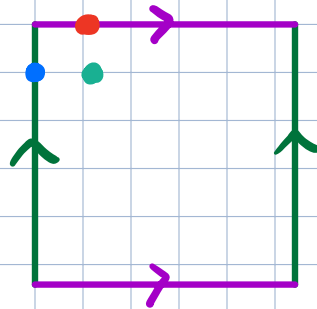
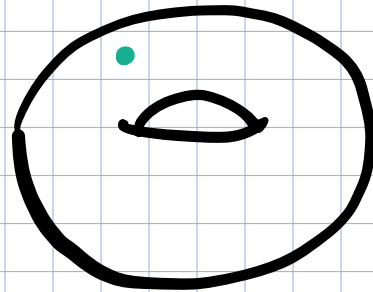
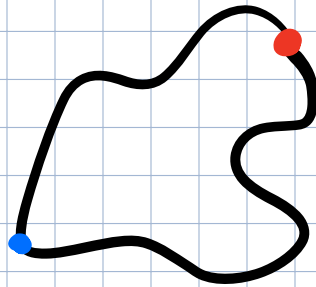


De ruimte van **geozdende** paren punten op de kromme

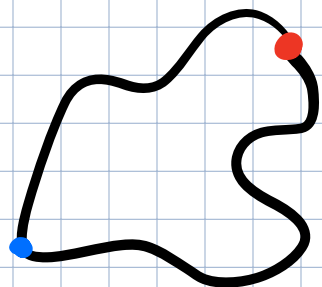
G_1 ruimte van één
punt op de kromme
is een cirkel



G_2 ruimte van twee
punten op de kromme
is een torus

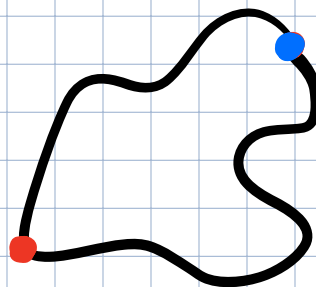


De ruimte van
op de kromme



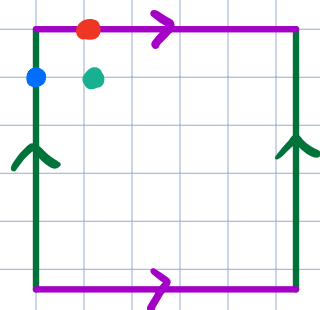
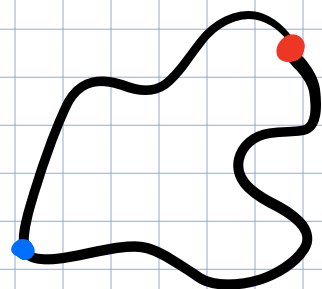
in
 G_2

paren punten



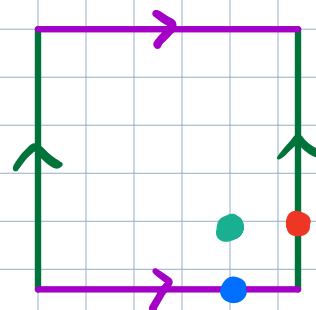
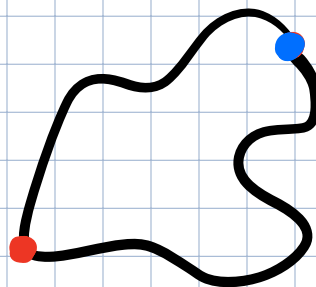
De ruimte van
op de kromme

paren punten



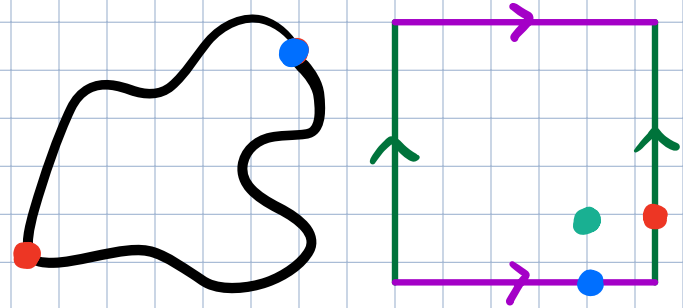
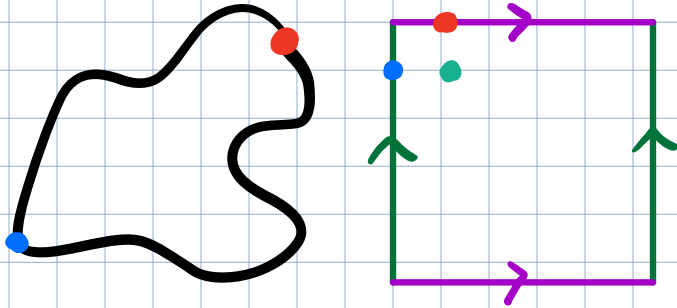
in
 G_2

verschillend

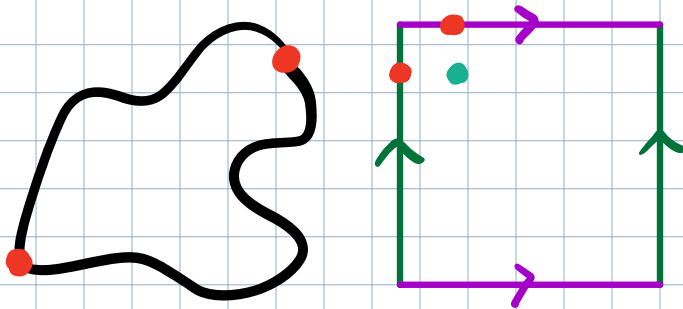


De ruimte van **ongeordene** paren punten
op de kromme

in
 G_2

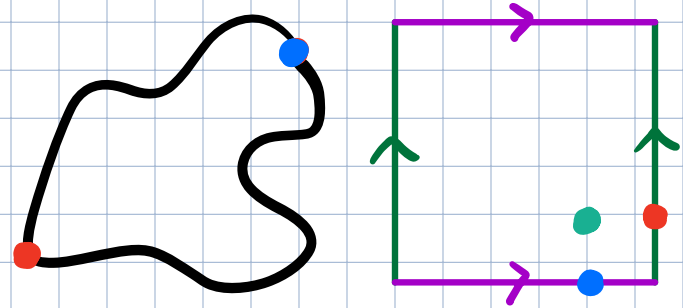
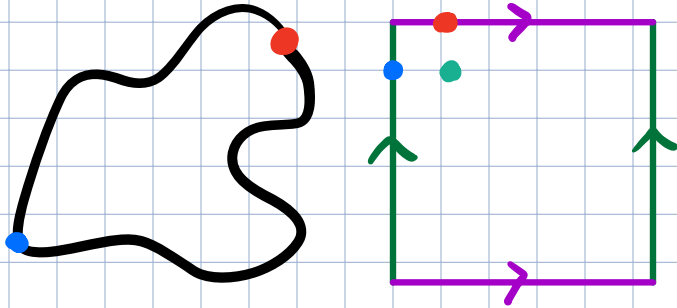


Ruimte van **ongeordene** paren O_2

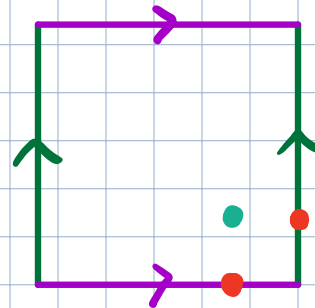
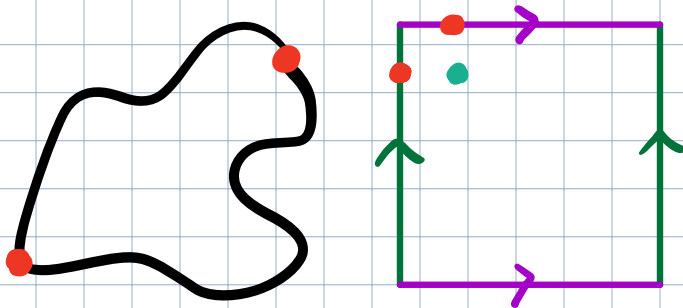


De ruimte van **ongeordene** paren punten
op de kromme

in
 G_2

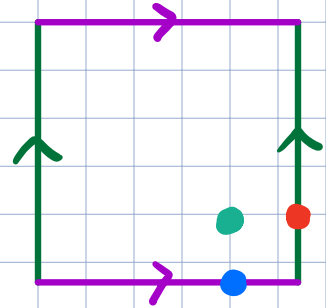
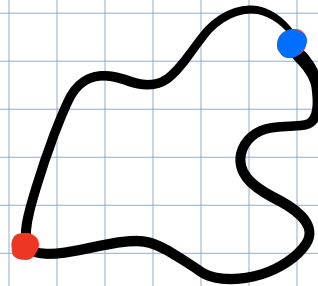
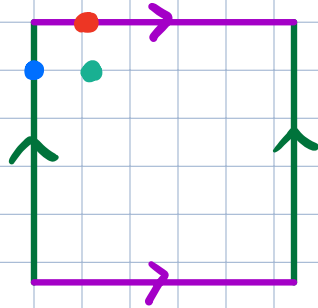
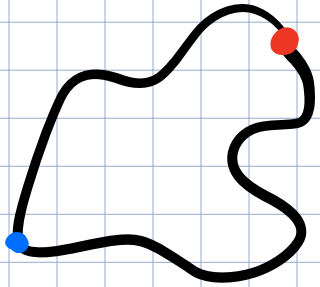


Ruimte van **ongeordene** paren O_2

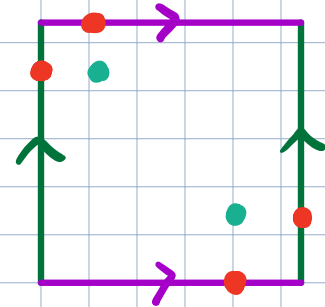
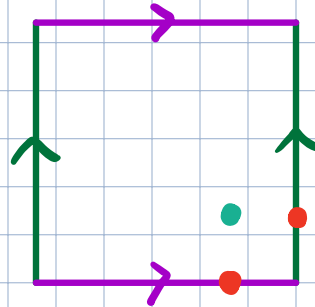
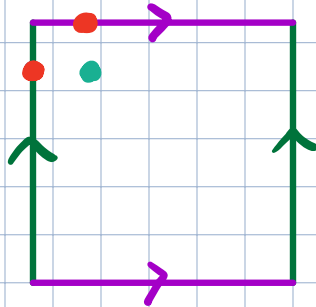
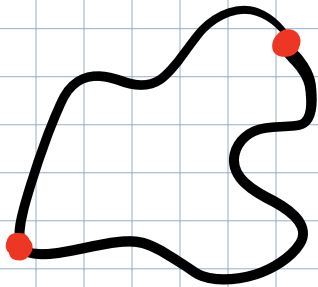


De ruimte van **ongeordene** paren punten
op de kromme

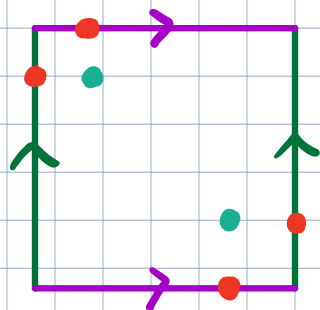
in
 G_2



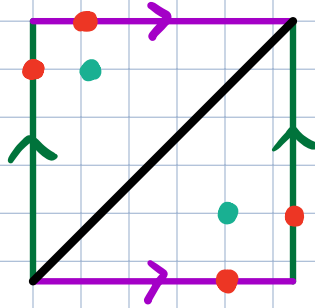
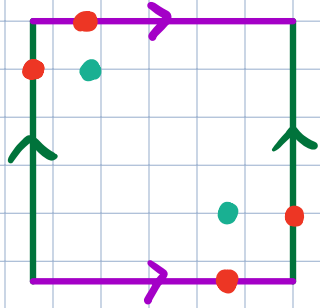
Ruimte van **ongeordene** paren O_2



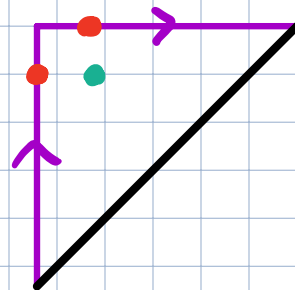
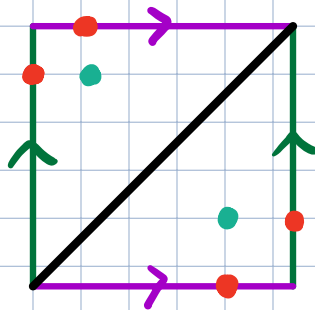
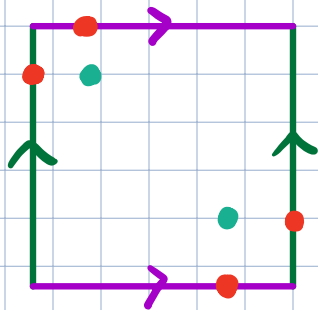
O_2



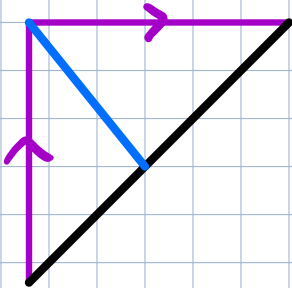
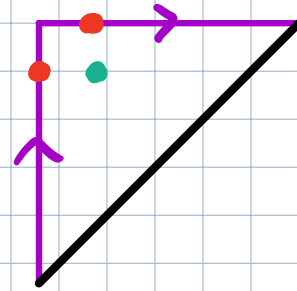
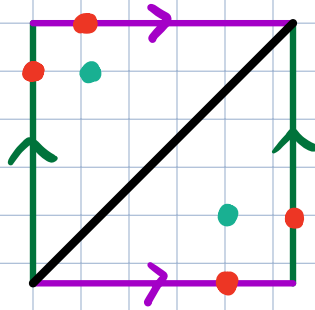
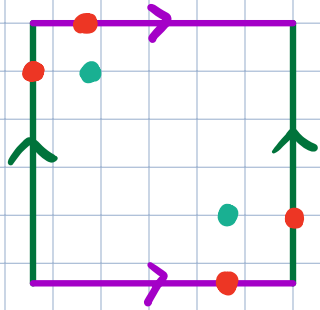
O_2



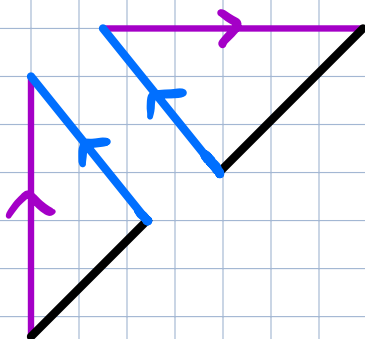
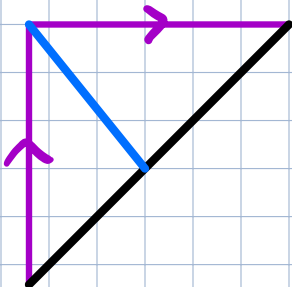
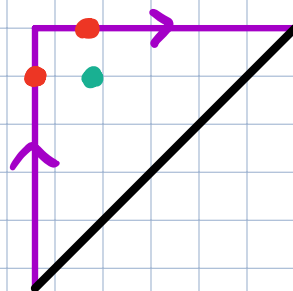
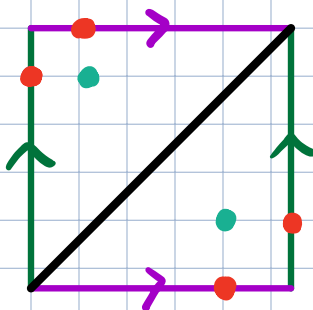
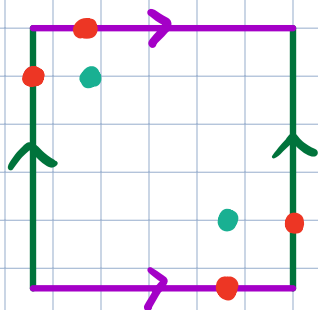
O_2



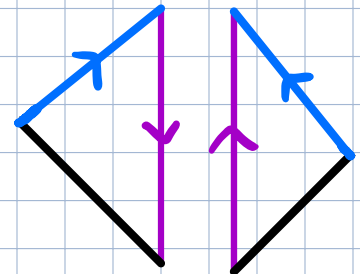
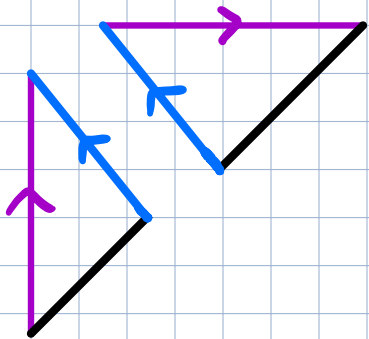
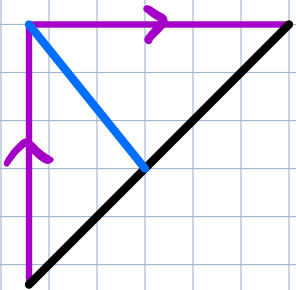
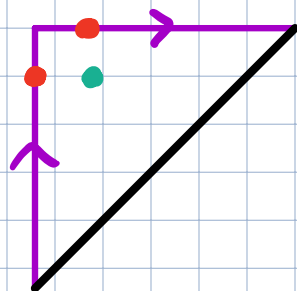
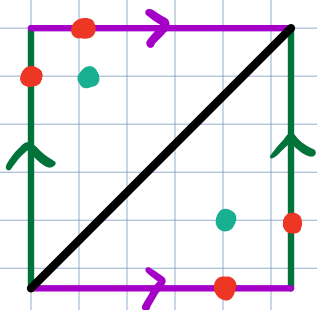
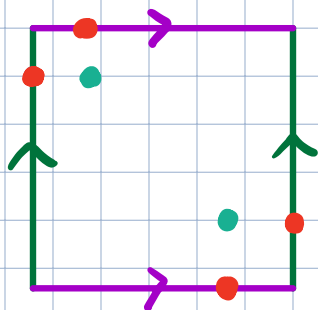
O_2



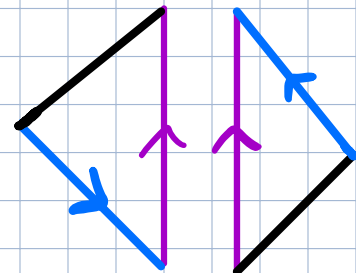
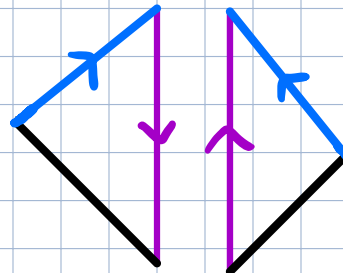
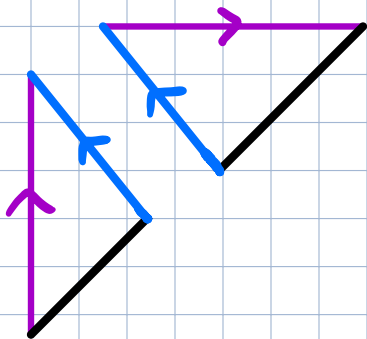
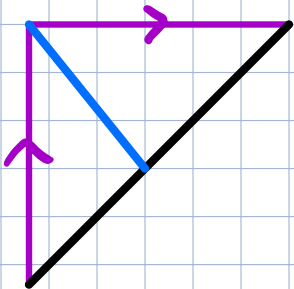
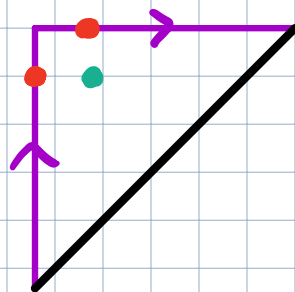
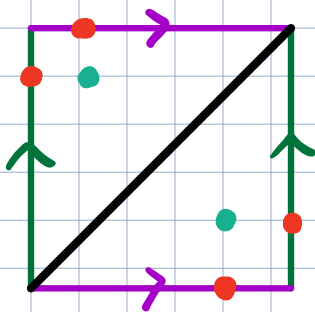
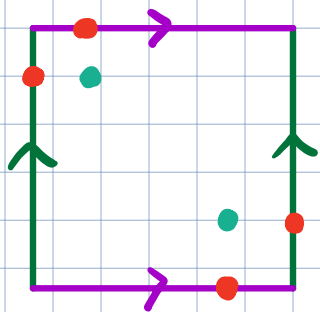
O_2



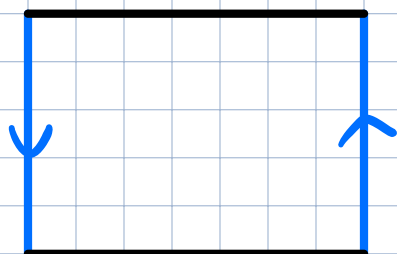
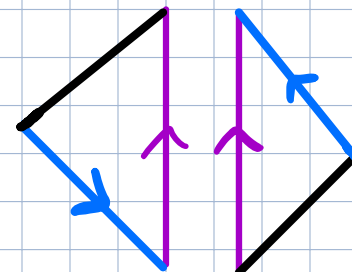
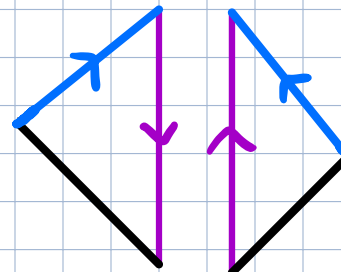
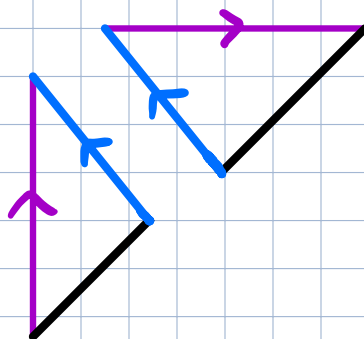
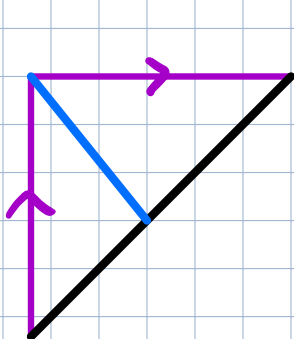
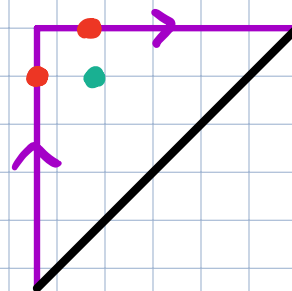
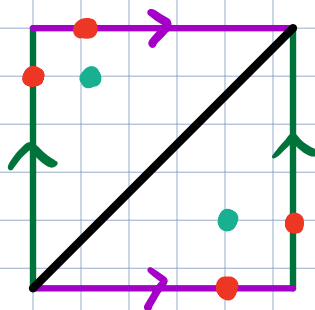
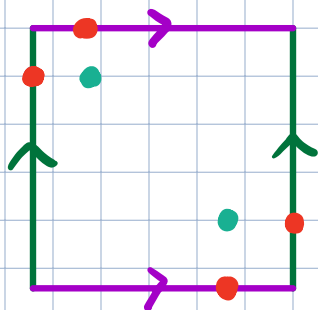
O_2



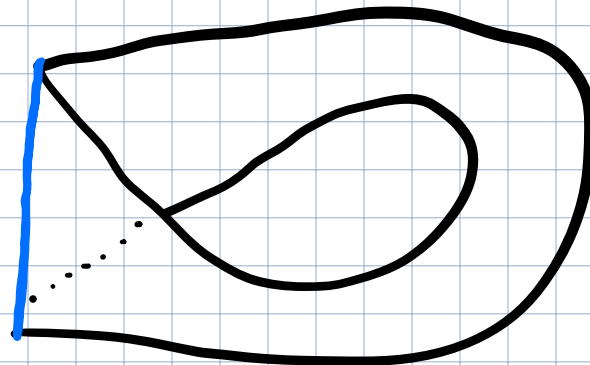
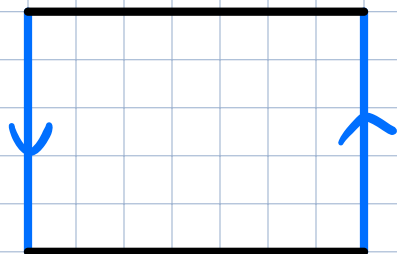
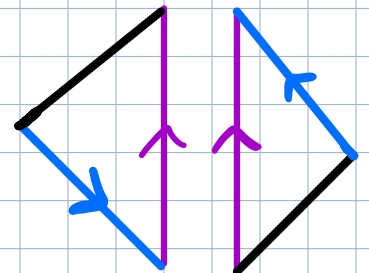
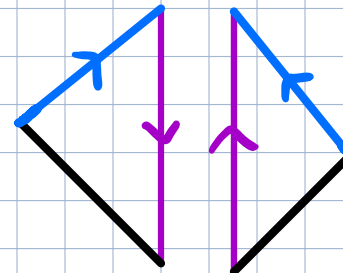
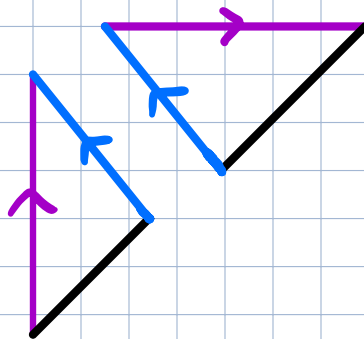
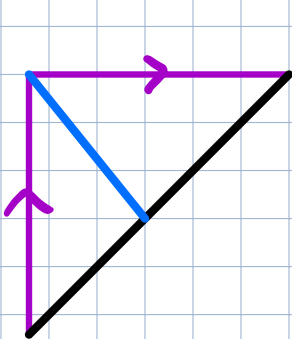
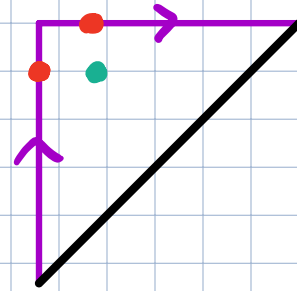
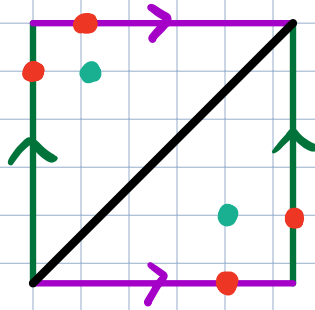
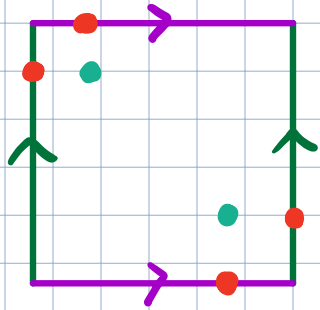
O_2



O_2

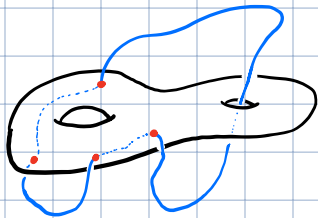


O_2 is een Möbius strip!

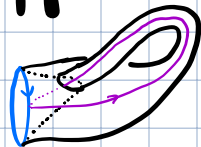


Een touw
heeft
even
aantal uiteinden



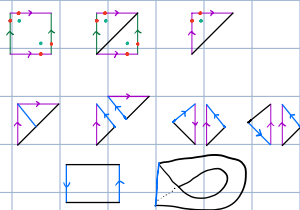
even

doorsnijdingen



onoriënteerbare
oppervlakken

passen niet in $\mathbb{R}^3$

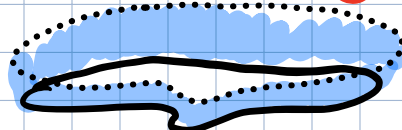


ruimte van
ongeordende paren

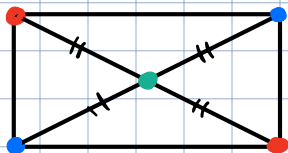


is Möbius strip.

Geen **onmogelijke**


Möbius strip

Euclidische
Meetkunde

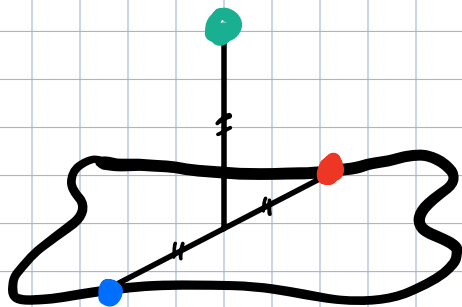


De logica

De afbeelding van geordende paren

$$f: G_2 \longrightarrow \mathbb{R}^3$$

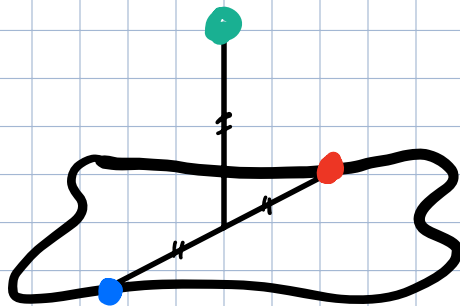
$$f((x_1, y_1), (x_2, y_2)) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \sqrt{\left(\frac{x_1 - x_2}{2} \right)^2 + \left(\frac{y_1 - y_2}{2} \right)^2} \right)$$



De afbeelding van geordende paren

$$f: G_2 \longrightarrow \mathbb{R}^3$$

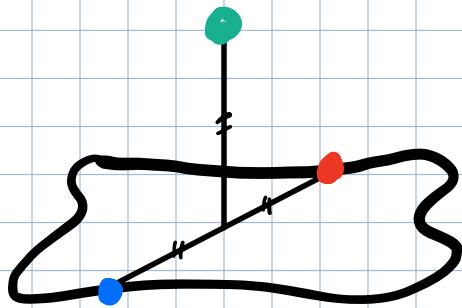
$$f((x_1, y_1), (x_2, y_2)) = \left(\underbrace{\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}}_{\text{Midden}}, \underbrace{\sqrt{\left(\frac{x_1 - x_2}{2}\right)^2 + \left(\frac{y_1 - y_2}{2}\right)^2}}_{\text{afstand tot midden}} \right)$$



De afbeelding van geordende paren

$$f: G_2 \longrightarrow \mathbb{R}^3$$

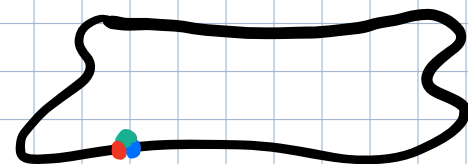
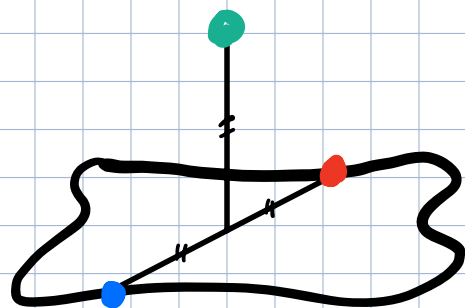
$$f((x_1, y_1), (x_2, y_2)) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \sqrt{\left(\frac{x_1 - x_2}{2} \right)^2 + \left(\frac{y_1 - y_2}{2} \right)^2} \right)$$



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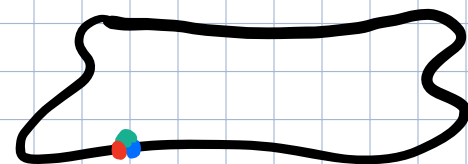
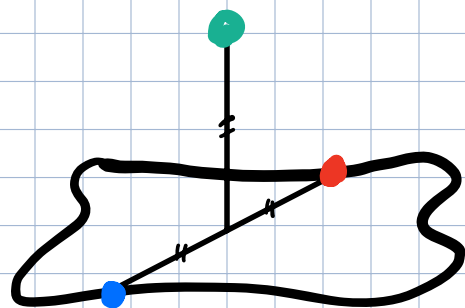
$$f((x_1, y_1), (x_2, y_2)) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \sqrt{\left(\frac{x_1 - x_2}{2} \right)^2 + \left(\frac{y_1 - y_2}{2} \right)^2} \right)$$



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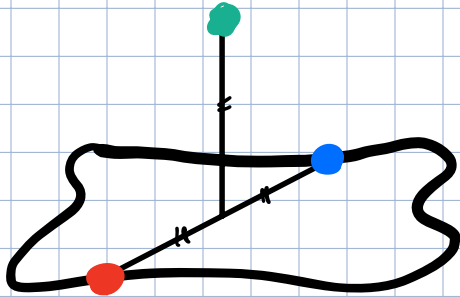
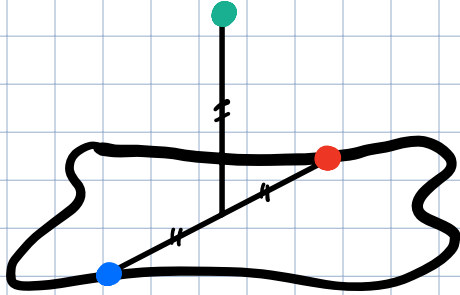
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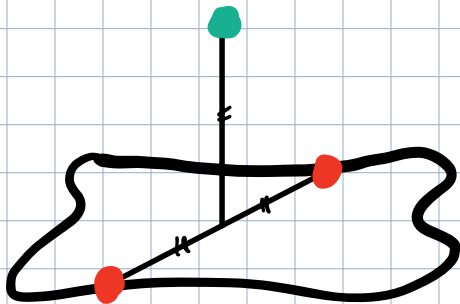
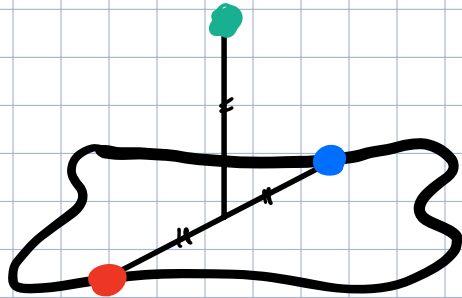
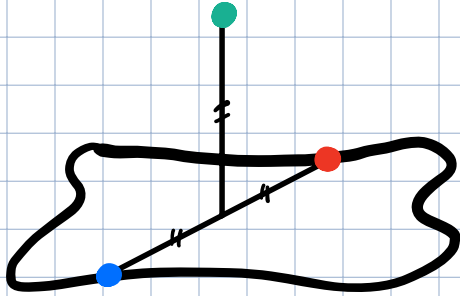


Eigenschap: Op de rand is de hoogte van f nul.

De afbeelding van ongeordende paren



De afbeelding van ongeordende paren



De symmetrie geeft een afbeelding

$$f: O_2 \longrightarrow \mathbb{R}^3$$

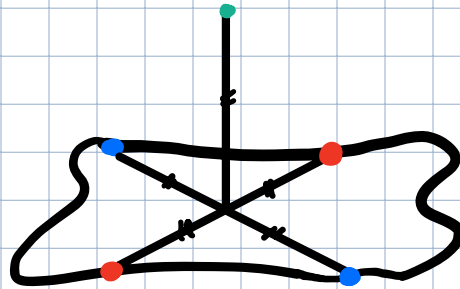
$$f[(x_1, y_1), (x_2, y_2)] = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \sqrt{\left(\frac{x_1 - x_2}{2} \right)^2 + \left(\frac{y_1 - y_2}{2} \right)^2} \right)$$

De tegenspraak.

- Als $f[(x_1, y_1), (x_2, y_2)] = f[(x_1, y_1), (x_2, y_2)]$

De tegenspraak.

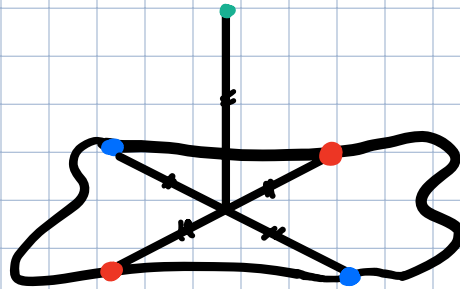
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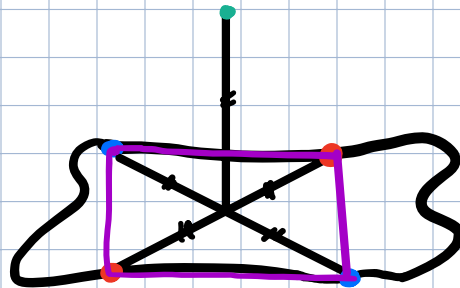
Dan is ez een recht hoek!

De tegenspraak.

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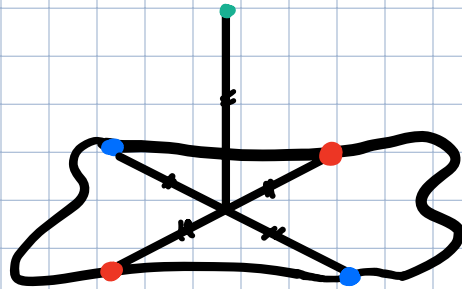


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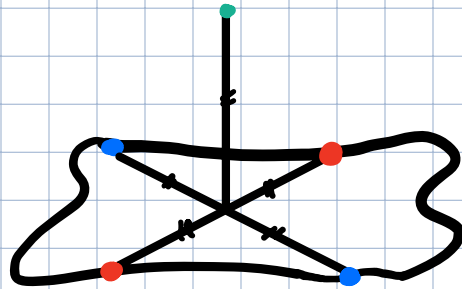


Dan is ez een recht hoek!

- Als dat nooit zo is dan definieert
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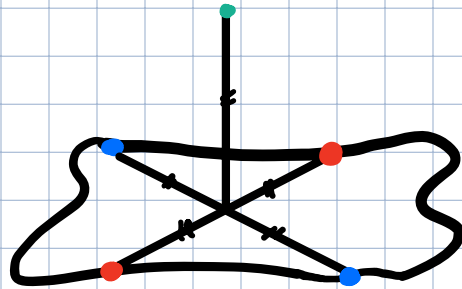


Dan is ez een recht hoek!

- Als dat nooit zo is dan definieert f een onmogelijke Möbius strip
- Dit bestaat niet, dus heeft de kromme een recht hoek

De tegenspraak.

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Dan is ez een recht hoek!

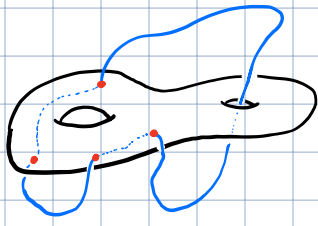
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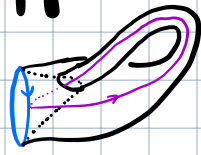
Q.E.D.

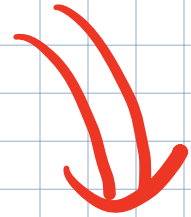
Een touw
heeft
even
aantal uiteinden



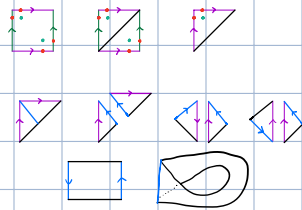
even

doorsnijdingen



onoriënteerbare
oppervlakken

passen niet in $\mathbb{R}^3$

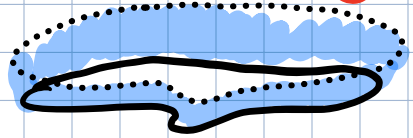


ruimte van
ongeordende paren

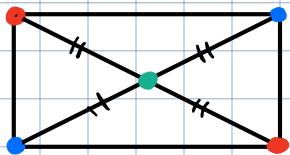


is Möbius strip.

Geen **onmogelijke**


Möbius strip

Euclidische
Meetkunde



Er bestaat



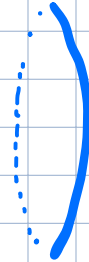
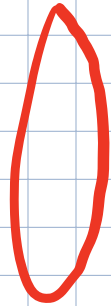
een rechthoek



De logica

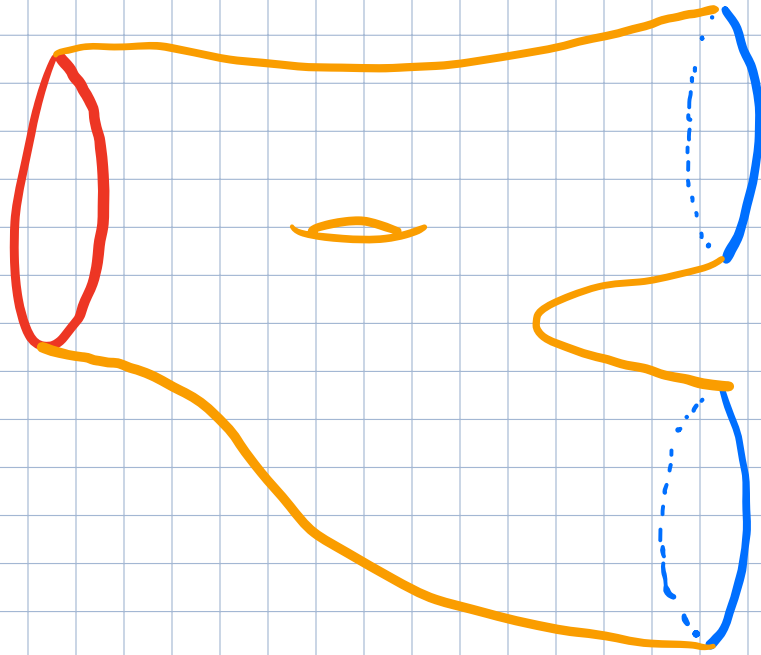
Extra: Hoger dimensionale cobordismen

Een cobordisme tussen twee n dimensionale
gesloten variëteiten N en M



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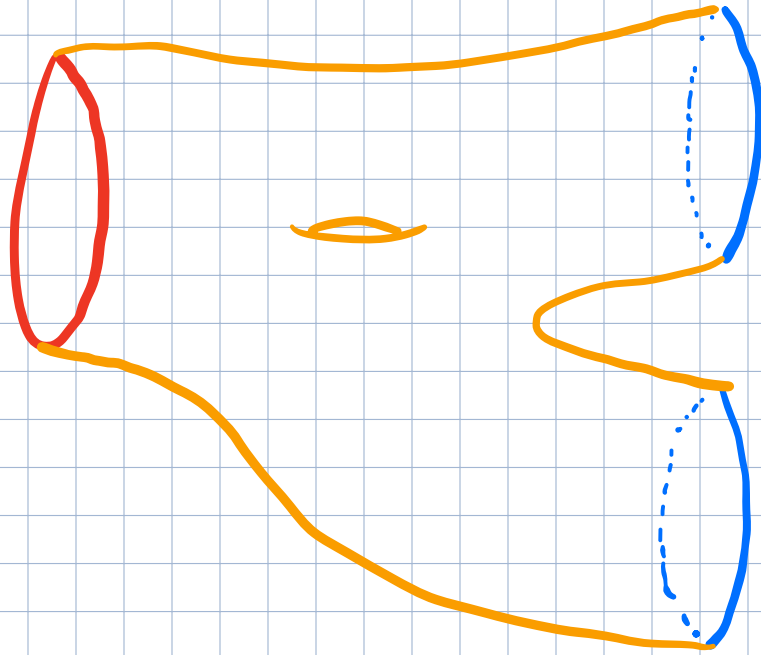
Een cobordisme tussen twee n dimensionale
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Is een compacte $n+1$ dimensionale variëteit W
met rand $N \cup M$

Extra: Hoger dimensionale cobordismen

Een cobordisme tussen twee n dimensionale
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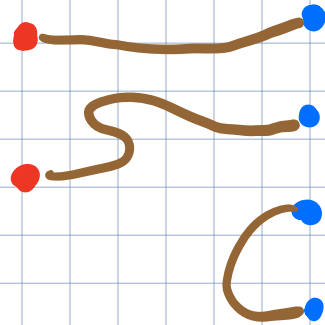
met rand $N \cup M$

~~Vraag:~~

Hoeveel variëteiten op cobordisme na zijn er?

Wat we weten

$n = 0$



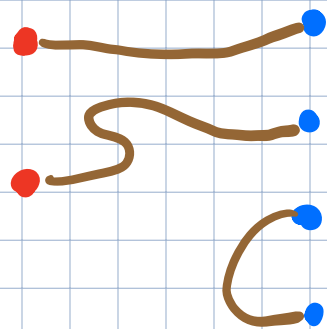
Er zijn twee klassen

even en oneven punten

(een touw heeft een even
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Wat we weten

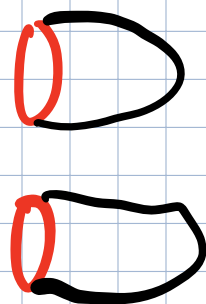
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even en oneven punten

$n=1$

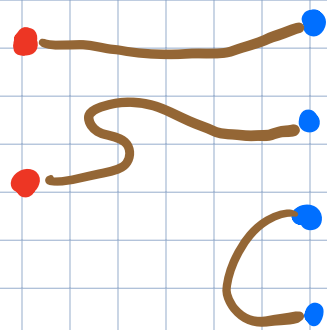


Er is één Klasse.

Alles is een rand

Wat we weten

$n=0$



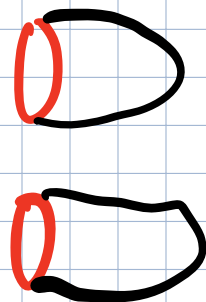
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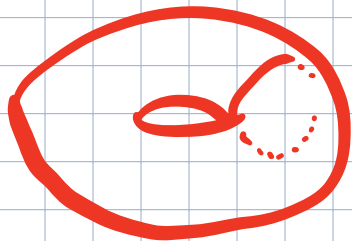
Er is één Klasse.

$n=1$



Alles is een rand

$n=2$



Twee Klassen.

- Georiënteerde oppervlakken zijn een rand.
 - De helft van de ongeoriënteerde oppervlakken zijn geen rand
- Kleinse fles wel $\mathbb{R}P^2$ niet

De stelling van Thom

Stelling $N_n \cong \mathbb{Z}_2[x_i : 0 < i \leq 2^n - 1]$

$$\cong \mathbb{Z}_2[x_2, x_4, x_5, x_6, x_8, x_9, \dots]$$

De stelling van Thom

cobordisme ring van variëteiten

Stelling

N_n

$$\cong \mathbb{Z}_2[x_i : 0 < i \leq 2^e - 1]$$

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Algebraïsche beschrijving ervan.

De stelling van Thom

cobordisme ring van variëteiten

Stelling

$$N_n \cong \mathbb{Z}_2[x_i : 0 \leq i \leq 2^n - 1]$$

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Algebraïsche beschrijving ervan.

$$\# N_0 = 2 \quad \# N_1 = 1 \quad \# N_2 = 2 \quad \# N_3 = ($$

$$\# N_4 = 4 \quad \# N_5 = 2, \dots$$

Bedankt voor uw aandacht

Referenties

Doolhof

: Differential topology
1974 Pollack & Guillemin

De Kleinse fles
past niet in $\mathbb{R}^3$

: Orientability of hypersurfaces in $\mathbb{R}^n$
1969 Samelson

Recht hoeken
op krommen

: Balancing acts
1981 Meyerzon (Bewijs van Vaughn)

ook op youtube: Who cares about topology

2017 3Brown1Blue

Vierkan ten
op krommen

: The rectangular peg problem
2020 Greene & Lobb

