Problem 6.
(Evans, page 313)
Let $U \subset \mathbf{R}^n$ be bounded, open set and let

$$B[u,v] = \int_U \Big[a_{ij} u_{x_i} v_{x_j} + b_i u_{x_i} v + cuv \Big] dx,$$

with a_{ij}, b_i and c constants and $a_{ij}\xi_i\xi_j \geq \theta|\xi|^2$. The function $u \in H^1(U)$ is a weak solution of the identity

$$B[u,v] = (f,v)_{L^2(U)}, \quad \forall v \in H^1_0(U),$$

with $f \in L^2(U)$.

Show that u satsifies the estimate

$$\|u\|_{H^1(W)} \leq C\big(\|f\|_{L^2(U)} + \|u\|_{L^2(U)}\big),$$

for any $W \subset\subset U$ (Hint: mimic the proof of Theorem 1 in Sect. 6.3.1 of Evans).