A Philosophical Foundation for Unification of Dynamic Modelling Methods Based on Higher-Order Potentialities and their Reducers *

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Abstract

In the development of disciplines addressing dynamics, a major role was played by the assumption that processes can be modelled by introducing state properties, called potentialities, anticipating in which respect a next state will be different. A second assumption often made is that these state properties can be related to other state properties, called reducers. The current paper proposes a philosophical framework in terms of potentialities and their reducers, which can be used to obtain a common philosophical foundation for methods in AI, Cognitive Science and beyond to model dynamics. Based on this framework a metamodel for dynamic modelling approaches is described. The philosophical framework and the metamodel together provide a unified foundation for numerical, symbolic, and hybrid dynamic modelling approaches used in a large variety of disciplines.

1. Introduction

In recent years, within Cognitive Science, dynamics has been recognised and emphasised as a central issue in describing cognitive processes (Port and Gelder, 1995). Van Gelder and Port (1995) propose the Dynamical Systems Theory (DST) as a new paradigm that is better suited to the dynamic aspects of cognition than symbolic modelling approaches. However, as DST (which subsumes neural networks and many other quantitative approaches to adaptive and control systems), commits to the use of quantitative methods (differential and difference equations), it is often considered less suitable to model higher cognitive processes such as reasoning and language processing. DST is based on the state-determined system assumption: properties of a given state fully determine the properties of future states; cf.

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(Ashby, 1960), p. 25; (Gelder and Port, 1995), p. 6. This means that for state properties that are different in a future state, compared to the given current state, other state properties in the given state can be found that somehow indicate or anticipate these differing (changed) properties. This idea closely relates to the concept of *potentiality* that goes back to Zeno and Aristotle (350 BC'): if a potentiality *p* for a state property *a* occurs in a given state, then in a next state, property *a* will occur. For example, if at some point in time *t* a given object at position *P* has a velocity *v* (described as state property *p*, the potentiality considered), then at a next point in time it will have a position *P'* at a distance from *P* which is proportional to *v* (this new position is described by the state property *a* for which *p* is a potentiality). Another example, if at some point in time *t* a force (described as state property *p*, the potentiality considered) is exerted on an object with a given velocity *v*, then the velocity in a next state will be *v* increased by an amount proportional with the exerted force (this increased velocity is described by the state property *a* for which *p* is a potentiality). A potentiality is a kind of anticipatory state property: a state property anticipating the different state property (in the changed state). As already discussed in (Treur, 2005), a basic assumption under DST is the existence of potentialities as state properties. Note that Ashby (1960)'s state-determined system assumption implies a deterministic system.

In the current paper the notion of potentiality is used as a basis for a more elaborated philosophical framework to analyse modelling methods that address dynamics. The two main concepts in this framework are the concept of 'potentiality' and that of 'reducer'. The latter concept, which was not addressed in detail in Treur (2005), is used to limit chains of higher-order potentialities, by relating the potentiality of some higher-order to lower-order potentialities. The framework is applicable to obtain philosophical foundations for both quantitative approaches (such as differential equations and neural networks, as in DST) and qualitative or symbolic approaches (such as BDI-modelling and transition and production systems) to modelling of dynamics. As part of the analysis it is shown, for example, how the notion of reducer of a higher-order potentiality is the more generalised philosophical notion underlying the technical notion 'differential equation'. More specifically, an *n*-th order differential equation is an equality that shows how the *n*-th order derivative of the considered function is equal to an expression in terms of lower-order derivatives. This means that the *n*-th order potentiality has a reducer defined by this expression in terms of lower order potentialities.

It is discussed that DST can work when, in addition to the assumption on the existence of potentialities as state properties mentioned above (expressed in the *state-determined system assumption*, explicated, for example, by Ashby (1960), also a second assumption is made, namely that for any potentiality considered, a natural number *n* exists, such that for the corresponding *n*-th order potentiality a *reducer exists* (and actually can be found). Given that the conceptual philosophical framework based on potentialities and reducers as developed is applicable to DST, which since long has proved its value for quantifiable areas within a wide variety of disciplines, also the scope of applicability of the proposed framework covers disciplines such as Physics, Chemistry, Biology, and Economics. For the areas of cognitive modelling and knowledge and reasoning system modelling, it is shown how the framework can be applied to provide a foundation for symbolic modelling methods such as production systems, BDI-models, and models for adaptive behaviour. Here the more general notion of reducer can be directly related (as a philosphical foundation) to transition or production rules (as technical notions). The framework easily describes both mental aspects and physical aspects of embodied cognitive agents (in terms of potentialities of a certain order) and their relationships (based on the notion of reducer).

Below, first in Section 2 the notions of potentiality and higher-order potentiality are discussed and illustrated by examples from Mathematics (higher-order derivatives and Taylor series). In Section 3 it is shown how higher-order potentialities play a role as a philosophical foundation of basic concepts in Physics (momentum, kinetic energy, force). Section 4 discusses the assumption that potentialities of some higher order can be reduced to lower level state properties. Sections 5 and 6 show how the framework can be applied to symbolic modelling methods, i.e., to analyse BDI-models (Section 5), and transition and production systems (Section 6). Section 7 shows how it can be applied to modelling of adaptive agents. In Section 8 the philosophical framework is related to the Dynamical Systems Theory, including neural networks and many other (quantitative) approaches to dynamic systems. In Section 9 the framework is extended by defining and formalising a metamodel for dynamic modelling approaches, which is illustrated in Section 10 in a number of cases. Section 11 is a discussion.

2. Potentialities of Different Orders as State Properties Anticipating Change

In this section, the notions of potentiality and higher-order potentiality are discussed, related to the notion of state-determined system for Dynamical Systems Theory (Ashby, 1960; Port and van Gelder, 1995) and formalisations from Mathematics are shown.

2.1 Potentialities

Given a particular state that just changed with respect to some of its state properties, it is natural to ask for an explanation of why these new state properties occurred. In a state-based approach, as a source for such an explanation, state properties found in the previous state form a primary candidate. A main question becomes how to determine for a certain state that it is going to change to a different state, and, more specifically, how to determine (on the basis of some of the state properties in the given state) those state properties for which the new state will differ from the given one. This poses the challenge to identify state properties occurring in a given state that anticipate the next state: anticipatory state properties. If such state properties (historically sometimes called potentialities; see Treur (2005) for a more extensive historical analysis) are given, anticipation to change is somehow encoded in a state.

Aristotle introduced such a type of concept; he called it *potentiality* (to move), or movable.[2] For example, following Zeno, he argued that the difference between an arrow at rest and the snapshot of a moving arrow at time t at position P is that the former has no potentiality to be at P', whereas the latter has. This explains why at a next instant t' the former arrow is still where it was, at P, while the latter arrow is (assuming no obstruction) at a different position P': Aristotle did not only consider changes of positions (due to locomotion), but also, for example, a young man becoming an old man, and a cold object becoming hot. For each of these types of changes a specific type of potentiality is considered; e.g., the potentiality to be at position P', the potentiality (of a cold object) to be hot. In general, if the potentiality p (occurring in a state S) to have state property a has led to a state S' where indeed a holds, then this state property a of state S' is called the *fulfilment* or

[2] 'We have now before us the distinctions in the various classes of being between what is full real and what is potential. (…) The fulfilment of what exists potentially, in so far as it exists potentially, is motion - namely, of what is alterable qua alterable, alteration: of what can be increased and its opposite what can be decreased (there is no common name), increase and decrease: of what can come to be and can pass away, coming to be and passing away: of what can be carried along, locomotion.' (Aristotle, 350 BC, Book III, Part 1)

actualisation of the potentiality *p* for *a,* occurring in state *S*. Notice that Aristotle considered both *absolute* potentialities, indicating a state property for the future state independent of this state property in the present state, and *relative* potentialities, indicating a difference (increase or decrease) in a future state property compared to the present state.

The assumption on the existence of such properties is the crucial factor for the validity of the assumptions underlying dynamic modelling methods based on the Dynamical Systems Theory (Ashby, 1960; Port and van Gelder, 1995): the *state-determined system assumption*. This assumption states that from a given state of the system the next state(s) can be determined in a effective and unique manner. Van Gelder and Port (1995), following Ashby (1952) explain what a dynamical system is in the following manner. A *system* is a set of changing aspects (or state properties) of the world. A *state* at a given point in time is the way these aspects or state properties are at that time; so a state is characterised by the state properties that hold. The set of all possible states is the *state space*. A *behaviour* of the system is the change of these state properties over time, or, in other words, a succession or sequence of states within the state space. Such a sequence in the state space can be indexed, for example, by natural numbers (*discrete* case) or real numbers (*continuous* case), and can also be called a *trace* or *trajectory*. Following Ashby, such a system is *state-determined* if:

> ‘A system is state-determined only when its current state always determines a unique future behaviour. Three features of such systems are worth noting.
>
> First, in such systems, the future behaviour cannot depend in any way on whatever states the system might have been in *before* the current state. In other words, past history is irrelevant (or at least, past history only makes a difference insofar as it has left an effect on the current state).
>
> Second, the fact that the current state determines future behaviour implies the existence of some *rule of evolution* describing the behaviour of the system as a function of its current state. (…)
>
> Third, the fact that future behaviours are uniquely determined means that state space sequences can never fork.’ (Gelder and Port, 1995, p. 6)

According to some, a dynamical system is just a state-determined system (Giunti, 1995). For some others a dynamical system is a state-determined system for which the state properties are described by assignments of numerical values to a given set of variables (van Gelder and Port, 1995). Ashby (1960), expresses the heuristics based on state-determined systems as follows:

> ‘As a working guide, the scientist has for some centuries followed the hypothesis that, given a set of variables, he can always find a larger set that (1) includes the given variables, and (2) is state-determined. Much research work consists of trying to identify such a larger set (…). The assumption that such a larger set exists is implicit in almost all science, but, being fundamental, it is seldom mentioned explicitly.’ (Ashby, 1960, p. 28).

2.2. Higher-Order Potentialities and Mathematical Formalisation

The effect of a potentiality on a future state can be described by relating its occurrence in the present state to the occurrence of a certain state property in the future state, usually under an additional *opportunity* condition (e.g., assuming no obstruction by influences otherwise). This indicates what it is a potentiality for. A more complicated question is how to specify when (under which past and present circumstances) a potentiality itself will occur. For the case of empty space, where an object is assumed to have no interaction with other objects, a potentiality to change position is present because it was present at an earlier point in time and persisted until t (inertia of motion). However, if the potentiality in a new state is different from the earlier one, a question becomes why this is so. This leads to the question addressed in this section of how a changed potentiality can be explained.

The use of higher-order potentialities is an answer to this question. The idea behind higher-order potentialities is simple; see also (Treur, 2005). To obtain an explanation of changed state properties over time, potentialities were introduced. Potentialities are also state properties that change over time. Therefore it would be reasonable to treat them just like any other state property that changes over time. This means that for a potentiality $p^{(1)}$ a socalled *second-order potentiality* $p^{(2)}$ is introduced to explain why $p^{(1)}$ may become changed over time. And of course this process can be repeated for $p^{(2)}$, and so on. This leads to an infinite sequence of *higher-order* potentialities

$$p^{(1)}, p^{(2)}, p^{(3)}, p^{(4)}, \ldots$$

where for each natural number n the potentiality $p^{(n)}$ is called an *n-th-order potentiality*. Using such higher-order potentialities, the idea is the following:

- for a certain point in time $t0$ the occurrence of a state property a is determined by the state at a previous time point $t1 < t0$, in particular, by the occurrence of the first-order potentiality $p^{(1)}$ for a at that time point $t1$,
- the occurrence of the first-order potentiality $p^{(1)}$ at $t1$ is determined by the state at a time $t2 < t1$, in particular by the occurrence of its own potentiality which is the second-order potentiality $p^{(2)}$ for a at $t2$, et cetera.

This process can be visualised as depicted in Figure 1.

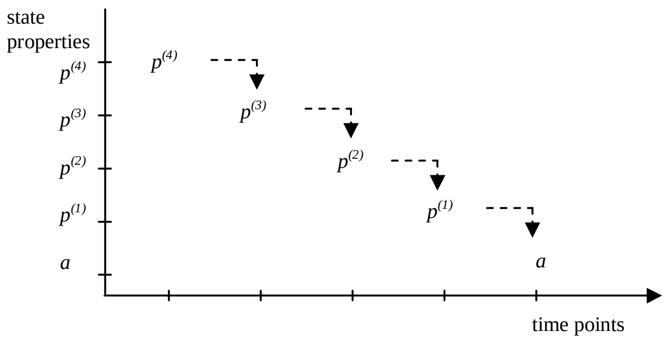

Figure 1. Dynamics based on higher-order potentialities

This shows how the concept of potentiality to explain change of a certain basic state property a can take the form of a large number of (higher-order) entities. Strange as the idea of an infinite number of higher-order potentialities may seem at first sight, in mathematical context (in particular in calculus) this has been worked out and formalised well (using infinite summations).

Higher-order potentialities have been formalised in the form of higher-order) derivatives of a function. The (first-order) derivative of a function at a time point t gives an estimated measure of how the function will change its value in a next time point. The well-known

Taylor series (Taylor, 1715) for sufficiently smooth functions (at least infinitely often differentiable to guantee the existence of the derivatives) shows how changes of the function value from t to t' (within some given neighbourhood of t) depend on all (higher-order) derivatives:

$$f(t') = f(t) + \Sigma_k\ f^{(k)}(t)(t' - t)^k / k!$$

This shows how the (relative) potentiality at t, defined by the combination of all higher-order potentialities, determines the changed state at the future time points t'.

3. Higher-Order Potentialities Underlying Physics

In later times, successors of Aristotle, such as René Descartes (1596-1650), Christiaan Huygens (1629-1695), Isaac Newton (1643-1727) and Gottfried Wilhelm Leibniz (1646-1716), among others, have addressed the question how to further develop the phenomenon of dynamics (or change), in particular within Physics; for a more extensive historical analysis, see (Treur, 2005). They developed classical mechanics based on concepts that can be philosophically founded as certain types of potentialities.

3.1 Momentum as Potentiality for Changed Position

Descartes (1633) took the product of mass and velocity of an object for its potentiality to be in a changed position, or 'quantity of motion'. Notice that this anticipatory state property 'quantity of motion' is a *relative* potentiality: the actualisation of a given quantity of motion entails being at another position as specified by this quantity relative to the current position. Descartes also expresses a law of conservation for this quantity of motion. In modern physics this 'quantity of motion' concept is called *linear momentum*, or just *momentum*, and expressed as

$$p = mv$$

The phenomenon that this quantity of motion is conserved (for example, during elastic collisions) is called the 'law of momentum conservation'. Newton (1729) incorporated this notion in his approach to motion.

Huygens (1629-1695), and later his student Leibniz (1646-1716), used a different way to exploit a concept potentiality for changed position. Leibniz sometimes called this concept *vis viva* (*living force)*. He claimed that this potentiaility was propertional not with velocity as in the case of Descartes' quantity of motion, but with the square of velocity. In this way Leibniz put the foundation for the law of conservation of energy, in this case involving kinetic energy, which was later taken

$$E = ½mv^2$$

and potential energy, and exchange between the two.

This shows how a concept 'potentiality' (for changed position) was used as a philosophical basis to introduce formalised concepts in physics, thus providing some of the cornerstones of classical mechanics.

3.2 Force as Second-Order Potentiality for Changed Momentum

As also the concept of 'quantity of motion', describing change of position, can change itself, this leads to a second-order potentiality. In his second law Newton (1729) uses the term 'impressed motive force' to express the change of motion.[3] This law expresses that the concept of force used by Newton directly relates to change of motion. For (quantity of) motion he gives the same definition as Descartes, i.e., momentum. For an impressed force a definition is given that refers to 'exerted action', and to the corresponding change of the object's state of motion.[4] He shows how this notion applies in the particular case of centripetal (i.e., directed to one point) force.[5] This shows that the concept 'force' used by Newton as an addition to state ontology can be given a definitional relationship to 'motion generated in a given time'. This 'motion generated in a given time' can be philosophical founded as a second-order potentiality for the first-order potentiality 'motion'. So, within classical mechanics, after the concepts 'momentum' and 'kinetic energy' which were added to the state ontology as specific types of concepts based on a (first-order) potentiality, the concept 'force' can be considered a third anticipatory state property added to the state ontology, this time based on a second-order potentiality. Newton and also Leibniz developed mathematical techniques of calculus, such as differentiation and integration. Using these techniques, Newton's second law is formulated as

$$F = dp/dt \quad \text{or} \quad F = d(mv)/dt.$$

For a mass which is constant over time this is equivalent with

$$F = ma$$

with a the acceleration dv/dt; in this - most known form - the law was formulated by Euler 65 years after the *Principia* appeared. In 20th century text books such as (Mach, 1942) the concept 'moving force' is *defined* in terms of acceleration, which is based on a second-order potentiality for change of position.[6]

As illustrated by the examples described above, the idea of using potentialities to analyse the change of states has successfully contributed to the development of well-respected disciplines such as Mathematics and Physics.

4. How to Limit Chains of Higher-Order Potentialities

Apparently, the use of potentialities may lead to an infinite-dimensional vector of higher-order potentialities. As this can be difficult to handle, it makes sense to look for ways to break off this chain of higher-order potentialities.

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[3] 'The change of motion is proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed.' (Newton, 1729)

[4] 'An impressed force is an action exerted upon a body, in order to change its state ...' (Newton, 1729)

[5] 'The motive quantity of a centripetal force is (…) proportional to the motion which it generates in a given time.' (Newton, 1729)

[6] 'Moving force is the product of the mass value of a body with the acceleration induced in that body.' (Mach, 1942, p. 304)

4.1 Limiting Chains by Vanishing Higher-Order Potentialities

One possible option to limit a chain of higher-order potentialities is to consider only changes that involve a finite number of higher-order potentialities. For example, for a falling object within a constant gravitation field, the second-order potentiality (the second-order derivative of the function measuring the distance) is constant (*9.8* m/sec^2), and hence no third- or higher-order potentiality is needed. Here the basic relation for the distance *x* depending on the time variable *t* is described by a quadratic function in *t*; for such a function the third and higher derivatives are zero. In general, if the basic relation of a state property *x*, depending on the time variable *t*, is described by a polynomial in *t* of degree *n-1*, then its *n*-th and higher derivatives are zero, so the potentialities of *x* of *n*-th order and higher are zero and hence they do not play a role in the dynamics of *x*.

However, more in general, and further away in the universe, if an object is approaching the earth, gravitation will increase over time, so the assumption of constancy of the second-order potentiality of distance is not always fulfilled. In that case, as is also shown by the Taylor series, often an adequate approximation can be obtained by taking into account only the terms up to some *n*-th order, as the terms substantially decrease in absolute size; for example, when all derivatives are bounded by some constant *M*, and $|t' - t| < d$, then the effect of the *n*-th term is less than

$$Md^n/n!$$

which rapidly vanishes for larger *n*. So, as an approximation, for higher *n* its contribution can be counted as zero.

4.2 Limiting Chains by Reducers of Higher-Order Potentialities

A more general way to get rid of the infinite vector of higher-order potentialities is when for some *n* the *n*-th-order potentiality is not vanishing, but is equivalent to a combination of lower level potentialities and/or basic state properties (this is called a *reducer* of the higher-order potentiality) and by means of this relation can be reduced to them. This is what happens in classical mechanics, and, in a more general context, in other cases where a differential equation can be found that relates a higher-order derivative to lower order derivatives and/or basic state properties. Note that limiting chains by vanishing higher-order potentialities can be considered a special case of this: reducing a higher-order potentiality to a constant or to zero.

The domain of Physics illustrates this. Analysing the motion of planets around the sun, Newton found out that they can only follow their orbit if a second-order potentiality is assumed, in the direction of the sun. Newton calculated (using his calculus under development) in detail that this motive force was proportional to 1 divided by the square of the distance. For example, for an object in space with mass *m* approaching earth (with mass *M*), Newton's law of gravitation for the motive force on the object is as follows (here *x* is the distance between the object and the earth, and *c* is a constant):

$$F = c\ mM/x^2$$

Such a relation between the second-order potentiality force and basic state properties mass and distance shows how a higher-order potentiality can be reduced: in this case

$$c\ mM/x^2$$

is a reducer for *F*.

As the example from Physics shows, a differential equation is a manner to reduce higher-order potentialities. This reveals another assumption underlying DST, in addition to the state-determined system assumption, namely that for any potentiality, for some *n* the related *n*-th order potentiality can be reduced to lower level potentialities and/or basic state properties. Without this assumption no differential equations can be found, whereas DST usually works on the basis of differential equations. Thus, by introducing reducers to the conceptual framework based on potentialities, a philosophical concept has been added underlying (at least) the notion of differential equation. The existence of reducers was found to be a crucial basic assumption for DST. Subsequent sections investigate the role played by reducers in other, non-quantitative dynamic modelling approaches, such as BDI-modelling, and transition and production systems.

5. Higher-Order Potentialities and Reducers in BDI-Models

Although traditionally only used within disciplines such as Mathematics and Physics, a natural question is whether the idea of higher-order potentialities is also suitable to obtain philosophical foundations for domains such as AI, Cognitive Science and Agent Systems. In this and subsequent sections this question is answered in an affirmative manner and illustrated.

5.1 Intentions as Potentialities

To describe the internal dynamics of agents, the concepts *beliefs*, *desires* and *intentions* have been introduced; e.g., (Aristotle, 350 BC'). From a historical perspective, the reason for introducing these concepts was not unlike the reason for introducing the concepts momentum and force within classical mechanics: they were needed as abstract notions to explain the change of states, in this case of living creatures. Aristotle describes how desire plays a role similar to that of the potentiality for an action. Here 'desire' is indicated as the source of motion of a living being. He shows how the occurrence of certain internal (mental) state properties (desires, 'the good') within the living being entail or cause the occurrence of an action in the external world, given an opportunity ('the possible') to actualise the potentiality for the action indicated by the desire.

In this section it is discussed how to philosophically found concepts such as desire and intention by potentialities. As a first step, the notion of intention can be founded by a potentiality for an action in the world (i.e., for a changed world state). A next question is: where do intentions come from? A common view is that, given some beliefs, intentions come from desires, by some form of selection process.

5.2 Desires as Second-Order Potentialities

Given the interpretation that intentions come from desires, a desire itself can be founded by a potentiality as well. However, in that case, the latter is not a potentiality for some state of the world, but a potentiality for an intention, which itself is also considered a potentiality, for an action (i.e., for a changed world state). Therefore this view identifies a desire as a second-order potentiality (for a changed world state). To the question where desires come from there seems to be no general answer. In some cases certain personality aspects may play the role of a potentiality for a desire (third-order potentiality for a changed world state). Examples of such personality aspects are personal norms or what Frankfurt (1971) calls second-order desires. A second-order desire is a desire to have certain desire. If it is assumed that within a cognitive system such a second-order desire is used to affect first-order desires a person has, then it can be considered a third order potentiality for a changed world state providing a source for desires.[7] A question then is where such third-order potentialities for a changed world state come from. In some cases they may be related to a reducer, for example, in terms of observations[8] (see Figure 2) and/or a person's physical properties, or they may have developed through a history of experiences.

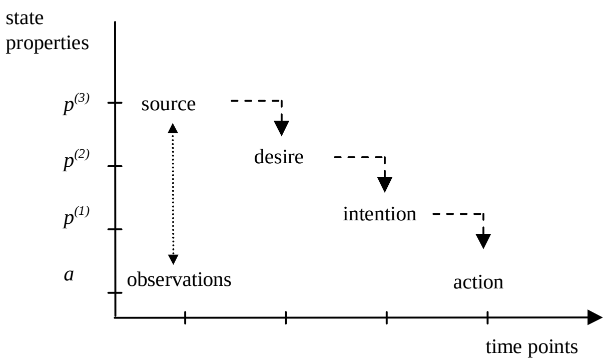

Figure 2. Third order potentialities as a source for desires.

Another option would be when a desire itself may have a reducer in terms of physical states (see Figure 3). For example, in literature on criminal behaviour, it is often claimed that for certain types of criminals, specific desires are related to their physical make up. For example, for a psychopath a desire for actions with strong stimuli relates to a high excitement threshold, and the absence of a desire for nonrisky actions relates to a high anxiety threshold. These thresholds are assumed to relate to physical aspects such as certain brain deviations and deviating levels of hormones and neurotransmitters, such as serotonin. Similarly, in the literature the desire for aggressive actions is often related to elevated testosterone levels; cf. (Raine, 1993; Moir and Jessel, 1995; Delfos, 2004).

A different account for certain types of desires (for example, the innate ones) could involve the history of ontological and/or evolutionary development. Considering such more long-term temporal perspectives in relation to the topic of potentialities is an interesting challenge on its own that requires a more extensive treatment, and is left out of the scope of this paper.

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[7] Note that this is a nontrivial assumption. It may very well be the case that a person has norms or second-order desires, but the desires themselves are generated in a way that they are not affected by these personality aspects, and indeed may just conflict with them; for example, the second-order desire not to desire to smoke, or 'I hate that I love you'.

[8] For example, observing an occasion to buy icecream and an empty stomach.

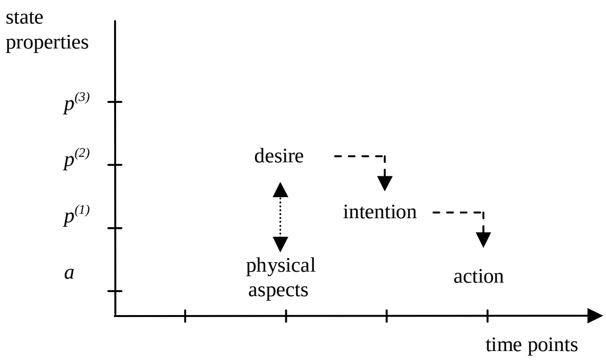

Figure 3. Reducing desires to physical aspects

6. Potentialities and Reducers in Transition and Production System Modelling

Often used dynamical modelling methods in Computer Science, AI and Cognitive Science to specify how a state in a system may change are transition systems or production systems. Viewed at an abstract level, these approaches are based on collections of rules, denoted as

$$\varphi \rightarrow \psi$$

with antecedent φ and consequent ψ ; here:

- φ indicates a (combined) state property for the current state
- ψ indicates one or more state properties for the next state

The idea is that if the combination of properties specified in φ holds in a (current) state, then in a next state the properties specified by ψ will hold. This is illustrated by a simple model of agent behaviour based on beliefs desires and intentions. Consider an agent walking down a street and observing an ice cream sign across the street he believes the supermarket sells ice cream. Based on this belief (b1) the agent generates a desire (d) for ice cream. Given this desire, and the belief (b2) that the supermarket is reachable (by crossing the street) the agent generates the intention (i) of having ice cream. Given this intention and the belief (b3) that no traffic is on the street he actually crosses the street and obtains the ice cream (e). In this case the state ontology is described by six basic state properties: b1, b2, b3, d, i, e. The dynamical model is specified by:

$$\text{b1} \twoheadrightarrow \text{d} \qquad \text{b2} \wedge \text{d} \twoheadrightarrow \text{i} \qquad \text{b3} \wedge \text{i} \twoheadrightarrow \text{e}$$

Based on this a trace of subsequent states is made:

- Given a current state S, take the production rules for which the antecedent holds in the current state. This is the set of applicable rules.
- Collect the consequents of all applicable rules and obtain the next state S' by modifying S so that all these consequents hold in S' (and the rest of S is persisting).

So, for example, the subsequent states for a given initial state for which the three beliefs hold are as follows:

0 [b1, b2, b3]
1 [b1, b2, b3, d]
2 [b1, b2, b3, d, i]
3 [b1, b2, b3, d, i, e]

How can this be interpreted in terms of potentialities? For example, consider state 1. As in the next state, state 2, state property i holds, in state 1 the potentiality for i to hold has to be present. On the other hand, i occurs in state 2 because of the second production rule. Taken together this means that this rule can be interpreted for state 1 as indicating that, due to the occurrence of both b2 and d in this state, also the potentiality p(i) for i occurs in state 1. Similarly the other rules can be interpreted as indications of which potentialities occur in a given state. In general, according to this interpretation, such a system specifies for each state which potentialities occur: for each rule $\varphi \twoheadrightarrow \psi$, if in a state S its antecedent φ holds, then in this state S also the potentiality $p(\psi)$ for ψ occurs. Thus a rule $\varphi \twoheadrightarrow \psi$ can be interpreted as an implication $\varphi \rightarrow p(\psi)$, describing a logical relationship between state properties in a given state, e.g., φ is a reducer for $p(\psi)$. Since the idea of transition or production rules is used in various other modelling approaches (e.g., knowledge-based and reasoning systems, and cognitive architectures such as ACT-R (Anderson and Lebiere, 1998) and SOAR (Laird *et al.*, 1987)), in this way it is possible to interpret such approaches in terms of potentialities as well. Note that some of these approaches show a monotonic pattern in the sense that generated facts persist over time. However, in general a transition system approach does not need to be limited to such monotonic patterns. For example, it may well be the case that transition rules $a \twoheadrightarrow b$, $b \twoheadrightarrow \text{not } a$, $\text{not } a \twoheadrightarrow \text{not } b$, and $\text{not } b \twoheadrightarrow a$ are present, which show a pattern in which both a and not a and b and not b alternate. From a temporal perspective there is no contradiction in that a holds at time t and not a at time point $t' \neq t$.

7. Higher-Order Potentialities and Adaptive Agents

Adaptive agents are often modelled in numerical and algorithmic manners. In this sense modelling approaches for adaptive agents are in general closer to the DST modelling approach than to symbolic modelling approaches as often applied for other types of cognitive agents such as BDI agents. The potentiality-based analysis framework is applicable to obtain a philosophical foundation for both types of modelling approaches. As in Section 5 and 6 it was shown how symbolic models can be founded by potentialities, in this section adaptive agents are addressed, illustrated by a case study of *Aplysia*. *Aplysia* is a sea hare that is often used to do experiments. It is able to learn; for example, it performs classical conditioning in the following manner. This (a bit simplified) description is mainly based on (Gleitman, 1999), pp. 155-156. Initially the following behaviour is shown: a tail shock leads to a response (contraction), and a light touch on its siphon is insufficient to trigger such a response. Now suppose the following experimental protocol is undertaken. In each trial the subject is touched lightly on its siphon and then, shocked on its tail (as a consequence it responds). It turns out that after a number of trials (assumed three in the current example) the behaviour has changed: the animal also responds (contracts) on a siphon touch. The cause of this change in behaviour is, in short, that the learning trials strengthen the internal connection between sensory nand motor neurons. To obtain a potentiality-based analysis of this adaptive agent the following steps can be made.

- Introduce the basic world state properties: siphon touch, tail shock, contraction, weak connection between sensory neuron and motor neuron.
- Introduce a potentiality *p* for the contraction (based on the opportunity that a siphon touch occurs).
- Introduce a potentiality *p'* for *p* (based on the opportunity that both a tail shock and siphon touch occur).
- Introduce a potentiality *p"* for *p'* (based on the opportunity that both a tail shock and siphon touch occur).
- Introduce a potentiality *p"'* for *p"* (based on the opportunity that both a tail shock and siphon touch occur).
- Specify a reducer for *p"'* based on the untrained state of the connection between the relevant sensory and motor neurons.

Notice that in the analysis of the example the intermediate states during the adaptation process were founded by first-, second- and third-order potentialities

p, p', p"

that were not reduced, similar to the not reduced first-order potentiality momentum in classical mechanics as an intermediate state between the second order potentiality force, which is reduced, and position. However, if more detailed information from the neurological area is incorporated (i.e., about the physiological states of the synapses between certain neurons, e.g., sensory neuron SN2 and motor neuron MN, during the adaptation process), then all of the potentialities can be reduced; e.g., (Gleitman, 1999), pp. 155-156, see also Figure 4. Within a potentiality-based analysis this is incorporated as follows. Suppose

s1, s2, s3

are reducers of

p, p', p"

respectively, then the (higher-order) potentialities

p', p", p"'

become (first-order) potentialities for the world state properties

s1, s2, s3

respectively.

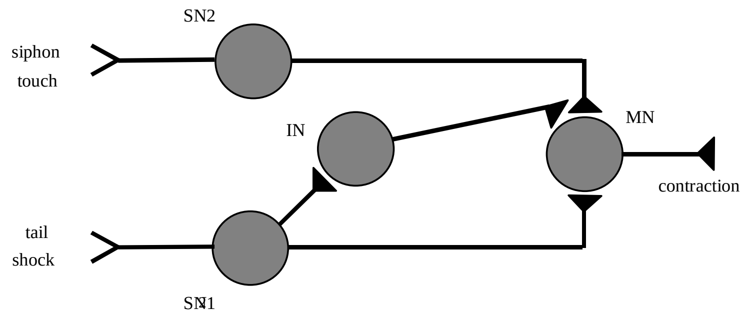

Figure 4. Aplysia from neurological perspective

This shows how potentialities cover the foundation of both internal agent states that are used as functional states without direct physical grounding, and internal agent states that are considered as embodied and embedded in the physical world. Moreover, the relationship between these functionalist and physicalist perspectives can easily be analysed within the potentiality-based philosophical framework. For reasons of presentation, the example of an adaptive agent discussed in this section was kept relatively simple. However, the case can be extended easily to include, for example, numerical aspects, or more precise timing aspects such as in trace conditioning.

8. Potentialities and Dynamical Systems Theory

Within Dynamical Systems Theory (DST; cf. (Ashby, 1960; Port and Gelder, 1995)), techniques used are difference and differential equations. The analysis based on potentialities covers the DST approach in the following manner.

- For the basic state properties, take value assignments to the basic variables used to describe a phenomenon in DST.
- For any n, for the nth-order potentiality for a basic state property, take the value assignment to the nth-order derivative of the variable.
- For the reduction relation of an nth-order potentiality, take the nth-order differential equation relating the value of the nth-order derivative to the values of lower-order derivatives and basic variables.

As an example, the gravitation case is covered as follows:

- Basic state properties: value assignments to x the position of the object, m mass of the object, M mass of the earth.
- First-order potentiality for basic state property mx: value assignment to first-order derivative p of mx (momentum p of the object):

$$p = dmx/dt$$

- Second-order potentiality for state property *mx*: value assignment to first-order derivative *f* of *p*, or, equivalently, second-order derivative *f* of *mx* (force *f*):

$$f = dp/dt = d^2mx/dt^2$$

- Reduction relation of the second-order potentiality *f*:

$$f = c*mM/x^2 \quad (\text{or } d^2mx/dt^2 = c*mM/x^2)$$

As neural networks can be considered as a specific use of DST, the relationship of the potentiality-based view to DST also covers the relationship to neural networks and many other quantitative modelling methods.

9. A Potentiality-Based Metamodel For Dynamic Modelling Methods

Higher-order potentialities and their reducers can be formalised to obtain a metamodel that can be used in a generic method to analyse dynamics in terms of higher-order potentialities. In this section the metamodel is defined and formalised. In Section 10 it is shown for a number of cases, how the metamodel can be used: how it can be instantiated and mapped onto different dynamical model formats.

9.1 Overview of the Metamodel

The potentiality-based metamodel provides an ontology consisting of a number of types of objects and relations to describe and conceptually analyse dynamics. Relations used are of type 'is potentiality for', 'is opportunity for' and 'is reducer of'. Moreover, two generic axioms (one on actualisation of a potentiality, and one on reduction of a potentiality) define the semantics of the relations. More details will be given in Section 9.2. Using the metamodel to conceptually analyse dynamics involves a number of steps (also see Figure 1), roughly expressed as follows:

1. pick a basic world state property *a* of which the change over time needs to be analysed.
2. introduce a potentiality $p^{(1)}$ that explains the change over time of *a*.
3. can a reducer be found for the introduced potentiality $p^{(n)}$ that has the highest order?
   3a. *yes* -> introduce a reducer x that connects $p^{(n)}$ to a lower-level state property: in terms of world state properties and lower level potentialities; then end.
   3b. *no* -> introduce a potentiality $p^{(n+1)}$ that describes the change over time of $p^{(n)}$; then go to step 3.

To illustrate the idea of the method, suppose one wants to explain the fact that an object approaching the earth changes position over time (due to gravitation). In that case the basic world state property a would be something like 'object *O* is in position *P*' (step 1). Then, to explain the change over time of this state property, for the first-order potentiality $p^{(1)}$ a concept 'momentum' can be introduced (step 2). However, since no reducer of this potentiality can be found, a second-order potentiality $p^{(2)}$ is introduced to explain the change over time of a momentum, i.e. a concept 'force' (step 3b). Finally, a reducer is introduced for this second-order potentiality force (step 3a), the expression

$c*mM/x^2$

To summarise, this example shows how the change of a world state property in the domain of Physics can be explained and analysed in terms of higher-order potentialities, using the metamodel. Below, the formalisation of this metamodel is addressed.

9.2 Formalisation of the Metamodel

This section introduces a formalisation of the metamodel in terms of a reified order-sorted predicate logic-based temporal language (Galton, 2003, 2006). Within this language, dynamics is considered as evolution of states over time, labeled by time points. The notion of state as used here is characterised on the basis of an ontology defining a set of properties that do or do not hold at a certain point in time. For a given (order-sorted predicate logic, including the use of real and integer numbers) ontology Ont, the set of all *state ground atoms* (or *atomic state properties*) based on Ont is denoted by APROP(Ont). The *state properties* based on a certain ontology Ont are formalised by the propositions that can be made (using conjunction, negation, disjunction, implication) from the ground atoms, denoted by PROP(Ont). A *state* S is an indication of which atomic state properties are true and which are false, i.e., a mapping S: APROP(Ont) → {true, false}. Moreover, a given ontology Ont may comprise specific sub-ontologies WorldOnt and PotOnt to describe, respectively, basic world state properties and (higher-order) potentialities. Within state properties real and integer numbers, with their ordering relations and arithmetical operations can be used. The predicate

holds_at(a, t)

is used to indicate that a state property a holds in the state at a certain time point t.

To analyse dynamics of a given process in terms of basic world state properties and higher-order potentialities, a number of relations are needed as part of the ontology used:

potentiality_for:	APROP(PotOnt) x PROP(Ont) x PROP(Ont) x DURATION
potentiality_for:	APROP(PotOnt) x PROP(Ont) x DURATION
opportunity_for:	PROP(Ont) x APROP(PotOnt)
reducer_of:	PROP(Ont) x APROP(PotOnt)

For example,

potentiality_for(p, a', a, d)

expresses that p is a relative potentiality for the change from a' to a after duration d, and

potentiality_for(p, a, d)

expresses that p is an absolute potentiality to obtain a after duration d. Moreover,

reducer_of(r, p)

expresses that state property r is a reducer of p.

Note that these relations use as arguments terms that indicate state properties in a reified form. Therefore the relations are second-order in the sense that they are meta-relations over the object language that conceptualises the world states. On the basis of these relations, the following generic dynamic properties are introduced as axioms of the metamodel:

AP (Actualisation of a Potentiality)

If Z' and Y hold, and Y is a relative potentiality for changing Z' into Z after duration d, and O holds, and O is an opportunity for Y, then Z will hold after duration d.

holds_at(Z'∧Y∧ O, t) & potentiality_for(Y, Z', Z, d) & opportunity_for(O,Y) ⇒ holds_at(Z, t+d)

If Y holds, and Y is an absolute potentiality to obtain Z after duration d, and O holds, and O is an opportunity for Y, then Z will hold after duration d.

holds_at(Y∧ O, t) & potentiality_for(Y, Z, d) & opportunity_for(O,Y) ⇒ holds_at(Z, t+d)

Here, the notion of *opportunity* is introduced to describe the fact that sometimes an additional condition is needed in order for a potentiality to have its effect. For example, the concept 'momentum' only acts as a potentiality for a position in certain situations, for objects not being obstructed in their movement. It is not uncommon that opportunities are only silently assumed, not mentioning them explicitly; for example, in physical laws usually idealised conditions are assumed: undesired interference from other phenomena outside is assumed absent.

RP (Reduction of a Potentiality)

If X holds, and X is a reducer of Y, then Y holds.

holds_at(X, t) & reducer_of(X, Y) ⇒ holds_at(Y, t)

Note that these dynamic properties are completely domain-independent. To analyse the dynamics within a specific domain, the ontology Ont (and its sub-ontologies) have to be filled with domain-specific ground atoms for the different sorts, and some specific instances of the relations potentiality_for, opportunity_for, and reducer_of have to be created. In terms of the method introduced in Section 9.1 the analysis works as follows:

1. pick basic world state properties *a* of which the change over time needs to be analysed:
   fill sort APROP(WorldOnt) with relevant atom instances a.
2. introduce a potentiality $p^{(1)}$ that explains the change within time duration d of relevant world state properties *a*:
   for all relevant instances a and a' in PROP(WorldOnt) so that a may change within time duration d into a' introduce an instance p in APROP(PotOnt) and choose an instance o in PROP(WorldOnt) and relations potentiality_for(p, a', a, d) (or potentiality_for(p, a, d)) and opportunity_for(o, p).
3. can a reducer be found for the introduced potentiality $p^{(n)}$ with the highest order?
   3a. *yes* -> introduce a reducer x that connects $p^{(n)}$ to a state property built from world state properties and/or lower level potentialities:

for potentially p in APROP(PotOnt) choose an instance r in PROP(Ont) built from world atoms and lower level potentialities and specify the relation reducer_of(r, p); then end.

3b. *no* -> introduce a potentiality $p^{(n+1)}$ that explains the change within time duration d of $p^{(n)}$:

for instances p' and p in APROP(PotOnt) so that p' may change into p within time duration d introduce an instance q in APROP(PotOnt) and choose an instance o in PROP(Ont) built from world atoms and lower level potentialities and relations potentiality_for(q, p', p, d) (or potentiality_for(q, p, d)) and opportunity_for(o, q); then go to step 3.

In Section 10 it is shown in detail how this metamodel can be used to analyse dynamical models.

10. Mapping Metamodel Instantiations onto Dynamical Modelling Approaches

In this section it is shown how the metamodel can be used – by instantiation – to analyse a number of models in a specific dynamic modelling language: a cognitive model for BDI-agents (Section 10.1), a model for adaptive agents (Section 10.2), and DST-models (Section 10.3). The general pattern of using the metamodel to obtain a specification in a specific dynamic modelling language is roughly as follows:

1. Instantiate in generic properties AP and RP the variables X, Y, Z, and O to obtain a number of instances of AP and RP, thereby respecting the relations potentiality_for and reducer_of.
2. Eliminate the relations potentiality_for, opportunity_for, and reducer_of.
3. Make a mapping of the obtained instances to the specific dynamic modelling language used.

This will be illustrated in more detail in the remainder of this section.

10.1 Instantiation of the Metamodel for a Cognitive Agent Model (BDI)

To make the above more concrete, this section describes a specific case study in the domain of BDI-modelling. Consider the example situation of a manager who deliberates after some negative experience(s) whether or not to fire his employee. To describe the situation, the following state properties are assumed (see Table 2):

internal state properties:	
doubt(fire)	The agent doubts about firing the employee
desire(fire)	The agent desires to fire the employee
intention(fire)	The agent intends to fire the employee
input state properties:	
observation_result(no_spec_circumstances)	The agent observes that there are no special circumstances in favour of the employee
observation_result(neg_experience)	The agent experiences a negative experience with the employee
observation_result(add_neg_experience)	The agent experiences an additional negative experience with the employee
observation_result(law_allows(fire))	The agent observes that the law allows firing
observation_result(employee_present)	The agent observes that the employee is present
output state properties:	
is_performed(fire)	The agent initiates the action of firing the employee

Table 1. State Properties for the BDI example

In this case, the internal mental state properties are part of the ontology PotOnt, and the input and output state properties are part of the ontology WorldOnt. Moreover, note that the state property doubt(fire) can be considered as a specific case of a source for desire (in terms of Figure 2). To analyse the dynamics of the case study, the following specific instances of the relations given in Section 9.2 are specified:

potentiality_for(intention(fire), is_performed(fire), d)
potentiality_for(desire(fire), intention(fire), d)
potentiality_for(doubt(fire), desire(fire), d)
opportunity_for(observation_result(employee_present), intention(fire))
opportunity_for(observation_result(law_allows(fire)), desire(fire))
opportunity_for(observation_result(add_neg_experience), doubt(fire))
reducer_of(and(observation_result(no_spec_circumstances), observation_result (neg_experience)), doubt(fire))

To specify simulation models in a declarative, logical manner, the LEADSTO language (Bosse et al., 2007) enables one to model direct temporal dependencies between two state properties in successive states, also called *local dynamic properties*. A specification of such dynamic properties in LEADSTO format has as advantages that it is executable and that it can often be depicted graphically in a causal graph like style. The format is defined as follows. Let α and β be state properties of the form 'conjunction of atoms or negations of atoms', and e, f, g, h non-negative real numbers. Then the notation $\alpha \twoheadrightarrow_{e, f, g, h} \beta$, means:

> *If state property α holds for a certain time interval with duration g*
> *then after some delay (between e and f) state property β will hold*
> *for a certain time interval of length h.*

For readability, in the description of the dynamic properties provided below the parameters e, f, g, h are often omitted. Within the state properties real and integer numbers, with their ordering relations and arithmetical operations can be used. Moreover, variables can be used that are considered as universally quantified over the whole formula.

Based upon the LEADSTO language, the LEADSTO environment (Bosse et al., 2007) comprises a dedicated piece of software to perform simulation. This software takes as input a number of executable dynamic properties and state properties (as initial facts), and generates simulation traces as output. An example of such a trace is given in Figure 5. Time is on the horizontal axis, the state properties are on the vertical axis. A dark box on top of the line indicates that the property is true during that time period, and a lighter box below the line indicates that the property is false. This particular trace is based on specific LEADSTO properties that can be described as mapped instances of AP and RP from the metamodel in the following manner:

1. *Instantiate the variables* X, Y, Z, *and* O *to obtain a number of instances of AP and RP, thereby respecting the relations* potentiality_for *and* reducer_of.

Thus the following instances are obtained:

holds_at(intention(fire) ∧ observed(employee_present), t) & potentiality_for(intention(fire), is_performed(fire), d) &
  opportunity_for(observed(employee_present), intention(fire))
    ⇒ holds_at(is_performed(fire), t+d)
holds_at(desire(fire) ∧ observed(law_allows(fire)), t) & potentiality_for(desire(fire), intention(fire), d) &
  opportunity_for(observed(law_allows(fire)), desire(fire))
    ⇒ holds_at(intention(fire), t+d)
holds_at(doubt(fire) ∧ observed(add_neg_experience), t) & potentiality_for(doubt(fire), desire(fire), d) &
  opportunity_for(observed(add_neg_experience), doubt(fire))
    ⇒ holds_at(desire(fire), t+d)
holds_at(and(observed(no_spec_circumstances), observed(neg_experience)), t) &
  reducer_of(and(observed(no_spec_circumstances), observed(neg_experience)), doubt(fire))
    ⇒ holds_at(doubt(fire), t)

2. *Eliminate the relations* potentiality_for, opportunity_for, *and* reducer_of.

This provides the following set TR_{BDI} of temporal relationships for this BDI case:

holds_at(intention(fire) ∧ observed(employee_present), t)
    ⇒ holds_at(is_performed(fire), t+d)
holds_at(desire(fire) ∧ observed(law_allows(fire)), t)
    ⇒ holds_at(intention(fire), t+d)
holds_at(doubt(fire) ∧ observed(add_neg_experience), t)
    ⇒ holds_at(desire(fire), t+d)
holds_at(and(observed(no_spec_circumstances), observed(neg_experience)), t)
    ⇒ holds_at(doubt(fire), t)

3. *Make a mapping of these instances to the specific dynamic modelling language used.*

The general mapping $\pi_{LEADSTO}$ of temporal relationships into the LEADSTO format is defined as follows

$\pi_{LEADSTO}$: [holds_at(a, t) ⇒ holds_at(b, t+d)] ⟶ [a $\twoheadrightarrow_{0, 0, d, d}$ b]

$\pi_{LEADSTO}$: [holds_at(a, t) ⇒ holds_at(b, t)] ⟶ [a $\twoheadrightarrow_{0, 0, 0.01d, 0.01d}$ b]

Here, within property AP from the metamodel in relation to the delay parameter d, the values 0, 0, d, d are chosen for the timing parameters e, f, g, h, and for RP from the metamodel (where in fact d=0), the values 0, 0, 0.01, 0.01 are chosen[9]. Applied to the

[9] Ideally, this would be 0, 0, 0, 0 (to simulate an instantaneous relation), but within the current version of the software this is not allowed.

instances in the set TR_{BDI} above, the following set LT_{BDI} of LEADSTO rules is obtained (i.e., $\pi_{LEADSTO}(TR_{BDI}) = LT_{BDI}$):

intention(fire) ∧ observation_result(employee_present) $\twoheadrightarrow_{0, 0, d, d}$ is_performed(fire)
desire(fire) ∧ observation_result(law_allows(fire)) $\twoheadrightarrow_{0, 0, d, d}$ intention(fire)
doubt(fire) ∧ observation_result(add_neg_experience) $\twoheadrightarrow_{0, 0, d, d}$ desire(fire)
observation_result(no_spec_circumstances) ∧ observation_result(neg_experience) $\twoheadrightarrow_{0, 0, 0.01d, 0.01d}$ doubt(fire)

As can be seen in Figure 5 (where d = 1), as soon as the agent experiences a negative experience with the employee, and meanwhile observes that there are no special circumstances, it starts to doubt about firing the employee (time point 4). Later, when the appropriate opportunities are present, this doubt leads to a desire (time point 7), then to an intention (time point 10), and eventually to the action of firing the employee (time point 14).

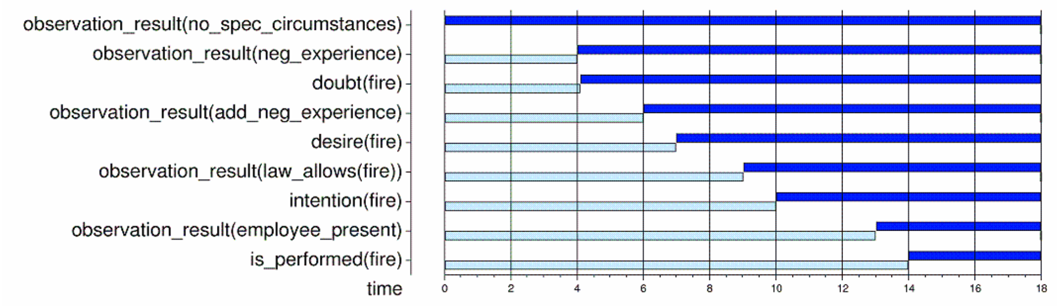

Figure 5. Example simulation trace for a BDI-model

Here doubt on firing is a source for the desire: is a mental state property that acts as a potentiality for a desire (thus, as a third-order potentiality for a world state). As shown in Figure 5, here an intention is interpreted as a potentiality for an action, a desire as a potentiality for the intention, and a source as a potentiality for the desire.

10.2 Instantiation of the Metamodel for an Adaptive Agent Model (Aplysia)

To obtain a potentiality-based analysis of the adaptive agent discussed in Section 7 the following steps can be made.

1. Introduce the basic world state properties:
   siphon_touch, tail_shock, contraction, weak_connection(sensory_neuron, motor_neuron)
2. Introduce a potentiality *p* for the contraction based on a siphon touch:
   potentiality_for(p, not contraction, contraction, d)
   opportunity_for(siphon_touch, p)
3. Introduce a potentiality *p'* for *p*:
   potentiality_for(p', not p, p, d)
   opportunity_for(and(tail_shock, siphon_touch), p')
4. Introduce a potentiality *p''* for *p'*:
   potentiality_for(p'', not p', p', d)
   opportunity_for(and(tail_shock, siphon_touch), p'')

5. Introduce a potentiality *p'''* for *p''*:
   potentiality_for(p''', not p'', p'', d)
   opportunity_for(and(tail_shock, siphon_touch), p''')
6. Specify a reducer for *p'''* based on the physical state of the untrained connection between the relevant sensory and motor neurons:
   reducer_of(weak_connection(sensory_neuron, motor_neuron), p''')

For this case in a way similar to 10.1 the set TR_{AA} of relevant instances of the dynamic properties AP and RP from the metamodel can be obtained and mapped by $\pi_{LEADSTO}$ onto the following set LT_{AA} of LEADSTO properties (i.e., $\pi_{LEADSTO}(TR_{AA}) = LT_{AA}$):

not contraction ∧ p ∧ siphon_touch	$\twoheadrightarrow_{0,\, 0,\, d,\, d}$	contraction
not p ∧ p' ∧ siphon_touch ∧ tail_shock	$\twoheadrightarrow_{0,\, 0,\, d,\, d}$	p
not p' ∧ p'' ∧ siphon_touch ∧ tail_shock	$\twoheadrightarrow_{0,\, 0,\, d,\, d}$	p'
not p'' ∧ p''' ∧ siphon_touch ∧ tail_shock	$\twoheadrightarrow_{0,\, 0,\, d,\, d}$	p''
weak_connection(sensory_neuron, motor_neuron)	$\twoheadrightarrow_{0,\, 0,\, 0.01d,\, 0.01d}$	p'''

Suppose *s1, s2, s3* are reducers of *p, p', p''* respectively, then the (higher-order) potentialities *p', p'', p'''* become (first-order) potentialities for the world state properties *s1, s2, s3* respectively. The formalisation of the set TR_{RAA} of instances for this adaptive agent case of the dynamic properties AP and RP from the metamodel that are obtained can now be mapped by $\pi_{LEADSTO}$ onto the following set LT_{RAA} of LEADSTO properties:

not contraction ∧ p ∧ siphon_touch	$\twoheadrightarrow_{0,\, 0,\, d,\, d}$	contraction
not s1 ∧ p' ∧ siphon_touch ∧ tail_shock	$\twoheadrightarrow_{0,\, 0,\, d,\, d}$	s1
not s2 ∧ p'' ∧ siphon_touch ∧ tail_shock	$\twoheadrightarrow_{0,\, 0,\, d,\, d}$	s2
not s3 ∧ p''' ∧ siphon_touch ∧ tail_shock	$\twoheadrightarrow_{0,\, 0,\, d,\, d}$	s3
weak_connection(sensory_neuron, motor_neuron)	$\twoheadrightarrow_{0,\, 0,\, 0.01d,\, 0.01d}$	p'''
s3	$\twoheadrightarrow_{0,\, 0,\, 0.01d,\, 0.01d}$	p''
s2	$\twoheadrightarrow_{0,\, 0,\, 0.01d,\, 0.01d}$	p'
s1	$\twoheadrightarrow_{0,\, 0,\, 0.01d,\, 0.01d}$	p

10.3 Instantiation of the Metamodel for DST Models

A formalisation of the example on a moving object attracted by the earth in empty space as discussed in Section 8 (in discretised form with step size d) is as follows; here v and w are variables over real numbers:

potentiality_for(has_value(p,v), has_value(x,w), has_value(x,w + vd), d)
potentiality_for(has_value(f,v), has_value(p,w), has_value(p,w + vd), d)
opportunity_for(empty_space, has_value(p,v))
opportunity_for(empty_space, has_value(f,v))
reducer_of(has_value($c*mM/x^2$,v), has_value(f,v))

The obtained instances of the generic meta-templates AP and RP from the metamodel for this gravitation case provides the following set TR_{GR} of temporal relationships.

holds_at(and(has_value(x, w), has_value(p, v), empty_space), t) $\Rightarrow$ holds_at(has_value(x, w+vd), t+d)
holds_at(and(has_value(p, w), has_value(f, v), empty_space), t) $\Rightarrow$ holds_at(has_value(p, w+vd), t+d)
holds_at(has_value(c*mM/x^2, v), t) $\Rightarrow$ holds_at(has_value(f, v), t)

As before, this set TR_{GR} can be mapped by $\pi_{LEADSTO}$ onto the following set LT_{GR} of LEADSTO properties:

has_value(x, w) $\wedge$ has_value(p, v) $\wedge$ empty_space $\twoheadrightarrow_{0, 0, d, d}$ has_value(x, w+vd)
has_value(p, w) $\wedge$ has_value(f, v) $\wedge$ empty_space $\twoheadrightarrow_{0, 0, d, d}$ has_value(p, w+vd)
has_value(c∗mM/x^2, v) $\twoheadrightarrow_{0, 0, 0.01d, 0.01d}$ has_value(f, v)

Alternatively, assuming that the empty space assumption is fulfilled, the set of instances TR_{GR} obtained from the potentiality-based analysis for the gravitation case can be mapped onto a dynamical model DE_{GR} based on differential equations in mathematical terms by the general mapping (here the capitals are meta-variables) π_{DST} defined by:

π_{DST} : [holds_at(and(has_value(X, W), has_value(P, V), C), t) $\Rightarrow$ holds_at(has_value(X, W+Vd), t+d)]
$\longrightarrow$ [$X(t+d) = X(t) + P(t)d$]
π_{DST} : [holds_at(has_value(E, V), t) $\Rightarrow$ holds_at(has_value(P, V), t)] $\longrightarrow$ [$P(t) = E(t)$]

Thus the following set $DE_{GR} = \pi_{DST}(TR_{GR})$ is obtained:

$$x(t+d) = x(t) + v(t)d$$
$$v(t+d) = v(t) + f(t)d$$
$$f(t) = c{\ast}mM/x(t)^2$$

This is a discrete dynamical model (a system of discretized differential equations or difference equations) with step size d. For very small d this can be further translated into the standard form of a system of differential equations:

$$dx/dt = v(t)$$
$$dv/dt = f(t)$$
$$f(t) = c{\ast}mM/x(t)^2$$

An example of an application within Cognitive Science is the study of the use of logistic and other difference equations to model growth (and in particular growth spurts) of various cognitive phenomena, e.g., the growth of a child's lexicon *L* between 10 and 17 months; cf. (Geert, 1991). The logistic difference equation used is:

$$L(n+1) = L(n)\ (1 + r - r\ L(n)/K)$$

Here *r* is the growth rate and *K* the carrying capacity. This equation can be analysed by the potentiality-based approach as follows (actually with d = 1):

potentiality_for(has_value(p, v), has_value(L, w), has_value(L, w + v), d)
reducer_of(has_value(Lr (1 - L/K), v), has_value(p, v))

Note that, as holds for the difference equation, the opportunity for the change is considered satisfied by assumption, hence left out. Another illustration in the cognitive area is Decision Field Theory (DFT) a dynamical model for decision making presented by (Busemeyer and Townsend, 1993). The core of their decision model for the preference *P* for an action is based on the differential equation:

$$dP(t)/dt = -s\,P(t) + c\,V(t)$$

where *s* and *c* are constants and *V* is a given evaluation function. Also this can be analysed in the potentiality-based approach along the lines discussed above (discretisation with step size d):

potentiality_for(has_value(p,u), has_value(P,w), has_value(P,w + ud), d)
reducer_of(has_value(-sP+cV,u), has_value(p,u))

11. Discussion

Dynamical Systems Theory (DST) has recently been put forward within Cognitive Science to model dynamics of cognitive processes, and proposed as an alternative for the symbolic/computational approach; for example, (Gelder and Port, 1995; Port and Gelder, 1995). DST subsumes many quantitative approaches such as neural networks and other adaptive system modelling approaches. The notion of a state-determined system (Ashby, 1960) is central for DST; such a system is based on the assumption that properties of a given state fully determine the properties of future states. Note that this assumption implies a deterministic system. Within DST the notion of dynamical system that is assumed is based on world states at different points in time that are conceptualised by (real number) value assignments to continuous variables. The way in which properties of a present state determine properties of a future state is expressed by difference or differential equations. This is one specific way to achieve the state-determined system assumption, where derivatives play the role of potentialities: at the underlying philosophical level this corresponds to the assumption on the existence of potentialities as state properties that indicate how the state will change. The existence of differential equations relates to another basic assumption at the underlying philosophical level: on the existence of reducers for higher-order potentialities for some order.

In other literature, such as (Giunti, 1995), the notion of dynamical system is more general, not (only) based on real number value assignments. Also in such systems a state-determined system assumption can be incorporated by introducing potentialities to the state ontology that indicate in which respect the state is going to change. The notion of potentiality goes back to Zeno and Aristotle and was exploited by Descartes, Newton, Huygens and Leibniz, among others, to develop fundamental areas within Physics and Mathematics (classical mechanics and calculus).

In this paper, a philosophical framework including a metamodel has been introduced to analyse modelling methods for dynamics by (first- and higher-order) potentialities and reducers for them. It was shown how this framework provides unified philosophical foundations for both symbolic and mathematical approaches, approaches that are often seen as mutually exclusive. As a result, hybrid modelling approaches can be philosophically founded, in which both symbolic and mathematical aspects are covered. Examples of such approaches that make some first steps in that direction are cognitive modelling frameworks such as ACT-R (Anderson and Lebiere, 1998), SOAR (Laird *et al.*, 1987), and (recently) LEADSTO (Bosse *et al.*, 2007).

Within Mathematics the same philosophical concept has been worked out in the *n*th-order derivatives of a function for all *n*, and the Taylor series to calculate changes of the function value. In many cases, after a few steps a reduction can be made in the sense that a higher-order potentiality is equivalent to a combination of basic state properties and/or lower level potentialities. Within DST this is where a differential equation comes in, reducing an *n*-th-order derivative to lower level properties: the assumption of reduction is another crucial assumption underlying DST.

The presented philosophical framework unifies the modelling of a wide variety of dynamic phenomena in the natural and artificial world, from cognitive phenomena to biological, chemical and physical phenomena. For example, in the *Aplysia* case study it was shown how modelling from a functionalist/mental perspective can easily be integrated at the underlying philosophical level with a perspective from the physical/neurological level. Thus it was shown how philosophical foundations of agents embodied in their physical environment are addressed in a unified manner.

Notice that the framework in principle implies determinism for that part of reality that is modelled, as the relation between a potentiality *p* and the property *a* for which is a potentiality is a deterministic relation. This is the same as for the state-determined system assumption. However, this determinism may imply partial determinism of reality if it is assumed that for some properties in reality no potentialities may be given and these properties may occur without any preceding indication. A further option could be to develop a probabilistic variant of the framework, where a probability is attached to the relation between between a potentiality *p* and the property *a* for which is a potentiality. This would be future work, and could cover unification of stochastic modelling methods.

An interesting challenge for further work is to use the unified conceptual framework and the metamodel to relate different modelling approaches to each other. This can be done by first obtaining mappings of the metamodel onto each of the modelling approaches considered, and then use these mappings to define a direct mapping from one modelling approach to the other one. Such a mapping, when formalised could be used as a basis of a software environment for automated transformation of models in one format into models in another format.

Another interesting challenge for further work could be to involve longer histories in relation to potentialities in the analysis made. Examples of such longer term histories are the history of ontological and/or evolutionary development of individuals or generations. Also the cognitive interactivist approach to representational content, which involves interaction histories in the notion of representational content of cognitive states could be taken into account in such an enterprise; cf. (Bickhard, 1993, 2000; Jonker and Treur, 2003).

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