Temporal Factorisation: a Unifying Principle for Dynamics of the World and of Mental States

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Abstract

Dynamics play a major role in many phenomena, addressed in a variety of disciplines. This paper contributes to the identification of common principles underlying approaches to dynamics used within a variety of cognitive and noncognitive disciplines, such as Cognitive Science, Physics, Mathematics, Chemistry, Computer Science, and Biology. More specifically, as a central, unifying principle, the temporal factorisation principle is introduced, formalised and illustrated. This principle expresses, that every temporal relationship of the form 'past pattern implies future pattern' can be factorised into a relationship of the form 'past pattern implies present state' and a relationship of the form 'present state implies future pattern'. To enable this, for every 'past pattern implies future pattern' relationship, the principle postulates the existence of certain mediating state properties in the present state. It provides a conceptual framework which unifies various approaches to dynamics within a variety of disciplines. In particular, it is shown how the principle can be applied in the case of cognitive states of agents.

1 Introduction

The physicalist perspective on cognition, which has become more and more predominant in Cognitive Science and Philosophy of Mind, views cognition as one of the phenomena of nature. A natural consequence of such a position is that it is a challenge to relate principles behind cognition to principles in nature, or even to search for common principles behind the physical world and cognition. In particular, for cognitive agent models the issue of grounding or embedding them in the physical world is challenging. Having common principles behind nature and cognitive agent models would give a new perspective on this issue. This paper addresses the search for such common principles, in particular principles that describe dynamics. A unifying principle called 'temporal factorisation' is identified and shown to play a crucial role in different disciplines such as Physics, Chemistry, Biology, Mathematics, Computer Science, and Cognitive Science. Roughly spoken, the temporal factorisation principle claims that if a certain (past) pattern of events leads to a certain (future) pattern of events, then there exists a state property *p* such that the past pattern leads to a (present) state where this property *p* holds, and any state where the state property *p* holds leads to the future pattern. This postulated state property *p* is called a mediating state property for the 'past pattern implies future pattern' relationship. It enables to split or factorise this relationship into two other, in general simpler, temporal relationships: a 'past pattern implies present state' relationship and a 'present state implies future pattern' relationship.

As an illustration of the use of the conceptual framework of temporal factorisation, a variety of examples are analysed showing how it can be used to describe dynamics of the world and of cognition. Examples from Physics include state properties such as velocity, momentum (which are obtained by temporal factorisation of past-future relationships between patterns in the position (and mass) of an object over time), and force (obtained by temporal

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factorisation of past-future relationships between patterns in momentum or velocity over time). In approaches in Cognitive Science, the notion of mental state is often considered crucial. This notion can be considered to be postulated by the temporal factorisation principle applied to the dependence of future behaviour patterns of an agent on patterns of past sensory inputs.

For the cognitive domain a number of foundational themes and approaches are discussed in the literature on Philosophy of Mind. As in this paper a unifying approach is put forward for cognitive and noncognitive domains, an interesting question is in how far such themes and approaches can be generalised beyond the cognitive area. Indeed, this turns out to be possible, at least for the following notions from the cognitive area: mental states, representation relations for a mental state, physical realisation of mental states, mental state properties as second-order world properties. For the first two of these it is shown in this paper how certain approaches described in the literature on Philosophy of Mind can be generalised beyond the cognitive domain and incorporated in a conceptual framework based on temporal factorisation. More specifically, the scope of the notions described above is widened, thus covering mediating state properties instead of mental state properties. As a result it is shown, for example, how a more general notion of representational content for mediating state properties can be specified based on a generalisation of Kim (1996, pp. 200-202)'s relational specification approach for representational content of mental state properties.

In Section 2, the temporal factorisation principle is introduced and explained in some detail. It is shown how its formulation does not commit to any determinism assumption, and how it relates to views on the dynamics of the world by Descartes and Laplace. Examples of temporal factorisation from the cognitive domain and from the physical domain are Section 3 discusses repeated application of the principle. In Section 4, it is shown how the relational specification approach to representational content of mental state properties, as introduced by Kim (1996, pp. 200-202), can be used to formulate the temporal factorisation principle in more detail, and to define a form of representational content for mediating state properties resulting from temporal factorisation. Moreover, it is shown in Section 5 how, using an appropriate formal language for temporal relational specification, the temporal factorisation principle can be formalised. This formalisation not only involves temporal relationships of different types, but also quantifiers over traces (or trajectories) and over (the existence of) state properties.

It is discussed in Section 6 how, within Cognitive Science, mental state propertiescan be viewed as mediating state properties, resulting from temporal factorisation. More specifically,first this is shown for approaches based on the functionalist or symbolic tradition, for mental state properties such as belief, desire and trust. Moreover, it is also shown how the temporal-interactivist approach to cognition (cf. Bickhard, 1993; Jonker and Treur, 2003), relates to the temporal factorisation principle. In addition, it is shown how the Dynamical Systems Theory (DST), (e.g., Port and Gelder, 1995; Kelso, 1995) relates to the principle of temporal factorisation as a basic assumption. In Section 7 it is shown how, in the area of calculus within Mathematics, the concept of derivative and certain theorems (concerning smoothness of a function around a point and the existence of a derivative at this point), are postulated by the principle. The paper finishes with a discussion: Section 8.

2 Temporal Factorisation

In this section, first some historical contributions are discussed (Section 2.1), and next the temporal factorisation principle is formulated (Section 2.2) and illustrated by a few examples (Section 2.3). More examples will follow in subsequent sections.

2.1 Relating Past, Present and Future

Descartes(1633) introduced a perspective on the world that sometimes is called the *clockwork universe*. This perspective claims that with sufficiently precise understanding of the world's dynamics at some starting time, the future can be predicted by applying a set of laws. He first describes how at some starting time matter came into existence in a diversity of form, size, and motion. From that time on, dynamics continues according to 'laws of nature'.

> 'From the first instant that they are created, He makes some begin to move in one direction and others in another, some faster and others slower (or indeed, if you wish, not at all); thereafter, He makes them continue their motion according to the ordinary laws of nature. For God has so wondrously established these laws that, even if we suppose that He creates nothing more than what I have said, and even if He does not impose any order or proportion on it but makes of it the most confused and most disordered chaos that the poets could describe, the laws are sufficient to make the parts of that chaos untangle themselves and arrange themselves in such right order that they will have the form of a most perfect world, in which one will be able to see not only light, but also all the other things, both general and particular, that appear in this true world.' (Descartes, The World, 1634, Ch 6: Description of a New World, and on the Qualities of the Matter of Which it is Composed)

Descartes emphasizes that after such a starting time nothing (even no God) except the laws of nature determines the world's dynamics:

> 'Know, then, first that by "nature" I do not here mean some deity or other sort of imaginary power. Rather, I use that word to signify matter itself, insofar as I consider it taken together with all the qualities that I have attributed to it, and under the condition that God continues to preserve it in the same way that He created it. For from that alone (i.e., that He continues thus to preserve it) it follows of necessity that there may be many changes in its parts that cannot, it seems to me, be properly attributed to the action of God (because that action does not change) and hence are to be attributed to nature. The rules according to which these changes take place I call the "laws of nature."' (Descartes, The World, 1634, Ch 7: On the Laws of Nature of this New World)

This view on the world's dynamics is often compared to a clockwork. The view assumes that systematic relationships (laws of nature) are possible between world states over time, in the sense that (properties of) past world states imply (properties of) future world states:

$$\text{past states} \rightarrow \text{future states}$$

The clockwork universe view has been developed further by Newton, Leibniz, Laplace and others. The following quotation taken from Laplace (1825) sketches how an intellect could be able to determine future world states from a present world state, that by itself is the effect of past world states:

> 'We may regard the present state of the universe as the effect of its past and the cause of its future. An intellect which at any given moment knew all of the forces that animate nature and the mutual positions of the beings that compose it, if this intellect were vast enough to submit the data to analysis, could condense into a single formula the movement of the greatest bodies of the universe and that of the lightest atom; for such an intellect nothing could be uncertain and the future just like the past would be present before its eyes.' (Laplace, 1825)

2.2 The Temporal Factorisation Principle

The view expressed by Laplace (1825) assumes that the dynamics of the world can be described in the form of (a) relationships between past world states and the present world state, and (b) relationships between the present world state and future world states:

$$\text{past states} \rightarrow \text{present state} \rightarrow \text{future states}$$

To analyse in more detail the temporal relationships pointed at by Descartes and Laplace, the *temporal factorisation principle* can be used. This principle, as introduced in this paper, is formulated in terms of temporal relationships between past patterns, present states, and future patterns. Here a *past pattern a* refers to a property of a series of states or events in the past, and a *future pattern b* refers to a property of a series of states or events in the future.[2]

To put it in a nuttshell, the temporal factorisation principle states that any systematic temporal 'past pattern implies future pattern' relationship

$$a \rightarrow b$$

between a past pattern a and a future pattern b can be factorised in the form of two temporal relationships

$$a \rightarrow p \text{ and } p \rightarrow b$$

for some state property p of the present world state.[3,4] More specifically, the principle claims that for any 'past pattern implies future pattern' relationship $a \rightarrow b$ *there exists* a world state property p (expressed in the ontology for state properties) such that temporal relationships 'past pattern implies present state property' $a \rightarrow p$ and 'present state property implies future pattern' $p \rightarrow b$ hold.[5,6] In short:

$$a \rightarrow b \quad \Rightarrow \quad \exists p \quad a \rightarrow p \quad \& \quad p \rightarrow b$$

The postulated state property p is called a *mediating state property* for the given 'past pattern implies future pattern' relationship. In other words, the principle claims that the description of the present world state contains sufficient information so that we can forget about the temporal pattern a in the past if we want to understand why the temporal pattern b occurs in the future; therefore it essentially is a claim that the state ontology is (or can be chosen) sufficiently rich to express all the relevant information on the past in some condensed form in one state description, and the same with respect to the future.

The principle can be viewed as a way to make temporal complexity of dynamics more manageable by relating it to state complexity, where an underlying assumption is that the state complexity

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[2] An example of a past pattern, referring to different points in time, is: at some state in the past c occurred and since then to the present it persisted. An example of a future pattern, referring to different time points in the future is: if in some future state c occurs, then in some later state d will occur. A special, simple case of a past (or future) pattern is the occurrence of a state property in some single past (or future) state.

[3] Here p may be a combination, for example a conjunction or proposition, of atomic state properties

[4] The word factorisation can be explained as follows. Consider the temporal past implies future relationship to be an operator f, which assigns future patterns to past patterns

f : past patterns $\rightarrow$ future patterns

Then the temporal factorisation principle claims that operators g and h exist such that h assigns present states to past patterns and g assigns future patterns to present states:

g: present states $\rightarrow$ future patterns

h: past patterns $\rightarrow$ present states

such that the operator f is factorised by g and h in the following algebraic manner: $f = gh$, where gh denotes the composition of two operators g and h (i.e., first apply h, then g).

[5] So, notice that the notation $\rightarrow$ is used here to indicate logical implication (between temporal properties).

[6] Sometimes such relationships are simply called 'past to present', present to future', or 'past to future' relationships.

needed can be kept limited. The mediating state property in the present state may be viewed in a way, to represent the past pattern and the future pattern in the present state. Indeed, in Section 4 it will be shown how the relational specification approach to representational content of mental state properties, as proposed by Kim (1996), can be extended beyond the cognitive area to the more general situation here.

Note that the temporal factorisation principle itself does not claim that any 'past pattern implies future pattern', 'past pattern implies present state' or 'present state implies future pattern' relationships can be found. Due to the conditional, it only claims that if a 'past pattern implies future pattern' relationship is available, then also 'past pattern implies present state' and 'present state implies future pattern' relationships can be found. To make this more precise, if Descartes' view is interpreted in the sense that

> dynamics can be described by 'past pattern implies future pattern' relationships (D)

and Laplace's view is interpreted in the sense that

> dynamics can be described by 'past pattern implies present state' and 'present state implies future pattern' relationships (L)

then the temporal factorisation principle (TFP) logically connects the two: Descartes' view interpreted as D and the temporal factorisation principle TFP together imply Laplace's view interpreted as L, i.e.,

$$\text{D} \;\&\; \text{TFP} \Rightarrow \text{L}$$

So, the temporal factorisation principle can be used to explain the shift in history, from Descartes's view to Laplace's view. While Descartes' and Laplace's views each can be considered to assume a deterministic world, the temporal factorisation principle is not based on such an assumption, due to the conditional. Temporal factorisation addresses those cases and those aspects of the world where 'past pattern implies future pattern' relationships can be found, but not in any way claims that such relationships can always be found for all aspects of the world. Thus, the principle supports all forms of partial determinism, or, in other words, any perspective between a fully deterministic world and a fully non-deterministic world. For a more extensive discussion about (non)determinism and how it can be considered from different perspectives (e.g., an external God's eye perspective or an internal agent perspective), see, for example, Earman (1986), Dennett (2003, pp. 25-96).

2.3 Some Examples of Temporal Factorisation

The following example illustrates the use of temporal factorisation as a conceptual framework to analyse the dynamics of various phenomena. Suppose in reality or in a virtual game context there is a locked door that only can be opened if the right key is available. Someone approaching the door can do so after many different histories, some of which (say those satisfying pattern *a*) lead to a future after entering the door (say with pattern *b*) and others (those not satisfying *a*) to a future after not entering (not satisfying pattern *b*).

The difference between these two types of past histories as can be seen in an intermediate (present) state is that some of them lead to approaching the door while carrying the key, whereas other histories do not lead to a state carrying the key at the door. Only these histories for which a time exists that the door is encountered while the key is present, lead to futures satisfying pattern *b*, or, in other words, futures that can occur after entering the door. Those histories for which no time point exists where the key is present at the door, will not provide the possibility to have a future with pattern *b* after entering the door. An explanation based on the temporal factorisation principle

$$a \rightarrow b \quad \Rightarrow \quad \exists p \;\; a \rightarrow p \;\;\&\;\; p \rightarrow b$$

here involves the occurrence at some point in time of the mediating world state property *p* describing that the key is combined with the locked door (thus resulting in an unlocked door). Mediating state property *p* provides a form of interpolation between past and future patterns, on the one hand reflecting the past pattern where the key was taken and on the other hand the pattern *b* of possible futures that can occur after entering the door. This example illustrates that for the histories which lead to a future with pattern *b*, from the past perspective there is a convergence to a state where *p* holds, whereas for the future perspective there is a divergence from that state (see Figure 1).

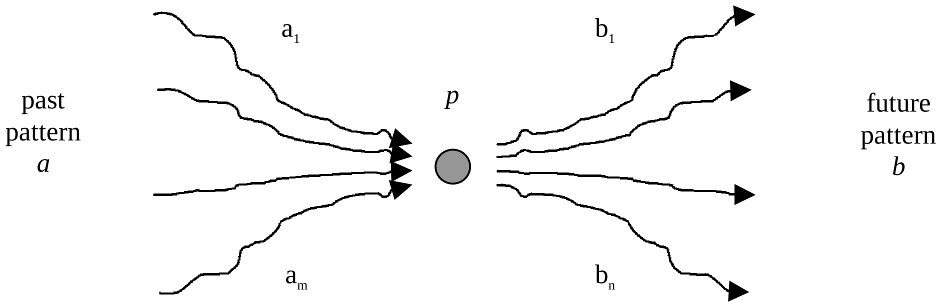

Figure 1. Mediating state property p for the relationships between patterns $a \rightarrow b$

Using this example as a metaphor, the state where *p* holds sometimes is called a *key state* for the pattern $a \rightarrow b$, and the mediating state property *p* is called a *key state property*. This key state plays the role of a gate, through which a certain type of future can be reached from the past. If, coming from the past, this gate is missed, that type of future is not reachable.

Another example illustrating the temporal factorisation principle is a mental state property such as a desire. Several circumstances or sources in the past can lead to the generation of a specific desire. However, as the assumption on the notion 'desire' is, once the desire is there, independent of how this desire was generated, the future behaviour is directed by it.

The idea of a desire state property is that it provides a way of reducing representational complexity by abstracting from a variety of possible histories, and thus enables to base dynamics of future behaviour on simple properties in the present state, instead of on one of all those possible histories. It is just more efficient to explain behaviour of an agent by referring to a desire state, then to enter all details of how this desire possibly was reached. For this specific case the picture in Figure 1 can be interpreted as follows: the pattern *a* describes a set of (possible) histories a_i of an agent, *p* the mental state of the agent and the pattern *b* a set of possible futures b_j in the behaviour of the agent.[7]

As an example from Physics, temporal factorisation can be illustrated by the notion 'momentum' of a moving object in classical mechanics as a key state property. Different histories of the object can lead to the same momentum in the present state. The future of the object only (besides the object's current position) depends on this momentum in the present state, not on the specific history. This was the criterion by which the concept momentum was introduced in Physics in history (see Treur, 2005, for a more detailed historical case study). Therefore the state property momentum can be understood as a mediating state property for past and future patterns in (change of) position of an object; the temporal factorisation principle postulates the existence of this state property (also see the analysis of the notion velocity in Section 7.2).

The state property momentum abstracts from the various histories that could have happened and would have resulted in the same future pattern. In the other time direction, a momentum indicates from what pattern it originated, no matter what future will arise, so it abstracts from futures. Similarly the concept 'force' from classical mechanics within Physics can be considered a postulated mediating state property obtained by temporal factorisation for past to future patterns in momentum: it mediates between a (past) state with some given momentum to a (future) state with a changed momentum.

[7] Temporal factorisation can be viewed as a way to reduce complexity in the following sense. For example, suppose a number of m specific instances of histories $a_1, \ldots, a_m$ all lead to (under different future circumstances) a number n of different instances of futures $b_1, \ldots, b_n$. This gives rise to m*n relationships $a_i \rightarrow b_j$ $(1 \leq i \leq m, 1 \leq j \leq n)$. A first step to reduce complexity is by postulating a common mediating state property p, thus factorising the m*n relationships to m+n relationships $a_i \rightarrow p$ $(1 \leq i \leq m)$, $p \rightarrow b_j$ $(1 \leq j \leq n)$. An additional step to reduce complexity is to describe the histories $a_1, \ldots, a_m$ and futures $b_1, \ldots, b_n$ by patterns *a* and *b* instead of by enumeration. Then two relationships $a \rightarrow p$ and $p \rightarrow b$ are obtained. This shows how temporal factorisation can be viewed as a way of modularisation of a more complex set of temporal relationships in two independent parts, with *p* as the connecting element, which provides enhanced representational and combinatorial efficiency. An illustration can be made for the notion of desire as a key state. It is more efficient to explain behaviour of an agent by referring to a desire state, then to enter all details of how this desire possibly was reached.

3 Repeated Temporal Factorisation

When temporal factorisation is applied to a 'past to future' relationship between patterns based on a given state ontology, new mediating state properties may be created, thus extending the state ontology. Once mediating state properties have been included in the state ontology, usually they also show dynamics by themselves. The new state properties can be used to describe new patterns; in particular, a temporal past to future relationship can exist based on the dynamics of a certain mediating state property. Also on such past to future relationships the temporal factorisation principle can be applied. This leads to the introduction of higher-order mediating state properties.

An example of a second-order mediating state property is for the notion of force in Physics. Independent of the way in which a given force on an object was obtained in the past, the force entails changed properties (i.e., changed momentum) in the future states of the object. The only relevant aspect for change of momentum is the force in the present; if this force is known (in addition to the present momentum), all other aspects of the world state and its history are irrelevant. The force abstracts a certain pattern from the possible histories. This is another way of expressing that a specific force can be considered to be a mediating state property (anticipating change of momentum) between a history pattern and a future pattern. As momentum itself is already a mediating state property (see Section 2.3), anticipating change of position, this makes force a second-order mediating state property.

The temporal factorisation principle can also be applied repeatedly, according to the following general refinement pattern:

$$a \rightarrow b \quad \Rightarrow \quad \exists\, p_1 \;\; a \rightarrow p_1 \;\;\&\;\; p_1 \rightarrow b \quad \Rightarrow$$
$$\exists\, p_1, p_2 \;\; a \rightarrow p_1 \;\;\&\;\; p_1 \rightarrow p_2 \;\;\&\;\; p_2 \rightarrow b \quad \Rightarrow$$
$$\ldots.$$

For example, in the case of a desire as a state property p_1 used as temporal factorisation between past history patterns and future behaviour patterns, the process from desire to future behaviour can be further factorised by introducing an intention state property p_2 as an intermediate step.[8] Iterated temporal factorisation can be pursued or refined until the mediating state properties are close to each other in the sense that they are related by simple relationships that can be considered basic mechanisms or causal steps in the process. Often simulation models that calculute 'runs' step by step can be based on such a refined factorisation.

4 Temporal Factorisation and Relational Specification

A mediating state property *p* for a 'past pattern *a* implies future pattern *b*' relationship, as postulated by the temporal factorisation principle can be considered to carry information both about the past pattern *a* and about the future pattern *b*; it in a way represents both the past pattern and the future pattern in the present state, as was also remarked in Section 2.2. Note that as the temporal factorisation principle is a quite general principle about the world, this form of representation, in principle is not related to any agent or cognitive system.

In the cognitive area, much literature can be found on the notion of representational content of a mental state property of a certain agent; see, for example, Kim (1996), Bickhard (1993), Jonker and Treur (2003). One of the approaches described within the Philosophy of Mind literature, is the relational specification approach to representational content for internal (mental) agent states; cf. Kim (1996, pp. 200-202). This approach turns out to provide a suitable approach, for the more general case, beyond the cognitive area, for mediating state properties. The temporal factorisation principle is formulated in terms of temporal relationships:

- temporal relationships between past and future patterns
- temporal relationships between past patterns and present states
- temporal relationships between present states and future patterns

Based on Kim (1996)'s relational specification approach to representational content, this section addresses in more detail the way in which such temporal relationships can be specified. The relational specification approach is briefly discussed in Section 4.1. In Section 4.2 it is shown how temporal relational specifications can be used to formulate temporal relationships for the mediating state properties postulated by the temporal factorisation principle, thus defining anotion of representational content for mediating state properties, generalising this notion beyond the cognitive domain.

4.1 Relational Specification of State Properties

In Philosophy of Mind, in the context of representational or mental content of (mental) state

[8] Notice that in the single step factorisation extra conditions (e.g., *c* and *d*, for triggering stimuli) for future behaviour that play a role can be incorporated as conditionals in the pattern *b* (e.g., *b* is of the form: *if* $c \wedge d$ *then b'*). In the case of the iterated factorisation, such conditions may be incorporated, for example, in the following manner: $p_1 \rightarrow$ (*if c then* p_2), and $p_2 \rightarrow$ (*if d then b'*).

properties, Kim (1996, pp. 200-202), puts forward the concept of relational specification of a state property:

> 'The third possibility is to consider beliefs to be wholly internal to the subjects who have them but consider their contents as giving *relational specifications* of the beliefs. On this view, beliefs may be neural states or other types of physical states of organisms and systems to which they are attributed. Contents, then, are viewed as ways of specifying these inner states; wide contents, then, are specifications in terms of, or under the constraints of, factors and conditions external to the subject, both physical and social, both current and historical.' (Kim, 1996, pp. 200-201); italics in the original.

In particular, concentrating on the temporal dimension, a temporal relational specification can be viewed as the specification of temporal relationships of a (mental) state to other patterns in past and future. Kim emphasizes that relational specifications in general may be crucial to be able to formulate laws and explanations.

> 'Consider physical magnitudes such as mass and length, which are standardly considered to be paradigm examples of intrinsic properties of material objects. But how do we *specify*, *represent*, or *measure* the mass or length of an object? The answer: relationally. To say that this rod has a mass of 5 kilograms is to say that it bears a certain relationship to the International Prototype Kilogram (it would balance, on an equal-arm balance, five objects each of which balances the Standard Kilogram). Likewise, to say that the rod has a length of 2 meters is to say that it is twice the length of the Standard Meter (or twice the distance travelled by light in a vacuum in a certain specified fraction of a second). These properties are intrinsic, but their specifications or representations are extrinsic and relational, involving relationships to other things and properties in the world. It may well be that the availability of such extrinsic representations are essential to the utility of these properties in the formulation of scientific laws and explanations.'
> (Kim, 1996, p. 201); italics in the original.

In Kim's proposal a mental state property of a subject itself is distinguished from its relationships to other items. This contrasts to some other approaches where the mental state property is considered to be ontologically constituted as one entity comprising both the subject and the related items, or where the mental state property is considered to be the relation between the subject and the other items (cf. Kim, 1996, pp. 200-202). Kim explains how a mental state property itself can be considered an intrinsic internal state property, whereas its relational specification expresses how it relates to other items in the world as follows.

> 'The approach we have just sketched has much to recommend itself over the other two. It locates beliefs and other intentional states squarely within the subjects; they are internal states of the persons holding them, not something that somehow extrudes from them. This is a more elegant metaphysical picture than its alternatives. What is "wide" about these states is their specifications or descriptions, not the states themselves.' (Kim, 1996, pp. 201-202).

4.2 Temporal Relational Specification for Temporal Factorisation

The concept of relational specification as just described, introduced by Kim (1996) in the context of mental states, offers a way to describe in a more general context the representational content of a mediating state property, postulated by the temporal factorisation principle. Addressing the future direction first, if *p* is a mediating state property related to some future pattern *b* (and some past pattern *a*), then the actual occurrence of *p* at some time point *t* leads to the actual occurrence of *b* in the future after *t*. Indeed, a relational specification may be identified expressing what the effect of this mediating state property *p* on the subsequent future is (i.e., that pattern *b* will occur). For example, if *p* is the mediating state property anticipating that the subject (at time *t* being at some position different from a position *P*), can be at position *P* in a subsequent state at time *t'*, then (assuming no intervention from elsewhere), this relational specification of *p* can be expressed as:

if at time point t state property p holds,
then at some time point t'>t the state property
'being at position P' holds

Based on this, state property *p* can be considered to represent the fact that 'at some future time point being at position P' holds. This notion of relational specification need not be limited to one future state. It can be extended to incorporate a future pattern *b* incorporating a series of states (and possibly also conditionals) at different points in time.

A similar analysis can be made for the past relationships. Given past pattern *a* that is assumed to lead to a mediating state property *p*, a relational specification can be identified to express this temporal relationship. Thus, state property *p* can be considered to represent in the present state the fact that the past pattern *a* occurred. Combining the past and future perspective, the fact that *p* is a mediating state property between future pattern *b* and past pattern *a*, can be relationally specified in a temporal manner by a scheme of the following type:

if before t, past pattern a occurs,
then at t, state property p holds

if at t, state property p holds,
then after t, future pattern b will occur

These two temporal relationships are a 'past pattern implies present state property' and a 'present state property implies future pattern' relationship, respectively.[9] Together they can be considered to

[9] Notice that the concept of relational specification as put forward by Kim is applicable, as soon as a state property *p* is given, but by itself provides no physical existence or realism of such a state property. This topic of realisation will be addressed a next paper.

provide a relational specification of the representational content of the mediating state property *p*, which takes into account both the past and the future.

5 Formalisation of Temporal Factorisation

In this section, it is shown how the temporal factorisation principle can be expressed in a formal language. First this language is briefly introduced (Section 5.1). Next it is shown how past and future patterns can be expressed in this language (Section 5.2). In Section 5.3 it is shown how temporal relationships between past patterns, future patterns and present states are expressed. Finally, in Section 5.4 it is shown how the temporal factorisation principle as a whole can be expressed in the language.

5.1 Specification and Formalisation

To specify and formalise temporal relationships that play a role in temporal factorisation, an expressive formal language is needed that allows to refer to patterns over time. Furthermore, it should be possible to express the existential quantifier for state properties, which occurs in the temporal factorisation principle. The Temporal Trace Language (TTL) is such a language (Jonker and Treur, 2002). TTL will be briefly introduced here; it has some similarities to situation calculus and event calculus, and is supported by a software environment for specification and verification.

The language TTL is based on traces (or trajectories), time points, and state properties as primitive notions. A state can be parameterised by a trace in which it occurs and a time point at which it occurs. The language is built up as follows. A *state ontology* is a specification (in sorted predicate logic) of a vocabulary (i.e., a signature). A state for ontology Ont is an assignment of truth-values {true, false} to the set At(Ont) of ground atoms expressed in terms of Ont. The *set of all possible states* for state ontology Ont is denoted by STATES(Ont). The set of *state properties* STATPROP(Ont) for state ontology Ont is the set of all propositions over ground atoms from At(Ont).[10]

A fixed *time frame* T is assumed, which is linearly ordered. Depending on the application, the time frame T may be dense (e.g., the real numbers), or discrete (e.g., the natural numbers), or any other form, as long as it has a linear ordering. A *trace* or *trajectory* γ over a state ontology Ont and time frame T is a mapping $\gamma : T \rightarrow$ STATES(Ont) (i.e., a time-indexed set of states γ_t ($t \in T$) in STATES(Ont)). The set of all traces over state ontology Ont is denoted by TRACES(Ont).

The set of *dynamic properties* DYNPROP(Ont) over state ontology Ont is the set of temporal statements that can be formulated with respect to traces based on the state ontology Ont in the following manner. Given a trace γ over state ontology Ont, a state of the world at time point t is syntactically denoted by state(γ, t). These states can be related to state properties via the formally defined (in TTL syntax) satisfaction relation $\models$, i.e.: state(γ, t) $\models$ p , which denotes that state property *p* holds in trace γ at time t (this has a similarity with the Holds-predicate in situation calculus).

Based on these statements, dynamic properties can be formulated in a formal manner in a sorted predicate logic with sorts TIME for time points, TRACES for traces and STATPROP for state formulae, using quantifiers, among others, over time, traces and state formulae, and the usual logical connectives such as $\neg$, &, $\vee$, $\Rightarrow$, $\forall$, $\exists$. Within TTL (real and integer) numbers can be used for time, but also within state properties.

5.2 Formalisation of Past and Future Patterns

To formalise the temporal factorisation principle, formalisations are needed for the temporal relationships between past patterns, present states and future patterns with respect to a given time point *t*. As a first step it is shown how past patterns and future patterns can be specified. The basic idea is that a pattern refers to a specific set of traces, for example a past pattern refers to a specific set of past traces (up to some time point *t*); see also Figure 1. The way in which this reference takes place is by expressing a pattern in the form of a (temporal) property that the traces in the set have in common, or, in other words, that characterises this set of traces. To express this property characterising a pattern, the language TTL is used.

Specification of a Past Pattern
A *past statement* for γ and t is a temporal statement $\varphi(\gamma, t)$ where γ and t are free variables, such that each time variable different from t is restricted to the time interval before t. In other words, for every time quantifier for a variable s a restriction of the form $s \leq t$, or $s < t$ is required within the statement. A *past pattern* is any past statement. A trace γ satisfies a past pattern $\varphi(\gamma, t)$ for t if $\varphi(\gamma, t)$ is true. The set of past statements over state ontology Ont with respect to time point t is denoted by PFOR(Ont, γ, t).

Specification of a Future Pattern
Similarly as the past statements, FFOR(Ont, γ, t) denotes the set of *future statements* over state ontology Ont with respect to trace γ and time point t: γ

[10] When no confusion is expected, the argument Ont will be left out: STATPROP.

and t are free variables and every time quantifier for a variable s is restricted by $s > t$ or $s > t$. A *future pattern* is any future statement.

5.3 Formalisation of the Temporal Relationships

Given the specification of past and future patterns defined above, the temporal relationships can be defined as follows.

Specification of a Past to Future Relationship
A 'past pattern implies future pattern' relationship for state ontology Ont at any given time point t is specified as an implication

$$\forall\gamma\ [\ \varphi(\gamma, t) \Rightarrow \psi(\gamma, t)\ ]$$

where γ ranges over the sort TRACES, $\varphi(\gamma, t) \in$ PFOR(Ont, γ, t), $\psi(\gamma, t) \in$ FFOR(Ont, γ, t).

Specification of a Past to Present Relationship
A 'past pattern implies present state' relationship for a state ontology Ont and time point t is specified as a logical implication

$$\forall\gamma\ [\ \varphi(\gamma, t) \Rightarrow \text{state}(\gamma, t) \models p\ ]$$

for a given state property $p \in$ STATPROP(Ont) and $\varphi(\gamma, t) \in$ PFOR(Ont, γ, t), whereas γ ranges over the sort TRACES.

Specification of a Present to Future Relationship
A 'present state implies future pattern' relationship for a state ontology Ont and time point t is specified as a logical implication

$$\forall\gamma\ [\ \text{state}(\gamma, t) \models p \Rightarrow \psi(\gamma, t)\ ]$$

for a given state property $p \in$ STATPROP(Ont) and $\psi(\gamma, t) \in$ FFOR(Ont, γ, t), whereas γ ranges over the sort TRACES.

5.4 Formalisation of Temporal Factorisation

Using the notions defined above, the temporal factorisation principle over state ontology Ont expresses that for any past and future formulae $\varphi(\gamma, t) \in$ PFOR(Ont, γ, t), $\psi(\gamma, t) \in$ FFOR(Ont, γ, t) with respect to t, for which for any trace γ and time point t

$$\varphi(\gamma, t) \Rightarrow \psi(\gamma, t)$$

holds, there exists a state property $p \in$ STATPROP(Ont) such that for all traces γ and time points t both

$$\varphi(\gamma, t) \Rightarrow \text{state}(\gamma, t) \models p$$
$$\text{state}(\gamma, t) \models p \Rightarrow \psi(\gamma, t)$$

hold, or in concise format[11]:

$$\forall\gamma, t\ [\ \varphi(\gamma, t) \Rightarrow \psi(\gamma, t)\ ] \Rightarrow$$
$$\exists p\ [\ \forall\gamma, t\ [\ \varphi(\gamma, t) \Rightarrow \text{state}(\gamma, t) \models p\ ]\ \&$$
$$\forall\gamma, t\ [\ \text{state}(\gamma, t) \models p \Rightarrow \psi(\gamma, t)\ ]\ ]$$

where γ ranges over the sort TRACES, t over sort TIME and p over sort STATPROP.

6 Temporal Factorisation and Mental States

One of the challenges in the cognitive domain is to describe how a (human or animal) agent's behaviour depends on its past experiences (e.g., sensing of stimuli). As relationships between future patterns of an agent's behaviour and past patterns of its experiences may be quite complex, the notion of mental state has been postulated as a mediating state between past stimuli and future behaviour. The mental state of an agent depends on its past, and the agent's future behaviour depends on its mental state. In this sense the temporal factorisation principle applies.

The postulated mental state properties play an important role in the explanation and prediction of behaviour. In Section 2.3, the mental state property 'desire' and its relation to temporal factorisation was briefly discussed. Two other simple examples concerning an agent's belief and an agent's trust state illustrate the case in some more detail in Section 6.1 below. In Section 6.1, it is also shown how temporal factorisation can be related to the notion of functional role as described in Philosophy of Mind and Cognitive Science (e.g., Kim, 1996, pp. 87). In Section 6.2, it is shown how temporal factorisation can be related to the temporal-interactivist approach to mental states (cf. Bickhard, 1993, 2000, and Jonker and Treur, 2003). In Section 6.3 it is discussed how temporal factorisation relates to the Dynamical Systems Theory approach to cognition.

6.1 Temporal Factorisation and Functional Roles

To illustrate how temporal factorisation relates to the notion of mental state property, an example of a belief state is adressed. Consider an agent's reaction on its observation of the presence of food at a position *P*:

if at any time t' ≤ t the agent observed food at position P
then if at some t'' ≥ t the agent observes the opportunity to go to position P,
then at some time point t' ≥ t'' the agent will go to position P

Here, it is assumed that sometimes position *P* is not visible. For example, when after observation of the food but before observation of the opportunity, a cup is placed upside down at *P* (i.e., the food is present but not visible anymore). Then, at the moment that the opportunity to go to *P* is observed, the food at *P* is not observed. The above specification describes a direct temporal relationship between past (observation) events and future behaviours, without taking into account internal, mental states. The

[11] Notice that for this formalisation the following three features of TTL are crucial: (1) it is based on an expressive first-order language for temporal relationships, (2) traces are first class citizens in the language, which means that variables and quantification over traces are possible, and (3) state properties are first class citizens, which means that variables and quantification over state properties are possible. For example, standard (modal) temporal logics do not have these features.

mental state property 'belief that food is present at position *P*' can be seen as a temporal factorisation of this temporal past to future relationship. Its temporal relational specification can be obtained in a simplified form in the following manner:

if at any time t' ≤ t the agent observed food at position P
then at t the agent has the belief that
food is present at position P

if at t the agent has the belief that
food is present at position P
then if at some t'' ≥ t the agent observes
the opportunity to go to position P,
then at some time point t' ≥ t'' the agent will
go to position P

This form corresponds to what is called the functional role of the mental state property 'belief that food is present at position *P*' (e.g., Kim, 1996, pp. 87).[12] Notice that this belief state property is rather simple. For example, if it is first observed that food is present at *t*, and later it is observed that no food is present at *P*, still the belief that food is present at *P* will persist. In Section 6.2, where the interactivist approach is addressed, the notion will be extended to avoid this. Formalisation of the example is as follows. For this case, the past pattern is described by $\varphi(\gamma, t)$, which is the past statement

$$\exists t1 \leq t \quad \text{state}(\gamma, t1) \models \text{observation\_result(food\_present\_at(P))}$$

which states that there exists a past time point in which the agent observed food at *P*.
Moreover, the future pattern is described by $\psi(\gamma, t)$, which is the statement

$$\forall t2 \geq t \; [\; \text{state}(\gamma, t2) \models \text{observation\_result(opportunity\_to\_go\_to(P))} \Rightarrow \exists t3 \geq t2 \; \text{state}(\gamma, t3) \models \text{to\_be\_performed(go\_to(P))} \;]$$

expressing that as soon as an opportunity is observed, the agent goes to *P*.
Temporal factorisation of

$$\forall \gamma, t \; [\; \varphi(\gamma, t) \Rightarrow \psi(\gamma, t) \;]$$

for this case is obtained by the following temporal relational specifications for the belief state:

$$\forall \gamma, t \; [\; \varphi(\gamma, t) \Rightarrow \text{state}(\gamma, t) \models \text{belief(food\_present\_at(P))} \;]$$
$$\&$$
$$\forall \gamma, t \; [\; \text{state}(\gamma, t) \models \text{belief(food\_present\_at(P))} \Rightarrow \psi(\gamma, t) \;]$$

This states that if there is a past observation of food at *P*, then a belief state concerning this fact is there, and if such a belief state is there, then the agent will go to *P* as soon as the opportunity is observed.

[12] Note that by repeated temporal factorisation, as discussed in Section 3, the example can be extended easily to involve multiple mental state properties.

6.2 Temporal Factorisation in the Temporal-Interactivist Approach

In this section it is shown how the interactivist approach to mental states can be described on the basis of the conceptual framework of temporal factorisation. First the general idea is discussed. Next to illustrate this idea, two examples are addressed, one for a belief state and one for a trust state.

6.2.1 Interactivist Temporal Factorisation

In some recent literature in the area of Cognitive Science and Philosophy of Mind, cognitive functioning is studied from an interactivist perspective (e.g., (Bickhard, 1993, 2000; Jonker and Treur, 2003). Bickhard (1993) emphasises the relation between the (mental) state of a system (or agent) and its past and future in the interaction with its environment:

> 'When interaction is completed, the system will end in some one of its internal states - some of its possible final states. (…) The final state that the system ends up in, then, serves to implicitly categorise together that class of environments that would yield that final state if interacted with. (...) The overall system, with its possible final states, therefore, functions as a *differentiator* of environments, with the final states implicitly defining the differentiation categories. (…) Representational content is constituted as indications of potential further interactions.' (Bickhard, 1993)

This suggests that mental states are related to interaction histories on the one hand, and to future interactions, on the other hand. Bickhard (1993, 2000), does not address the question how to formalise the interactivist approach, but in (Jonker and Treur, 2003) a formalisation is proposed which takes into account the temporal aspects of this interactivist perspective.

The general idea is as follows. Suppose for an agent a mental state property *p* is given, which relates to a pattern of past interaction traces (from a given time point *t*), on the one hand and to a pattern of future interaction traces on the other hand. Let $\varphi(\gamma, t)$ be a specification of this pattern of past interaction traces and $\psi(\gamma, t)$ a specification of the pattern of future interaction traces. The temporal-interactivist approach considers the mental state property *p* holding in the present can mediate in this process as follows:

$$\varphi(\gamma, t) \Rightarrow \text{state}(\gamma, t) \models p \quad \&$$
$$\text{state}(\gamma, t) \models p \Rightarrow \psi(\gamma, t)$$

For this case the temporal past to future relationship is specified in the form $\varphi(\gamma, t) \Rightarrow \psi(\gamma, t)$. The above specifications form a temporal factorisation of this past to future relationship.

6.2.2 Interactivist Temporal Factorisation Based on a Belief State

An illustration of the temporal-interactivist approach by an extension of the belief example above is as follows. Consider, as in Section 6.1, *p* to be the mental state property describing the belief that food is present at position *P*. Now it is assumed that this state property will not hold anymore when it is observed that no food is present at *P*. Then, taking

$\varphi(\gamma, t)$:
at some time t' ≤t the agent observed food at position P
and from t' to t it did not observe that no food was present at position P

$\psi(\gamma, t)$:
if at some t'' ≥ t the agent observes the opportunity to go to position P,
then at some time t' ≥''t the agent will go to position P

the temporal factorisation of $\varphi(\gamma, t) \Rightarrow \psi(\gamma, t)$ can be described as follows:

The past to present relationship

$$\varphi(\gamma, t) \Rightarrow \text{state}(\gamma, t) \models p$$

if *at some time t' ≤t the agent observed food at position P*
and *from t' to t it did not observe that no food was present at position P*
then *at t the agent has the belief that food is present at position P*

The present to future relationship

$$\text{state}(\gamma, t) \models p \Rightarrow \psi(\gamma, t)$$

if *at t the agent has the belief that food is present at position P*
then *if at some t'' ≥ t the agent observes the opportunity to go to position P,*
then at some time point t' ≥''t the agent will go to position P

The past to future relationship

$$\varphi(\gamma, t) \Rightarrow \psi(\gamma, t)$$

if *at some time t' <t the agent observed food at position P*
and *from t' to t, it did not observe that no food was present at position P*
then *if at some t'' ≥ t the agent observes the opportunity to go to position P,*
then at some time point t' ≥''t the agent will go to position P

Formalisation can be done as a variant of the one in Section 6.1.2 above. For this case, $\varphi(\gamma, t)$ is the past statement

$$\exists t1 \leq t\ [\ \text{state}(\gamma, t1) \models \text{observation\_result(food\_present\_at(P))}\ \&\ \forall t2\ [\ t1 \leq t2 \leq t \Rightarrow \neg\ \text{state}(\gamma, t2) \models \text{observation\_result(not\_food\_present\_at(P))}\ ]]$$

Moreover, $\psi(\gamma, t)$ is the same future statement as before

$$\forall t2 \geq t\ [\ \text{state}(\gamma, t2) \models \text{observation\_result(opportunity\_to\_go\_to(P))} \Rightarrow \exists t3 \geq t2\ \ \text{state}(\gamma, t3) \models \text{to\_be\_performed(go\_to(P))}\ ]$$

6.2.3 Interactivist Temporal Factorisation Based on a Trust State

Another example illustrating temporal factorisation in the temporal-interactivist approach is the concept 'trust'. This is an example of a mental state property that heavily relies on histories of experiences; e.g. (Jonker and Treur, 1999; Jonker, Schalken, Theeuwes, and Treur, 2004). By abstracting from these histories in the form of a trust state, the future dynamics can be described on the basis of the present mental state in a simple manner. To illustrate this, consider the following example, concerning agent A and a specific shop. The behaviour of agent A considered is as follows:

- agent A can go to the shop or avoid it
- when meeting somebody, agent A can tell that it is a bad shop or that it is a good shop

The following types of events determine the behaviour of agent A.

negative events:
- an experience that a product bought in this shop was of bad quality
- somebody else tells A that it is a bad shop
- passing the shop, A observes that there are no customers in the shop

positive events:
- an experience that a product bought in this shop was of good quality
- somebody else tells A that it is a good shop
- passing the shop, A observes that there are customers in the shop

Assume for the sake of simplicity that only the last two experiences count for the behaviour of A, and that the past pattern *a* considered are histories in which the last two experiences are negative events. The future pattern *b* considered are the futures in which the agent avoids the shop and, when meeting somebody tells that it is a bad shop. It is assumed that past pattern *a* leads to future pattern *b*.

For this case the past pattern *a* can be taken to be the past statement $\varphi(\gamma, t)$:

$$\exists t1 < t2 \leq t\ [\ \text{state}(\gamma, t1) \models \text{observation\_result(e1)}\ \&\ \text{state}(\gamma, t2) \models \text{observation\_result(e2)}\ \&\ \text{negative(e1)}\ \&\ \text{negative(e2)}\ \&\ \forall t3\ \ t1 \leq t3 \leq t \Rightarrow \neg \exists e3\ [\ \text{positive(e3)}\ \&\ \text{state}(\gamma, t3) \models \text{observation\_result(e3)}\ ]$$

Moreover, for the future pattern *b*, take the future statement $\psi(\gamma, t)$:

$$\exists t1 \geq t\ \ \text{state}(\gamma, t1) \models \text{avoiding\_shop}\ \&\ \forall t2 \geq t\ [\ \text{state}(\gamma, t2) \models \text{meeting(A, B)} \Rightarrow \exists t3 \geq t2\ \ \text{state}(\gamma, t3) \models \text{speaking\_bad\_about\_shop\_to(A, B)}\ ]$$

Temporal factorisation of

$$\forall \gamma, t\ [\ \varphi(\gamma, t) \Rightarrow \psi(\gamma, t)\ ]$$

for this case is obtained by the following temporal relational specifications of the trust state

$$\forall\gamma, t\ [\ \varphi(\gamma, t) \ \Rightarrow\ \text{state}(\gamma, t) \models \text{trust(shop, negative)}\ ]\ \ \&$$
$$\forall\gamma, t\ [\ \text{state}(\gamma, t) \models \text{trust(shop, negative)} \ \Rightarrow\ \psi(\gamma, t)\ ]$$

6.3 Temporal Factorisation and Dynamical Systems Theory

Dynamics in domains such as Physics, Chemistry, and Biology, has been addressed in history by the development of the Dynamical Systems Theory (DST). In recent times, it has been proposed to apply the DST approach to cognition as well (e.g., Port and van Gelder, 1995). One of the assumptions underlying DST is the assumption on state-determined systems (cf. van Gelder and Port, 1995; Ashby, 1952). In this section the state-determined system assumption is discussed in relation to the temporal factorisation principle. In section 7 it is analysed in more detail how in quantitative domains, where states are described by continuous variables, DST relates to the temporal factorisation principle.

Van Gelder and Port (1995), following Ashby (1952) explain what a dynamical system is in the following manner. A *system* is a set of changing aspects (or state properties) of the world. A *state* at a given point in time is the way these aspects or state properties are at that time; so a state is characterised by the state properties that hold. The set of all possible states is the *state space*. A *behaviour* of the system is the change of these state properties over time, or, in other words, a succession or sequence of states within the state space. Such a sequence in the state space can be indexed, for example, by natural numbers (*discrete* case) or real numbers (*continuous* case), and can also be called a *trace* or *trajectory*. Following Ashby, such a system is *state-determined* if:

> 'A system is state-determined only when its current state always determines a unique future behaviour. Three features of such systems are worth noting.
> First, in such systems, the future behaviour cannot depend in any way on whatever states the system might have been in *before* the current state. In other words, past history is irrelevant (or at least, past history only makes a difference insofar as it has left an effect on the current state).
> Second, the fact that the current state determines future behaviour implies the existence of some *rule of evolution* describing the behaviour of the system as a function of its current state. (…)
> Third, the fact that future behaviours are uniquely determined means that state space sequences can never fork.'
> (Gelder and Port, 1995, p. 6)

According to some, a dynamical system is just a state-determined system (Giunti, 1995). For some others a dynamical system is a state-determined system for which the state properties are described by assignments of numerical values to a given set of variables (van Gelder and Port, 1995). Ashby (1960), expresses the heuristics based on state-determined systems as follows:

> 'Because of its importance, science searches persistently for the state-determined. As a working guide, the scientist has for some centuries followed the hypothesis that, given a set of variables, he can always find a larger set that (1) includes the given variables, and (2) is state-determined. Much research work consists of trying to identify such a larger set, for when it is too small, important variables will be left out of account, and the behaviour of the set will be capricious. The assumption that such a larger set exists is implicit in almost all science, but, being fundamental, it is seldom mentioned explicitly.' (Ashby, 1960, p. 28).

Ashby refers to Temple (1942) and Laplace (1825) to support his claims. He distinguishes phenomena at a macroscopic level for which his claim is assumed to hold from phenomena at the atomic level, for which the claim turns out not to hold.

> 'Temple, though, refers to '… the fu ndamental assumption of macrophysics that a complete knowledge of the present state of a system furnishes sufficient data to determine definitely its state at any future time or its response to any future influence.' Laplace made the same assumption about the whole universe when he stated that, given its state at one instant, its future progress should be calculable. The definition given above makes this assumption precise and gives it in a form ready for use in the later chapters.
> The assumption is now known to be false at the atomic level. We, however, will seldom discuss events at this level; and as the assumption has proved substantially true over great ranges of macroscopic science, we shall use it extensively.'
> (Ashby, 1960, p. 28).

Thus, according to Ashby, following Temple and Laplace, a main question for a scientist is how to obtain an appropriate state ontology such that based on this ontology for a certain state it can be found out how it is going to change to a different state, according to a certain rule of evolution. The hypothesis is that such a state ontology always can be found. At first sight, this seems to be close to the consequent

$$\exists\, p\ \ a \rightarrow p\ \ \&\ \ p \rightarrow b$$

of the temporal factorisation principle, especially in the claim that certain state properties exist. However, in Ashby's formulation much emphasis is put on the relationship $p \rightarrow b$, almost remaining silent about how p is brought about based on past events. Therefore it might be more fair to state that his position is expressed most sincerely by only part of the consequent:

$$\exists\, p\ \ p \rightarrow b$$

In contrast to Ashby's bias on the 'present to future' relationship, in the formulation of the consequent of the temporal factorisation principle an equal balance between past and future has been achieved.

A second difference between Ashby's state-determined system assumption and the temporal factorisation principle is the conditional $a \rightarrow b$ used in the latter. This conditional may also be expected to be a silent assumption in Ashby's view.

The temporal factorisation principle makes this assumption explicit in the form of a conditional. This conditional makes a crucial difference in the sense that temporal factorisation does not assume a deterministic system, whereas Ashby's notion of state-determined system is deterministic, and therefore his notion is more limited (see also the discussion about determinism at the end of Section 2.2).

7 Temporal Factorisation for Continuous State Properties

The analysis of how the temporal factorisation principle relates to DST, as shown in Section 6.3, was made at a conceptual level, abstracting from how exactly state properties are shaped. In DST states are usually described by continuous variables and the assignment of numerical values to them, and techniques are exploited from the area of calculus within Mathematics, in particular difference and differential equations. In this section it is analysed in more detail how in such cases DST relates to temporal factorisation, and how the temporal relationships based on continuous variables as involved in DST can be specified as temporal relational specifications.

From a more general perspective, it is shown how the conceptual framework based on temporal factorisation can be used in the analysis of dynamics for continuous state properties *x* (as an example, patterns in positions in space in past and future as well as a mediating state property between them may be taken in mind; however, the description is more general). Furthermore, it is shown how based on this mathematical analysis the introduction of a number of concepts of Physics can be understood from the conceptual framework based on the temporal factorisation principle: velocity, momentum, force.

A comparable perspective is put forward by Nagel (1961) who claims that as an outcome of his analysis, state properties such as instantaneous accelerations and velocities 'presuppose nothing more' than state properties concerning spatial relations at certain (other) time points, since they can be defined in terms of mathematical operations on these state properties:

> '..the notion of instantaneous acceleration (..) is defined as the limit of a series, each of whose terms is the ratio of the difference of two instantaneous velocities and a time interval; and an instantaneous velocity is defined as the limit of a series, each of whose terms is the ratio of a distance along a straight line and a time. (..) in any event instantaneous accelerations and velocities presuppose nothing more than certain mathematical operations upon the measures of spatial and temporal relations.' (Nagel, 1961, p.167)

In the spirit of Nagel's analysis, in this section, a more detailed analysis is made of how, in retrospect, the notion of the derivative (e.g., velocity or change rate) of a continuous variable can be viewed as a mediating state property, and can be described by relational specifications. First, to get the idea, this is done in a simplified discrete case (Section 7.1), second the continuous case involving limits is discussed, in Section 7.2. Note that the analysis is done for a continuous variable x in general. In Section 7.3 it is shown how this can be applied in the context of Physics, for x denoting the position of an object, and the mediating state property velocity, or for x denoting momentum, and the mediating state property the force on the object.

7.1 Simplified Discrete Analysis

For a simplified analysis, for any $t' < t$ and value w

$$(x(t') - x(t)) / (t' - t) = w \;\rightarrow\; p(t) = w$$

is a temporal relational specification in the form of a 'past pattern implies present state' relationship $a \rightarrow p$ of mediating state property $p(t)$ at t. More specifically, the left hand side

$$(x(t') - x(t)) / (t' - t) = w$$

characterizes the set of all traces satisfying the past pattern that the difference quotient for x between t' and t is w. The right hand side characterizes all traces satisfying the criterion for the present state at time point t expressed by the state property that state parameter p has value w. The implication expresses that any trace in the former set (i.e., satisfying the past pattern that the difference quotient for x from t' to t is w) is also in the latter set (i.e., satisfies the present state property that p has value w).[13]

In a similar way, for any $t'' > t$ and value w

$$p(t) = w \;\rightarrow\; (x(t'') - x(t)) / (t'' - t) = w$$

is a temporal relational specification in the form of a 'present state implies future pattern' relationship $p \rightarrow b$ of mediating state property $p(t)$ at t.

[13] Note that in terms of the formalisation described in Section 5, this can be expressed in the language TTL, as follows (the specification of 'present state implies future pattern' is similar).

- past pattern φ(γ, t):
  ∃u, u' state(γ, t) |= has_value(x, u) &
  state(γ, t') |= has_value(x, u') & t'<t & (u' - u) / (t' - t) = w
- present state criterion:
  state(γ, t) |= has_value(p, w)
- past pattern implies present state:
  ∃u, u' state(γ, t) |= has_value(x, u) &
  state(γ, t') |= has_value(x, u') & t'<t & (u' - u) / (t' - t) = w
  ⇒ state(γ, t) |= has_value(p, w)

The following 'past pattern implies future pattern' relationship $a \rightarrow b$ can be formulated[14], where $t' < t < t''$:

$$(x(t') - x(t)) / (t' - t) = w \;\rightarrow\; (x(t'') - x(t)) / (t'' - t) = w$$

This expresses that the differences in values of variable x before t and after t are in proportion, which can be considered a kind of smoothness condition. So, for this discrete perspective, the temporal factorisation principle expresses the mathematical fact that if the differences in values of variable x before t and after t are in proportion (the past to future relationship), then at t a value w for the variable p exists, such that this value on the one hand relates to the differences of x in the past (past to present relationship) and on the other hand to differences of x in the future (present to future relationship). In the next subsection this will be made more precise from the continuous perspective.

7.2 Analysis Based on Limits

A more advanced analysis of velocity or change rate in a continuous process involves the notion of limit. A temporal relational specification in the form of a 'past pattern implies present state' relationship $a \rightarrow p$ is given by

$$\lim_{t'\uparrow t} ((x(t') - x(t)) / (t' - t)) = w \;\rightarrow\; p(t) = w$$

which relates past state properties at $t' < t$ to the mediating state property at t. Here, the right hand side expressed a criterion for the present state, as in Section 7.1. The left hand side expresses a past pattern using $\lim_{t'\uparrow t}$, which means the upward limit (i.e., restricted to $t' < t$); in mathematical formalisation this past pattern is characterized by:

$$\forall\varepsilon{>}0\ \exists\delta{>}0\ \forall t'\ \ [\ 0 < t - t' < \delta \;\rightarrow\; |\ ((x(t') - x(t)) / (t' - t)) - w\ | < \varepsilon\ ]$$

This expression[15] describes the past pattern a satisfied by all traces for which the graph of the values for x over time has a tangent from the left hand side at t with slope w. In a mathematical sense this condition expresses that the function x from t is left differentiable with derivative w.

Similarly, a temporal relational specification in the form of a 'present state implies future pattern' relationship $p \rightarrow b$ is given by

$$p(t) = w \;\Rightarrow\; \lim_{t''\downarrow t} ((x(t'') - x(t)) / (t'' - t)) = w$$

which relates the mediating state property at t to future state properties at $t'' > t$. Here the right hand side expresses a future pattern using $\lim_{t''\downarrow t}$, which means the downward limit (i.e., restricted to $t'' > t$); in mathematical formalisation this future pattern is characterized by:

$$\forall\varepsilon{>}0\ \exists\delta{>}0\ \forall t''\ \ [\ 0 < t'' - t < \delta \;\Rightarrow\; |\ (x(t'') - x(t)) / (t'' - t) - w\ | < \varepsilon\ ]$$

This expression[16] describes future pattern b satisfied by all traces for which the graph of the values for x over time has a tangent from the right hand side at t with slope w. In a mathematical sense this condition expresses that the function x from t is right differentiable with derivative w.

The temporal relational specifications in the form of a 'past pattern implies future pattern' relationship $a \rightarrow b$ are as follows:

$$\lim_{t'\uparrow t}\ (x(t') - x(t)) / (t' - t) = w \;\Rightarrow\; \lim_{t''\downarrow t}\ (x(t'') - x(t)) / (t'' - t) = w$$

In mathematical formalisation[17] of $\lim_{t'\uparrow t}$ and $\lim_{t''\downarrow t}$:

[14] Note that this 'past pattern implies future pattern' can be expressed in the language TTL by
∃u, u' state(γ, t) |= has_value(x, u) &
state(γ, t') |= has_value(x, u') & (u' - u) / (t' - t) = w ⇒ ∃u, u'' state(γ, t) |= has_value(x, u) &
state(γ, t'') |= has_value(x, u'') & (u'' - u) / (t'' - t) = w

[15] In terms of the formalisation described in Section 5 based on the language TTL the 'past pattern implies present state' relationship can be expressed as follows. The past formula φ(γ, t) is given by
∀ε>0 ∃δ>0 ∀t',v',v [0 < t - t' < δ & state(γ, t) |= has_value(x, v) &
state(γ, t') |= has_value(x, v') ⇒
w - ε < (v' – v) / (t' – t) < w + ε]
The present state formula is given as before by has_value(p, w). Then the temporal 'past pattern implies present state' relationship $a \rightarrow p$ takes the form:
[∀ε>0 ∃δ>0 ∀t' v',v
[0 < t - t' < δ & state(γ, t) |= has_value(x, v) &
state(γ, t') |= has_value(x, v') ⇒ w - ε < (v' – v) / (t' – t) < w + ε]
⇒ state(γ, t) |= has_value(p, w)]

[16] In terms of the formalisation based on TTL, this 'present state implies future pattern' relationship can be expressed as follows. The future formula ψ(γ, t) is given by:
∀ε>0 ∃δ>0 ∀t'' , v'',v
[0 < t'' - t < δ & state(γ, t) |= has_value(x, v) &
state(γ, t'') |= has_value(x, v'') ⇒
w - ε < (v'' – v) / (t'' – t) < w + ε]
As before, the present state formula is given by has_value(p, w). Then the temporal 'present state implies future pattern' relationship $p \rightarrow b$ takes the form:
state(γ, t) |= has_value(p, w) ⇒
∀ε>0 ∃δ>0 ∀t'', v'',v [0 < t'' - t < δ & state(γ, t) |= has_value(x, v) & state(γ, t'') |= has_value(x, v'') ⇒
w - ε < (v'' – v) / (t'' – t) < w + ε]

[17] In terms of the TTL formalisation, this 'past pattern implies future pattern' relationship φ(γ, t) ⇒ ψ(γ, t) takes the form:
[∀ε>0 ∃δ>0 ∀t',v',v [0 < t - t' < δ & state(γ, t) |= has_value(x, v) & state(γ, t') |= has_value(x, v') ⇒
w - ε < (v' – v) / (t' – t) < w + ε]] ⇒
[∀ε>0 ∃δ>0 ∀t'', v'',v [0 < t'' - t < δ & state(γ, t) |= has_value(x, v) & state(γ, t'') |= has_value(x, v'') ⇒
w - ε < (v'' – v) / (t'' – t) < w + ε]]

$$
\begin{array}{l}
\forall w \\
\quad [\ \forall \varepsilon > 0\ \exists \delta > 0\ \forall t'\ \ [\ 0 < t - t' < \delta \ \Rightarrow \\
\qquad\qquad | (x(t') - x(t)) / (t' - t) - w | < \varepsilon\ ]\ \Rightarrow \\
\quad \forall \varepsilon > 0\ \exists \delta > 0\ \forall t''\ \ [\ 0 < t'' - t < \delta \ \Rightarrow \\
\qquad\qquad | (x(t'') - x(t)) / (t'' - t) - w | < \varepsilon\ ]\ ]
\end{array}
$$

This 'past pattern implies future pattern' relationship expresses a mathematical smoothness condition on the function of x depending on t. Roughly spoken it states that the slope of the graph of x depending on t is, in a (small) past interval for a given t is the same as the slope of the graph at a (small) future interval for t. Within mathematics, a standard example of a nonsmooth function is the absolute value function $x(t) = |t|$ considered at $t = 0$; it is not smooth at $t = 0$ because the slope left of $t = 0$ is -1 whereas the slope right of $t = 0$ is $+1$.[18] For this function, as a consequence of the lack of smoothness, in $t = 0$ no derivative exists.

Under the smoothness condition as expressed, a common value w exists that connects past and future. This value provides the existence of a mediating state property in the state at time t relating both to past and future. From a mathematical perspective, the smoothness condition is precisely the condition under which the (both left and right) derivative of x as a function of t exists at time t (i.e., the implication from the smoothness condition to the existence of the derivative in the state at t is a mathematical theorem).

From the perspective of the temporal factorisation principle, application of this principle precisely yields the existence of a mediating state property for time t that plays the role of a both left (past to present relationship) and right (present to future relationship) derivative at t. In other words, in this setting, application of the temporal factorisation principle postulates, if the smoothness condition is fulfilled, the existence of a mediating state property as a state property at time t relating to past and to future as relationally specified above, which specification is equal to the specification of a both left and right derivative at t. Therefore, application of the temporal factorisation principle in a sense entails the mathematical theorem that if the smoothness condition is fulfilled, then at t a derivative exists for the function x of t (i.e., the function is differentiable at t).[19]

7.3 Velocity, Momentum and Force in Physics as Mediating State Properties

This analysis of a continuous state variable and its derivative as a mediating state property can be applied to obtain the concepts velocity, momentum, and force in Physics by application of the temporal factorisation principle. First, if for the continuous variable x the position (on a line) of an object with mass m is taken, then the temporal factorisation provides the mediating state property dx/dt, which is the velocity of the object. Momentum of the object is obtained by $p = mv$, or by temporal factorisation of the variable mx. Furthermore, Newton's second law $F = ma$ (with a the acceleration) can be fomulated as

$$m\, d^2x/dt^2 = F$$

or

$$m\, dv/dt = F$$

or

$$dp/dt = F$$

This shows in more detail (compared to Section 3) how force can be obtained as a second-order mediating state property. For more details about the historic analysis, see Treur (2005).

8 Discussion

The more popular, physicalist views on cognition in Philosophy of Mind, consider cognition as a phenomenon of nature. A challenge then is to relate principles behind cognition to principles in nature, or even to search for common principles. One of the well known arguments against (a too bold form of) physicalism adresses this issue to the negative by claiming that some of the laws behind cognition (such as coherence of beliefs) are not corresponding to physical laws (cf. Davidson, 1993, on the nonexistence of psychophysical laws; see also Kim, 1996, pp. 132-139). Notwithstanding, from the physicalist perspective, the challenge remains to find out why and how the principles behind nature give rise to such special effects as cognition. How can physical architectures, functioning on the basis of principles valid in the physical world, show cognition; which principles make that possible?

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[18] The smoothness condition is violated in any case that a common number a does not exist, whereas still one value w_1 for the past and one w_2 (a distinct one) for the future In other words, it may be the case that for the first clause above a value w_1 exists and for the second clause a value w_2 such that both clauses hold, but with w_1 not equal to w_2:

$$
\begin{array}{l}
\exists w_1, w_2\ [\ \forall \varepsilon > 0\ \exists \delta > 0\ \forall t'\ \ [\ 0 < t - t' < \delta \ \Rightarrow \\
\qquad\qquad | (x(t') - x(t)) / (t' - t) - w_1 | < \varepsilon\ ]\ \& \\
\qquad \forall \varepsilon > 0\ \exists \delta > 0\ \forall t''\ \ [\ 0 < t'' - t < \delta \ \Rightarrow \\
\qquad\qquad | (x(t'') - x(t)) / (t'' - t) - w_2 | < \varepsilon\ ]\ ]
\end{array}
$$

In such a case, the value w_2 has a specified effect (by the second clause) on future states, but the origin of this mediating state property in the past is different from the one specified by clause 1. Therefore, it can not be considered a mediating state property. In this case apparently some novel influence or interaction occurred at time t, which did not play a role at any time point $t' < t$. Also, the value w_1 does not count as a mediating state property because it has not the specified relationship to state properties occurring in future states.

[19] Within application domains, the smoothness condition incorporates the assumption that the change proportion (shortly) before t persists (shortly) after t.

8.1 The Temporal Factorisation Principle

A central principle was identified and discussed, which deals with dynamics both in the physical world and in cognitive processes. From a historic perspective, this temporal factorisation principle seems rather fundamental in scientific development (e.g., the development of areas within Mathematics and Physics such as calculus, differential equations and classical mechanics). It postulates the existence of mediating state properties that can be used to decompose any temporal 'past pattern implies future pattern' relationship into two simpler temporal relationships: a 'past pattern implies present state' relationship and a 'present state implies future pattern' relationship.

In this paper, in addition, a formalisation of this temporal factorisation principle was put forward. The temporal factorisation principle has been shown to be a basic assumption underlying standard approaches to dynamics in disciplines such as Physics, Chemistry, and Biology, but also in Mathematics, Computer Science, and Cognitive Science; such approaches include Dynamical Systems Theory (DST), transition systems and functionalist approaches to cognition.

In the context of DST it was shown how the principle relates to the notion of state-determined system (cf. Ashby, 1952; van Gelder and Port, 1995). Furthermore, it was shown how the principle is exploited in Mathematics (calculus). In particular, it was shown how application of the temporal factorisation principle in a sense entails the mathematical theorem that if a smoothness condition for a function is fulfilled around some time point t, then at t a derivative exists for this function (i.e., the function is differentiable at t). Furthermore, it was shown to be a basic assumption underlying different approaches in Cognitive Science: besides the Dynamical Systems approach, also functionalist approaches and the interactivist approach (cf. Kim, 1996, Bickhard, 1993).

8.2 The Temporal Factorisation Principle vs the Locality Principle in Physics

Within Physics, the *principle of locality* claims that objects can only have direct influence on one another when they are close: an object is not influenced directly by objects not in its immediate surroundings; e.g., Einstein (1948)[20]. This means that when distant objects influence each other, this can only happen in an indirect manner, for example, via some causal chain where each of the causal steps takes place in one local environment. This has in common with the temporal factorisation principle, that it also shows a form of temporal interpolation.

However, the locality principle in Physics is meant for the physical context only, and explicitly takes locality aspects of states and events into account (as, indeed, the name of the principle already indicates), whereas the temporal factorisation principle has a much wider scope, beyond the area of Physics, and abstracts from any locality aspects of state properties, and therefore is much more general in that respect. This shows that the temporal factorisation principle is not implied by the locality principle in Physics.

In how far, on the other hand, the locality principle and its formalisation put forward here, can be seen as a specialisation of the temporal factorisation principle (maybe in iterated form, as described in Section 3) in the formal sense is an interesting question that requires some more research. One issue here is that by applying the temporal factorisation principle in its general form, an intermediate state is found, but since no locality information for the mediating state property is provided, the intermediate state may even be more remote than the original starting point. So it is not simply the case that the temporal factorisation principle implies the locality principle.

Another interesting issue is: for a given case of remote influence, how fine-grained should the intermediate steps postulated by the temporal factorisation principle be taken to obtain a situation that every single step can be viewed as local?

8.3 What the Temporal Factorisation Principle Unifies

As the temporal factorisation principle was shown to unify in one conceptual framework various approaches to the dynamics in different disciplines, it can be used as a conceptual framework to analyse quite a large variety of dynamic phenomena. More specifically, the perspective put forward in this paper addresses within one conceptual framework the following aspects that are often addressed separately:

- dynamics of phenomena in various disciplines; in particular, of cognitive and noncognitive phenomena
- the past view in relation to the present and the future view in relation to the present
- deterministic approaches and nondeterministic approaches

[20] 'The following idea characterises the relative independence of objects far apart in space (A and B): external influence on A has no direct influence on B; this is known as the Principle of Local Action, which is used consistently only in field theory.' Einstein (1948).

Unification with respect to cognitive and noncognitive phenomena

The factorisation principle has been shown to relate to views on the dynamics of the physical world, and equally well to dynamics of cognitive phenomena. For the cognitive domain, the principle subsumes the notion of a mental state in relation to histories and futures of an agent. For the domain of Physics, for example, a number of concepts that are crucial in the area of classical mechanics are postulated by the temporal factorisation principle.

This unification of the cognitive and the noncognitive shows throughout the paper. Many times the focus switched from dynamics of the physical world to dynamics of cognition and conversely, thereby keeping and further developing the same generic conceptual framework. For example, in Section 2 dynamics in the physical world was addressed first (Descartes and Laplace's universe), but after that also the dynamics of behaviour in relation to a desire state was discussed. Section 3 starts by focussing on mental state properties such as desires and intentions, but returns to the physical area again, addressing, for example the concept force. Section 4 adresses concepts and methods from the cognitive domain (representational content of a mental state property), which are applied in a generalised form beyond the cognitive domain in Section 5. Similarly in the remaining sections the focus was going back and forth between the cognitive and the noncognitive domain, in the meantime further developing the unifying conceptual framework.

It may be argued that by van Gelder and Port (1995) it is put forward that also the Dynamical Systems Theory (DST) provides a unifying framework for cognitive and noncognitive domains. This unification is at a different description level, however. In the way it is put forward, DST is at the more specific level of a given mathematical modelling approach (which, for example, has some limitations for modelling higher-level cognition): mainly difference and differential equations. The unifying conceptual framework discussed here lies at a more foundational level. It is underlying DST, but also other types of modelling approaches, such as more logical Belief-Desire-Intention (BDI) modelling or transition systems.

Unification with respect to the past view and the future view

The picture shows that when occurring in some state, a mediating state property, as postulated by the temporal factorisation principle, plays two roles at the same time:

- as an adequate *summary of the past*, compiling aspects from the past that are relevant for the future, thereby abstracting from irrelevant detail, and
- as a *synopsis for the future*, indicating what state properties will occur in a future state, and under which conditions

The combination of these two roles makes that mediating state properties connect past and future in the present state in an effective manner. Looking from the past, instead of dealing with each history separately, in order to find properties of the future, a mediating state property provides (in the present state) a convenient substitute of a whole set of histories: a set that collects all histories that have a specific effect on the future in common. Looking from the future a similar view obtains: the present state with its mediating state properties provides a substitute for the set of futures that have a certain history pattern in common. In this sense the principle shows a symmetric treatment of past and future.

In other approaches, this is not always the case. For example, the historic analysis of literature about potentialities or anticipatory state properties to explain dynamics, as given in (Treur, 2005), shows that this literature usually focuses on the present-to-future relationship thereby more or less neglecting the description of the past-to-present relationship. As an example, in the notion of state-determined system described by Ashby (1952) and van Gelder and Port (1995), this unbalance is visible in the emphasis on the rule of evolution from present to future states (see also Section 6.3 above).

Also in the cognitive domain the two views (past and future) are often treated in isolation. For example, some mental states such as sensory representations are considered from the past view (how such a sensory representation is created), whereas some other types of mental states (e.g., affective or motivational states) are considered from the future view. For example, an intention is related to the corresponding future action that will take place as soon as the opportunity is there. The temporal factorisation principle unifies these two views on mental states.

In the paper the notion of representational content as known from literature on Philosophy of Mind was one the one hand generalised beyond the usual cognitive area, and on the other hand was extended to a two-sided notion, relating to past and future at the same time. More specifically, it was shown how Kim (1996)'s relational specification approach to representational content of mental state properties can be extended to this more general case of mediating state properties, and thereby taking into account both past and future.

Unification of deterministic and nondeterministic approaches

In contrast to deterministic views as those from, for example, Descartes (1644), Laplace (1825) and Ashby (1952), due to the conditional formulation, the temporal factorisation principle can be used both for approaches assuming determinism and approaches assuming nondeterminism. For a more extensive discussion about (non)determinism and how it can be considered from an external God's eye perspective or an internal agent perspective, see, for example, Earman (1986), Dennett (2003, pp. 25-96).

The temporal factorisation principle takes into account as a premise the existence of some 'past pattern implies future pattern' relationships. A description of the world based on such past to future relationships can take any position on the scale from nondeterminism or incomplete determinism to complete determinism: for example, a specification of dynamics by 'past pattern implies future pattern' relationships can vary from a specification of more partial determination of the future by the past, to a specification of more complete determination of the future by the past.

8.4 What is Gained by Mediating State Properties

If the number of allowed (additional) state properties is arbitrary, then there is one trivial way to obtain mediating state properties, namely by introducing a new mediating state property p_h for each specific history h, and thus encoding the history in the present state by postulating this new state property p_h for the specific history h at hand. However, although in theory this is possible, in practice such a trivial trick will not be of help, due to the combinatorial complexity of such a solution. Temporal factorisation will only work in a practical method if it is assumed that only a limited number of state properties for the present state can be used to relate them to relevant sets of histories and relevant sets of futures. These state properties relate to characterising patterns for the relevant sets of histories and futures as a whole, instead of considering each of the histories and futures separately.

How well this works depends on how homogeneous such a set of histories or futures is. If the description of the pattern for the past and/or for the future is only possible as a disjunction of a large number of cases, this gives a heterogeneous situation that may still be not simple to handle. For classical mechanics it seems to work as a result of nature's dynamics, as the effectiveness of classical mechanics in applications shows. In the cognitive domain, to describe behaviour, using such a factorisation by mental state properties, it seems a reasonable option as well.

Note that for the notion of force in classical mechanics the future pattern has a rather homogeneous characterisation: the effect of a force on an object. However, the past pattern is not homogeneous at all; it seems a disjunction of a large number of possibilities to change the world in a way that a certain force on a certain object is exerted (e.g., pulling or pushing, electrical force, gravitation, magnetic force, deformation force); cf. Nagel (1961). Yet the area of classical mechanics was quite successful, probably mainly because each of the disjuncts of the past pattern was developed separately so that more specific context-dependent patterns became possible (in a way comparable to Kim (1996)'s local or context-dependent reduction). This shows how even in more heterogeneous cases a succesful approach can be developed.

8.5 About the Status of Mediating State Properties

The temporal factorisation principle postulates the existence of mediating state properties. What does this step of postulating mean? Does this mean that such properties were already present in a given state ontology, and these existing state properties are just given the role of mediating state properties? Or were they not present yet and just added to obtain an extension of the state ontology? Both is possible. During development of a theory such as classical mechanics, for example, notions such as momentum or instantanous velocity were added to extend the state ontology as mediating state properties based on temporal factorisation. Once such state properties have been added, they already are there for next applications of the temporal factorisation principle.

An interesting question in practice, however, is in how far such added state properties relate to already existing state properties. In case a first-order differential equation is given for some velocity or change rate, for example, such a relation for the concept of change rate can be found on the basis of this differential equation: this equation just expresses how the change rate relates to other state properties. This issue will be further addressed in a next paper on realisation of mediating state properties.

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References

Ashby, W.R. (1952). *Design for a Brain*. Chapman & Hall, London. Revised edition 1960.

Bickhard, M.H. (1993). Representational Content in Humans and Machines. *Journal of Experimental and Theoretical Artificial Intelligence*, vol. 5, pp. 285-333.

Bickhard, M.H. (2000). Information and representation in autonomous agents. *Journal of Cognitive Systems Research*, vol. 1, pp. 285-333.

Davidson, D. (1993), Thinking Causes. In: J. Heil & A. Mele (eds.), *Mental Causation*. Clarendon Press, Oxford.

Dennett, D.C. (2003), *Freedom Evolves*. Penguin Books.

Descartes, R. (1634). *The World or Treatise on Light*. Translated version by M.S. Mahoney: URL: http://www.princeton.edu/~hos/mike/texts/descartes/world/world.htm

Descartes, R. (1998). *The World and Other Writings*. (edited by S. Gaukroger) Cambridge Texts in the History of Philosophy. Cambridge University Press.

Descartes, R. (1644). *Principles of Philosophy* Translation by V.R. Miller, R.P. Miller, Reidel Publisher, 1983.

Earman, J. (1986). *A Primer on Determinism*. Reidel Publishing Company, Kluwer, Dordrecht.

Einstein, A. (1948). Quantum Mechanics and Reality (Quanten-Mechanik und Wirklichkeit), *Dialectica* vol. 2, 1948, pp. 320-324.

Gelder, T.J. van (1999) Defending the dynamical hypothesis. In W. Tschacher & J.-P. Dauwalder (ed.), *Dynamics, Synergetics, Autonomous Agents: Nonlinear Systems Approaches to Cognitive Psychology and Cognitive Science*. Singapore: World Scientific, pp. 13-28

Gelder, T.J. van, and Port, R.F., (1995). It's About Time: An Overview of the Dynamical Approach to Cognition. In: Port and van Gelder, 1995, pp. 1-43.

Giunti, M. (1995). Dynamical Models of Cognition. In: (Port and Gelder, 1995), pp. 549-572.

Jonker, C.M., Schalken, J.J.P., Theeuwes, J., and Treur, J, (2004). Human Experiments in Trust Dynamics. In: Jensen, C., Poslad, S., & Dimitrakos, T. (eds.), *Trust Management, Proc. of the Second International Conference on Trust Management, iTrust 2004*. Lecture Notes in Computer Science, vol. 2995, Springer Verlag, pp. 206-220.

Jonker, C.M., and Treur, J., (2002). Compositional Verification of Multi-Agent Systems: a Formal Analysis of Pro-activeness and Reactiveness. *International Journal of Cooperative Information Systems*, vol. 11, pp. 51-92.

Jonker, C.M., and Treur, J., (2003). A Temporal-Interactivist Perspective on the Dynamics of Mental States. *Cognitive Systems Research Journal*, vol. 4, pp. 137-155.

Jonker, C.M., Treur, J., and Wijngaards, W.C.A., (2002). Reductionist and Antireductionist Perspectives on Dynamics. *Philosophical Psychology Journal*, vol. 15, pp. 381-409.

Kelso, J.A.S. (1995). *Dynamic Patterns: the Self-Organisation of Brain and Behaviour*. MIT Press, Cambridge, Mass.

Kim, J. (1996). *Philosophy of Mind*. Westview press.

Kim, J. (1998). *Mind in a Physical world: an Essay on the Mind-Body Problem and Mental Causation*. MIT Press, Cambridge, Mass.

Laplace, P.S. (1825). *Philosophical Essays on Probabilities*. Springer-Verlag, New York, 1995. Translated by A.I. Dale from the 5th French edition of 1825.

Nagel, E. (1961). *The Structure of Science*. Routledge and Kegan Paul; Harcourt, Brace and World, London.

Port, R.F., Gelder, T. van (eds.) (1995). *Mind as Motion: Explorations in the Dynamics of Cognition*. MIT Press, Cambridge, Mass.

Putnam, H. (1975). The Nature of Mental States. In: H. Putnam, *Collected Papers II*. Cambridge University Press.

Temple, G. (1942). *General Principles of Quantum Theory*, 2^{nd} ed. Methuen, London, 1942.

Treur, J., (2005). States of Change: Explaining Dynamics by Anticipatory State Properties. *Philosophical Psychology Journal*. *Philosophical Psychology Journal*, vol. 18, 2005, pp. 441-471. Shorter version in: Mira, J., Alvarez, J.R. (eds.), *Proc. of the First International Work-conference on the Interplay between Natural and Artificial Computation, IWINAC' 05* Lecture Notes in Artificial Intelligence, vol. 3561. Springer Verlag, 2005, pp. 27-37.