

Problem 7.

Consider the parabolic initial value problem

$$u_t + \Delta^2 u = f \quad \text{in } U_T = U \times (0, T], \quad U \text{ bounded,}$$

with

$$u|_{\partial U \times (0, T]} = \frac{\partial u}{\partial n}|_{\partial U \times (0, T]} = 0, \quad \text{and} \quad u(x, 0) = g(x) \quad \text{on } U.$$

- (a) Give the weak formulation of this equation.
- (b) Write down the Galerkin approximation.
- (c) What is the natural orthogonal basis in order to get a diagonalized system?