Temporal Factorisation: Realisation of Mediating State Properties for Dynamics

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Abstract

Temporal factorisation is a principle underlying approaches to dynamics used within many disciplines. According to this principle any temporal relationship of the form 'past pattern implies future pattern' can be factorised into a relationship of the form 'past pattern implies present state' and a relationship of the form 'present state implies future pattern'. To enable this, the principle postulates the existence of certain mediating state properties in the present state. In this paper the question is addressed whether and how a postulated mediating state property relates to other state properties in the (present) state in which they occur. In particular, the situation is analysed that realisers exist: other state properties or combinations thereof that co-occur with the mediating state property in states. This analyses provides a conceptual framework covering various concepts and themes that usually are considered totally different and unrelated, such as, the notion of differential equations in Mathematics, the notions of transition system and rule-based system in Computer Science, Cognitive Science and Artificial Intelligence, and the notion of reduction in Cognitive Science and Philosophy of Science.

1 Introduction

In (Treur, 2007) the temporal factorisation principle was identified, formalised, and shown to play a crucial role for modelling dynamics in different disciplines such as Physics, Chemistry, Biology, Mathematics, Computer Science, and Cognitive Science. The temporal factorisation principle claims that if a certain (past) pattern of events leads to a certain (future) pattern of events, then there exists a state property *p* such that the past pattern leads to a (present) state where this property *p* holds, and any state where the state property *p* holds leads to the future pattern. This postulated state property *p* is called a mediating state property for the 'past pattern implies future pattern' relationship. For some of the foundational themes and approaches discussed in the literature in the cognitive domain, it has been shown in (Treur, 2007) how they can be generalised beyond the cognitive area, and incorporated in the more general conceptual framework based on temporal factorisation; in particular this has been addressed for the notion of mental state, and the notion of representational content of a mental state property. For two other themes, namely, physical realisation of mental state properties, and mental state properties as second-order world properties, it will be considered in the current paper, how they can be generalised beyond the cognitive domain and added to the framework based on the temporal factorisation principle. Thus the conceptual framework for dynamics based on 'temporal factorisation' as introduced in (Treur, 2007) will be extended to a conceptual framework based on 'temporal factorisation and realisation'.

To obtain this extension, the question of realism is addressed for the mediating state properties postulated in temporal factorisation. The corresponding type of question for the cognitive area, as addressed within Philosophy of Mind, namely the question of how real mental state properties are, will be a source of inspiration, in particular, the perspective of *physicalism* which aims at relating mental state properties in one way or the

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other to physical state properties; e.g., (Kim, 1996, pp. 9-13). By itself, the temporal factorisation principle does not give any suggestion on whether and how, within a state, a mediating state property relates to other state properties. It could have a purely synthetic nature, isolated from other, more genuine, state properties. Indeed, for a number of historical cases, the state properties obtained by temporal factorisation seem to have no relationship to other state properties in the same state whatsoever.

This holds, for example, for state properties within Physics in free space such as velocity and momentum (inertia of motion). In other cases, such as the state property ‘force’ within Physics, such relationships to specific other state properties in the same state are assumed to exist (e.g., laws for specific types of forces, such as gravitational, electrical or magnetical forces), and they are often exploited in scientific practice. These relationships are not systematic, however, but differ from context to context, so they are rather heterogeneous, as, for example, pointed out by Nagel (1961), pp. 190-192.

For the cognitive domain, one of the advantages of a strict one-to-one relation between mental state properties and physical state properties is that in such a case causation by a mental state property (mental causation; cf. Kim, 1996, pp. 125-154) can be explained as causation by the realising physical state property. For mediating state properties it can also be questioned how they can cause physical state properties in successive states. A one-to-one relationship of a postulated mediating state property with a more genuine physical state property may provide an explanation of causation by the mediating state property similar to the one in the case of mental causation by a realised mental state property.

In this paper it is shown that for mediating state properties realisation is often but not always possible, and whenever it is possible it may concern context-dependent multiple realisation. So, the first bad news is that sometimes for the state properties postulated by the temporal factorisation principle, only the temporal relationships to properties of states at other points in time are available, and no realisation (in the same state) is possible at all. However, the good news is that historical case studies show that still temporal factorisation can play a quite fruitful role for scientific development. The second bad news is that if realisation is possible, often no unique realiser can be found. It is discussed that this situation also incorporates some good news: it provides the possibility to unify a number of processes into one description, which has genericity as an advantage and provides a higher level explanation of dynamic patterns, abstracting from details that are less essential, and more context-specific.

The paper is structured as follows. Section 2 presents a brief introduction of the temporal factorisation principle and its formalisation, adopted from Treur (2007). Section 3 discusses how mediating state properties can co-occur with other state properties, and introduces the notion of *realisation relation* for a mediating state property. In Section 4 it is shown how within the area of calculus in Mathematics, in quantitative mathematical dynamic modelling approaches based on continuous state properties, realisation relations are expressed in the form of difference equations (discrete time) or differential equations (continuous time). Section 5 shows how for qualitative dynamic modelling approaches based on discrete state properties, such as transition systems in Computer Science or rule-based systems in Cognitive Science or Artificial Intelligence, a certain qualitative format is used to express realisation relations. Section 6 discusses how within Physics realisation relations for the second-order mediating state property ‘force’ play an important role.

In many cases realisation occurs in the form of *multiple realisation*; that is, in different circumstances (or contexts) different state properties are co-occurring with a given mediating state property. The notions ‘supervenience’ and ‘local reduction’, used in Philosophy of Mind (Kim, 1996) to describe multiply realised (mental) concepts are discussed in Section 7 as a possible way to describe a realisation relation between a mediating state property and realisers in case of multiple realisation. In Section 8 it is discussed that the situation of multiple realisation has as advantage that a more *generic* description can be obtained subsuming different variants of phenomena on a higher level of abstraction.

Section 9 discusses the status of the conceptual framework of temporal factorisation. It is clearly not a basic (first-order) law of nature: as it expresses the existence of (mediating) state properties it has a second-order character. It is shown in Section 9 how the temporal factorisation principle itself but also the mediating state properties it postulates in its applications can be viewed as *second-order world properties*. This is shown to be similar to how in Philosophy of Mind mental state properties are considered as second-order properties (e.g. Kim, 1998, pp. 19-21). As an illustration in the area of Physics, inspired by the analysis in Nagel (1961), it is shown how the notion of force can be described as a second-order world property. The description of the temporal factorisation principle principle itself, and of the mediating state properties it postulates in its applications, as a second-order world property unifies the second-order descriptions of mental states in Cognitive Science and of state properties from Physics, such as force. The paper finishes with a discussion in Section 10.

2 Temporal Factorisation and its Formalisation

In this section a brief introduction is given of the temporal factorisation principle and its formalisation.

2.1 The Temporal Factorisation Principle

The temporal factorisation principle is formulated in terms of temporal relationships between past patterns, present states, and future patterns. Here a *past pattern a* refers to a property of a series of states or events in the past, and a *future pattern b* refers to a property of a series of states or events in the future. The temporal factorisation principle states that any systematic temporal 'past pattern implies future pattern' relationship $a \rightarrow b$ between a past pattern a and a future pattern b can be factorised in the form of temporal relationships $a \rightarrow p$ and $p \rightarrow b$ for some state property p of the present world state. More specifically, the principle claims that for any 'past pattern implies future pattern' relationship $a \rightarrow b$ *there exists* a world state property p (expressed in the ontology for state properties) such that temporal relationships 'past pattern implies present state property' $a \rightarrow p$ and 'present state property implies future pattern' $p \rightarrow b$ hold.[2] In short:

$$a \rightarrow b \quad \Rightarrow \quad \exists\, p \quad a \rightarrow p \quad \& \quad p \rightarrow b$$

Notice that the notation → is used here to indicate logical implication (between temporal properties). The postulated state property p is called a *mediating state property* for the given 'past pattern implies future pattern' relationship. The principle claims that the state ontology is (or can be chosen) sufficiently rich to express all the relevant information on the past in some condensed form in one state description, and the same with respect to the future. The principle can be viewed as a way to make temporal complexity of dynamics more manageable by relating it to state complexity, where an underlying assumption is that the state complexity needed can be kept limited.

As an example from Physics, temporal factorisation can be illustrated by the notion 'momentum' of a moving object in classical mechanics as a mediating state property. Different histories of the object can lead to the same momentum in the present state. The future of the object only (besides the object's current position) depends on this momentum in the present state, not on the specific history. This was the criterion by which the concept momentum was introduced in Physics in history (see Treur, 2005, for a more detailed historical case study). Therefore the state property momentum can be understood as a mediating state property for past and future patterns in (change of) position of an object; the temporal factorisation principle indicates the existence of this state property.

The state property momentum abstracts from the various histories that could have happened and would have resulted in the same mediating state property for the future. In the other time direction, no matter what future will arise, a momentum indicates from what pattern it originated, so it abstracts from futures. In some more detail, if for the variable x the position (on a line) of an object with mass m is taken, then the temporal factorisation of temporal relationships between positions before and after a certain time point t provides as a mediating state property the instantaneous velocity (or change rate of position) v of the object at time point t. Momentum of the object is obtained by $p = mv$, or by temporal factorisation of the variable mx. In addition, Newton's second law $F = ma$ (with a the acceleration) can be fomulated $m\, dv/dt = F$ or $dp/dt = F$. This shows how force can be obtained as a second-order mediating state property (the change rate of momentum, which itself is a first-order mediating state property). This shows some examples of temporal factorisation within Physics, in particular in classical mechanics. For more details and more extensive examples, also in the cognitive domain, see Treur (2007).

2.2 Formalisation of Temporal Factorisation

To specify and formalise temporal relationships that play a role in temporal factorisation, an expressive formal language is needed that allows to refer to patterns over time. Furthermore, it should be possible to express the existential quantifier for state properties, which occurs in the temporal factorisation principle. The Temporal Trace Language (TTL) is such a language (Jonker and Treur, 2002). TTL will be briefly introduced here.

The language TTL is based on traces (or trajectories), time points, and state properties as primitive notions. A state can be parameterised by a trace in which it occurs and a time point at which it occurs. The language is built up as follows. A *state ontology* is a specification (in sorted predicate logic) of a vocabulary (i.e., a signature). A state for ontology Ont is an assignment of truth-values {true, false} to the set At(Ont) of ground atoms expressed in terms of Ont. The *set of all possible states* for state ontology Ont is denoted by STATES(Ont). The set of *state properties* STATPROP(Ont) for state ontology Ont

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[2] Sometimes such relationships are simply called 'past to present', present to future', or 'past to future' relationships.

is the set of all propositions over ground atoms from At(Ont).[3]

A fixed *time frame* T is assumed, which is linearly ordered. Depending on the application, the time frame T may be dense (e.g., the real numbers), or discrete (e.g., the natural numbers), or any other form, as long as it has a linear ordering. A *trace* or *trajectory* γ over a state ontology Ont and time frame T is a mapping $\gamma : T \rightarrow$ STATES(Ont) (i.e., a time-indexed set of states γ_t ($t \in T$) in STATES(Ont)). The set of all traces over state ontology Ont is denoted by TRACES(Ont).

The set of *dynamic properties* DYNPROP(Ont) over state ontology Ont is the set of temporal statements that can be formulated with respect to traces based on the state ontology Ont in the following manner. Given a trace γ over state ontology Ont, a state of the world at time point t is syntactically denoted by state(γ, t). These states can be related to state properties via the formally defined (as an infix predicate in TTL syntax) satisfaction relation |=, i.e.: state(γ, t) |= p , which denotes that state property *p* holds in trace γ at time t. Based on these statements, dynamic properties can be formulated in a formal manner in a sorted predicate logic with sorts TIME for time points, TRACES for traces and STATPROP for state formulae, using quantifiers, among others, over time, traces and state formulae, and the usual logical connectives such as $\neg$, &, $\vee$, $\Rightarrow$, $\forall$, $\exists$. Within TTL numbers can be used for time, but also within state properties.

To formalise the temporal factorisation principle, formalisations are needed for the temporal relationships between past patterns, present states and future patterns with respect to a given time point *t*. As a first step it is shown how past patterns and future patterns can be specified. The basic idea is that a pattern refers to a specific set of traces, for example a past pattern refers to a specific set of past traces (up to some time point *t*). The way in which this reference takes place is by expressing a pattern in the form of a (temporal) property that the traces in the set have in common, or, in other words, that characterises this set of traces. To express this property the language TTL is used.

Specification of a Past Pattern

A *past statement* for γ and t is a temporal statement $\varphi(\gamma, t)$ where γ and t are free variables, such that each time variable different from t is restricted to the time interval before t. In other words, for every time quantifier for a variable s a restriction of the form $s \leq t$, or $s < t$ is required within the statement. A *past pattern* is any past statement. A trace γ satisfies a past pattern $\varphi(\gamma, t)$ for t if $\varphi(\gamma, t)$ is true. The set of past statements over state ontology Ont with respect to time point t is denoted by PFOR(Ont, γ, t).

Specification of a Future Pattern

Similarly as the past statements, FFOR(Ont, γ, t) denotes the set of *future statements* over state ontology Ont with respect to trace γ and time point t: γ and t are free variables and every time quantifier for a variable s is restricted by $s \geq t$ or $s > t$. A *future pattern* is any future statement.

Formalisation of the Principle

Using the notions defined above, the temporal factorisation principle over state ontology Ont expresses that for any past and future formulae $\varphi(\gamma, t) \in$ PFOR(Ont, γ, t), $\psi(\gamma, t) \in$ FFOR(Ont, γ, t) with respect to t, for which for any trace γ and time point t

$$\varphi(\gamma, t) \Rightarrow \psi(\gamma, t)$$

holds, there exists a state property $p \in$ STATPROP(Ont) such that for all traces γ and time points t both

$$\varphi(\gamma, t) \Rightarrow \text{state}(\gamma, t) \models p$$
$$\text{state}(\gamma, t) \models p \Rightarrow \psi(\gamma, t)$$

hold, or in concise format:

$$\begin{aligned} &\forall \gamma, t\ [\ \varphi(\gamma, t) \Rightarrow \psi(\gamma, t)\ ] \Rightarrow \\ &\quad \exists p\ [\ \forall \gamma, t\ [\ \varphi(\gamma, t) \Rightarrow \text{state}(\gamma, t) \models p\ ]\ \& \\ &\quad\quad \forall \gamma, t\ [\ \text{state}(\gamma, t) \models p \Rightarrow \psi(\gamma, t)\ ]\ ] \end{aligned}$$

where γ ranges over the sort TRACES, t over sort TIME and p over sort STATPROP.

Notice that for this formalisation the following three features of TTL are crucial: (1) it is based on an expressive first-order language for temporal relationships, (2) traces are first class citizens in the language, which means that variables and quantification over traces are possible, and (3) state properties are first class citizens, which means that variables and quantification over state properties are possible. For example, standard (modal) temporal logics do not have these features. These features will also turn out useful later on, when further quantification over state properties is needed.

3 Realisation of Mediating State Properties

To embed mediating state properties more intensively in the states in which they occur, their relationship to other, more properties of these states is considered. If in states, a mediating state property *p* always co-occurs with a certain state property *c* (or combination of state properties), such a co-ocurring property *c* is called a *realiser* (see also Figure 1; the vertical double arrow indicates the *realisation relation* between *p* and *c*).

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[3] When no confusion is expected, the argument Ont will be left out: STATPROP.

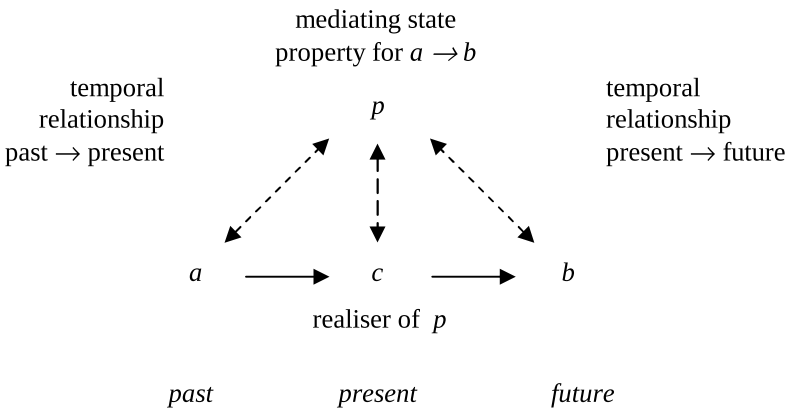


Figure 1. Realisation of a mediating state property

If a mediating state property has a realiser, it can be created or prevented by manipulating the state property that realises the mediating state property. For example, a mental state property can be created or prevented by affecting its realiser by physical means (e.g., by using drugs). Similarly, for example, a mediating state property to fall can be created or prevented by either putting or keeping an object close to the earth, or by taking it far away.

A natural question arising in the context of realisation is whether a mediating state property, which co-occurs with some other state property, should be considered identical to this state property. This would solve the problem, how mediating state properties can be genuine state properties and how they can cause other state properties.

Within physicalism, a mental state property is assumed to co-occur with its physical realiser; for mental state properties indeed such proposals to make identifications have been put forward in the form of different types of reductionism; e.g., (Kim, 1996, pp. 125-154, pp. 211-240; Bickle, 1998); see also (Bennett and Hacker, 2003; Bickle, 2003; Kim, 2005).

One of the advantages of such a mental-physical state property identification is that for relationships between states at different points in time, the causation relation from Physics can describe, for example, how a mental state property such as an intention affects the world state. However, the phenomenon of multiple realisation discussed in Section 7 obstructs such a direct one-to-one identification. Examples of the occurrence of realisers of mediating state properties are the following:

- An open tap will lead to the presence of water in a glass in a next state, where the size of the opening determines how much water will be present at this next point in time. So the mediating state property between the past and the future (i.e., from opening the tap to water present in the glass) is in a sense hardwired in the physical configuration of the opened tap, which is described by state properties in the present state.
- The (second-order) mediating state property anticipating an object to start moving or falling in free space co-occurs with a combination of state properties including the presence of other objects in the neighbourhood and not being limited or obstructed by anything (this realised mediating state property is gravitation).
- Colliding billiard balls show a deformation at the moment of collision. This deformation realises the second-order mediating state property for change of momentum during their interaction (this realised mediating state property is force).

In general, an interaction between objects, characterised by a specific configuration in the world state, is a manner to bring about a mediating state property in an object for which one of its state properties are to be changed. Specifying this configuration (and thus characterising the type of interaction) may provide a realiser of this mediating state property. As different configurations may lead to the same effect, more than one realiser may be obtained in this way. In this section, the case of unique realisers is addressed; the case of multiple realisers will be addressed in Section 7. The following notion of (unique) realiser will be used.

A mediating state property p is (*uniquely*) *realised* by a state property or combination of state properties q, if and only if in states p and q always co-occur. In other words:
(1) Always, if in a state p occurs, q occurs as well,
(2) In every state where q occurs, p occurs as well.

This relation between p and q is called a *realisation relation*.

This can be formalised in TTL form by

$$\forall \gamma, t \;\; \text{state}(\gamma, t) \models p \;\; \Leftrightarrow \;\; \text{state}(\gamma, t) \models q$$

In a logical sense, this definition can also be expressed in a bi-implication form: a state *S* has property *p* if and only if *S* has property *q*. That is for all states $p \leftrightarrow q$ holds; formalised:

$$\forall \gamma, t \;\; \text{state}(\gamma, t) \models p \leftrightarrow q$$

The realisation relation specifying the co-occurrence of the mediating state property and the realising state property, provides a useful means for modelling, calculation and acting. An objection against such a one-to-one correspondence between mediating state properties and realisers, is that this will not always be possible. These are two ways in which such an impossibility may show. One way is that there are in a sense too many realisers, i.e., no unique realiser exists satisfying the criteria (1) and (2) above, but there are (more than one) state properties satisfying (2) but not (1), although always in a state if *p* occurs, then at least one of the *q*'s occurs. In Section 7, this case of multiple realisation is addressed.

Another case in which it is not possible to satisfy (1) and (2) is the case that no realiser exists at all. For example, this happens for a billiard ball, or for a freely moving object in space, where a mediating state property (momentum) occurs that is independent of the other properties of the present state (inertia of motion). Assuming neglectable interaction with other objects, for such objects the persistence of the mediating state property (i.e., momentum) is the dominant factor. So, the mediating state property at time t depends purely on the history and is independent of other properties of the world state at t. To be able to cover such inertia of motion effects, it seems that the only way to relate mediating state properties to other state properties is by relating them to properties of states at time points different from t (i.e., by using temporal relationships). A direct, instantaneous form of realisation relation is not possible.

4 Realisation Relations in Quantitative Dynamic Modelling: Difference and Differential Equations

In this section it is shown how quantitative dynamic modelling methods from the area of calculus within Mathematics can be described by realisation relations. In Treur (2007, Section 7) it is discussed in which sense the derivative of a continuous variable at a certain time point can be viewed as a mediating state property for a past to future relationship in the form of a smoothness condition. In this section it is discussed in which form a realisation relation of such a mediating state property can occur. Let p be such a mediating state property (i.e., change rate) for variable x. How can this mediating state property be related to other state properties? As a special case, the relationship of (the value of) p to other state properties can focus on properties that can be expressed in terms of (the value of) x. A plain case of this idea is when a value v of p in a state is considered always to co-occur with this value v for some expression or function F in the value of x in the same state:

$$has\_value(p, v) \;\; \leftrightarrow \;\; \exists w \;\; has\_value(x, w) \;\; \& \;\; v = F(w)$$

or

$$has\_value(p, F(w)) \;\; \leftrightarrow \;\; has\_value(x, w)$$

This shows a bi-conditional form for the co-occurrence of the two properties in states, where the right hand side of the 'if and only if' is the realiser of the mediating state property at the left hand side (cf. Nagel, 1961, pp. 345-358; Kim, 1996, pp. 212-216). An alternative way to express the same biconditional relationship is:

$$p(t) = F(x(t))$$

Keeping in mind that the mediating state property p is the derivative of x, sometimes denoted by dx/dt the last way of expressing can be also written as

$$dx/dt = F(x(t))$$

This is the usual notation for a *differential equation*. These formats allow to relate a mediating state property at time *t* to other state properties of the state at t. As an example, take the function F defined by:

$$F(x) = \alpha x(1 - x/C)$$

For this example, mediating state property *p* is related to another state property as follows:

$$has\_value(p, \alpha w(1 - w/C)) \;\; \leftrightarrow \;\; has\_value(x, w)$$

Or, alternatively expressed:

$$p(t) = \alpha x(t)(1 - x(t)/C)$$

In the usual notation for a differential equation this is also formulated as

$$dx/dt = \alpha x(t)(1 - x(t)/C)$$

The differential equation based on this example function F describes (with the parameters α and C within a certain range) a logistic growth pattern with asymptotic value C (carrying capacity) and initial growth rate α.

It turns out that (first-order) differential equations can be understood from the conceptual framework based on 'temporal factorisation and realisation' as realisation relations for mediating state properties. The differential equation format

$$dx/dt = F(x(t))$$

expresses for a variety of cases how a mediating state property relates to another state property. Moreover, this can easily be extended to a system of differential equations, such as

$$dx/dt = F(x(t), y(t))$$
$$dy/dt = G(x(t), y(t))$$

where each of the mediating state properties dx/dt and dy/dt has a realisation relation to a combination of the state properties x and y, defined by F and G, respectively.

In a discretised form a *difference equation* is obtained:

$$x(t') - x(t) \quad = \quad F(x(t))\,(t' - t)$$

or

$$\Delta x = F(x(t))\Delta t$$

with

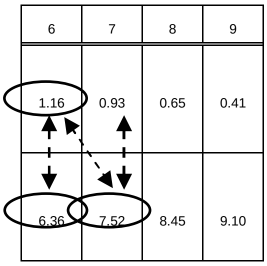
$\Delta x = x' - x$ and $\Delta t = t' - t$

For the example function F above with $\alpha = 0.5$ and $C = 10$, the resulting trace for the difference equation is depicted in Table 1 below (with $\Delta t = 1$).

Table 1. Simulation table for logistic growth described by mediating state properties

t	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
F(x)	0.45	0.62	0.82	1.03	1.19	1.25	1.16	0.93	0.65	0.41	0.23	0.12	0.06	0.03	0.02	0.01	0.00	0.00	0.00
x	1.00	1.45	2.07	2.89	3.92	5.11	6.36	7.52	8.45	9.10	9.51	9.74	9.87	9.93	9.97	9.98	9.99	10.00	10.00

To be able to compare the pattern of relationships within this table to other cases in Section 5, the pattern will be made more explicit. For every time point t, based on a given value for x, the mediating state property $p = \Delta x$ for x is determined by calculating $F(x)$ (the middle row in Table 1). The value for x at the next time point $t + 1$ is calculated as $x + p$ (the lower row in Table 1). The patterns of relations used in this calculation are depicted in Figure 2 (a picture similar to the one in Figure 1). Here, the lower line shows the successive states, with state properties a_0, a_1, a_2 and a_3. Above each of these state properties, the mediating state property is depicted that is realised by it in the same state. As before, this mediating state property in a certain (present) state relates to properties of the previous (past) and next (future) state. These relationships depicted by dotted arrows relate to Table 1 as shown in the following fragment:



6	7	8	9
1.16	0.93	0.65	0.41
6.36	7.52	8.45	9.10

The vertical bi-arrow is exploited by calculating $F(x)$ from x. The arrow to the next state by adding the resulting mediating state property to the present x, thus obtaining, the state property a_{i+1} as a successor for a_i. In Section 5 it will be shown that a similar pattern of relations occurs in the context of qualitative dynamic modelling techniques such as transition systems, causal models, logical models and rule-based systems.

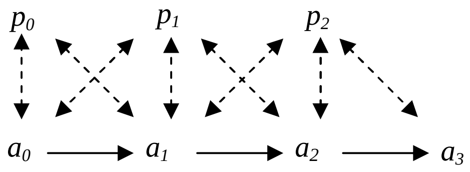


Figure 2. Mediating state properties realised in successive states

In summary, quantitative dynamic modelling approaches based on differential equations can be understood from the conceptual framework for dynamics based on temporal factorisation and realisation in the sense that they express realisation relations for mediating state properties. A characteristic of this area is that all state properties are based on variables and numerical values assigned to them. This excludes modelling approaches where dynamics is analysed based on qualitative state properties. In the next section such qualitative approaches are addressed.

5 Realisation Relations in Qualitative Dynamic Modelling: Causal, Rule-Based, Logical Systems

In this section qualitative dynamic modelling methods are considered. Examples of such methods are transition systems (e.g., Arnold, 1994), production or rule-based systems (e.g., Anderson, 1996; Buchanan and Shortliffe, 1984), causal models (e.g., Bosse etal., 2005), and executable temporal logic (e.g., Barringer et al., 1996; Fisher, 2005). These methods specify in a qualitative manner how a state in a system may change.

Table 2. Simulation table for a transition system described by mediating state properties

	0	1	2	3
mediating state properties	*p(hot_water_present)* *p(cup_present)*	*p(tea_bag_present)*	*p(tea_present)* *p(not tea_bag_pesent)*	
other state properties	*not hot_water_present* *not cup_present* *not tea_bag_present*	*hot_water_present* *cup_present* *not tea_bag_present*	*hot_water_present* *cup_present* *tea_bag_present*	*hot_water_present* *cup_present* *not tea_bag_present* *tea_present*

These methods can be analysed from the perspective of temporal factorisation and realisation as described below. Viewed from an abstract perspective, the following format is used in such methods. In a rule[4] $c \twoheadrightarrow d$ with antecedent c and consequent d:

- the first description c indicates a combination of state properties for the current state
- the second description d indicates one or more state properties for the next state

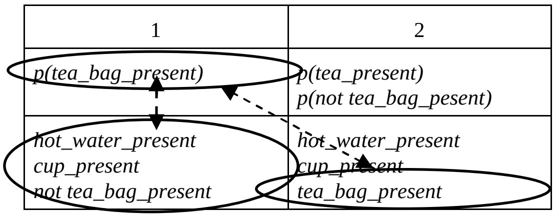
As an illustration, a simple example scenario describes the following rules for preparing a cup of tea: getting hot water, getting a cup, getting a tea bag, and finally, by combining these, getting the tea.

not hot_water_present
  ↠ *hot_water_present*
not cup_present
  ↠ *cup_present*
hot_water_present ∧ *cup_present* ∧ *not tea_bag_present*
  ↠ *tea_bag_present*
hot_water_present ∧ *cup_present* ∧ *tea_bag_present*
  ↠ *tea_ present* ∧ *not tea_bag_present*

From the conceptual framework based on ‘temporal factorisation and realisation’ such a rule can be understood as a relationship between mediating state properties and other state properties in the following manner. For example, for the second rule: if in a state not *cup_present* occurs, also the mediating state property *p(cup_present)* anticipating for *cup_present* occurs in this state. Thus, a rule $c \twoheadrightarrow d$ can be interpreted as an implication $c \rightarrow p(d)$, describing a logical relationship between state properties in a given state. In the current context, c can be considered a (single) realiser for $p(d)$. In the given specification this is the case because also the converse implication $p(d) \rightarrow c$ holds (i.e., always if $p(d)$ occurs in a state, also c occurs). Thus the following biconditional realisation relations for mediating state properties occur in the scenario of making tea:

p(hot_water_present) ↔ *not hot_water_present*
p(cup_present) ↔ *not cup_present*
p(tea_bag_present) ↔ *hot_water_present* ∧ *cup_present* ∧ *not tea_bag_present*
p(tea_present) ↔ *hot_water_present* ∧ *cup_present* ∧ *tea_bag_present*
p(not tea_bag_pesent) ↔ *hot_water_present* ∧ *cup_present* ∧ *tea_bag_present*

In the example scenario, the subsequent states can be described (in a similar table as in Table 1, for the difference equation); see Table 2. The relationships between the cells in this table are similar to those in Table 1:



1	2
p(tea_bag_present)	*p(tea_present)* *p(not tea_bag_pesent)*
hot_water_present *cup_present* *not tea_bag_present*	*hot_water_present* *cup_present* *tea_bag_present*

In a qualitative dynamic modelling approach, it is not always the case that all realisers are unique. For example, in the chosen specification, the mediating state property to have a tea bag present is based on the presence of hot water and cup. However, there may be alternative ways to achieve the presence of the tea bag, for example right at the start, based on the absence of a tea bag and absence of a cup; this would provide two different realisers for the tea bag being present:

p(tea_bag_present) ↔ *hot_water_present* ∧ *cup_present* ∧ *not tea_bag_present*
p(tea_bag_present) ↔ *not cup_present* ∧ *not tea_bag_present*

It is clear that in this case interpretation of the realisation relations as bi-implications leads to contradictions. By applying both bi-implications it can be established that the cup not being present is equivalent to the cup being present, which cannot be true. In Section 7, it is discussed how in case of multiple realisers such contradictions can be avoided.

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[4] So, what is called a rule here can have the form of a transition in a transition system, a production rule, a causal relation, or an executable temporal formula.

6 Realisation Relations in Physics: Forces

In Section 2 the notion of force in Physics was briefly discussed as an example of a (second-order) mediating state property for the notion of momentum. In Treur (2007, Section 7) more details were given to show how a derivative can be understood as a mediating state property, and how this can be applied to obtain the concepts velocity, momentum and force in Physics as mediating state properties. The analysis of force as a second-order mediating state property can be extended to obtain a case of realisation of a second-order mediating state property. A specific example, showing how a specific force F can be realised, is the following. For the trajectory of an object in space, with mass m, approaching earth (with mass M), Newton's second law can be used in conjunction with his law of gravitation, for the interaction between the two objects involved (here x is the distance between the object and the earth, and c is a constant):

$$m\, d^2x/dt^2 = F$$
$$F = c\, mM/x^2$$

An assumption here is that no other interaction plays a role. In this case, the force-function identified is $F(x) = c\, mM/x^2$, as described by Newton's gravitation law. This provides a second-order differential equation for the distance x of the object to the earth, depending on time t:

$$m\, d^2x/dt^2 = c\, mM/x^2$$

Here, the left hand side provides the second-order mediating state property F and the right hand side is a realiser of this mediating state property: the equality guarantees that both properties will always co-occur in states. Note that in this case the (realised) second-order mediating state property leads to a not realised first-order mediating state property (a changed momentum) in a next state. In turn, this first-order mediating state property momentum leads to a next state in which the changed state property position occurs.

7 Multiple Realisation of a Mediating State Property

A complicating issue for realisation of mediating state properties is that there may sometimes be a co-occurrence with one other state property and sometimes with another one: *multi-realisability*. Mental state properties usually have a large variety of realisers, for example in different animal species. Relating a mental state property in a biconditional manner to all of these mutually distinct (non-equivalent) realisers will lead to a contradiction, as already indicated in Section 5. If *p* is equivalent to each of two realisers q_1 and q_2, then it follows that q_1 is equivalent to q_2, and thus they always co-occur. In a multiple realisation case where in different states sometimes one, sometimes another realiser co-occurs with *p*, this is a contradiction.

A solution could be to differentiate the mediating state property into a (possibly large) number of distinct variants, thus creating a number of biconditional relationships. However, then the unifying and generic aspect of this mediating state property may be lost; (cf. Kim, 1996, pp. 233-236). Therefore, for the case of multi-realisability a broader definition of the notion of realiser is desirable.

In the case of multiple realisers, the relation between mediating state property *p* and its realisers can be described by a supervenience relation (e.g., Kim, 1998).

> 'Mental properties supervene over physical properties in that for every mental property M that occurs at some point in time t, there exists some physical property P that also occurs at t, such that always if P occurs at some point in time t', also M occurs at t'.' (Kim, 1998, p. 9)

This can be formalised in TTL by

$$\forall\gamma, t\ \ \mathrm{state}(\gamma, t) \models M \ \Rightarrow\ \exists P\ \ \mathrm{physical}(P)\ \&\ \mathrm{state}(\gamma, t) \models P\ \&$$
$$\forall\gamma', t'\ \ [\,\mathrm{state}(\gamma, t') \models P \ \Rightarrow\ \mathrm{state}(\gamma', t') \models M\,]$$

Following this line, the following can be defined (see also Kim, 1996, Ch. 9):

A set *Q* of state properties (or combinations thereof) is a *complete set of realisers* of *p* if and only if:
(1) if *p* occurs in a state then one of the *q* in *Q* occurs in this state, and
(2) if one of the q in *Q* occurs in a state then *p* occurs in this state
The elements *q* of *Q* are called the (*non-unique*) *realisers* of *p*. The relations between *p* and the elements of *Q* are called *(multiple) realisation relations*.

This can be formalised by

$$\forall\gamma, t\ \ [\,\mathrm{state}(\gamma, t) \models p \ \Rightarrow\ \exists q\ \ [\,\mathrm{in}(q, Q)\ \&\ \mathrm{state}(\gamma, t) \models q\,]\ \&$$
$$\forall\gamma', t', q\ \ [\mathrm{in}(q, Q)\ \&\ \mathrm{state}(\gamma', t') \models q \ \Rightarrow\ \mathrm{state}(\gamma', t') \models p\,]\,]$$

Sometimes, this definition can be expressed in a disjunctive bi-implication form: a state *S* has property *p* if and only if *S* has one of the properties *q* in *Q*, or:

$$\forall\gamma, t\ \ \ \mathrm{state}(\gamma, t) \models p \leftrightarrow \bigvee Q$$

where $\vee Q$ stands for the disjunction of the *q* in *Q* (assuming that *Q* is finite). Notice that the set *Q*, from which multiple realisers come, is a not defined itself by the above definitions. Indeed, in practice this set may be hard to define in a precise manner.

Supervenience applied to mediating state properties, expresses that mediating state properties are always realised in one way or the other.

However, this can happen in a nonsystematic, ad hoc manner: for every situation a different realiser. This may entail a branching of the mediating state property into a multitude of variants, thus loosing the unifying and generic aspect of the mediating state property. Sometimes, this situation is avoided by introducing as conditionals for each context strong, context characterising assumptions excluding all but one of the realisers.

As an example, for a qualitative dynamic modelling approach, multiple realisation can be incorporated easily. Suppose two rules

$$c_1 \twoheadrightarrow d \qquad\qquad c_2 \twoheadrightarrow d$$

are given. Then, in continuation of the line in Section 5, the set of state properties Q = $\{c_1 , c_2\}$ can be considered a set of realisers of the mediating state property $p(d)$ leading to d.

In Nagel (1961, pp. 186-192), the multiple realisation (in our terms) of the notion of force (which can be considered a second-order mediating state property; see also Sections 2 and 6) is discussed. In line with what was stated above, his analysis asserts that for various different situations specific force-functions (in his terms), specifying how force relates to other properties of the state (and/or first-order mediating state properties) are needed. Forces can occur due to state properties involving, for example:

- the presence of an object pushing or pulling
- deformation such as caused by collisions (e.g., billiard balls)
- the presence of objects with electrical charge,
- the presence of magnetic objects,
- the presence of other masses (gravitation),
- atmospheric pressures

For each of these circumstances, a different expression in terms of the world state ontology (a force-function) describes the force that occurs. Only if for a given situation such a force-function has been identified, something can be done using the laws of classical mechanics. In this sense, this case shows a heterogeneous situation, where past patterns and present states relating to a force mediating state property are described in some heterogeneous disjunctive form with at least, say, up to 5 to 10 essentally different contexts of the origin of the force. In this heterogeneous situation, in different contexts different force-functions (and hence realisers) are identified, which still allows succesful use of the notion of force in applications. This shows an example of an approach to multiple realisation, comparable to the notion of local or context-dependent reduction as described by Kim (1996), pp. 211-240.

Notice that an additive property for this mediating state property force holds in the following sense: the combined effect of any number of different contributions to the second-order mediating state property can be obtained by adding their values. So, any value w for this second-order mediating state property can be obtained as the combined effect from, for example, gravitation, electrical charge, and deformation by collision. For example, considering one dimension where all effects work along the same axis, this can occur in the form of an infinite number of possible sums $w = w_1 + w_2 + w_3$, where the terms are the contribution of one of the three effects (e.g., w_1 by gravitation, w_2 by electrical charge, w_3 by collision). This shows that the complete set of realisers Q defined in Section 7 can be infinite, and also that in reality contexts can not always be separated.

8 Obtaining Genericity by Multiple Realisation

Multiple realisation is often viewed from the negative side: it spoils the simple picture as provided in the unique realiser case. Within Philosophy of Mind, the fact that multiple realisation occurs is considered as a main argument against identification of mental and physical states, as in the mind-brain identity perspective; cf. Kim (1996). However, multiple realisation also has a positive side: it allows unification of different phenomena in one more abstract concept. For example, the power of interpreting the concept ‘force’ as a unified concept for the (heterogeneous) class of force-functions is emphasized by Nagel’s discussion of what is called Newton’s second law $F = ma$:

> ‘… what the second axiom can be taken to assert is that there are determinants for changes in momenta which can be formulated in a relatively simple manner and can be specified in terms of the spatial configurations and certain physical properties of the bodies. Accordingly, if the class of functions to which F is restricted is designated by ‘K’, then instead of stating the axiom in the form that gives it the appearance of a definitional equivalence (namely, ‘The force F is equal to the product of the mass and the acceleration’), it is more clear and less misleading to formulate it on the present interpretation of the axiom as ‘For every change in momentum of a body, there is a force F such that F is a member of K and F = ma’
> (…)
> From this perspective, therefore, there is no fundamental difference in ultimate outcome between the present interpretation of the second axiom and the view that the latter is only a nominal definition of the word ‘force’. However, it is undoubtedly convenient to retain the word in the exposition of the general theory of mechanics. For it is useful to have an expression which covers the various force-functions that may be employed in different problems, especially since the class of such functions is only vaguely delimited and cannot be exhaustively enumerated. With its help, moreover, it is possible to establish many general theorems that are valid for large classes of physical systems to which the theory of mechanics is applicable, quite irrespective of the particular character of the assumed forces.’ (Nagel, 1961, pp. 190-191).

Nagel claims that a unified concept for force plays an important role in formulating many theorems for classes of physical systems, abstracting from the particular character of the forces. Nagel's analysis may suggest to consider the concept force as a disjunction or sum of all possible force-functions. However, for the case of multiply realised mental state properties, this suggestion has been severely criticised due to the fact that such a property often is 'wildly heterogeneous':

> 'Now, most philosophers who believe in multiple realizability of mental properties will deny that mental properties are disjunctive properties – disjunctions of their realisers – for the reason that the first-order realizing properties are extremely diverse and heterogeneous, so much so that their disjunctions cannot be considered well-behaved properties with the kind of systematic unity required for propertyhood.
> (…)
> Functionalists have often touted the phenomenon of multiple realization as a basis for the claim that the properties studied by cognitive science are formal and abstract – abstracted from the material compositional details of the cognitive systems. What our considerations appear to show is that cognitive science properties so conceived threaten to turn out to be heterogeneous disjunctions of properties after all. And these disjunctions seem not to be suitable as nomological properties, properties in terms of which laws and causal explanations can be formulated.' (Kim , 1996, p. 117)

In a similar direction as the one discussed by Nagel (1961), Jackson and Pettit (1988, 1990) developed a notion of higher-level explanation, exploiting a metaphor from Computer Science, meant to be suitable for special sciences such as Biology, Cognitive Science, and Social Sciences: *program explanation*. According to this type of explanation, 'G occurred because F occurred' for higher-level properties F and G, can be an adequate explanation in the following way: F ensures ('programs for') some lower-level property P, which causes G. In other words: F ensures ('programs for') some lower-level property P, which causes a lower-level property Q, for which G is a higher-level description. For example, the question 'Why was the vase breaking?' can be answered by: 'Because it was fragile'. Here, the higher-level property of being fragile ensures or programs for the lower-level property of having one of a number of possible specific types of molecular structure. Jackson and Pettit explain the name 'program explanation' as follows:

> 'The property-instance does not figure in the productive process leading to the event but it more or less ensures that a property-instance which is required for that process does figure. A useful metaphor for describing the role of the property is to say that its realization programs for the appearance of the productive property and, under a certain description, for the event produced. The analogy is with a computer program which ensures that certain things will happen – things satisfying certain descriptions – though all the work of producing those things goes on at a lower, mechanical level' (Jackson and Pettit, 1990), p. 114

They discuss the value of such a higher-level explanation, using the example of explaining radiation from the decay of atoms, as follows:

> 'According to (Lewis, 1988), to explain something is to provide information on its causal history (…) A program explanation provides a different sort of information (…) A program account tells us what the history might have been. It gives modal information about the history, telling us for example that in any relevantly similar situation, as in the original situation itself, the fact that some atoms are decaying means that there will be a property realized - that involving the decay of such and such particular atoms - which is sufficient in the circumstances to produce radiation. In the actual world it was this, that and the other atom which decayed and led to radiation, but in possible worlds where their place is taken by other atoms, the radiation still occurs. ' (Jackson and Pettit, 1990, p. 117).

Jackson and Pettit emphasize that such a form of higher-level explanation has advantages over a lower-level causal explanation, in the sense that other information is provided, which implies *increased genericity*: it not only applies to the actual world, but also to other ('relevantly similar') possible worlds. A similar argument is made in Jonker, Treur, and Wijngaards (2002), where the development of higher-level languages in Computer Science is analysed from the same perspective: of obtaining better manageable and more generic descriptions of dynamics.

What Nagel expresses about the concept force seems to emphasize a similar advantage. For the case of functionalism in Philosophy of Mind, where the aim is to obtain general, mental state properties, in terms of which general (cognitive) laws are formulated, and which can be multiply realised, in a similar manner the unifying aspect is emphasized (e.g., Kim, 1996, pp. 73-124 and pp. 233-236). The unifying aspect of the concept force is that many ways are possible to have a specific force occur (e.g., based on gravitation, pushing or pulling, electrical charge, or a combination thereof). Independent of the way in which a given force is realised, the force (as a mediating state property) entails changed properties (i.e., changed momentum) in the future states.

9 Realisable Mediating State Properties as Second-Order Properties

From the formulation of the temporal factorisation principle it shows that it is not a law of nature. The existential quantifier over state properties in the consequent gives it a second-order character. As for a specific application of the temporal factorisation principle, this also will not always lead to plain laws, because of the in principle nonempirical status of the mediating state property. In the case that a mediating state property p for $a \to b$ is postulated with a single realiser c, it may seem that the relationships $a \to p$ and $p \to b$ can easily be replaced by the more basic relationships $a \to c$ and $c \to b$, which in some circumstances could be viewed as plain (first-order) laws. However, if the mediating state property p is left out completely, for the analyst it may provide a picture of the situation that lacks understandable interpretation at a more abstract level.

Even if the single realiser situation would be considered to lead to laws in a satisfactory manner, for the multiple realisation case it certainly does not work this way. For example, according to Nagel, the status of Newton's second law has an debatable status as a law:

> 'In the second place, although on the present interpretation the second law does have an empirical content, it is nevertheless incapable of ever being decisively refuted by any conceivable experiment. For the axiom does not specify a definite force which will account for some particular acceleration; it merely asserts that *there is* a force which satisfies certain tacitly assumed conditions and which it is the business of the physicist to specify in detail. But a statement of the form 'There is a force F such that …' can be shown to be false only if it is possible to establish its contradictory, namely a statement of the form 'For all forces F, it is not the case that ...'; and in general one could establish the latter only if one could examine exhaustively all possible force-functions satisfying the stipulated conditions. It is clear, however, that such an examination could never be completed, for the number of abstractly possible force-functions is not fixed and may exceed any finite limit. Accordingly, although the axiom may be confirmed through the discovery of appropriate force-functions which successfully account for the acceleration of bodies, it can never be shown false.' (Nagel, 1961), p. 191-192; italics in the original.

Nagel takes one further step by claiming that Newton's second law actually is not a law but should be considered a methodological rule:

> 'Now it is for this reason that the second axiom is frequently regarded not primarily as an assertion about the conditions under which accelerations occur but rather as a compact formulation of a specialized *guide* for research, as a *methodological rule* which directs the physicalist what to look for when he is analyzing the motions of bodies. (…) In point of fact, the second axiom viewed as a regulative principle has been eminently fruitful in guiding the construction of a systematic body of warranted knowledge' (Nagel, 1961, p. 192; italics in the original).

Nagel's claim that Newton's second law actually has the form of a statement 'There is an F such that if F occurs, the subsequent change in momentum is given by F', brings the concept of force in the context of what Kim (1998) calls a *second-order property* over more specific (first-order) physical state properties:

> 'Functionalism takes mental properties and kinds as functional properties, properties specified in terms of their roles as causal intermediaries between sensory inputs and behavioural outputs, and the physicalist form of functionalism takes physical properties as the only potential occupants, or "realizers", of these causal roles. To use a stock example, for an organism to be in pain is for it to be in some internal state that is typically caused by tissue damage, and that typically causes groans, winces, and other characteristic pain behaviour. In this sense being in pain is said to be a second-order property: for a system x to have this property is for x to have some first order property P that satisfies a certain condition D, where in the present case D specifies that P has pain's typical causes and typical effects. More generally, we can explain the idea of a second-order property in the following way. Let B be a set of properties; these are our first-order (or "base") properties. (…) We then have this:
>
>> F is a second-order property over set B of base (or first-order) properties iff F is the property of having some property P in B such that D(P) where D specifies a condition on members of B.
>
> Second-order properties therefore are second-order in that they are generated by quantification - existential quantification in the present case - over the base properties. We may call the base properties satisfying condition D the realizers of second-order property F.' (Kim, 1998, pp. 19-20).

For example, the mental state property 'pain' can be functionally characterised in a simplified form in the following manner[5]:

tissue damage leads to pain
pain leads to moving

As a second-order property, the state property 'pain' can be formulated as:

being in pain at some point in time t means that
a physical state property X exists such that X
occurs at time t and D(X) holds, where
D(X): tissue damage leads to X
X leads to moving

As an example, *X* may have the form 'C-fiber-activation', if it is assumed that this is a physical state property that fulfills *D(X)*. Kim (1998, pp. 19-21) uses this concept of second-order property to clarify the status of mental state properties in case of multiple realisation. Notice that in this example the second-order property is defined not as a state property but as a dynamic property referring to states at different time points. The part *D(X)* of the property refers to relationships between properties of states at different points in time. So, in principle, if *X* occurs at *t*, then tissue damage occurred at some time point

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[5] Note that, the relationships, in which the mental state property plays a role, are indicated here by 'leads to'. This is to avoid a commitment on whether the relationships involved are assumed to be causal relationships; they may be or may not be.

$t' < t$ and moving will occur at some time point $t'' > t$.[6] A formalisation in TTL can be obtained as follows. Here the variable X ranges over physical state properties.

$$\begin{aligned}&D(X):\\ &\forall\gamma, t\ \ [\ [\ \exists t'{<}t\ \ \text{state}(\gamma, t') \models \text{tissue damage}\ ] \Rightarrow\ \text{state}(\gamma, t) \models X\ \ ]\\ &\&\ \ \forall\gamma', t'\ \ [\ \text{state}(\gamma', t') \models X\ \ \Rightarrow\ \exists t''{>}t'\ \ \text{state}(\gamma', t'') \models \text{moving}\ \ ]\end{aligned}$$

Given this *D(X)*, ‘being in pain at some point in time *t*’ can be formalised by the second-order world property:

$$\exists X\ \ [\ \text{state}(\gamma', t) \models X\ \ \&\ \ D(X)\ ]$$

Any realisable mediating state property can be addressed from the perspective of second-order properties. For example, the concept ‘force’ can be considered to have a status comparable to the status of mental state properties according to this type of functionalism. In the pattern as put forward by Kim (1998, p. 20), the property ‘There is an *F* such that if *F* occurs, the subsequent change in momentum is given by *F*’ in a simplified, discrete form can be reformulated as[7]:

A force of size f occurs at time point t if
a physical state property X exists such that X occurs at t and $D_f(X)$ holds, where
$D_f(X)$:
if X occurs at time t
then the change in momentum from t to a next time point t'' > t is given by f,
and the change in momentum from a previous time point t' > t to t is given by f.

This can be formalised in a slightly simplified form as:

$$\exists X\ \ [\ \text{state}(\gamma', t) \models X\ \ \&\ \ D_f(X)\ ]$$

with

$$\begin{aligned}D_f(X):\ \ \forall\gamma, t\ \ [\ [\ &\text{state}(\gamma, t) \models X\ \ \&\ \text{state}(\gamma, t) \models \text{momentum}(p)\ ]\\ &\Rightarrow\ \exists t'{>}t\ \ \text{state}(\gamma, t') \models \text{momentum}(p{+}f)\ \ ]\ ]\end{aligned}$$

This again shows how a mediating state property can be considered a second-order world property. This can be made more specific in the following manner. As an example, in a particular circumstance (e.g., an object in space with mass m approaching earth with mass M) this physical state property X may have the form of the property that states that: the product of a constant c, mass m, and mass M divided by the square of the distance x is equal to f:

$$c\, mM/x^2 = f$$

Notice that this approach to a mediating state property as a second-order world property in principle (as far as world is considered as physical world) assumes the realisability of the mediating state property involved.

10 Discussion

Mediating state properties obtained by temporal factorisation provide a quite general concept to describe dynamics. Special cases can be found, for example, in mental state properties in Cognitive Science and several concepts in Physics. In many (but not all) cases mediating state properties can be grounded in the (present) state in which they occur by relating them to realisers: other state properties or combinations thereof that co-occur with the mediating state property. These realisers often are useful in modelling and calculation of trajectories or traces over time, for example, by using diffferential equations to specify realisation relations. However, apart from this convenience, how necessary are these realisations?

On the one hand, a mediating state property may not be defined by relating it to another state property, but solely by its temporal relational specification. In this case, in principle mediating state properties can be left out as they do not add powers over and above the power of the direct ‘past pattern implies future pattern’ temporal relationships.

On the other hand, there is no harm in assuming them: what they are used for can also be done using temporal relationships without mediating state properties as state properties. So, a possible view is that the temporal relationships actually justify the postulation and use of mediating state properties as a convenient shorthand for more complex temporal specifications. If, in addition, realisation relations are known, then the mediating state property can become more than a convenient shorthand: a concept like any other concept, occurring in a network of relations to other concepts. In particular, this applies to mental state properties as a special case of mediating state properties.

Within Physics, but also in disciplines such as Biochemistry, it is common to investigate phenomena in an idealised isolation. In experimental setups most of the possible interactions with other objects or phenomena are excluded by ‘screening them off’, and allowing only the precisely prescribed objects in the experiments. However, often this is only possible to a certain degree. Assuming that the other interactions are completely absent, finally is an idealising assumption, which goes hand in hand with the care taken to exclude the ‘undesired’ effects in experiments.

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[6] The time restriction for the tissue damage can be made more precise, for example, by stating that the time point t' should be in the recent past, for example, requiring t-e1 < t' < t for some small e1. Similarly the moving can be restricted in time, for example by t < t' < t+e2 for some small e2.

[7] Note that ‘There is an *F* such that the subsequent change in momentum is *F*’ is defined as a dynamic property and not as a state property, since in its part ‘change of momentum’ the property refers to momentum at two different points in time: before *F* had its effect and after.

For example, the laws of classical mechanics (as most laws in physics), are only valid under such restrictive assumptions (which are usually not explicitly formulated). Taking into account the gravitation induced by interaction with all objects in the universe, to determine the orbit of a falling apple on earth is not do-able. Fortunately, quite accurate approximations are obtained when all these interactions except one are assumed inexistent.

One of the laws in classical mechanics that assume complete isolation is the inertia of motion. For heterogeneous and dense environments on earth, the isolation assumptions often are not satisfied. For some cases, this opens the possibility to make the opposite assumption, namely that there is no inertia of motion at all. Using this assumption, history can be neglected; it can be assumed that all change is induced by the present state.

For example, this is assumed for the standard approach to the kinetics of chemical reactions based on differential equations, where changes in concentrations are considered to just depend on the present concentrations of the substances involved, and not on the history of these substances or their concentrations. By characterising the properties occurring in the present state, it can be predicted which properties will occur in a next state. This implies that for specific application areas working with dense media, such as intracellular processes or mental/neurophysiological processes, other means are available than, for example, in physical domains with nondense media.

Within the study of moving physical objects, an important step is to characterise the effect of an interaction between objects, based on the occurring state properties at the time of the interaction. It turned out that within classical mechanics (in nondense media), all second-order mediating state properties occurring in the types of interactions considered, can be characterised adequately in terms of other state properties (that serve as their realisers). Due to the law of interia of motion, it is clear that in general the first-order mediating state properties cannot be characterised by other state properties: the past plays an important role that cannot be eliminated. In domains dealing with dense media, (e.g., in chemical reactions as modelled by differential equations for their 'kinetics'), it seems that the first-order mediating state properties can be characterised in terms of other state properties (e.g., concentrations of chemical substances).

The temporal factorisation principle is not a (first-order) law. An existential quantifier over state properties in its consequent gives it a second-order character. In literature on Philosophy of Mind, usually unification between cognition and nature is aiming at the identification of common laws (e.g., Kim, 1996; Davidson, 1993). It is often attempted to defend this possibility of unification of laws in the two areas by some form of reductionism, by which the cognitive laws can be (logically) related to physical laws, using bridge principles to relate concepts from one theory to concepts from the other theory (e.g., Nagel, 1961; Kim, 1996, 2005; Bickle, 1998, 2003). However, this route for unification of cognition and nature sometimes seems quite difficult if not impossible; see, for example, Davidson (1993)'s argument on why psychophysical laws cannot exist (also see Kim, 1996).

The temporal factorisation principle contributes another route in the direction of unification of cognition and nature, not based on common first-order laws, but on common second-order principles. Such second-order principles may play an important role in bridging the gap between different disciplines, as shown in this paper, but also within the discipline of Physics it turns out that they can play an important role, as Nagel (1961) has pointed out concerning Newton's laws (although there they sometimes mistakenly are called laws).

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