

**Problem 3.**

(a) (Problem 6 in Evans, page 290) Show for  $u \in W^{1,p}(0,1)$ , for some  $1 < p < \infty$ , that

$$|u(x) - u(y)| \leq |x - y|^{1 - \frac{1}{p}} \left( \int_0^1 |u'(t)|^p dt \right)^{\frac{1}{p}},$$

a.e. for  $x, y \in [0, 1]$  (Hint:  $u(x) - u(y) = \int_x^y u'(t) dt$ ).

(b) (Problem 8 in Evans, page 290) Prove the interpolation inequality:

$$\int_U |Du|^2 dx \leq C \left( \int_U u^2 dx \right)^{\frac{1}{2}} \left( \int_U |D^2 u|^2 dx \right)^{\frac{1}{2}},$$

for all  $u \in C_c^\infty(U)$  (Hint: integration by parts of  $\int_U |Du|^2 dx$ ).