

# A MULTI-AGENT MODEL FOR MUTUAL ABSORPTION OF EMOTIONS

Tibor Bosse<sup>1</sup>, Rob Duell<sup>2</sup>, Zulfiqar A. Memon<sup>1</sup>, Jan Treur<sup>1</sup>, and C. Natalie van der Wal<sup>1,2</sup>

<sup>1</sup>Vrije Universiteit Amsterdam, Department of Artificial Intelligence  
De Boelelaan 1081, 1081 HV Amsterdam, The Netherlands

<sup>2</sup>Force Vision Lab, Barbara Strozilaan 362a, 1083 HN Amsterdam, The Netherlands  
{tbosse, rduell, zamemon, treur, cw1210}@few.vu.nl

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## ABSTRACT

In recent times researchers have initiated investigating emotion as a collective property of groups, emphasizing the influence of combined emotions among group members on group processes. Within groups humans recognize and react emotionally to expressions of emotions of other group members. This paper uses a multi-agent-based approach to formalize and simulate such emotion contagion within groups.

## 1. INTRODUCTION

Within psychology, emotion is often defined as a state or process that plays a role in various cognitive processes, among which decision making and action preparation. Emotions are elicited by a particular stimulus, often include physiological reactions, and are relatively intense and short-lived (Frijda 1986). In addition, emotions have a social function: humans may experience situations where expressions of emotion by one individual shape the emotions, thoughts and behavior of others; others' reactions can then influence their future interactions with the individual expressing the original emotion, as well as that individual's future emotions and behaviors; e.g. (Hareli, Rafaeli 2008), (Hatfield, Cacioppo, Rapson 1994). This everyday, continuous, automatic process of emotion contagion has been described as a tendency to mimic the nonverbal behavior of others, to "synchronize facial expressions, vocalizations, postures, and movements" with others, in order to "converge emotionally" (Hatfield, Cacioppo, Rapson 1994).

Modeling group emotion can be done at the level of the group or at the level of the individuals, which has been named respectively the top-down and bottom-up approach (Barsade, Gibson 1998). The bottom-up perspective sees group emotion as the sum of its parts, affected by the homogeneity or heterogeneity of the group and the mean emotions of the group members. Individual differences play an important role, such as specific personality traits and the underlying brain mechanisms. There is consensus that the basic structure of personality incorporates five superordinate factors (McCrae, Costa 1987): extraversion, agreeableness, conscientiousness, emotionality and intellect. Extraversion is characterized as a tendency towards sociability, activeness, dominance and craving excitement. Agreeableness is thought of as being concerned with the maintaining of relationships.

Conscientiousness reflects qualities of planning, persistence, and purposeful striving towards goals. Emotionality (or neuroticism) concerns the ease and frequency with which a person becomes upset and distressed. Intellect (or openness) reflects openness to experience and intelligence-related sub-traits such as intellect, creativity and curiosity. Personality dimensions have been linked to specific aspects of brain functioning. The functional approach considers two sets of structures in the brain, the behavioral approach system (BAS), and behavioral inhibition system (BIS) (Gray 1990). BAS causes animals and humans to move towards incentives: things they desire. Besides managing the approach, BAS also creates excitement and positive feelings (Carver, Scheier 2004). The system BIS can be thought of as a stopping system, a system responsive to threat. BIS is responsive to cues of punishment and danger, not incentives. The neurotransmitter linked to BIS is serotonin (Gray 1990). The feeling BIS creates is anxiety. The two systems have been linked with personality traits: BIS is critical to neuroticism and BAS to extraversion (seeking social incentives) (Carver, Scheier 2004).

Personality also has been related to the functioning of the nervous system in a variable called sensation seeking (Zuckerman 1994). People who score high on sensation seeking are in search for new and exciting experiences. They are more likely to do high-risk sports, such as skydiving, are more sexually experienced and sexually responsive and compared to people low on this trait, are faster drivers. Zuckerman (Zuckerman 1994) has suggested that there is a particular brain chemical for sensation seeking (an enzyme called monoamine oxidase, MAO), which regulates several neurotransmitters, including serotonin and dopamine. Sensation seeking has also been related to the previously mentioned BIS and BAS systems (Carver, Scheier 2004). Persons high on sensation seeking correspond to a high BAS and low (sensitive) BIS.

In the literature, the area of emotion contagion in groups has been studied from the theoretical side (Barsade, Gibson 1998) but no computational models of emotion contagion have been developed. This paper is an attempt to develop a computational model of emotion contagion in groups incorporating the concepts given in the literature. The goal of this paper is to introduce a multi-agent-based modeling approach that formalizes and simulates the dynamics of emotion contagion within groups. The proposed approach makes use of standard numerical simulation software for simulation purposes, and the high-level declarative temporal modeling

language TTL (Bosse, Jonker, Meij, Sharpanskykh, Treur 2008) for analysis purposes. This modeling language is well suited for the current purposes, since it allows the modeler to combine qualitative, logical aspects with quantitative, numerical aspects (such as real numbers, and mathematical operations).

Below, in Section 2, a detailed model of group emotion contagion is explained and formalized. Next, in Section 3, simulation results are presented and in Section 4, the model is mathematically analyzed. Section 5 addresses formal verification of the emotion contagion model and the simulation results. Section 6 concludes the paper with a discussion.

## 2. A MULTI-AGENT EMOTION ABSORPTION MODEL

The model introduced in this paper has been designed as an interpretation of the bottom-up approach where group emotion can be seen as the sum of its parts (Barsade, Gibson 1998); therefore it is named absorption model<sup>1</sup>. It distinguishes multiple factors that influence emotion contagion processes. The model incorporates individual differences in personality traits (McCrae, Costa 1987): neuroticism and extraversion (BIS and BAS, (Gray 1990)). A number of aspects of the proposed computational model are distinguished that play a role in the contagion, varying from aspects related to an agent  $S$  sending the emotion, an agent  $R$  receiving the emotion, and the channel between sender  $S$  and receiver  $R$ ; see Table 1.

Table 1 Aspects related to a sender  $S$ , receiver  $R$ , or both

level of the sender's emotion	$q_S$
level of the receiver's emotion	$q_R$
sender's emotion expression	$\varepsilon_S$
openness for received emotion	$\delta_R$
the strength of the channel from sender to receiver	$\alpha_{SR}$

The aspect  $\varepsilon_S$  depends on how introvert or extravert, expressive, and/or active or energetic the person is. These aspects correspond to the personality trait extraversion and sensation seeking and the underlying neural system BAS. It represents in how far a person transforms internal emotion into external expression. In this sense, an introvert person will induce a weaker contagion of an emotion than an extravert person. The aspect  $\alpha_{SR}$  depends on the type and intensity of the contact between the two persons (e.g., distance vs attachment). The aspect  $\delta_R$  indicates the degree of susceptibility of the receiver. This represents in how far the receiver allows the emotions received from others to affect the own emotion, and how flexible/persistent the person is emotionally.

The parameter  $\alpha_{SR}$  may be related to a combination of more specific aspects such as the directness of the

emotion contagion, and the relations between sender and receiver. Emotion contagion is direct if the persons infecting each other with emotions are together in the same room and pay attention to each other (Hareli, Rafaeli 2008). Indirect contagion can happen when for instance the contagion between others is observed. Direct contagion is propagated stronger than indirect emotion contagion..

The aspects shown in Table 1 have been formalized numerically by numbers in the interval  $[0, 1]$ . In addition, the parameter  $\gamma_{SR}$  is used to represent the strength by which an emotion is received by  $R$  from sender  $S$ , modeled as:  $\gamma_{SR} = \varepsilon_S \alpha_{SR} \delta_R$ . The stronger the channel, the higher  $\alpha_{SR}$  and the more contagion will take place. The model works as follows: if  $\gamma_{SR}$  is 0, there will be no contagion, if it is 1, there will be a maximum strength of contagion. If  $\gamma_{SR}$  is not 0, there will be contagion and the higher the value, the more contagion will take place. In a way  $\gamma_{SR}$  expresses the energy level with which an emotion is being expressed, transferred and received. The overall strength by which emotions from all the other group members are received by  $R$  in a group  $G$ , indicated by  $\gamma_R$ , is defined as  $\gamma_R = \sum_{S \in G \setminus \{R\}} \gamma_{SR}$ . Suppose  $G$  is a group of agents. Take weights proportional to  $\varepsilon_S \alpha_{SR}$  defined by  $w_{SR} = \varepsilon_S \alpha_{SR} / \sum_{C \in G \setminus \{R\}} \varepsilon_C \alpha_{CR}$  and for any  $R \in G$  let  $q_R^* = \sum_{S \in G \setminus \{R\}} w_{SR} q_S$  be the weighted combined emotion from the other agents. The set of differential equations for emotion contagion in group  $G$  is

$$dq_R/dt = \gamma_R (q_R^* - q_R)$$

for all  $R \in G$ . Here  $\gamma_R = \sum_{S \in G \setminus \{R\}} \gamma_{SR}$  with  $\gamma_{SR} = \varepsilon_S \alpha_{SR} \delta_R$ .

## 3. SIMULATION RESULTS

A large number of simulations have been performed, using standard numerical simulation software, resulting in a variety of interesting patterns. In this section some of the simulation results are discussed. More simulation results can be found in Appendix A at <http://www.few.vu.nl/~tbosse/emotion/ECMS09.pdf>. Section 4 presents results of a mathematical analysis; the occurrence of most patterns under certain conditions was proven. Simulations shown here are for a group of 3 agents.

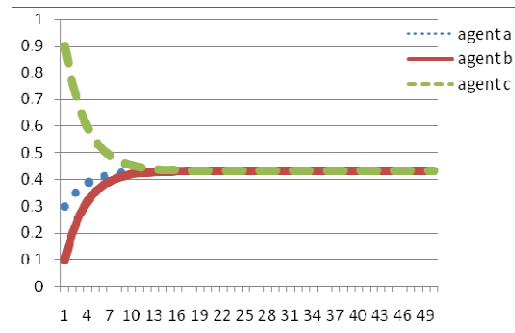


Figure 1. Simulation trace 1 (all  $\gamma_R = 1$ )

A first pattern found is that when the  $\gamma_R$  for all agents are not 0, in this case they are all 1, the emotion levels

<sup>1</sup> Note that it is not claimed that this model can describe the emotional dynamics of all persons in all situations. For some situations, an “amplification approach” seems to be more appropriate (see Sec. 6).

of all of them will approximate their average initial emotion level, with speed depending on the  $\delta_R$  (susceptibility) and  $\varepsilon_S \alpha_{SR}$ ; see Figure 1. The occurrence of this pattern has been proved mathematically; see Theorem 3 in the next section.

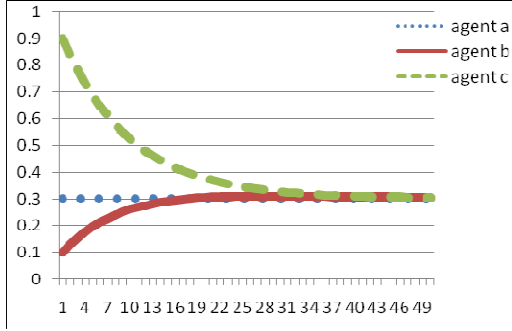


Figure 2. Simulation trace 2 ( $\delta_a = 0$ )

The opposite of this pattern happens when all  $\gamma_R$  are 0, then all agents will have equilibria that are equal to their initial emotional levels. In other words: the emotional levels of all agents will not change at all; see Theorem 1 in Section 4. Another situation (see Figure 2) occurs when agent a has  $\delta_a$  set to 0 and the other agents have this parameter  $\neq 0$ . This situation represents that agent a is not open to receive emotions, but can send emotions. As a result agent a's initial emotion level will remain the same. Furthermore, the agents b and c will eventually reach the equilibrium of agent a, which is equal to his initial emotion level.

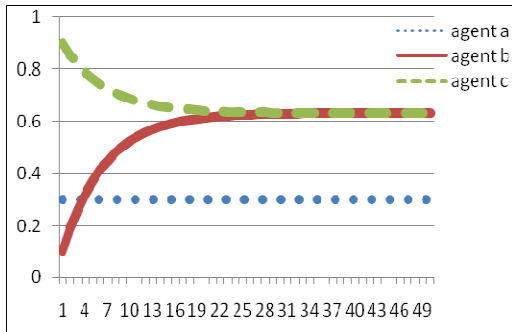


Figure 3. Simulation trace 3 ( $\delta_R(a, b, c) = (0, 0, 0.5)$ )

In Figure 3 it is shown that when agent a and b both have  $\delta_R$  set to 0, agent c will reach a value in between a and b's initial emotion values. The actual value agent c reaches depends on the settings of the parameter settings for all agents. This situation represents a case where two agents do not change their emotional level because they are only open to sending emotions, but not receiving emotions. As a result the third agent is forced to reach a value in between the emotional levels of the others. A next situation (see Figure 4) is one where  $\delta_a$  and  $\varepsilon_a$  are set to 0. This represents agent a being bidirectionally excluded from emotion contagion: (s)he can not receive or send emotions. The agents b and c are forced to go to a certain average in between their initial emotion values.

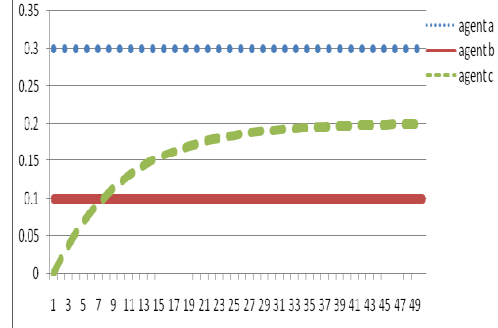


Figure 4. Simulation trace 4 ( $\delta_a = \varepsilon_a = 0$ )

The exact value they reach depends on the settings of their  $\delta_R$  (susceptibility) and  $\varepsilon_S \alpha_{SR}$ .

#### 4. MATHEMATICAL ANALYSIS

This section presents some of the results of a mathematical analysis of the model that has been made. Note that  $\gamma_A = 0$  iff  $\sum_{B \in G \setminus \{A\}} \varepsilon_B \alpha_{BA} \delta_A = 0$  iff  $\varepsilon_B \alpha_{BA} \delta_A = 0$  for all  $B \neq A$ . This means that  $\gamma_A = 0$  can only occur when for each  $B \neq A$  either  $\varepsilon_B = 0$  or  $\alpha_{BA} = 0$  or  $\delta_A = 0$ . This can be interpreted in the sense that  $A$  is isolated from emotional impact of all group members. In such a special case  $q_A$  will always be in an equilibrium state.

##### Theorem 1 (No change when $\gamma_A = 0$ )

If  $\gamma_A = 0$  then the emotion value for  $A$  will be in an equilibrium right from the start.

Next, conditions on monotonicity are addressed. Assuming  $\gamma_A > 0$ , from the equations it follows that  $dq_A/dt \geq 0$  if and only if  $q_{A*} \geq q_A$ . In particular, for  $A$  with the lowest  $q_A$  it holds  $q_B \geq q_A$  for all  $B \neq A$ , and therefore via  $q_{A*} = \sum_{B \in G \setminus \{A\}} w_{BA} q_B \geq \sum_{B \in G \setminus \{A\}} w_{BA} q_A = q_A$  it follows that  $q_A$  is monotonically increasing. Similarly the highest  $q_A$  is monotonically decreasing.

##### Theorem 2 (Monotonicity Conditions)

Suppose  $\gamma_A > 0$ . Then the following hold:

- (a)  $q_A$  is monotonically increasing iff  $q_{A*} \geq q_A$   
 $q_A$  is strictly monotonically increasing iff  $q_{A*} > q_A$
- (b)  $q_A$  is monotonically decreasing iff  $q_{A*} \leq q_A$   
 $q_A$  is strictly monotonically decreasing iff  $q_{A*} < q_A$
- (c) If  $q_B \geq q_A$  for all  $B \neq A$ , then  $q_A$  is monotonically increasing.  
 If in addition  $q_B > q_A$  for at least one  $B \neq A$ , then  $q_A$  is strictly increasing.
- (d) If  $q_B \leq q_A$  for all  $B \neq A$ , then  $q_A$  is monotonically decreasing.  
 If in addition  $q_B < q_A$  for at least one  $B \neq A$ , then  $q_A$  is strictly decreasing. ■

Next, equilibria are addressed for  $\gamma_A > 0$ . When at some point in time all  $q_A$  are the same, then from Theorem 2(c) and (d) it follows that they are both (non-strictly)

monotonically increasing and decreasing, so they are in an equilibrium. Moreover, from Theorem 2(c) and (d) it follows that as long as the values of the  $q_A$  are different, then the lowest and highest values keep on changing (strictly increasing, resp. decreasing), so are not in an equilibrium. This implies the following identification of equilibria.

**Theorem 3 (Equilibria when  $\gamma_A > 0$  for all  $A$ )**

Suppose  $\gamma_A > 0$  for all  $A$ . Then the equilibria are the cases where all  $q_A$  are equal. Equilibria are reached between the lowest and highest initial value. ■

In some cases the equilibria are the average of the initial values, due to preservation of the (overall) sum of the emotion levels:  $\sum_{A \in G} q_A(t') = \sum_{A \in G} q_A(t)$  for all  $t$  and  $t'$  or  $\sum_{A \in G} q_A(t + \Delta t) = \sum_{A \in G} q_A(t)$  for all  $t$  and  $\Delta t$ .

Taking the sum of the equations, the criterion for preservation is

$$\sum_{A \in G} \gamma_A (q_{A*} - q_A) = 0 \quad \text{or} \quad \sum_{A \in G} \gamma_A q_{A*} = \sum_{A \in G} \gamma_A q_A$$

Now

$$\begin{aligned} \gamma_A w_{BA} &= \sum_{C \in G \setminus \{A\}} \gamma_C w_{BA} = \sum_{C \in G \setminus \{A\}} \varepsilon_C \alpha_{CA} \delta_A w_{BA} \\ &= (\sum_{C \in G \setminus \{A\}} \varepsilon_C \alpha_{CA}) \delta_A \varepsilon_B \alpha_{BA} / \sum_{C \in G \setminus \{A\}} \varepsilon_C \alpha_{CA} \\ &= \varepsilon_B \alpha_{BA} \delta_A = \gamma_{BA} \end{aligned}$$

Therefore

$$\gamma_A q_{A*} = \sum_{B \in G \setminus \{A\}} \gamma_A w_{BA} q_B = \sum_{B \in G \setminus \{A\}} \gamma_{BA} q_B$$

and taking the sum

$$\sum_{A \in G} \gamma_A q_{A*} = \sum_{A \in G} \sum_{B \in G \setminus \{A\}} \gamma_{BA} q_B =$$

$$\sum_{B \in G} \sum_{A \in G \setminus \{B\}} \gamma_{BA} q_B = \sum_{B \in G} (\sum_{A \in G \setminus \{B\}} \gamma_{BA}) q_B$$

It follows that the criterion for overall emotion preservation is equivalent to

$$\sum_{A \in G \setminus \{B\}} \gamma_{BA} = \gamma_B = \sum_{A \in G \setminus \{B\}} \gamma_{AB} \quad \text{for all } B$$

which in terms of the basic parameters is equivalent to

$$\sum_{A \in G \setminus \{B\}} \varepsilon_B \alpha_{BA} \delta_A = \sum_{A \in G \setminus \{B\}} \varepsilon_A \alpha_{AB} \delta_B \quad \text{for all } B.$$

**Theorem 4 (Preservation of overall emotion)**

The following are equivalent:

- (i) The overall emotion in the group is preserved
- (ii)  $\sum_{A \in G \setminus \{B\}} \gamma_{BA} = \sum_{A \in G \setminus \{B\}} \gamma_{AB}$  for all  $B$ .
- (iii)  $\sum_{A \in G \setminus \{B\}} \varepsilon_B \alpha_{BA} \delta_A = \sum_{A \in G \setminus \{B\}} \varepsilon_A \alpha_{AB} \delta_B$  for all  $B$ .

When these conditions are satisfied, an equilibrium is reached where each emotion level is the average of the initial emotion levels.

The conditions are satisfied in particular when all  $\gamma_{BA}$  are equal, or when, more specifically, all  $\varepsilon_A$  are equal, all  $\alpha_{AB}$  are equal and all  $\delta_B$  are equal. ■

Finally it is analyzed under which conditions the emotion values stay within the interval  $[0, 1]$  (*closure property*). It can easily be verified that the expression describing change reaches its maximum for  $\varepsilon_A = \alpha_{AB} = \delta_B = q_S = 1$  and  $q_R = 1$ . Similarly, this function reaches its minimum for  $\varepsilon_A = \alpha_{AB} = \delta_B = q_R = 1$  and  $q_S = 0$ . Using this the following equations for upper and lower bounds are obtained:

$$(1 - (\#(G) - 1))\Delta t = q_{\min} \leq q_R(t + \Delta t) \leq q_{\max} = (\#(G) - 1)\Delta t$$

In order to maintain the closure property for emotion contagion in the absorption model,  $q_{\max}$  has to be constrained to 1 and  $q_{\min}$  to 0. Therefore, respectively:

$$(\#(G) - 1)\Delta t \leq 1 \quad \text{and} \quad (1 - (\#(G) - 1))\Delta t \geq 0$$

which both lead to the same constraint  $\Delta t \leq 1/(\#(G) - 1)$ . So, as long as this constraint is maintained, the closure property holds for the absorption model:

**Theorem 5 (Closure property)**

The emotion values generated remain in the interval  $[0, 1]$  if  $\Delta t \leq 1/(\#(G) - 1)$ .

## 5. FORMAL VERIFICATION

This section addresses analysis of the emotion contagion model by verification of dynamic properties. The purpose of this type of verification is to check whether the model behaves as it should. A typical example of a property that may be checked is whether no unexpected situations occur, such as a variable running out of its bounds (e.g.,  $q_A(t) > 1$ , for some  $t$  and  $A$ ), or whether eventually an equilibrium value is reached. Other, more complex examples can be found in the theorems presented in the previous section. To analyze the resulting simulation traces in more detail, the TTL Checker tool (Bosse, Jonker, Meij, Sharpanskykh, Treur 2008) has been used. This tool takes as input a temporal predicate logical language TTL formula and a set of traces, and verifies automatically whether the formula holds for the traces. For the emotion absorption model, a number of such dynamic properties have been formalized in the language TTL (Bosse, Jonker, Meij, Sharpanskykh, Treur 2008) varying from properties addressing limit behavior (equilibria reached) to properties of the process from initial values to the equilibria. Below, a number of these properties are introduced, both in semi-formal and in informal notation (where  $\text{state}(\gamma, t) \models p$  denotes that  $p$  holds in trace  $\gamma$  at time  $t$ ). Note that the properties are all defined for a particular trace  $\gamma$  and sometimes for a particular time interval between  $t_b$  and  $t_e$ .

**P1a - Emotional Stability for Agent a**

For all time points  $t_1$  and  $t_2$  between  $t_b$  and  $t_e$  in trace  $\gamma$ , if at  $t_1$  the level of emotion of agent  $a$  is  $x_1$ , then at  $t_2$  the level of emotion of agent  $a$  is between  $x_1 - \alpha$  and  $x_1 + \alpha$ .

$\text{P1a}(\gamma: \text{TRACE}, t_b, t_e: \text{TIME}, a: \text{AGENT}, \alpha: \text{REAL}) \equiv$

$\forall t_1, t_2: \text{TIME} \forall x_1, x_2: \text{REAL}$

$\text{state}(\gamma, t_1) \models \text{emotion}(\text{agent}(a), x_1) \ \&$

$\text{state}(\gamma, t_2) \models \text{emotion}(\text{agent}(a), x_2) \ \&$

$t_b \leq t_1 \leq t_e \ \& \ t_b \leq t_2 \leq t_e \Rightarrow x_1 - \alpha \leq x_2 \leq x_1 + \alpha$

This property can be used to verify in which situations a certain agent's level of emotion does not fluctuate much. It has been found, for example, that for the trace shown in Figure 1 and for  $\alpha = 0.00001$ , the emotion of agent  $a$  remains stable between time point 28 and 50. In other words, checking  $\text{P1a}(\text{traceFig1}, 28, 50, a, 0.00001)$  was successful, where  $\text{traceFig1}$  is the trace of Figure 1.

**P1b - Emotional Stability for Agent a around Value  $x$**

For all time points  $t$  between  $t_b$  and  $t_e$  in trace  $\gamma$

the level of emotion of agent  $a$  is between  $x - \alpha$  and  $x + \alpha$  (where  $\alpha$  is a constant).

$P1b(\gamma:TRACE, tb, te:TIME, x:REAL, a:AGENT, \alpha:REAL) =$   
 $\forall t:TIME \forall y:REAL$

$state(\gamma, t) \models emotion(agent(a), y) \ \& \ tb \leq t \leq te \Rightarrow x - \alpha \leq y \leq x + \alpha$

As a variant of P1a, property P1b can be used to check whether an agent's level of emotion stays around a certain (given) value. For example, for  $\alpha = 0.0001$ , property  $P1b(traceFig1, 25, 50, 0.4333, b, 0.0001)$  was true. One step further, P1a and P1b can be used as building blocks to check the propositions and theorems related to equilibria presented in Section 4 against the generated traces. For example, property P1c checks whether Theorem 3 holds:

#### P1c - Equal Equilibria

If for all agents  $A$  and  $B$ ,  $\gamma_{AB}$  is nonzero in trace  $\gamma$  then eventually the same equilibrium  $q$  (between 0 and 1) will occur for all agents

$P1c(\gamma:TRACE, \alpha:REAL) =$

$[\forall a1, a2:AGENT \ [a1 \neq a2 \Rightarrow \exists g:REAL > 0$

$[state(\gamma, 1) \models has\_gamma\_for(agent(a1), agent(a2), g)]]$

$\Rightarrow [\exists q:REAL \ \forall a:AGENT \ P1b(\gamma, 40, 50, q, a, \alpha)]]$

This property, which has been proven in the mathematical analysis, has been checked for  $\alpha = 0.07$  for all generated traces, and indeed was confirmed. In addition, similar properties have been formulated that make claims about the equilibria on the basis of the initial settings. Due to space limitations, details of these properties are not shown here. However, some examples (in informal notation) are:

- In case  $\gamma_{SR} = 0$  for all agents, then each agent ends up in an equilibrium that is equal to its initial emotion value.
- In case  $\delta_R = 0$  for exactly 1 agent  $A$  (i.e.,  $\delta_A = 0$ ), and other  $\delta_R$  are nonzero, and all  $\alpha_{SR}$  and  $\varepsilon_S$  are nonzero for all agents, then each agent ends up in an equilibrium that is equal to the initial emotion value of agent  $A$ .
- In case  $\delta_R = 0$  and  $\varepsilon_S = 0$  for exactly 1 agent  $A$  (i.e.,  $\delta_A = \varepsilon_A = 0$ ), and other  $\delta_R$  and  $\varepsilon_S$  are nonzero, and all  $\alpha_{SR}$  are nonzero for all agents, then agent  $A$  ends up in an equilibrium that is equal to its initial emotion value, and all other agents end up in an equilibrium that is in between their initials emotion values.
- In case  $\delta_R = 0$  for exactly 2 agents  $A$  and  $B$  (i.e.,  $\delta_A = \delta_B = 0$ ), and other  $\delta_R$  are nonzero, and all  $\alpha_{SR}$  and  $\varepsilon_S$  are nonzero for all agents, then agent  $A$  and  $B$  end up in an equilibrium that is equal to their initial emotion value, and all other agents end up in an equilibrium that is in between the initial emotion values of  $A$  and  $B$ .

#### P2a - Monotonic Increase of Emotion<sup>2</sup>

For all time points  $t1$  and  $t2$  between  $tb$  and  $te$  in trace  $\gamma$ , if at  $t1$  the level of emotion of agent  $a$  is  $x1$ , and at  $t2$  the level of emotion of agent  $a$  is  $x2$  and  $t1 < t2$ , then  $x1 \leq x2$ .

$P2a(\gamma:TRACE, tb, te:TIME, a:AGENT) =$

$\forall t1, t2:TIME \forall x1, x2:REAL$

$state(\gamma, t1) \models emotion(agent(a), x1) \ \&$

$state(\gamma, t2) \models emotion(agent(a), x2) \ \&$

$tb \leq t1 \leq te \ \& \ tb \leq t2 \leq te \ \& \ t1 < t2 \Rightarrow x1 \leq x2$

Property P2a and the variant P2b addressing monotonic decrease (by replacing  $\leq$  in the consequent by  $\geq$ ) can be used to check whether an agent's level of emotion increases or decreases monotonically over a certain interval. Such monotonicity, for example, occurs for agent  $c$  during the whole trace shown in Figure 1 (i.e., property  $P2b(traceFig1, 1, 50, c)$  succeeded). Furthermore, these properties can be used as building blocks to check the propositions and theorems related to monotonicity presented in Section 4 against the generated traces. For example, property P2c checks whether part (c) and (d) of Proposition 1 hold:

#### P2c - Conditional Monotonicity

For all agents  $A$ , if  $q_{A*} \geq q_A$  between  $tb$  and  $te$  in trace  $\gamma$ , then  $q_A$  is monotonically increasing during this interval, and if  $q_{A*} \leq q_A$  between  $tb$  and  $te$  in trace  $\gamma$ , then  $q_A$  is monotonically decreasing during this interval.

$P2c(\gamma:TRACE, tb, te:TIME) =$

$\forall a1:AGENT$

$[[\forall t:TIME \exists a2, a3:AGENT \exists x1, x2, x3, w2, w3:REAL$

$state(\gamma, t) \models emotion(agent(a1), x1) \ \&$

$state(\gamma, t) \models emotion(agent(a2), x2) \ \&$

$state(\gamma, t) \models emotion(agent(a3), x3) \ \& \ a2 \neq a3 \ \& \ tb \leq t \leq te \ \&$

$state(\gamma, 1) \models has\_w\_for(agent(a2), agent(a1), w2) \ \&$

$state(\gamma, 1) \models has\_w\_for(agent(a3), agent(a1), w3) \ \&$

$w2 \cdot x2 + w3 \cdot x3 \geq w1] \Rightarrow p2a(\gamma, tb, te, a1)] \ \&$

$[[\forall t:TIME \exists a2, a3:AGENT \exists x1, x2, x3, w2, w3:REAL$

$state(\gamma, t) \models emotion(agent(a1), x1) \ \&$

$state(\gamma, t) \models emotion(agent(a2), x2) \ \&$

$state(\gamma, t) \models emotion(agent(a3), x3) \ \& \ a2 \neq a3 \ \& \ tb \leq t \leq te \ \&$

$state(\gamma, 1) \models has\_w\_for(agent(a2), agent(a1), w2) \ \&$

$state(\gamma, 1) \models has\_w\_for(agent(a3), agent(a1), w3) \ \&$

$w2 \cdot x2 + w3 \cdot x3 \leq w1] \Rightarrow p2b(\gamma, tb, te, a1)]$

Here,  $q_{a1*}$  is explained in Section 2. This property has been confirmed for all possible intervals in all generated traces.

#### P3 - Emotion between Boundaries

For all time points  $t$  between  $tb$  and  $te$  in trace  $\gamma$

if at  $t$  the level of emotion of agent  $a$  is  $x$ , then  $\min < x < \max$ .

$P3(\gamma:TRACE, tb, te:TIME, \max, \min:REAL, a:AGENT) =$

$\forall t:TIME \forall x:REAL$

$state(\gamma, t) \models emotion(agent(a), x) \ \& \ tb \leq t \leq te \Rightarrow \min < x < \max$

This property can be used to check whether the emotion of an agent stays between certain boundaries. For example, no emotional value should ever become lower than 0 or higher than 1. This turned out to be the case for all generated traces where  $\Delta t \leq 1/(\#(G) - 1)$ . That is, property  $P3(trace, 1, 50, 0.0, 1.0, X)$  succeeded for all traces trace with these settings and agents  $X$ , which confirms Theorem 5 of the previous section. In addition, it was found that the property failed for some traces that do not have these settings. E.g., for a trace with  $\Delta t = 0.7$ , all  $\gamma_{SR} = 1$ , and initial values  $q_A = 0.3$ ,  $q_B = 0.1$ , and  $q_C = 0.9$ , the emotion values eventually run out of their boundaries.

#### P4 - Emotion Agent a1 above Agent a2

For all time points  $t$  between  $tb$  and  $te$  in trace  $\gamma$ , if at  $t$  the level of emotion of agent  $a1$  is  $x1$

and the level of emotion of agent  $a2$  is  $x2$ , then  $x1 \geq x2$ .

$P4(\gamma:TRACE, tb, te:TIME, a1, a2:AGENT) =$

$\forall t:TIME \forall x1, x2:REAL$

$state(\gamma, t) \models emotion(agent(a1), x1) \ \&$

<sup>2</sup> A strict variant of such properties can be created by replacing  $\leq$  by  $<$ .

$\text{state}(\gamma, t) \models \text{emotion}(\text{agent}(a_2), x_2) \ \& \ t_b \leq t \leq t_e \Rightarrow x_1 \geq x_2$

Property P4 can be used to check whether an agent's emotion level stays above (or below) another agent's level during a specified interval. For example, in the trace of Figure 1, agent c always has a higher emotion than agent a (i.e., property  $P4(\text{traceFig1}, 1, 50, c, a)$  succeeded). However, in the end the difference becomes very small, and if the simulation were continued longer, eventually this property would fail.

#### P5 - Emotion Approaches Value x with Speed s

For all time points  $t_1$  and  $t_2$  between  $t_b$  and  $t_e$  in trace  $\gamma$ , if at  $t_1$  the level of emotion of agent a is  $x_1$ , and at  $t_2$  the level of emotion of agent a is  $x_2$ , and  $t_2 = t_1 + 1$ , then  $s * |x - x_1| \geq |x - x_2|$  (where  $s$  is a constant  $< 1$ ).

$P5(\gamma: \text{TRACE}, t_b, t_e: \text{TIME}, x: \text{REAL}, a: \text{AGENT}) =$

$\forall t_1, t_2: \text{TIME} \ \forall x_1, x_2: \text{REAL}$

$\text{state}(\gamma, t_1) \models \text{emotion}(\text{agent}(a), x_1) \ \&$

$\text{state}(\gamma, t_2) \models \text{emotion}(\text{agent}(a), x_2) \ \&$

$t_b \leq t_1 \leq t_e \ \& \ t_b \leq t_2 \leq t_e \ \& \ t_2 = t_1 + 1 \Rightarrow |x - x_1| * s \geq |x - x_2|$

Property P5 can be used to check whether an agent's emotion level approaches a given value  $x$ , and to determine the speed  $s$  with which this happens (where  $0 < s < 1$ , and a high  $s$  denotes a slow speed). For example, for the trace shown in Figure 2, it turned out that agent b approaches emotion level 0.3 with a speed of approximately 0.9991.

## 6. DISCUSSION

Usually computational models of emotion consider a single agent (Gratch, Marsella 2001). However, since a couple of decades, researchers have started to investigate emotions as a collective property of groups, emphasizing the influence of collective emotions among group members on group processes (Barsade, Gibson 1998). Inspired by these developments, the current paper proposed a multi-agent-based approach to formalize and simulate the dynamics of emotion absorption within groups. A dynamical systems style multi-agent model has been developed and simulated. Although an extensive empirical validation is left for future work, these experiments pointed out that the model is able to produce various interesting emerging patterns as described (informally) in the psychological literature. Moreover, by a mathematical analysis a number of properties of the model have been determined, among which conditional monotonicity properties and the occurrence of equilibria. Using the TTL checking software (Bosse, Jonker, Meij, Sharpanskykh, Treur 2008), several of these properties have been verified and confirmed against generated simulation traces, which is an important additional indication that the model behaves as expected. As a next step, a more detailed validation of the model in laboratory experiments is planned. The idea is to create a setting in which various humans interact in a room, while continuously being subject to (physiological) measurements (e.g., using emotion recognition approaches as discussed in (Goldman, Sripada 2005)) to assess their emotion. The obtained data can then be used to fine-tune the model.

Concerning further work, other possible models are being examined as well. For instance, another model of emotion contagion could represent patterns where emotion is amplified through contagion instead of being absorbed as in the current model. This amplification model could correspond to so called upwards and downwards emotional spirals that are hypothesized by the authors in (Frederickson, Barbara, Joiner, Thomas 2002), based on Frederickson's broaden-and-build theory. A corresponding personality trait that could model this difference between absorbing or amplifying emotions in the emotion contagion process could be the 'sensation seeking' personality trait (Zuckerman 1994).

Besides extending the presented model or examining other models, it is worthwhile to investigate the possibilities to apply it in real-world case studies. An interesting application is to exploit it in an Ambient Intelligence setting. The idea is to develop an Ambient Agent that is able to estimate the group emotion of a team in a particular environment (e.g., a team of operators on a naval vessel, or a sports team), and to provide support in case the group emotion negatively affects the team performance. This direction will be further explored in future research.

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