Problem 4.
(Problem 18 in Evans, page 292) The Fourier transform is defined by:

$$\widehat{u}(\xi) := \frac{1}{(2\pi)^{n/2}} \int_{\mathbf{R}^n} e^{-i\xi\cdot x} u(x)dx, \quad u \in L^1(\mathbf{R}^n).$$

Let $0 < s < \infty$ and define the spaces

$$H^s(\mathbf{R}^n) := \big\{ u \in L^2(\mathbf{R}^n) \mid (1 + |\xi|^s)\widehat{u} \in L^2(\mathbf{R}^n) \big\},$$

with norm $\|u\|_{H^s(\mathbf{R}^n)} := \|(1 + |\xi|^s)\widehat{u}\|_{L^2(\mathbf{R}^n)}$.

Show that for $s > \frac{n}{2}$,

$$\|u\|_{L^\infty(\mathbf{R}^n)} \leq C\|u\|_{H^s(\mathbf{R}^n)}, \quad \forall\, u \in H^s(\mathbf{R}^n).$$