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**Assignment I**  
**Introduction Partial Differential Equations**  
**for students Mathematics and Physics**

Afdeling Wiskunde  
Faculteit der Exacte Wetenschappen  
Vrije Universiteit Amsterdam

Due: Thursday October 13, 2005.

Instructions: include input and output; *motivate all answers*.

Individual assignment, written in English.

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Consider the functions

$$f(x) = \begin{cases} -x, & -\pi \leq x \leq 0, \\ x, & 0 \leq x \leq \pi, \end{cases} \quad g(x) = \begin{cases} -1, & -\pi \leq x < 0, \\ 1, & 0 < x \leq \pi. \end{cases}$$

1. Compute the Fourier series of  $f$  and  $g$  (give a closed formula of both series).

2. Use MAPLE, or MATHEMATICA to plot

(a)  $FS_{10}(f)$  and  $FS_{10}(g)$ ,

(b)  $FS_{20}(f)$  and  $FS_{20}(g)$ ,

(c)  $FS_{100}(g)$ ,

(d) and compare these plots to the graphs of  $f$  and  $g$ .

3. What conclusion can you draw concerning the convergence of both Fourier series?

4. (a) Approximate the limit

$$\lim_{n \rightarrow \infty} FS_n(g)\left(\frac{\pi}{2n}\right),$$

using the above calculations (Hint: for example compute the value at  $n = 100$ , and  $n = 1000$ ).

(b) Compare this value to the jump in  $g$  at  $x = 0$ .

5. Use the above limit to argue that the Fourier series of  $g$  does not converge uniformly on  $[-\pi, \pi]$ .

*Good luck*