

Problem 2. Consider the partial differential operator

$$L = \Delta^2.$$

- (a) Derive the Green's identities in local and integral form for the partial differential operator L .
- (b) Compute the adjoint operator L^* .
- (c) Find the fundamental solution $\Phi(x)$ for L in dimensions $n \geq 5$.

Let $U \subset \mathbf{R}^n$ be an open set with smooth boundary ∂U , and consider the boundary value problem

$$\Delta^2 u = f, \quad u|_{\partial U} = g, \quad \frac{\partial u}{\partial \mathbf{n}} \Big|_{\partial U} = h,$$

where \mathbf{n} is the outward pointing unit normal to ∂U .

- (d) Let $u \in C^4(\overline{U})$ be a solution of the above boundary value problem. Derive the representation formula for u and describe the Green's function.