Temporal Factorisation and Realisation in Cognitive Dynamics and Beyond

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Abstract

Temporal factorisation is a principle underlying approaches to dynamics used within many disciplines. According to this principle any temporal relationship of the form 'past pattern implies future pattern' can be factorised into a relationship of the form 'past pattern implies present state' and a relationship of the form 'present state implies future pattern'. To enable this, the principle postulates the existence of certain mediating state properties in the present state. In this paper the question is addressed whether and how a postulated mediating state property relates to other state properties in the (present) state in which it occurs. This analyses provides a conceptual framework covering concepts and themes that usually are considered unrelated, such as, the notions of differential equations in Mathematics, of transition system and rule-based system in Computer Science, Cognitive Science and Artificial Intelligence, and of reduction and realisation of mental states in Cognitive Science and Philosophy of Science.

Introduction

In (Treur, 2006) the temporal factorisation principle was identified, formalised, and shown to play a crucial role for modelling dynamics in different disciplines such as Physics, Chemistry, Biology, Mathematics, Computer Science, and Cognitive Science. The temporal factorisation principle claims that if a certain (past) pattern of events leads to a certain (future) pattern of events, then there exists a state property *p* such that the past pattern leads to a (present) state where this property *p* holds, and any state where the state property *p* holds leads to the future pattern. This postulated state property *p* is called a mediating state property for the 'past pattern implies future pattern' relationship. For some of the foundational themes and approaches discussed in the literature in the cognitive domain, it has been shown in (Treur, 2006) how they can be generalised beyond the cognitive area, and incorporated in the more general conceptual framework based on temporal factorisation; in particular this has been addressed for the notion of mental state, and the notion of representational content of a mental state property. For another theme, namely, physical realisation of mental state properties, it is considered in the current paper, how it can be generalised beyond the cognitive domain and added to the framework based on the temporal factorisation principle. Thus the conceptual framework for dynamics based on 'temporal factorisation' as introduced in (Treur, 2006) will be extended to a conceptual framework based on 'temporal factorisation and realisation'.

To obtain this extension, the question of realism is addressed for the mediating state properties postulated in temporal factorisation. The corresponding type of question for the cognitive area, as addressed within Philosophy of Mind, namely the question of how real mental state properties are, will be a source of inspiration, in particular, the perspective of *physicalism* which aims at relating mental state properties in one way or the other to physical state properties; e.g., (Kim, 1996, pp. 9-13). By itself, the temporal factorisation principle does not give any suggestion on whether and how, within a state, a mediating state property relates to other state properties. It could have a purely synthetic nature, isolated from other, more genuine, state properties. Indeed, for a number of historical cases, the state properties obtained by temporal factorisation seem to have no relationship to other state properties in the same state whatsoever.

This holds, for example, for state properties within Physics in empty space such as velocity and momentum (inertia of motion). In other cases, such as the state property 'force' within Physics, such relationships to specific other state properties in the same state are assumed to exist (e.g., laws for specific types of forces, such as gravitational, electrical or magnetical forces), and they are often exploited in scientific practice. These relationships are not systematic, however, but differ from context to context, so they are rather heterogeneous, as, for example, pointed out by Nagel (1961), pp. 190-192.

For the cognitive domain, one of the advantages of a relation between mental state properties and physical state properties is that in such a case causation by a mental state property (mental causation; cf. Kim, 1996, pp. 125-154) can be explained as causation by the realising physical state property. For mediating state properties it can also be questioned how they can cause physical state properties in successive states. A one-to-one relationship of a postulated mediating state property with a more genuine physical state property may provide an explanation of causation by the mediating state property similar to the one in the case of mental causation by a realised mental state property. In this paper it is shown that for mediating state properties realisation is often but not always possible, and whenever it is possible it may concern multiple realisation.

The paper is structured as follows. First a brief introduction of the temporal factorisation principle and its formalisation is presented, adopted from (Treur, 2006). Next it is discussed how mediating state properties can co-occur with other state properties, and introduces the notion of (*multiple*) *realisation relation* for a mediating state property. Furthermore, it is shown how in quantitative mathematical dynamic modelling approaches based on continuous state properties within the area of calculus in Mathematics, realisation relations are expressed in the form of difference equations (discrete time) or differential equations (continuous time). In addition it is shown how for qualitative dynamic modelling approaches based on discrete state properties, such as transition systems in Computer Science or rule-based systems in Cognitive Science or Artificial Intelligence, a qualitative format is used to express realisation relations. Finally, it is discussed how within Physics realisation relations for the second-order mediating state property 'force' play an important role.

The Temporal Factorisation Principle

The temporal factorisation principle as introduced in Treur, 2006) is formulated in terms of temporal relationships between past patterns, present states, and future patterns. Here a *past pattern a* refers to a property of a series of states or events in the past, and a *future pattern b* refers to a property of a series of states or events in the future. The temporal factorisation principle states that any systematic temporal 'past pattern implies future pattern' relationship $a \rightarrow b$ between a past pattern *a* and a future pattern *b* can be factorised in the form of temporal relationships $a \rightarrow p$ and $p \rightarrow b$ for some state property *p* of the present world state. More specifically, the principle claims that for any 'past pattern implies future pattern' relationship $a \rightarrow b$ *there exists* a world state property *p* (expressed in the ontology for state properties) such that temporal relationships 'past pattern implies present state property' $a \rightarrow p$ and 'present state property implies future pattern' $p \rightarrow b$ hold.[1] In short:

$$a \rightarrow b \quad \Rightarrow \quad \exists p \quad a \rightarrow p \quad \& \quad p \rightarrow b$$

Notice that the notation $\rightarrow$ is used here to indicate logical implication (between temporal properties). The postulated state property *p* is called a *mediating state property* for the given 'past pattern implies future pattern' relationship. The principle claims that the state ontology is (or can be chosen to be) sufficiently rich to express all the relevant information on the past in some condensed form in one state description, and the same with respect to the future. The principle can be viewed as a way to make temporal complexity of dynamics more manageable by relating it to state complexity, where an underlying assumption is that the state complexity needed can be kept limited.

As an example from Physics, temporal factorisation can be illustrated by the notion 'momentum' of a moving object in classical mechanics as a mediating state property. Different histories of the object can lead to the same momentum in the present state. The future of the object only (besides the object's current position) depends on this momentum in the present state, not on the specific history. This was the criterion by which the concept momentum was introduced in Physics in history (see Treur, 2005, for a more detailed historical case study). Therefore the state property momentum can be understood as a mediating state property for past and future patterns in (change of) position of an object; the temporal factorisation principle indicates the existence of this state property. For more details of this case within Physics, see Section 6.

Formalisation of the principle in the predicate logical temporal language TTL is given by:

$$\begin{array}{l} \forall \gamma, t \; [\; \varphi(\gamma, t) \;\Rightarrow\; \psi(\gamma, t) \;] \;\Rightarrow \\ \quad \exists p \; [\; \forall \gamma, t \; [\; \varphi(\gamma, t) \;\Rightarrow\; \text{state}(\gamma, t) \models p \;] \;\& \\ \quad\quad \forall \gamma, t \; [\; \text{state}(\gamma, t) \models p \;\Rightarrow\; \psi(\gamma, t) \;] \;] \end{array}$$

where $\varphi(\gamma, t)$ is a past pattern specification and $\psi(\gamma, t)$ a future pattern specification in TTL. Moreover, γ ranges over traces, t over time and p over state properties, and $\text{state}(\gamma, t) \models p$ denotes that state property p holds in the state of trace γ at time t. For more details, and more extensive examples, also in the cognitive domain, see (Treur, 2006).

Realisation of Mediating State Properties

To embed mediating state properties more intensively in the states in which they occur, their relationship to other properties of these states is considered. If in states, a mediating state property *p* always co-occurs with a certain state property *c* (or combination of state properties), such a co-ocurring property *c* is called a *realiser* (see also Figure 1; the vertical double arrow indicates the *realisation relation* between *p* and *c*).

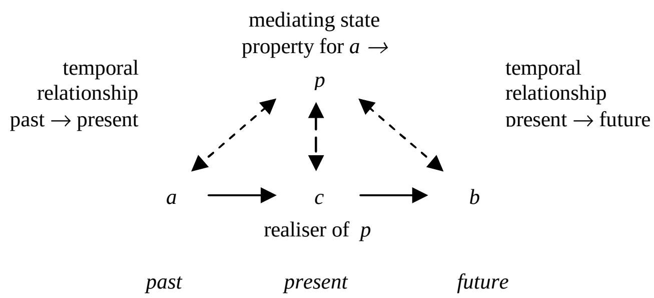


Figure 1. Realisation of a mediating state property

An example of the occurrence of realisers of mediating state properties are the following. An open a tap will lead to the presence of water in a glass in a next state, where the size of the opening determines how much water will be present at this next point in time. So the mediating state property between the past and the future (i.e., from opening the tap to water present in the glass) is in a sense hardwired in the physical configuration of the opened tap, which is described by state properties in the present state.

If a mediating state property has a realiser, it can be created or prevented by manipulating the state property that realises the mediating state property. For example, a mental

[1] Sometimes such relationships are simply called 'past to present', present to future', or 'past to future' relationships.

state property can be created or prevented by affecting its realiser by physical means (e.g., by using drugs). Similarly, for example, a mediating state property to fall can be created or prevented by either putting or keeping an object close to the earth, or by taking it far away.

A natural question arising in the context of realisation is whether a mediating state property, which co-occurs with some other state property, should be considered identical to this state property. This would solve the problem of how mediating state properties can be genuine state properties and how they can cause other state properties. Within physicalism, a mental state property is assumed to co-occur with its physical realiser; for mental state properties indeed such proposals to make identifications have been put forward in the form of different types of reductionism; e.g., (Kim, 1996, pp. 125-154, pp. 211-240; Bickle, 1998); see also (Bennett and Hacker, 2003; Bickle, 2003; Kim, 2005). One of the advantages of such a mental-physical state property identification is that for relationships between states at different points in time, the causation relation from Physics can describe, for example, how a mental state property such as an intention affects the world state. However, the phenomenon of multiple realisation obstructs a direct one-to-one identification.

The complicating issue of *multi-realisability* of mediating state properties is that there may sometimes be a co-occurrence with one other state property and sometimes with another one. Mental state properties usually have a large variety of realisers, for example in different animal species. Relating a mental state property in a biconditional manner to all of these mutually distinct (non-equivalent) realisers will lead to a contradiction: if p is equivalent to each of two realisers q_1 and q_2, then it follows that q_1 is equivalent to q_2, and thus they always co-occur. In a multiple realisation case where in different states sometimes one, sometimes another realiser co-occurs with p, this is a contradiction.

In the case of multiple realisers, the relation between mediating state property p and its realisers can be described by a supervenience relation (e.g., Kim, 1998).

> 'Mental properties supervene over physical properties in that for every mental property M that occurs at some point in time t, there exists some physical property P that also occurs at t, such that always if P occurs at some point in time t', also M occurs at t'.' (Kim, 1998, p. 9)

This can be formalised in TTL by

$$\forall\gamma, t \;\; \text{state}(\gamma, t) \models M \;\Rightarrow\; \exists P \;\; \text{physical}(P) \;\&\; \text{state}(\gamma, t) \models P \;\& \;\; \forall\gamma', t' \;\; [\, \text{state}(\gamma, t') \models P \;\Rightarrow\; \text{state}(\gamma', t') \models M \,]$$

Following this line, the following can be defined (see also Kim, 1996, Ch. 9): A set Q of state properties (or combinations thereof) is a *complete set of realisers* of p if and only if: (1) if p occurs in a state then one of the q in Q occurs in this state, and (2) if one of the q in Q occurs in a state then p occurs in this state. The elements q of Q are called the (*non-unique*) *realisers* of p. The relations between p and the elements of Q are called *(multiple) realisation relations*. This can be formalised by

$$\forall\gamma, t \;\; [\, \text{state}(\gamma, t) \models p \;\Rightarrow\; \exists q \;\; [\, \text{in}(q, Q) \;\&\; \text{state}(\gamma, t) \models q \,] \;\& \;\; \forall\gamma', t', q \;\; [\, \text{in}(q, Q) \;\&\; \text{state}(\gamma', t') \models q \;\Rightarrow\; \text{state}(\gamma', t') \models p \,] \,] \,]$$

Supervenience applied to mediating state properties, expresses that mediating state properties are always realised in one way or the other. However, this can happen in a nonsystematic, ad hoc manner: for every situation a different realiser. Sometimes, variants are introduced for each context by context characterising assumptions: local reduction; cf. (Kim, 1996, pp. 211-240).

Realisation Relations in Quantitative Cases: Differential Equations

In this section it is shown how quantitative dynamic modelling methods from the area of calculus within Mathematics can be described by realisation relations. In Treur (2006, Section 7) it is discussed in which sense the derivative of a continuous variable at a certain time point can be viewed as a mediating state property for a past to future relationship in the form of a smoothness condition. In this section it is discussed in which form a realisation relation of such a mediating state property can occur. Let p be such a mediating state property (i.e., change rate) for variable x. How can this mediating state property be related to other state properties? As a special case, the relationship of (the value of) p to other state properties can focus on properties that can be expressed in terms of (the value of) x. A plain case of this idea is when a value v of p in a state is considered always to co-occur with this value v for some expression or function F in the value of x in the same state:

$$has\_value(p, v) \;\;\leftrightarrow\;\; \exists w \;\; has\_value(x, w) \;\&\; v = F(w)$$

or

$$has\_value(p, F(w)) \;\leftrightarrow\; has\_value(x, w)$$

This shows a bi-conditional form for the co-occurrence of the two properties in states, where the right hand side of the 'if and only if' is the realiser of the mediating state property at the left hand side (cf. Nagel, 1961, pp. 345-358; Kim, 1996, pp. 212-216). An alternative way to express the same biconditional relationship is:

$$p(t) = F(x(t))$$

Keeping in mind that the mediating state property p is the derivative of x, sometimes denoted by dx/dt the last way of expressing can be also written as

$$dx/dt = F(x(t))$$

This is the usual notation for a *differential equation*. These formats allow us to relate a mediating state property at time t to other state properties of the state at t. As an example, take the function F defined by:

$$F(x) = \alpha x(1 - x/C)$$

For this example, mediating state property p is related to another state property as follows:

$$has\_value(p, \alpha w(1 - w/C)) \;\;\leftrightarrow\;\; has\_value(x, w)$$

Or, alternatively expressed:

$p(t) = \alpha x(t)(1 - x(t)/C)$

In the usual notation for a differential equation this is also formulated as

$dx/dt = \alpha x(t)(1 - x(t)/C)$

The differential equation based on this example function F describes (with the parameters α and C within a certain range; e.g., $\alpha = 0.5$ and $C = 10$) a logistic growth pattern with asymptotic value C (carrying capacity) and initial growth rate α.

It turns out that (first-order) differential equations can be understood from the conceptual framework based on 'temporal factorisation and realisation' as realisation relations for mediating state properties. The differential equation format

$dx/dt = F(x(t))$

expresses in a variety of cases how a mediating state property relates to another state property. Moreover, this can easily be extended to a system of differential equations, such as

$dx/dt = F(x(t), y(t))$
$dy/dt = G(x(t), y(t))$

where each of the mediating state properties dx/dt and dy/dt has a realisation relation to a combination of the state properties x and y, defined by F and G, respectively. In a discretised form a *difference equation* is obtained:

$x(t') - x(t) \;=\; F(x(t))\,(t' - t)$

or

$\Delta x = F(x(t))\Delta t$

with

$\Delta x = x' - x$ and $\Delta t = t' - t$

The patterns of relations used in the calculation based on this difference equation are depicted in Figure 2 (a picture similar to the one in Figure 1). Here, the lower line shows the successive states, with state properties indicating values for x by a_0, a_1, a_2 and a_3. Above each of these state properties, the mediating state property is depicted that is realised by it in the same state.

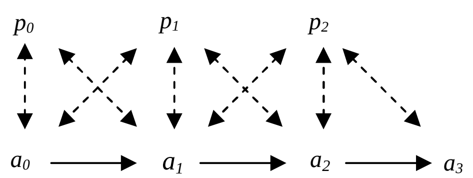


Figure 2 Mediating state properties realised in successive states

The vertical bi-arrow is exploited by calculating $F(a_i)$ from a_i. The arrow to the next state by adding the resulting mediating state property to the present a_i, thus obtaining, the state property a_{i+1} as a successor for a_i. In Section 5 it will be shown that a similar pattern of relations occurs in the context of qualitative dynamic modelling techniques such as transition systems, causal models, logical models and rule-based systems.

In summary, quantitative dynamic modelling approaches based on differential equations can be understood from the conceptual framework for dynamics based on temporal factorisation and realisation in the sense that they express realisation relations for mediating state properties. A characteristic of this area is that all state properties are based on variables and numerical values assigned to them. This excludes modelling approaches where dynamics is analysed based on qualitative state properties. In the next section such qualitative approaches are addressed.

Realisation Relations in Qualitative Cases: Causal, Rule-Based and Transition Systems

In this section qualitative dynamic modelling methods are considered. Examples of such methods are transition systems (e.g., Arnold, 1994), production or rule-based systems (e.g., Anderson, 1996; Buchanan and Shortliffe, 1984), causal models (e.g., Bosse et al., 2005), and executable temporal logic (e.g., Barringer et al., 1996; Fisher, 2005). These methods specify in a qualitative manner how a state in a system may change. These methods can be analysed from the perspective of temporal factorisation and realisation as described below. Viewed from an abstract perspective, the following format is used in such methods. In a rule[2] $c \twoheadrightarrow d$ with antecedent c and consequent d:

- the first description c indicates a combination of state properties for the current state
- the second description d indicates one or more state properties for the next state

As an illustration, a simple example scenario describes the following rules for preparing a cup of tea: getting hot water, getting a cup, getting a tea bag, and finally, by combining these, getting the tea.

not hot_water_present $\twoheadrightarrow$ *hot_water_present*
not cup_present $\twoheadrightarrow$ *cup_present*
hot_water_present $\wedge$ *cup_present* $\wedge$ *not tea_bag_present*
$\twoheadrightarrow$ *tea_bag_present*
hot_water_present $\wedge$ *cup_present* $\wedge$ *tea_bag_present*
$\twoheadrightarrow$ *tea_ present* $\wedge$ *not tea_bag_present*

From the conceptual framework based on temporal factorisation and realisation such a rule can be understood as a relationship between mediating state properties and other state properties in the following manner. For example, for the second rule: if in a state *not cup_present* occurs, also a specific mediating state property *p1* occurs in this state which anticipates on *cup_present*. Thus, any rule $c \twoheadrightarrow d$ can be interpreted as an implication $c \rightarrow p$, describing a logical relationship between state properties in a given state. In the current context, c can be considered a (single) realiser for p.

[2] So, what is called a rule here can have the form of a transition in a transition system, a production rule, a causal relation, or an executable temporal formula.

In the given specification this is the case because also the converse implication $p \rightarrow c$ holds (i.e., always if p occurs in a state, also c occurs). Thus the following biconditional realisation relations for mediating state properties *p1* to *p5* occur in the tea scenario:

p1 $\leftrightarrow$ *not hot_water_present*

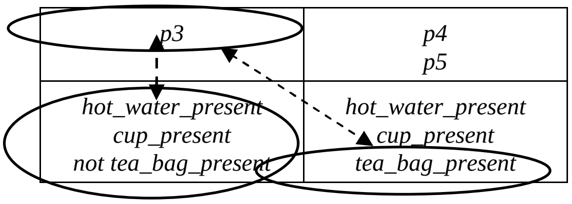
p2 $\leftrightarrow$ *not cup_present*

p3 $\leftrightarrow$ *hot_water_present* $\wedge$ *cup_present* $\wedge$ *not tea_bag_present*

p4 $\leftrightarrow$ *hot_water_present* $\wedge$ *cup_present* $\wedge$ *tea_bag_present*

p5 $\leftrightarrow$ *hot_water_present* $\wedge$ *cup_present* $\wedge$ *tea_bag_present*

Here the mediating state properties *p1* to *p5* anticipate on respectively *hot_water_present, cup_present, tea_bag_present, tea_present, not tea_bag_present.* Some of the relationships as depicted in Figure 2 are for this case:



In a qualitative dynamic modelling approach, it is not always the case that all realisers are unique. For example, in the chosen specification, the mediating state property to have a tea bag present is based on the presence of hot water and cup. However, there may be alternative ways to achieve the presence of the tea bag, for example right at the start, based on the absence of a tea bag and absence of a cup; this would provide two multiple realisers for a mediating state property leading to the tea bag being present:

hot_water_present $\wedge$ *cup_present* $\wedge$ *not tea_bag_present*

and

not cup_present $\wedge$ *not tea_bag_present*

Realisation Relations in Physics: Forces

In Section 2 the notion of momentum in Physics was mentioned as an example of a mediating state property for the notion of position. The state property momentum abstracts from the various histories that could have happened and would have resulted in the same mediating state property for the future. In the other time direction, no matter what future will arise, a momentum indicates from what pattern it originated, so it abstracts from futures. In some more detail, if for the variable x the position (on a line) of an object with mass m is taken, then the temporal factorisation of temporal relationships between positions before and after a certain time point t provides as a mediating state property the instantaneous velocity (or change rate of position) v of the object at time point t. Momentum of the object is obtained by $p = mv$, or by temporal factorisation of the quantity mx. In addition, Newton's second law $F = ma$ (with a the acceleration) can be fomulated as

$$m\, dv/dt = F \quad \text{or} \quad dp/dt = F$$

This shows how force can be obtained as a second-order mediating state property (the change rate of momentum, which itself is a first-order mediating state property).

A specific example, showing how a specific force can be realised, is the following. For the trajectory of an object in space, with mass m, approaching earth (with mass M), Newton's second law can be used in conjunction with his law of gravitation, for the interaction between the two objects involved (here x is the distance between the object and the earth, and c is a constant):

$$m\, d^2x/dt^2 = F$$
$$F = c\, mM/x^2$$

An assumption here is that no other interaction plays a role. In this case, the force-function identified is $F(x) = c\, mM/x^2$, as described by Newton's gravitation law. This provides a second-order differential equation for the distance x of the object to the earth, depending on time t:

$$m\, d^2x/dt^2 = c\, mM/x^2$$

Here, the left hand side provides the second-order mediating state property F and the right hand side is a realiser of this mediating state property: the equality guarantees that both properties will always co-occur in states. Note that in this case the (realised) second-order mediating state property leads to a not realised first-order mediating state property (a changed momentum) in a next state. In turn, this first-order mediating state property momentum leads to a next state in which the changed state property position occurs.

In Nagel (1961, pp. 186-192), the multiple realisation of the notion of force is discussed. In line with what was stated above, his analysis asserts that for various different situations specific force-functions, specifying how force relates to other properties of the state (and/or first-order mediating state properties) are needed. Forces can occur due to state properties involving, for example: the presence of an object pushing or pulling, deformation such as caused by collisions (e.g., billiard balls), the presence of objects with electrical charge, the presence of magnetic objects, the presence of other masses (gravitation), atmospheric pressures. For each of these circumstances, a different expression in terms of the world state ontology (a force-function) describes the force that occurs. Only if for a given situation such a force-function has been identified, something can be done using the laws of classical mechanics. In this sense, this case shows a heterogeneous situation, where past patterns and present states relating to a force mediating state property are described in some heterogeneous disjunctive form with at least, say, up to 5 to 10 essentally different contexts of the origin of the force. In this heterogeneous situation, in different contexts different force-functions (and hence realisers) are identified, which still allows succesful use of the notion of force in applications. This shows an example of an approach to multiple realisation, comparable to the notion of local or context-dependent reduction as described by Kim (1996), pp. 211-240.

Notice that an additive property for this mediating state property force holds in the following sense: the combined effect of any number of different contributions to the second-order mediating state property can be obtained by adding their values. So, any value w for this second-order mediating state property can be obtained as the combined effect from, for example, gravitation, electrical charge, and deformation by collision. For example, considering one dimension where all effects work along the same axis, this can occur in the form of an infinite number of possible sums $w = w_1 + w_2 + w_3$, where the terms are the contribution of one of the three effects (e.g., w_1 by gravitation, w_2 by electrical charge, w_3 by collision). This shows that the complete set of realisers Q can be infinite, and that in contexts can not always be separated.

Discussion

Mediating state properties obtained by temporal factorisation provide a quite general concept to describe dynamics. Special cases can be found, for example, in mental state properties in Cognitive Science and several concepts in Physics. In many (but not all) cases mediating state properties can be grounded in the (present) state in which they occur by relating them to realisers: other state properties or combinations thereof that co-occur with the mediating state property. These realisers often are useful in modelling and calculation of trajectories or traces over time, for example, by using diffferential equations to specify realisation relations. However, apart from this convenience, how necessary are these realisations?

On the one hand, a mediating state property may not be defined by relating it to another state property, but solely by its temporal relational specification. In this case, in principle mediating state properties can be left out as they do not add powers over and above the power of the direct 'past pattern implies future pattern' temporal relationships.

On the other hand, there is no harm in assuming them: what they are used for can also be done using temporal relationships without mediating state properties as state properties. So, a possible view is that the temporal relationships actually justify the postulation and use of mediating state properties as a convenient shorthand for more complex temporal specifications, in line with the position put forward in (Jonker, Treur and Wijngaards, 2002) that higher levels of conceptualisation often may pay off. If, in addition, realisation relations are known, then the mediating state property can become more than a convenient shorthand: a concept like any other concept, occurring in a network of relations to other concepts. In particular, this applies to mental state properties as a special case of mediating state properties.

Notice that the temporal factorisation principle is not a (first-order) law of nature. Due to its reference to and quantification over state properties it is a second-order principle. Therefore the temporal factorisation principle contributes another route in the direction of unification of cognition and nature, not based on common first-order laws, but on common second-order principles. Such second-order principles may play an important role in bridging the gap between different disciplines, as shown in this paper, but also within the discipline of Physics it turns out that they can play an important role, as Nagel (1961) has pointed out concerning Newton's laws (although there they sometimes mistakenly are called laws).

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