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Abstract

In the literature classical conditioning is usually described and analysed informally. If formalisation is used, this is often based on mathematical models based on difference or differential equations. This paper explores a formal description and analysis of the process of trace conditioning, based on logical specification and analysis methods of dynamic properties of the process. Specific types of dynamic properties are global dynamic properties, describing properties of the process as a whole, or local dynamic properties, describing properties of basic steps in a conditioning process. If the latter type of properties are specified in an executable format, they provide a temporal declarative specification of a simulation model. By a software environment these local properties can be used to actually perform simulation. Global properties can be checked automatically for simulated or other traces. Using these methods the properties of conditioning processes informally expressed by Los and Van Den Heuvel (2001) have been formalised and verified against a specification of local properties based on Machado (1997)'s differential equation model.

Formal Analysis of Trace Conditioning

Introduction

A common approach to describe dynamics of cognitive processes is by relating sensory, cognitive or behavioural states to previous or subsequent states. For example, this is shown in approaches from what in Philosophy of Mind is called the functionalist perspective, where the functional role of a mental state property is defined by its predecessor and successor states. Also in Dynamical Systems Theory (DST), a relatively new approach to describe the dynamics of cognitive processes, which subsumes connectionist modelling, e.g., (Port and Gelder, 1995), relations of a state with previous and subsequent states are central. Gelder and Port (1995) briefly explained what a dynamical system is in the following manner. A *system* is a set of changing aspects (or state properties) of the world. A *state* at a given point in time is the way these aspects or state properties are at that time; so a state is characterised by the state properties that hold. The set of all possible states is the *state space*. A *behaviour* of the system is the change of these state properties over time, or, in other words, a succession or sequence of states within the state space. Such a sequence in the state space can be indexed, for example, by natural numbers (*discrete* case) or real numbers (*continuous* case), and is also called a *trace* or *trajectory*. Given these notions, the notion of *state-determined system*, adopted from (Ashby, 1960) is taken as the basis to describe what a dynamical system is:

‘A system is state-determined only when its current state always determines a unique future behaviour. (..) the future behaviour cannot depend in any way on whatever states the system might have been in *before* the current state. In other words, past history is irrelevant (or at least, past history only makes a difference insofar as it has left an effect on the current state). (..) the fact that the current state determines future behaviour implies the existence of some *rule of evolution* describing the behaviour of the system as a function of its current state.’ (Gelder and Port, 1995, p. 6).

The assumption of state-based systems, which is fundamental for DST, entails a *local modelling perspective* where states are related to immediate predecessor and successor states. This local modelling perspective, which DST has in common with the functional perspective based on causal relations mentioned earlier, is especially useful for simulation purposes. However, more indirect or *global temporal relations* between states are beyond such a modelling approach.

In more sophisticated cognitive processes, a cognitive state or a behaviour can be better understood in a more global manner, e.g., in the way in which it depends on a longer history of experiences or inputs. In experimental research, examples of such phenomena are usually indicated as inter-trial or adaptive effects. Approaches to model such more sophisticated cognitive processes require means to express a more advanced *temporal complexity* than for the less sophisticated processes, where direct succession relations between states suffice as appropriate means.

Various types of adaptive or learning behaviour are known and have been studied in some depth. For example, for various forms of learned stimulus-response behaviours it has been studied how they are determined by an attained cognitive state of the organism. Still, the question remains how such attained cognitive states themselves depend on the previous history, for example on a particular training session extending over a longer time period. Often insight is obtained by formulating such more complex temporal relationships. However, usually such complex temporal relationships are expressed purely informally, due to the lack of modelling techniques that reach beyond the local perspective.

In order to temporally relate states to states at other points in time in a more global manner, modelling approaches to dynamical systems of a different type have recently been proposed.

Within the areas of Computer Science and Artificial Intelligence techniques have been developed to analyse the dynamics of phenomena using logical means. Examples are dynamic and temporal logic, and event and situation calculus; e.g., (Eck et al., 2001; Kowalski and Sergot, 1986; Reiter, 2001). These logical techniques allow to consider and relate states of a process at different points in time, and in this sense reach beyond the local perspective. The form in which these relations are expressed can cover qualitative aspects, but also quantitative aspects.

This paper addresses temporal aspects of conditioning and illustrates the usefulness of a logical approach for the analysis and formalisation of such processes both at a local and at a more global level. First a local perspective model for temporal conditioning in a high-level executable format is presented. This executable model can be compared to (and was inspired by) Machado (1997)'s differential equation model. Some simulation traces are shown and compared to traces of Machado (1997)'s model.

Next, as part of a non-local perspective analysis, a number of relevant dynamic properties of the conditioning process are identified and formalised. These dynamic properties were obtained by formalising the informally expressed properties to characterise temporal conditioning processes, as put forward by Los and Van Den Heuvel (2001). It has been automatically verified that (under reasonable conditions) these global dynamic properties are satisfied by the simulation traces.

Temporal Dynamics of Conditioning

Below, first some basic terminology used in the area of conditioning is introduced. Next, based on this terminology, Machado (1997)'s differential equation model for conditioning is briefly explained.

Basic Concepts of Conditioning

The aim of research into conditioning is to reveal the principles that govern associative learning. To this end, several experimental procedures have been developed. In *classical conditioning*, an organism is presented with an initially neutral conditioned stimulus (e.g., a bell) followed by an unconditioned stimulus (e.g., meat powder) that elicits an innate or learned unconditioned response in the organism (e.g. saliva production for a dog). After acquisition, the organism elicits an adaptive conditioned response (also saliva production in the example) when the conditioned stimulus is presented alone. In *operant conditioning*, the production of a certain operant response that is part to the volitional repertoire of an organism (e.g., bar pressing for a rat) is strengthened after repeated reinforcement (e.g., food presentation) contingent on the operant response.

In their review, Gallistel and Gibbon (2000) argued that these different forms of conditioning have a common foundation in the adaptive timing of the conditioned (or operant) response to the appearance of the unconditioned stimulus (or reinforcement). This feature is most apparent in an experimental procedure called *trace conditioning*, in which a blank interval (or 'trace') of a certain duration separates the conditioned and unconditioned stimulus (in classical conditioning) or subsequent reinforcement phases (in operant conditioning). In either case, the conditioned (or operant) response obtains its maximal strength, here called *peak level*, at a moment in time, called *peak time*, that closely corresponds to the moment the unconditioned stimulus (or reinforcement) occurs.

For present purposes, we adopt the terminology of an experimental procedure that is often used to study adaptive timing and the possible role of conditioning in humans. In this procedure, a trial starts with the presentation of a *warning stimulus* (S1; comparable to a conditioned

stimulus). After a blank interval, called the *foreperiod* (FP), an *imperative stimulus* (S2, comparable to an unconditioned stimulus) is presented to which the participant responds as fast as possible. The *reaction time* (RT) to S2 is used as an estimate of the conditioned state of preparation at the moment S2 is presented.

In this type of research, FP is usually varied at several discrete levels. That is, S2 can be presented at several moments since the offset of S1, which are called *critical moments*. The moment that is used for the presentation of S2 on any given trial is called the *imperative moment* of that trial. A final distinction concerns the way the different levels of FP are presented to the participant. In a *pure block*, the same FP is used across all trials of that block (and varied between different pure blocks). That is, in a pure block there is one critical moment that corresponds to the imperative moment on each trial. In a *mixed block*, all levels of FP occur randomly across trials. That is, a mixed block has several critical moments, but on any specific trial, only one of the moments is the imperative moment.

Modelling Conditioning by Differential Equations

Machado (1997) presented a basic model of the dynamics of a conditioning process. The structure of this model, with an adjusted terminology as used by Los, Knol, and Boers (2001), is shown in Figure 1. The model posits a layer of *timing nodes* (Machado calls these *behavioral states*) and a single *preparation node* (called *operant response* by Machado). Each timing node is connected both to the next (and previous) timing node and to the preparation node. The connection between each timing node and the preparation node (called *associative link* both by Machado and within the current paper) has an adjustable weight associated to it. Upon the presentation of a warning stimulus, a cascade of activation propagates through the timing nodes according to a regular pattern. Owing to this regularity, the timing nodes can be likened to an

internal clock or pacemaker. At any moment, each timing node contributes to the activation of the preparation node in accordance with its activation and its corresponding weight. The activation of the preparation node reflects the participant's preparatory state, and is as such related to reaction time for any given imperative moment.

The weights reflect the state of conditioning, and are adjusted by learning rules, of which the main principles are as follows. First, *during* the foreperiod extinction takes place, which involves the decrease of weights in real time in proportion to the activation of their corresponding timing nodes. Second, *after* the presentation of the imperative stimulus a process of reinforcement takes over, which involves an increase of the weights in accordance with the current activation of their timing nodes, to preserve the importance of the imperative moment. In Machado (1997) the more detailed dynamics of the process are given by a mathematical model (based on linear differential equations), representing the (local) temporal relationships between the variables involved. For example, $d/dt X(t,n) = \lambda X(t,n-1) - \lambda X(t,n)$ expresses how the activation level of the n-th timing node $X(t+dt,n)$ at time point $t+dt$ relates to this level $X(t,n)$ at time point t and the activation level $X(t,n-1)$ of the (n-1)-th timing node at time point t . Similarly, as another example, $d/dt W(t,n) = -\alpha X(t,n)W(t,n)$ expresses how the n-th weight $W(t+dt,n)$ at time point $t+dt$ relates to this weight $W(t,n)$ at time point t and the activation level $X(t,n)$ of the n-th timing node at time point t .

Modelling in terms of Dynamic Properties

As discussed above, mathematical models based on differential equations can be used to model (1) in a quantitative manner, (2) local temporal relationships within conditioning processes. However, conditioning processes can also be characterised by temporal relationships of a less local form. As an example, taken from Los and Van Den Heuvel (2001), a dynamic property can be formulated expressing the monotonicity property that 'the response level

increases before the critical moment is reached and decreases after this moment'. This is a more global property, relating response levels at any two points in time before the critical moment (or after the critical moment). Therefore it is useful to explore formalisation techniques, as an alternative to differential equations, to express not only for local properties, but also for non-local properties. A second limitation of differential equations is that they are based on quantitative (calculational) relationships, whereas also non-quantitative aspects may play a role (for example, the monotonicity property mentioned above). This suggests that it may be useful to explore alternative formalisation techniques for dynamic properties of conditioning processes that both allow to express quantitative and non-quantitative aspects.

As already mentioned in the Introduction, the approach presented in this paper indeed introduces alternative formalisation languages to express dynamic properties of conditioning processes, both for local and nonlocal properties and both for quantitative and non-quantitative aspects. These formalisation languages are briefly introduced in the next section. After this introduction, in a subsequent section first a model based on local properties of a conditioning process is presented (comparable to and inspired by Machado's model).

Languages to Model Dynamic Properties

The domain of reasoning about dynamical systems in disciplines such as the Behavioural Sciences requires an abstract modelling form yet showing the essential dynamic properties. A high-level language is needed to characterise and formalise dynamic properties of such a dynamical system. To this end the *Temporal Trace Language* TTL is used as a tool; for previous applications of this language to the analysis of (cognitive) processes, (see Jonker and Treur, 2002; Jonker and Treur, 2003a,b; Jonker, Treur, and Vries, 2002). Using this language, dynamic properties can be expressed in informal, semi-formal, or formal format. Moreover, to perform

simulations, models are desired that can be formalised and are computationally easy to handle. These executable models are based on the so-called *LEADSTO* format which is defined as a sublanguage of TTL (Bosse et al., 2005); for a previous application of this format for simulation of cognitive processes, see (Jonker, Treur, and Wijngaards, 2003). The Temporal Trace Language TTL is briefly defined as follows.

A *state ontology* is a specification (in order-sorted logic) of a vocabulary to describe a state of a process. A state for ontology Ont is an assignment of truth-values true or false to the set $\text{At}(\text{Ont})$ of ground atoms expressed in terms of Ont . The *set of all possible states* for state ontology Ont is denoted by $\text{STATES}(\text{Ont})$. The set of *state properties* $\text{STATPROP}(\text{Ont})$ for state ontology Ont is the set of all propositions over ground atoms from $\text{At}(\text{Ont})$. A fixed *time frame* T is assumed which is linearly ordered, for example the natural or real numbers. A *trace* γ over a state ontology Ont and time frame T is a mapping $\gamma: T \rightarrow \text{STATES}(\text{Ont})$, i.e., a sequence of states γ_t ($t \in T$) in $\text{STATES}(\text{Ont})$. The set of all traces over state ontology Ont is denoted by $\text{TRACES}(\text{Ont})$. The set of *dynamic properties* $\text{DYNPROP}(\text{Ont})$ is the set of temporal statements that can be formulated with respect to traces based on the state ontology Ont in the following manner.

These states can be related to state properties via the formally defined satisfaction relation \models , comparable to the Holds-predicate in the Situation Calculus (cf. Reiter, 2001): $\text{state}(\gamma, t) \models p$ denotes that state property p holds in trace γ at time t . Based on these statements, dynamic properties can be formulated, using quantifiers over time and the usual first-order logical connectives \neg (not), \wedge (and), \vee (or), \Rightarrow (implies), \forall (for all), \exists (there exists); to be more formal: formulae in a sorted first-order predicate logic with sorts T for time points, Traces for traces and F for state formulae.

To model basic mechanisms of a process at a lower aggregation level, direct temporal dependencies between two state properties, the simpler LEADSTO format is used. This executable format can be used for simulation and is defined as follows. Let α and β be state properties. In LEADSTO specifications the notation $\alpha \rightarrow_{e, f, g, h} \beta$, means:

if state property α holds for a certain time interval with duration g , then after some delay (between e and f) state property β will hold for a certain time interval h .

For a more formal definition, see (Jonker, Treur, and Wijngaards, 2003) and (Bosse et al., 2005).

Types of Dynamic Properties

Dynamic properties of a conditioning process can be specified at different levels of aggregation. At the highest level, *global dynamic properties*, i.e., properties of the conditioning process as a whole, can be expressed, for example indicating how a certain pattern of behaviour has been changed by a conditioning process. At the lowest level of aggregation, *local properties* are dynamic properties of the basic mechanisms of the conditioning process. Based on these local properties, and certain conditions of the environment, the global properties of the system emerge. Such conditions of the environment (of the subject) are characterised by *environmental properties*. In laboratory circumstances, these properties are usually guaranteed by a specific experimental design.

Local properties are logically related to global properties in the sense that the local properties together with the relevant environmental properties entail the global properties. To clarify such logical *interlevel relations*, it is often useful to specify dynamic properties at intermediate levels of aggregation: *intermediate properties*. Thus the overall picture shown in Figure 2 is obtained.

Local Dynamic Properties

This section presents the local properties (LPs) that were defined in order to describe the conditioning process from a local perspective. These properties are executable dynamic properties (in LEADSTO format) of the elements of this model. Within the dynamic properties the following state properties are used:

$X(n,u)$	Timing node n has activation level u . In the current simulation, n ranges over the discrete domain $[0,5]$. Thus, our model consists of six timing nodes. The activation level u can take any continuous value in the domain $[0,1]$.
$W(n,v)$	Associative link n has weight v . Again, n ranges over the discrete domain $[0,5]$. The weight v can take any continuous value in the domain $[0,1]$.
$R(r)$	The preparation node has response strength r (a continuous value in the domain $[0,1]$).
$S1(s)$	Warning stimulus $S1$ occurs with strength s . Within our example, s only takes the values 0.0 and 1.0. However, the model could be extended by allowing any continuous value in-between.
$S2(s)$	Imperative stimulus $S2$ occurs with strength s .
$Xcopy(n,u)$	Timing node n had activation level u at the moment of the occurrence of the last imperative stimulus ($S2$). See dynamic property LP4 and LP6.
$instage(ext)$	The process is in a stage of extinction. This stage lasts from the occurrence of $S1$ until the occurrence of $S2$.
$instage(reinf)$	The process is in a stage of reinforcement. This stage starts with the occurrence of $S2$, and lasts during a predefined reinforcement period, (e.g. 3 seconds).
$instage(pers)$	The process is in a stage of persistence. This stage starts right after the reinforcement stage, and lasts until the next occurrence of $S1$.

As Machado (1997)'s model was used as a source of inspiration, for some of the properties presented below the comparable differential equation within Machado's model is given as well. However, since Machado's mathematical approach differs at several points from the logical approach presented in this paper, there is not always a straightforward 1:1 mapping between both formalisations. For instance, state property $X(n,u)$ within our LEADSTO formalisation has a slightly different meaning than the corresponding term $X(t,n)$ in Machado's differential equations.

In the former, n stands for the timing node, u stands for the activation level, and $X(n,u)$ stands for the fact that timing node n has activation level u . In the latter, t stands for a time point, n stands for the timing node, and $X(t,n)$ as a whole stands for the activation level.

LP1 Initialisation

The first local property LP1 expresses the initialisation of the values for the timing nodes and the associative links. Formalisation (for n ranging over $[0,5]$):

$$\text{start} \rightarrow X(n, 0) \wedge W(n, 0)$$

LP2 Activation of initial timing nodes

Local property LP2 expresses the activation (and adaptation) of the 0th timing node. Immediately after the occurrence of the warning stimulus (S1), this state has full strength. After that, its value decreases until the next warning stimulus. Together with LP3, this property causes the spread of activation across the timing nodes. Here, $\lambda > 0$ is a rate parameter that controls the speed of this spread of activation, and step is a constant indicating the smallest time step in the simulation. For the simulation experiments presented in the next section, λ was set to 10 and step was set to 0.05.

Formalisation:

$$X(0, u) \wedge S1(s) \rightarrow X(0, u*(1-\lambda*\text{step})+s)$$

Comparable differential equation in Machado (1997)'s model: $d/dt X(t,0) = -\lambda X(t,0)$.

LP3 Adaptation of timing nodes

LP3 expresses the adaptation of the n^{th} timing node (for n ranging over $[1,5]$), based on its own previous state and the previous state of the $n-1^{\text{th}}$ timing node. Together with LP2, this property causes the spread of activation across the timing nodes. Here, λ is a rate parameter that controls the speed of this spread of activation (see LP2).

Formalisation (for n ranging over $[1,5]$):

$$X(n, u1) \wedge X(n-1, u0) \rightarrow X(n, u1+\lambda*(u0-u1)*\text{step})$$

Comparable differential equation in Machado (1997)'s model: $d/dt X(t,n) = \lambda X(t,n-1) - \lambda X(t,n)$.

LP4 Storage of timing nodes at moment of reinforcer

LP4 is needed to store the value of the n^{th} timing node at the moment of the occurrence of the imperative stimulus (S2). These values are used later on by property LP6.

Formalisation (for n ranging over $[0,5]$):

$$X(n, u) \wedge S2(1.0) \rightarrow X_{\text{copy}}(n, u)$$

LP5 Extinction of associative links

LP5 expresses the adaptation of the associative links during extinction, based on their own previous state and the previous state of the corresponding timing node. Here, α is a learning rate parameter. For the simulation experiments presented in the next section, the value 2 was chosen for α . Formalisation (for n ranging over $[0,5]$):

$$\text{instage}(\text{ext}) \wedge X(n, u) \wedge W(n, v) \rightarrow W(n, v*(1-\alpha*u*\text{step}))$$

Comparable differential equation in Machado (1997)'s model: $d/dt W(t,n) = -\alpha X(t,n)W(t,n)$

LP6 Reinforcement of associative links

LP6 expresses the adaptation of the associative links during reinforcement, based on their own previous state and the previous state of Xcopy. Here, β is a learning rate parameter. For the simulation experiments presented in the next section, the value 2 was chosen for β .

Formalisation (for n ranging over $[0,5]$):

$$\text{instage}(\text{reinf}) \wedge X\text{copy}(n, u) \wedge W(n, v) \rightarrow W(n, v*(1-\beta*u*\text{step}) + \beta*u*\text{step})$$

Comparable differential equation in Machado (1997)'s model: $d/dt W(t,n) = \beta X(T,n)[K-W(t,n)]$.

LP7 Persistence of associative links

LP7 expresses the persistence of the associative links at the moments that there is neither extinction nor reinforcement.

Formalisation (for n ranging over $[0,5]$):

$$\text{instage}(\text{pers}) \wedge W(n, v) \rightarrow W(n, v)$$

LP8 Response function

LP8 calculates the response by adding the discriminative function of all states, i.e., their associative links multiplied by the degree of activation of the corresponding state.

Formalisation:

$$W(1, v1) \wedge W(2, v2) \wedge W(3, v3) \wedge W(4, v4) \wedge W(5, v5) \wedge X(1, u1) \wedge X(2, u2) \wedge X(3, u3) \wedge X(4, u4) \wedge X(5, u5) \rightarrow R(v1*u1 + v2*u2 + v3*u3 + v4*u4 + v5*u5)$$

LP9 Initialisation of stage pers

LP9 expresses that the initial stage of the process is pers.

Formalisation:

$$\text{start} \rightarrow \text{instage}(\text{pers})$$

LP10 Transition to stage ext

LP10 expresses that the process switches to stage ext when a warning stimulus occurs.

Formalisation:

$$S1(1.0) \rightarrow \text{instage}(\text{ext})$$

LP11 Persistence of stage ext

LP11 expresses that the process persists in stage `ext` as long as no imperative stimulus occurs.

Formalisation:

$$\text{instage}(\text{ext}) \wedge S2(0.0) \rightarrow \text{instage}(\text{ext})$$
LP12 Transition to stage reinf and pers

LP12 expresses that the process first switches to stage `reinf` for a while, and then to stage `pers` when an imperative stimulus occurs. Notice that LP12a and LP12b must have different timing parameters to make sure both stages do not occur simultaneously.

Formalisation:

$$S2(1.0) \rightarrow \text{instage}(\text{reinf}) \quad (\text{LP12a})$$

$$S2(1.0) \rightarrow \text{instage}(\text{pers}) \quad (\text{LP12b})$$
LP13 Persistence of stage pers

LP13 expresses that the process persists in stage `pers` as long as no warning stimulus occurs.

Formalisation:

$$\text{instage}(\text{pers}) \wedge S1(0.0) \rightarrow \text{instage}(\text{pers})$$

Simulation

Simulation of executable models is performed by a special software environment. This software environment generates simulation traces of the conditioning process based on an input consisting of dynamic properties in LEADSTO format (Bosse et al., 2005). A large number (about 20) of such traces have been generated, with different parameters for foreperiod (50, 100, 150, 200, 300, 350, and 500 msec), selected on the basis of (Los, Knol, and Boers, 2001). An example of such a trace can be seen in Figure 3. Here, time is on the horizontal axis. Each time unit corresponds to 50 msec. The relevant state properties (`S1`, `S2`, `instage(ext)`, `instage(pers)`, `instage(reinf)` and `R`) are on the vertical axis. A dark box on top of the line indicates that the property is true during that time period.

This trace is based on all local properties presented above. For almost all properties, the timing parameters (0,0,1,1) were used. Exceptions are the properties LP4, LP12a and LP12b. For

these properties, the timing parameters were respectively (0,0,1,3), (0,0,1,3) and (3,3,1,1), where 3 corresponds to the reinforcement duration (i.e., 150 msec). Figure 3 describes the dynamics *during* (not after) a conditioning process. To be specific, this trace describes the dynamics of a person that is subject to conditioning in a pure block with a foreperiod of 6 time units (i.e., 300 msec). As can be seen in the trace, the level of response-related activation increases on each trial. Initially, the subject is not prepared at all: at the moment of the imperative stimulus (S2), the level of response is 0. However, already after two trials a peak in response level has developed that coincides exactly with the imperative moment.

Figure 4 describes the dynamics of the same pure block (with foreperiod of 300 msec) *after* the conditioning has taken place. At this moment, the internal model has evolved in such a way that the subject is maximally prepared (response strength $r > 0.4$) at the critical moment (i.e., after 300 msec), even without the actual occurrence of an imperative stimulus S2.

In contrast to Figure 3 and 4 (describing the dynamics of a pure block), Figure 5 and 6 are examples of a trace where a mixed block is considered. In this case, two types of foreperiod (FP=100 msec and FP=500 msec) are randomly presented during the learning trials. Figure 5 depicts the dynamics during conditioning, and Figure 6 depicts the dynamics after the conditioning has occurred. As can be seen in both figures, the curves that plot the response level have two peaks: one for each critical moment. The trace depicted in Figure 6 shows two trials: one in which the imperative moment corresponds to the first critical moment, and one in which it corresponds to the second critical moment. A detailed explanation of the shape of both curves will be given in the next sections.

As mentioned above, a number of similar experiments have been performed, with different parameters for foreperiod and block type. The results were consistent with the data produced by Machado.

Analysis of Nonlocal Dynamic Properties

This section addresses analysis of the process of conditioning from a nonlocal perspective. In (Los and Van Den Heuvel, 2001), the following dynamic properties of the overall conditioning process are put forward:

‘Corresponding to each critical moment there is a state of conditioning, the adjustment of which is governed by learning rules of trace conditioning (specified subsequently).’

- (1) ‘The state of conditioning implicates an increase and decay of response-related activation as a critical moment is bypassed in time.’
- (2) ‘the conditioned response takes more time to build up and decay and its corresponding asymptotic value is lower when its corresponding critical moment is more remote from the warning signal.’
- (3) ‘on any trial, the strength of the conditioned response corresponding to a critical moment is reinforced (i.e., increased toward its asymptote) if and only if that critical moment coincides with the imperative moment.’
- (4) ‘on any trial the strength of the conditioned response is extinguished (i.e., driven away from its asymptote) if and only if its corresponding critical moment occurs before the imperative moment, whereas it is left unaffected if its corresponding critical moment occurs later than the imperative moment.’

(Los and Van Den Heuvel, 2001, p. 372.)

These properties have an informal and non-mathematical nature. Below it is first shown how these properties can be formalised in TTL. In contrast to the earlier presented local properties, we will call these properties global properties (GPs). In the next section it is shown how these global properties can be checked against the generated traces.

GP1 has_global_hill_prep(γ , t1, t2, s1, a, u)

The first global property GP1 is a formalisation of informal property (1) presented above. It describes the following: If at t_1 a stimulus s_1 starts, then the preparation level for action a will increase from t_1 until t_2 and decrease from t_2 until $t_1 + u$, under the assumption that no stimulus occurs too soon (within u time) after t_1 . Formally (in TTL):

$$\begin{aligned} & \forall t', t'', s', p', p'', x, x' \\ & \text{stimulus_starts_at}(\gamma, t_1, s_1, x) \quad \& \\ & \neg \text{stimulus_starts_within}(\gamma, t_1, t_1+u, s', x') \quad \& \\ & \text{has_preparation_level_at}(\gamma, t', p', a) \quad \& \\ & \text{has_preparation_level_at}(\gamma, t'', p'', a) \\ \Rightarrow & [t_1 \leq t' < t'' \leq t_2 \quad \& \quad t'' \leq t_1 + u \Rightarrow p' < p''] \quad \& \\ & [t_2 \leq t' < t'' \leq t_1 + u \quad \Rightarrow \quad p' > p''] \end{aligned}$$

GP2 pending_peak_versus_critical_moment($\gamma_1, \gamma_2, t_1, t_2, c_1, c_2$)

Global property GP2 is a formalisation of informal property (2). It describes that: If for trace γ_2 at time t_2 peak time c_2 is more remote than peak time c_1 for γ_1 at time t_1 , then at t_2 in γ_2 the pending peak level is lower than the pending peak level at t_1 in γ_1 . Formally:

$$\begin{aligned} & \forall s_1, a, p_1, p_2 \\ & \text{has_pending_peak_level}(\gamma_1, t_1, c_1, p_1, s_1, a) \quad \& \\ & \text{has_pending_peak_level}(\gamma_2, t_2, c_2, p_2, s_1, a) \\ \Rightarrow & [c_1 < c_2 \quad \Rightarrow \quad p_1 > p_2] \end{aligned}$$

GP3 dynamics_of_pending_preparation($\gamma, t_1, t_2, c, v, p, p', s_1, s_2, a, d, \epsilon$)

GP3 is a formalisation of both informal property (3) and (4) together. It describes that:

If $t_1 < t_2$

- and at t_1 the pending preparation level for time t_1+v , action a , and stimuli s_1 and s_2 is p ,
- and at t_2+d the pending preparation level for time t_2+d+v , action a , and stimuli s_1 and s_2 is p' ,
- and in trace γ at time t_1 a stimulus s_1 starts,
- and in trace γ at time t_2 a stimulus s_2 starts,
- and in trace γ the maximum peak level for a is p_{\max} ,
- and in trace γ the minimum preparation level for a is p_{\min} ,

then:

$$\begin{aligned} t_2 \in [t_1 + c - \epsilon, t_1 + c + \epsilon] & \quad \text{iff} \quad p' > p \quad (\text{reinforcement, given that } p < p_{\max}) \\ t_2 > t_1 + c + \epsilon & \quad \text{iff} \quad p' < p \quad (\text{extinction, given that } p > p_{\min}) \\ t_2 < t_1 + c - \epsilon & \quad \text{iff} \quad p' = p \quad (\text{persistence}) \end{aligned}$$

Parameter d refers to the time needed to process the events ($d > 0$), and c refers to a critical moment. Formally:

$$\begin{aligned} & \text{dynamics_of_pending_preparation}(\gamma, t_1, t_2, c, v, p, p', s_1, s_2, a, d, \epsilon) \Leftrightarrow \\ & \text{reinforcement}(\gamma, t_1, t_2, c, v, p, p', s_1, s_2, a, d, \epsilon) \quad \& \end{aligned}$$

extinction($\gamma, t1, t2, c, v, p, p', s1, s2, a, d, \epsilon$) &
 persistence($\gamma, t1, t2, c, v, p, p', s1, s2, a, d, \epsilon$)

reinforcement($\gamma, t1, t2, c, v, p, p', s1, s2, a, d, \epsilon$) \Leftrightarrow
 $\forall x1, x2, pmin, pmax$
 two_stimuli_occur($\gamma, t1, t2, c, v, p, p', s1, s2, a, d$)
 $\Rightarrow [p < pmax \Rightarrow [t2 \in [t1 + c - \epsilon, t1 + c + \epsilon] \Leftrightarrow p' > p]]$

extinction($\gamma, t1, t2, c, v, p, p', s1, s2, a, d, \epsilon$) \Leftrightarrow
 $\forall x1, x2, pmin, pmax$
 two_stimuli_occur($\gamma, t1, t2, c, v, p, p', s1, s2, a, d$)
 $\Rightarrow [p > pmin \Rightarrow [t2 > t1 + c + \epsilon \Leftrightarrow p' < p]]$

persistence($\gamma, t1, t2, c, v, p, p', s1, s2, a, d, \epsilon$) \Leftrightarrow
 $\forall x1, x2, pmin, pmax$
 two_stimuli_occur($\gamma, t1, t2, c, v, p, p', s1, s2, a, d$)
 $\Rightarrow [t2 < t1 + c - \epsilon \Leftrightarrow p' = p]$

two_stimuli_occur($\gamma, t1, t2, c, v, p, p', s1, s2, a, d$) \Leftrightarrow
 $t1 < t2$ & has_pending_preparation_level($\gamma, t1, t1+v, p, s1, s2, a$) &
 has_pending_preparation_level($\gamma, t2+d, t2+d+v, p', s1, s2, a$) &
 stimulus_starts_at($\gamma, t1, s1, x1$) &
 stimulus_starts_at($\gamma, t2, s2, x2$) &
 target_action_for($a, s2$) &
 is_a_critical_moment(c) &
 maximum_peak_level($\gamma, pmax, a$) &
 minimum_preparation_level($\gamma, pmin, a$)

The concepts used in this last expression (such as `stimulus_starts_at`) are abbreviations that can be related to basic state properties, for example in the following manner:

stimulus_starts_at($\gamma, t, s1, x$) \Leftrightarrow
 $state(\gamma, t) \models s1(x)$

Checking Nonlocal Dynamic Properties on Traces

In addition to the software described in the Simulation section, other software has been developed that takes traces and formally specified properties as input and checks whether a property holds for a trace. Using automatic checks of this kind, the four formalised properties based on (Los and Van Den Heuvel, 2001) have been checked against the traces depicted in Figure 3 to 6. This section discusses the results of these checks.

GPI has_global_hill_prep(γ , t_1 , t_2 , s_1 , a , u)

This property has been checked against several traces. An interesting observation concerning these checks was the fact that the property does not hold for traces that describe the dynamics *during* conditioning (like the traces in Figure 3 and 5). From this, we may conclude that in these traces it is not the case that each critical moment corresponds to a single peak in response level.

However, when checking against traces describing the dynamics *after* conditioning (e.g. Figure 4 and 6), the property does hold, as long as the parameters are well chosen. In particular, the parameters should meet the following conditions:

γ = a trace describing the dynamics after a conditioning process (pure block or mixed block)

t_1 = a time point when s_1 occurs

t_2 = $t_1 + \text{FP}$ (where FP is the foreperiod during the preceding conditioning process)

s_1 = the warning stimulus

a = the action for which the subject prepares after observing s_1

u = iti (the intertrial interval during the preceding conditioning process)

To give an example, the following property meets these conditions: has_global_hill_prep(γ_1 , 20, 27, s_1 , a , 20), where γ_1 is the trace provided in Figure 4. Thus, for this trace the following holds: if at time point 20 a stimulus s_1 starts, then the preparation level for action a increases

from 20 until 27 and decreases from 27 until 40, under the assumption that no stimulus occurs between 20 and 40.

GP2 pending_peak_versus_critical_moment(γ_1 , γ_2 , t_1 , t_2 , c_1 , c_2)

Checking property GP2 involves comparing two traces. Basically, it states that in traces where the foreperiod is longer, the level of response is lower. In order to check GP2, several traces have been generated that are similar to the traces in Figure 4 and 6, but each with a different foreperiod. For all combinations of traces, the property turned out to hold, but again, the choice of the parameters was limited by certain constraints:

γ_1 = a trace describing the dynamics after a conditioning process (pure block or mixed block)

γ_2 = a trace describing the dynamics after a conditioning process (pure block or mixed block)

t_1 = a time point

$t_2 = t_1$

c_1 = the peak time for trace γ_1 at time t_1

c_2 = the peak time for trace γ_2 at time t_2

As an extra condition, note that the number of trials and the block type of γ_1 should be similar to those of γ_2 . Otherwise, it makes no sense to compare both traces. E.g., the following property holds: pending_peak_versus_critical_moment(γ_1 , γ_2 , 20, 20, 6, 7), where γ_1 is the trace provided in Figure 4, and γ_2 is a similar trace with FP=7 (i.e., 350 msec). This means that, if for trace γ_2 at time 20 peak time 7 is more remote than peak time 6 for γ_1 at time 20 (which is indeed the case), then at 20 in γ_2 the pending peak level is lower than the pending peak level at 20 in γ_1 (which is also the case).

GP3 dynamics_of_pending_preparation(γ , t_1 , t_2 , c , v , p , p' , s_1 , s_2 , a , d , ϵ)

Property GP3 combines property (3) and (4) as mentioned in the previous section. Basically, the property consists of three separate statements that relate the strength of the conditioned response (p) to the critical moment (t_1+c) and the imperative moment (t_2), by stating that:

p increases iff $t_2 = t_1+c$ (GP3a)

p decreases iff $t_2 > t_1+c$ (GP3b)

p remains the same iff $t_2 < t_1+c$ (GP3c)

Also for this property, it is important to choose the parameters with care. They should meet the following conditions in order for the property to make sense:

γ = a trace describing the dynamics after a conditioning process (pure block or mixed block)

t_1 = a time point when S_1 occurs

t_2 = a time point when S_2 occurs

c = a foreperiod within γ (such that $c = t_2-t_1$)

$v = c$

s_1 = the warning stimulus

s_2 = the imperative stimulus

a = the action for which the subject prepares after observing s_1

d = long enough to process the events (e.g. iti)

$\epsilon = 0$

An example of a property that meets these criteria is: dynamics_of_pending_preparation(γ , 10, 12, 10, 10, p , p' , s_1 , s_2 , a , 18, 0), where γ is the trace depicted in Figure 6. However, even with these parameter settings the property turned out not to hold. A close examination of Figure 6 will reveal the cause of this failure. This trace describes a mixed block with two types of foreperiod (FP=100 msec and FP=500 msec). At time point 10, a warning stimulus (S_1) occurs. At this time point, the pending preparation level for the latest critical moment (time point 20) has a certain

value. And since this critical moment occurs after the occurrence of S2 (the imperative moment: time point 12), the pending preparation level for the latest critical moment should remain the same, according to statement C above. However, in the trace in question this is not the case (see Figure 6: in the second curve the second peak is slightly lower than in the first curve). Hence, it may be concluded that statement C (sub-property persistence presented earlier) does not hold for the chosen parameters.

Summarising, automated checks have pointed out that property GP3 does not always hold for traces generated by our simulation model. In particular, in traces where the critical moment occurs after the imperative moment, the strength of the conditioned response does not stay exactly the same. Additional tests have indicated that this value sometimes decreases, and sometimes increases a bit, depending on the specific settings. Fortunately, an explanation of this finding can be found in a later section of (Los and Van Den Heuvel, 2001), where the authors revise their original model as follows:

‘According to the original model, extinction and reinforcement affect each state of conditioning in an all-or-none way, thereby excluding a coupling between states of conditioning corresponding to adjacent critical moments. According to the revised model, extinction and reinforcement affect the states of conditioning more gradually across the time scale, resulting in a coupling between adjacent states.’

(Los and Van Den Heuvel, 2001), p. 383.

The revision of the model also implies a revision of property GP3. To be precise, sub-property persistence can be changed into the following:

$$\begin{aligned}
 &\text{persistence}(\gamma, t1, t2, c, v, p, p', s1, s2, a, d, \epsilon) \Leftrightarrow \\
 &\quad \forall x1, x2, pmin, pmax \\
 &\quad \text{two_stimuli_occur}(\gamma, t1, t2, c, v, p, p', s1, s2, a, d) \\
 &\quad \Rightarrow [t2 < t1 + c - \epsilon \Leftrightarrow p' \in [p - \delta, p + \delta]]
 \end{aligned}$$

Here, δ is a tolerance factor allowing a small deviation from the strength of the original response. After adapting GP3 accordingly, the property turned out to hold.

Discussion

Two software environments have been developed to support the research reported here. First a simulation environment has been used to generate simulation traces as shown. Second, checking software has been used that takes traces and formally specified properties and checks whether a property holds for a trace.

In comparison to Executable Temporal Logic (Barringer et al., 1996) our simulation approach has possibilities to incorporate (real or integer) numbers in state properties, and in the timing parameters e , f , g , h . Similarly, our approach to analysis has higher expressiveness than approaches in temporal logic such as (Fisher and Wooldridge, 1997).

The present paper has confirmed that the assumptions of the informal conditioning model proposed by Los and Van den Heuvel (2001) are global properties of the formal model developed by Machado (1997), given certain restrictions of the parameter values, and given slight adaptations of the persistence rule given by GP3C. This is an important finding, because the global properties have proved to be highly useful in accounting for key findings in human timing (Los and Van den Heuvel, 2001; Los et al., 2001).

One crucial finding the global properties can deal with effectively is the occurrence of sequential effects of FP. These effects entail that on any given trial, RT is longer when the FP of that trial is shorter than the FP of the preceding trial relative to when it is as long as or longer than the FP of the preceding trial. Stated differently, RT is longer when the imperative moment was bypassed during the FP on the preceding trial than when it was not bypassed during FP on the preceding trial (e.g., Los and Van den Heuvel, 2001; Niemi and Naatanen, 1981). This

finding is well accounted for by the learning rules formulated as GP3. According to GP3B, the state of conditioning (p) associated with a critical moment is subject to extinction when a critical moment is bypassed during FP (i.e., $t_2 > t_1 + c$), which is neither the case for the imperative moment, where according to GP3A reinforcement occurs (i.e., $t_2 > t_1 + c$), nor for critical moment beyond the imperative moment, where the state of conditioning persists according to GP3C (i.e., $t_2 < t_1 + c$). Note that the adjustment of GP3C suggested by the present check of nonlocal properties does not compromise the effectiveness of these learning rules, because the tolerance factor δ is small relative to the extinction described by GP3B.

In fact, the addition of the tolerance area δ to GP3C, might prove to be helpful in accounting for a more subtle effect in the extant literature. This concerns the finding that the FP-RT functions obtained in pure and mixed blocks cross over at the latest critical moment. Specifically, in pure blocks, the FP – RT function has been found to be upward sloping, given a minimal FP of about 250 – 300 msec. By contrast, in mixed blocks, the RT is slowest at the shortest critical moment (due to the influence of sequential effects described in the previous paragraph) and decreases as a negatively accelerating function of FP. At the latest critical moment the pure and mixed FP – RT functions come together, presumably because this moment is never bypassed during FP on the preceding trial, allowing the state of conditioning to approach its asymptotic value in either case. Sometimes, though, a cross-over of the two FP – RT functions is reported, which has been shown to be particularly pronounced in certain clinical populations, such as people diagnosed with schizophrenia (see Rist and Cohen, 1991, for a review). This finding may be related to the failure to confirm GP3C without the allowance of a tolerance area δ . Thus, it could be that, for certain parameter settings, the state of conditioning corresponding to the latest critical moment approaches its asymptotic value more closely when a shorter FP occurred on the

preceding trial (which is often the case in mixed blocks) than when the same FP occurred on the preceding trial (as is always the case in pure blocks).

Finally, the present exploration holds the promise of deriving other global properties from the formal model proposed by Machado (1997), which allows powerful new predictions. For instance, one global property that appears to hold is that not only FP, but rather the total duration of WS+FP influences the reaction time. Empirical work is currently in progress to test this prediction.

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Author Note

Lourens van der Meij provided the technical support for the software environments used in this project.

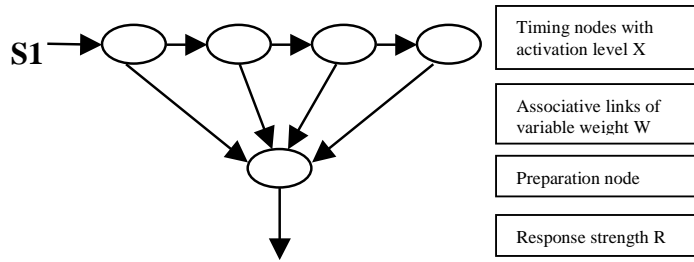


Figure 1: Structure of Machado's conditioning model (adjusted from Machado, 1997).

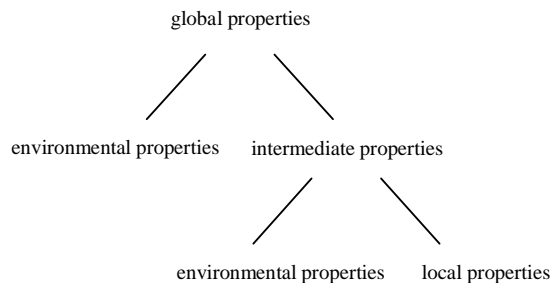


Figure 2: Interlevel relations between dynamic properties at different levels of aggregation.

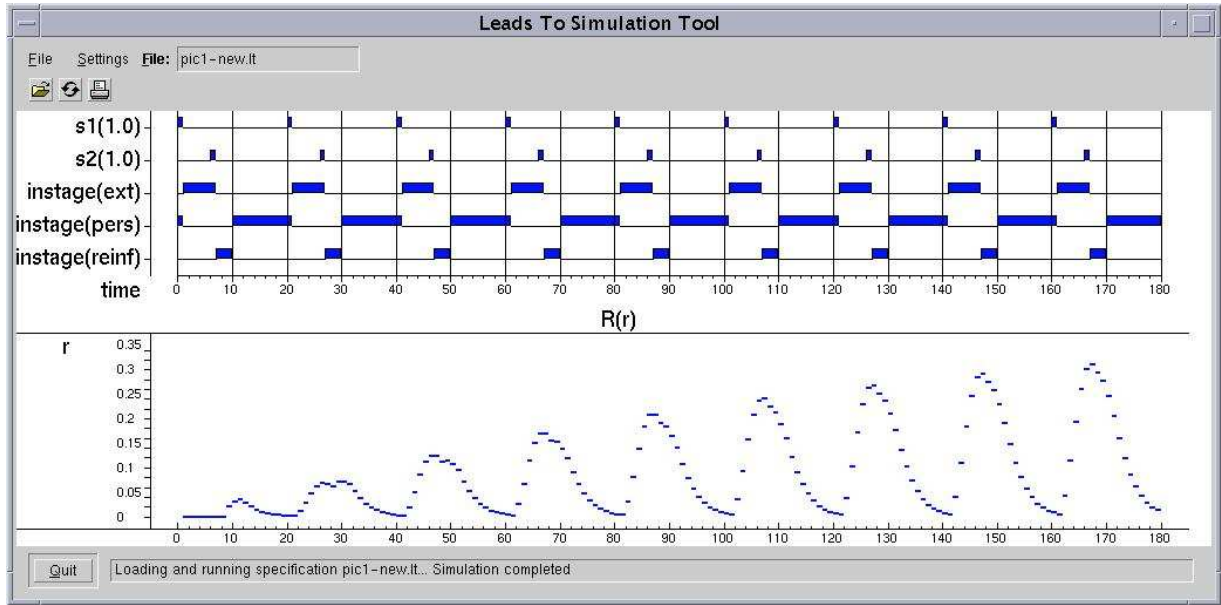


Figure 3: Simulation trace of the dynamics during conditioning (pure block, FP=300 msec).

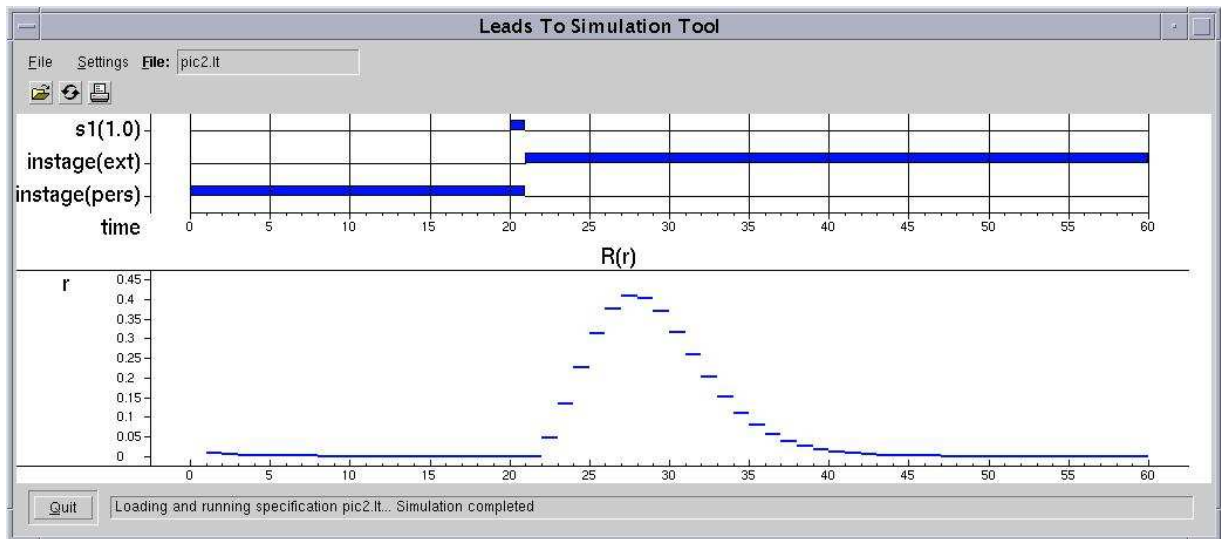


Figure 4: Simulation trace of the dynamics after conditioning (pure block, FP=300 msec).

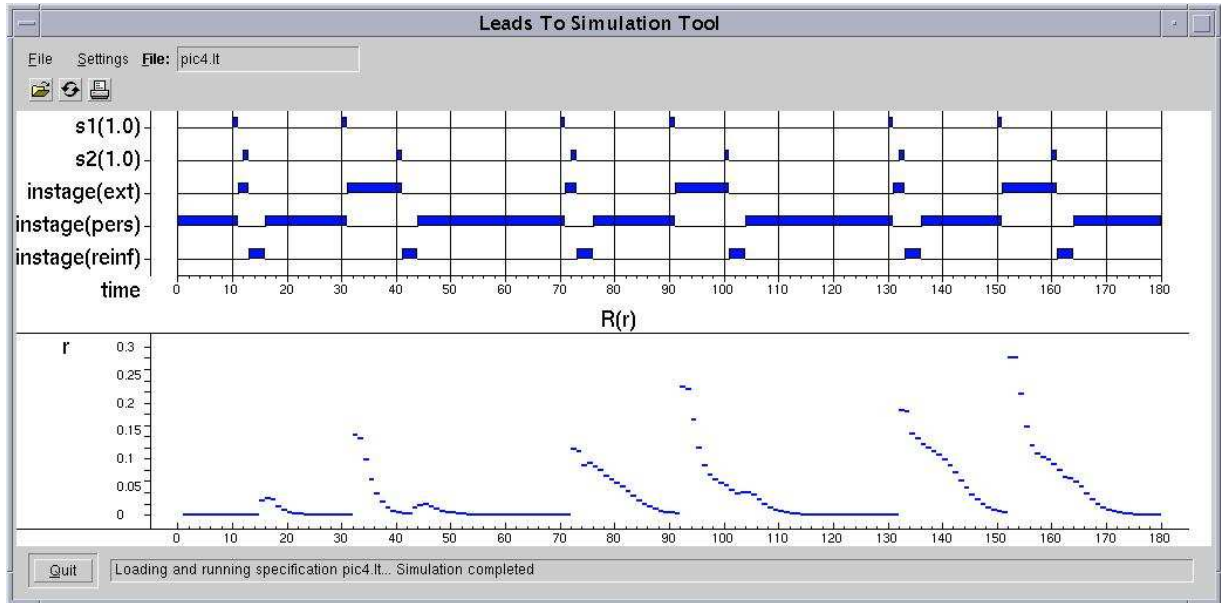


Figure 5: Simulation trace of the dynamics during conditioning (mixed block, FP=100 msec and FP=500 msec).

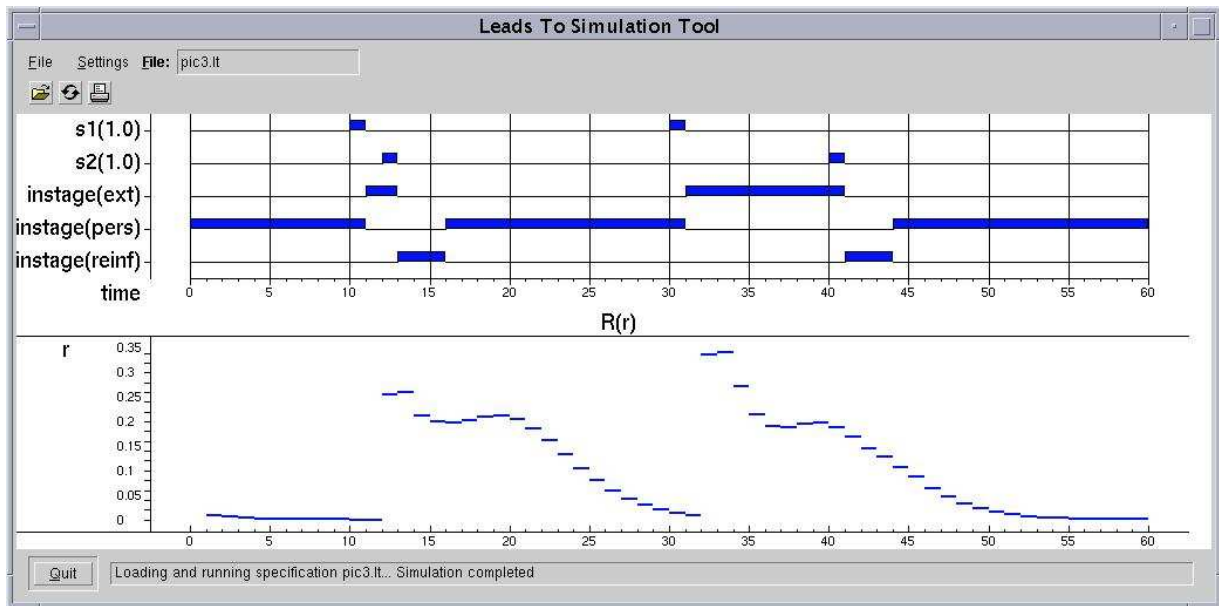


Figure 6: Simulation trace of the dynamics after conditioning (mixed block, FP=100 msec and FP=500 msec).