Comparison of Agent-Based and Population-Based Simulations of Displacement of Crime

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Abstract

Within Criminology, the process of crime displacement is usually explained by referring to the interaction of three types of agents: criminals, passers-by, and guardians. Most existing simulation models of this process are agent-based. However, when the number of agents considered becomes large, population-based simulation has computational advantages over agent-based simulation. This paper presents both an agent-based and a population-based simulation model of crime displacement, and reports a comparative evaluation of the two models. In addition, an approach is put forward to analyse the behaviour of both models by means of formal techniques.

1. Introduction

Within Criminology one of the main research interests is the emergence of so-called criminal hot spots. These hot spots are places where many crimes occur [14]. After a while the criminal activities shift to another location, for example, because the police has changed its policy and increased the numbers of officers at the hot spot. Another reason may be that the passers by move away when a certain location gets a bad reputation. Such a shift between locations is called the displacement of crime. The reputation of specific locations in a city is an important factor in the spatio-temporal distribution and dynamics of crime [8]. For example, it may be expected that the amount of assaults that take place at a certain location affect the reputation of this location. Similarly, the reputation of a location affects the attractiveness of that location for certain types of individuals. For instance, a location that is known for its high crime rates will attract police officers, whereas most citizens will be more likely to avoid it. As a result, the amount of criminal activity at such a location will decrease, which will affect its reputation again.

The classical approaches to simulation of processes in which groups of larger numbers of agents and their interaction are involved are population-based: a number of groups is distinguished (populations) and each of these populations is represented by a numerical variable indicating their number or density (within a given area or location) at a certain time point. The simulation model takes the form of a system of difference or differential equations expressing temporal relationships for the dynamics of these variables. Well-known classical examples of such population-based models are systems of difference or differential equations for predator-prey dynamics (e.g., [10], [15], [5], [11], [16]) and the dynamics of epidemics (e.g., [1], [5], [7], [9], [12]). Such models can be studied by simulation and by using analysis techniques from mathematics and dynamical systems theory.

From the more recently developed agent system area it is often taken as a presupposition that simulations based on individual agents are a more natural or faithful way of modelling, and thus will provide better results (e.g., [2], [6], [13]). Although for larger numbers of agents such agent-based modelling approaches are more expensive computationally than population-based modelling approaches, such a presupposition may provide a justification of preferring their use over population-based modelling approaches, in spite of the computational disadvantages. However, for larger numbers of agents (in the limit), agent-based simulations may equally well approximate population-based simulations. In such cases agent-based simulations just can be replaced by population-based simulations. In this paper, for the application area of crime displacement these considerations are explored in more detail. Comparative simulation experiments have been conducted based on different simulation models, both agent-based (for different numbers of agents), and population-based. The results are analysed and related to the assumptions discussed above.

This paper is organised as follows. First, Section 2 introduces the population based model which has been defined for this domain. Thereafter, this model is mathematically analysed in Section 3, and simulation results are presented in Section 4. Section 5 introduces the agent-based model of which simulation results are shown in Section 6. A comparison of the two different models by means of a formal analysis method is described in Section 7. Finally, Section 8 is a discussion.

2. A population-based model

In this section the population-based model is defined. Hereby, a number of variable names are used as shown in Table 1.

Table 1. Variables in population-based model

Name	Explanation
C	Total number of criminals
G	Total number of guardians
P	Total number of passers by
c(L, t)	Density of criminals at location L at time t.
g(L, t)	Density of guardians at location L at time t.
p(L, t)	Density of passers-by at location L at time t.
β(L, a, t)	Attractiveness of location L at time t for type a agents: c (criminals), p (passers-by), or g (guardians))
assault_rate(L, t)	Number of assaults taking place at location *L* per time unit.

The calculation of the number of agents at the various locations is done by determining the movement of agents that takes place based upon the attractiveness of the location. For instance, for the criminals the formula is specified as follows:

$$c(L, t + \Delta t) = c(L, t) + \eta_1 \cdot (\beta(L, c, t) - c(L, t)/ c)\, \Delta t$$

This expresses that the density $c(L, t + \Delta t)$ of criminals at location L on $t + \Delta t$ is equal to the density of criminals at the location at time t plus a constant η_1 (expressing the rate at which criminals move per time unit) times the movement of criminals from t to $t+\Delta t$ from and to location L multiplied by Δt. Here, the movement of criminals is calculated by determining the relative attractiveness $\beta(L, c, t)$ of the location (compared to the other locations) for criminals. From this, the density of criminals at the location at time t divided by the total number c of criminals (which is constant) is subtracted, resulting in the change of the number of criminals for this location. For the guardians and the passers-by similar formulae are used:

$$g(L, t + \Delta t) = g(L, t) + \eta_2 \cdot (\beta(L, g, t) - g(L, t)/ g)\, \Delta t$$
$$p(L, t + \Delta t) = p(L, t) + \eta_3 \cdot (\beta(L, p, t) - p(L, t)/ p)\, \Delta t$$

The attractiveness of a location can be expressed based on some form of reputation of the location for the respective type of agents. Several variants of a reputation concept can be used. The only constraint is that it is assumed to be normalized such that the total over the locations equals 1. An example of a simple reputation concept is based on the densities of agents, as expressed below.

$$\beta(L, c, t) = p(L, t) / p \quad \text{for criminals}$$
$$\beta(L, g, t) = c(L, t) / c \quad \text{for guardians}$$
$$\beta(L, p, t) = g(L, t) / g \quad \text{for passers-by}$$

This expresses that criminals are more attracted to locations with higher densities of passers-by, whereas guardians are attracted more to locations with higher densities of criminals, and passers-by to locations with higher densities of guardians. As a more general format, linear combinations of densities can be used:

$$\beta(L, p, t) = \beta_{11} \cdot c(L, t) / c + \beta_{12} \cdot g(L, t) / g + \beta_{13} \cdot p(L, t) / p + \delta_1$$
$$\beta(L, c, t) = \beta_{21} \cdot c(L, t) / c + \beta_{22} \cdot g(L, t) / g + \beta_{23} \cdot p(L, t) / p + \delta_2$$
$$\beta(L, g, t) = \beta_{31} \cdot c(L, t) / c + \beta_{32} \cdot g(L, t) / g + \beta_{33} \cdot p(L, t) / p + \delta_3$$

A natural setting of these values for criminals would be to have β_{23} positive since criminals need victims to assault, and to have β_{22} negative because criminals try to avoid guardians. For the guardians, β_{31} is likely to be positive since criminals attract guardians, whereas β_{32} is positive as well. Finally, for the passers-by the β_{11} can be taken negative as passers-by prefer not to meet criminals, and β_{12} (and possibly also β_{13}) positive because guardians (and other passers-by) protect the passers-by. Besides such linear variants, more complex variants can be used in the simulation model presented here as well.

In order to measure the assaults that take place per time unit, also different variants of formulae can be used; for example:

$$assault\_rate(L, t) = \min(\gamma_1 \cdot c(L, t) - \gamma_2 \cdot g(L, t), p(L, t))$$

Here, the assault rate at a location at time t is calculated as the minimum of the possible assaults that can take place and the number of passers-by. Here the possible number of assaults is the capacity per time step of criminals (γ1) multiplied by the number of criminals at the location minus the capacity of guardians to avoid an assault (γ2) times the number of guardians. In theory this can become less than 0 (the guardians can have a higher capacity to stop assaults than the criminals have to commit them), therefore the maximum can be taken of 0 and the outcome described above.

3. Analysis of population-based model

Before performing simulations using the population-based model, a formal analysis is conducted to identify certain characteristics of the model. To obtain such a formal analysis, it is assumed that the attractivenesses of a given location are linear functions of the densities of the different populations at that location, as described in Section 2. When the densities are normalised by taking, for example $c_n(L, t) = c(L, t)/c$ instead of $c(L, t)$, then the following (homogeneous) system of linear differential equations is obtained.

$$\frac{dc_n(L,t)}{dt} = \eta_{11}\, c_n(L, t) + \eta_{22}\, g_n(L, t) + \eta_{23}\, p_n(L, t)$$

$$\frac{dg_n(L,t)}{dt} = \eta_{21}\, c_n(L, t) + \eta_{22}\, g_n(L, t) + \eta_{23}\, p_n(L, t)$$

$$\frac{dp_n(L,t)}{dt} = \eta_{31}\, c_n(L, t) + \eta_{12}\, g_n(L, t) + \eta_{33}\, p_n(L, t)$$

So $p_n(L, t)$ et cetera denote the fraction of the overall population p that is at location L at time t. In linear algebra notation this system can be written as d**x**/dt = **Ax**, with **A** represented by a 3x3 matrix:

$$\begin{pmatrix} \frac{dcn(L,t)}{dt} \\ \frac{dgn(L,t)}{dt} \\ \frac{dpn(L,t)}{dt} \end{pmatrix} = \begin{pmatrix} \eta 11 & \eta 12 & \eta 13 \\ \eta 21 & \eta 22 & \eta 23 \\ \eta 31 & \eta 32 & \eta 33 \end{pmatrix} \begin{pmatrix} cn(L,t) \\ gn(L,t) \\ pn(L,t) \end{pmatrix}$$

Equilibria can be found by the system of linear equations **Ax = 0**:

$$\eta_{11}\, c_n(L, t) + \eta_{12}\, g_n(L, t) + \eta_{13}\, p_n(L, t) = 0$$
$$\eta_{21}\, c_n(L, t) + \eta_{22}\, g_n(L, t) + \eta_{23}\, p_n(L, t) = 0$$
$$\eta_{31}\, c_n(L, t) + \eta_{32}\, g_n(L, t) + \eta_{33}\, p_n(L, t) = 0$$

Behaviour around an equilibrium can be analysed by determining the eigen values of matrix **A** as follows. The eigen value equation is the determinant of the matrix **A** - λ**I** which is:

$$\begin{pmatrix} \eta 11-\lambda & \eta 12 & \eta 13 \\ \eta 21 & \eta 22-\lambda & \eta 23 \\ \eta 31 & \eta 32 & \eta 33-\lambda \end{pmatrix}$$

This equation is:

$$- (\lambda-\eta_{11})(\lambda-\eta_{22})(\lambda-\eta_{33}) + \eta_{12}\eta_{23}\eta_{31} + \eta_{21}\eta_{32}\eta_{13} - (-\eta_{31}\eta_{13}(\lambda-\eta_{22}) - \eta_{12}\eta_{21}(\lambda-\eta_{33}) - \eta_{23}\eta_{32}(\lambda-\eta_{11}))$$
$$= - (\lambda-\eta_{11})(\lambda-\eta_{22})(\lambda-\eta_{33}) + (\eta_{31}\eta_{13}(\lambda-\eta_{22}) + \eta_{12}\eta_{21}(\lambda-\eta_{33}) + \eta_{23}\eta_{32}(\lambda-\eta_{11})) + \eta_{12}\eta_{23}\eta_{31} + \eta_{21}\eta_{32}\eta_{13}$$
$$= - \lambda^3 + (\eta_{11}+\eta_{22}+\eta_{33})\, \lambda^2 + (-(\eta_{11}\eta_{22}+\eta_{22}\eta_{33}+\eta_{33}\eta_{11}) + (\eta_{31}\eta_{13} + \eta_{12}\eta_{21}+\eta_{23}\eta_{32}))\, \lambda + (\eta_{11}\eta_{22}\eta_{33} - (\eta_{31}\eta_{13}\eta_{22} + \eta_{12}\eta_{21}\eta_{33}+\eta_{23}\eta_{32}\eta_{11}) + \eta_{12}\eta_{23}\eta_{31} + \eta_{21}\eta_{32}\eta_{13}$$
$$= - \lambda^3 + b\lambda^2 + c\, \lambda + d$$

with

$$b = (\eta_{11}+\eta_{22}+\eta_{33})$$
$$c = (\eta_{31}\eta_{13} + \eta_{12}\eta_{21}+\eta_{23}\eta_{32}) - (\eta_{11}\eta_{22}+\eta_{22}\eta_{33}+\eta_{33}\eta_{11})$$
$$d = \eta_{11}\eta_{22}\eta_{33} - (\eta_{31}\eta_{13}\eta_{22} + \eta_{12}\eta_{21}\eta_{33}+\eta_{23}\eta_{32}\eta_{11}) + \eta_{12}\eta_{23}\eta_{31} + \eta_{21}\eta_{32}\eta_{13}$$

In general it is not easy to express how the eigen values depend on the many parameters involved. However, for the special case that criminals are (only) attracted to passers-by, guardians are attracted to criminals and passers by are attracted to guardians, a number of the parameters can be taken zero, or equal:

$\eta_{12} = 0$ $\quad$ $\eta_{13} = -\eta_{11}$
$\eta_{21} = -\eta_{22}$ $\quad$ $\eta_{23} = 0$
$\eta_{31} = 0$ $\quad$ $\eta_{32} = -\eta_{33}$

$$\mathbf{A} = \begin{pmatrix} \eta 11 & 0 & -\eta 11 \\ -\eta 22 & \eta 22 & 0 \\ 0 & -\eta 33 & \eta 33 \end{pmatrix}$$

Note that η_{11}, η_{22}, η_{33} are negative here. An equilibrium is determined by

$$\eta_{11}\, c_n(L, t) - \eta_{11}\, p_n(L, t) = 0$$
$$\eta_{22}\, g_n(L, t) - \eta_{22}\, c_n(L, t) = 0$$
$$\eta_{33}\, p_n(L, t) - \eta_{33}\, g_n(L, t) = 0$$

This is equivalent to $p_n(L, t) = c_n(L, t) = g_n(L, t)$. The eigen values can be determined by the equation: $- \lambda^3 + b\lambda^2 + c\lambda + d$ with

$$b = \eta_{11}+\eta_{22}+\eta_{33}$$
$$c = -(\eta_{11}\eta_{22}+\eta_{22}\eta_{33}+\eta_{33}\eta_{11})$$
$$d = 0$$

For this equation one eigen value is $\lambda = 0$, and the other two are the solutions of the quadratic equation

$$\lambda^2 - (\eta_{11}+\eta_{22}+\eta_{33})\lambda + (\eta_{11}\eta_{22}+\eta_{22}\eta_{33}+\eta_{33}\eta_{11}) = 0$$
$$\lambda = ((\eta_{11}+\eta_{22}+\eta_{33}) +/- \surd((\eta_{11}+\eta_{22}+\eta_{33})^2 - 4\, (\eta_{11}\eta_{22}+\eta_{22}\eta_{33}+\eta_{33}\eta_{11}))/2$$

When the square root gives real numbers (positive discriminant), then both solutions will be negative, as the root is less than $|\eta_{11}+\eta_{22}+\eta_{33}|$. When the square root gives imaginary numbers (negative discriminant), the real part of both solutions will be negative. In all cases attraction to the equilibrium will take place, in the first case monotonic, in the second case nonmonotonic. Hence, given the set of assumptions as described above, the model will eventually stabilise.

4. Population-based simulations

The model described in Section 2 and analysed in Section 3 has been used to generate simulation results. (using the Matlab programming environment). Hereby, the functions that represent the attractiveness of different locations have been varied.

4.1. Simple attractiveness function

In this section the results using the simple attractiveness function presented in Section 3 are shown. The simulation results described below used the parameter settings as shown in Table 2 and 3. The settings of the parameters that correspond to the number of passers-by, criminals, and guardians have been determined in cooperation with a team of criminologists.

The resulting simulation trace is depicted in Figure 1. The first three graphs depict the movement of, respectively, criminals, guardians and passers-by over the different locations. The last graph depicts the amount of assaults performed.

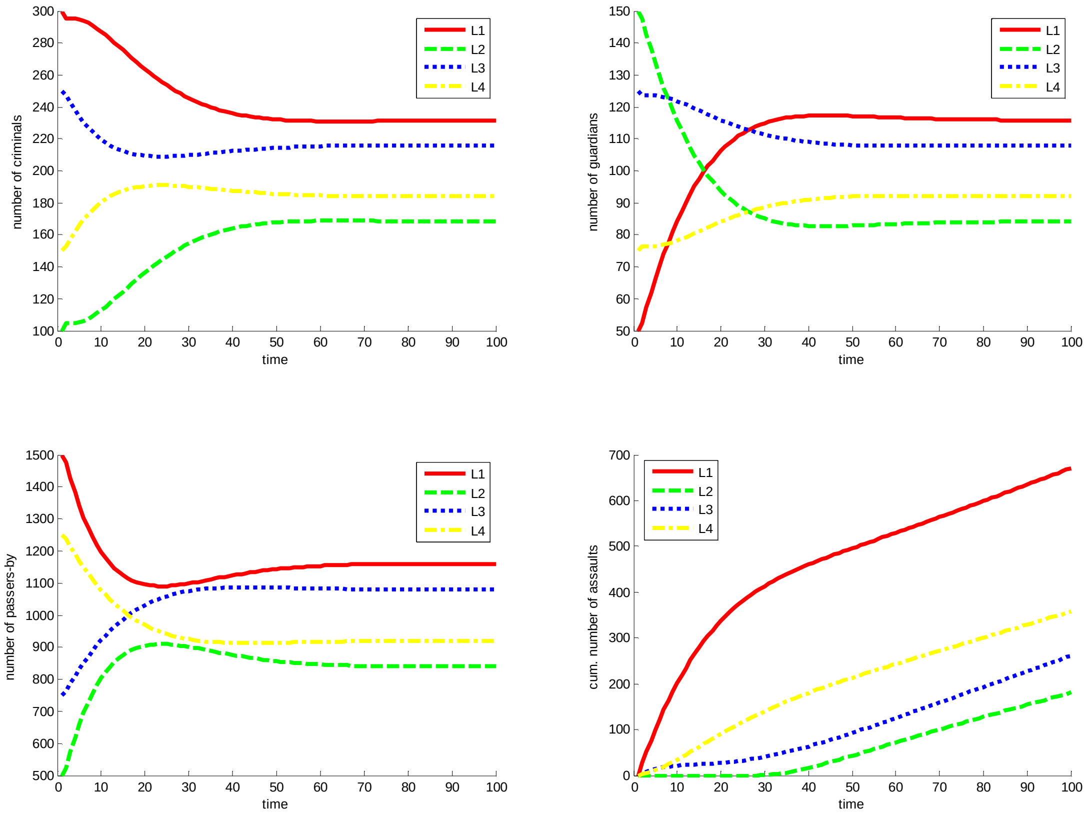

Figure 1. Population-based model - simulation results with simple function

Table 2. Parameter settings

SIMULATION LENGTH	100
LOCATIONS	4
PASSERS-BY	4000
CRIMINALS	800
GUARDIANS	400
β	1
Η	0.5
Δt	0.1

Table 3. Population distribution

	L1	L2	L3	L4
PASSERS-BY	1500	500	750	1250
CRIMINALS	300	100	250	150
GUARDIANS	50	150	125	75

As shown in Figure 1, from the beginning of the simulation many passers-by move away from location 1 (where there are many criminals and few guardians), and towards location 2 (where there are many guardians and few criminals). The guardians follow the opposite pattern: they move away from location 2, and towards location 1. As soon as the number of guardians at location 1 has increased, this location becomes more attractive for the passers-by. The criminals first move away from location 1, towards location 2, but as soon as the passers-by come back to location 1, a significant part of the criminals stays there. Eventually, all populations stabilise as expected after the mathematical analysis of the model. The total computational time needed to generate the results shown is less than one second. Besides this particular run, runs with different settings of parameters (not determined by criminologists) such as the value of β, η, and Δt have been conducted as well. Thereby similar trends are observed as shown in the graphs in Figure 1.

4.2. Complex attractiveness function

In addition, simulation runs have been generated with more complex attractiveness functions, namely the following:

$$\beta(L, c, t) = 0.5 * p(L, t) / p + 0.5 * (1 - p(L, t) / p)$$
$$\beta(L, g, t) = c(L, t) / c$$
$$\beta(L, p, t) = 1 - c(L, t) / c$$

Again, the parameters shown in Table 2 and 3 have been used. The simulation results are shown in Figure 2. The figure shows the same trends (namely an equilibrium) as have been observed before, except that the precise distribution of the various agent types is slightly different.

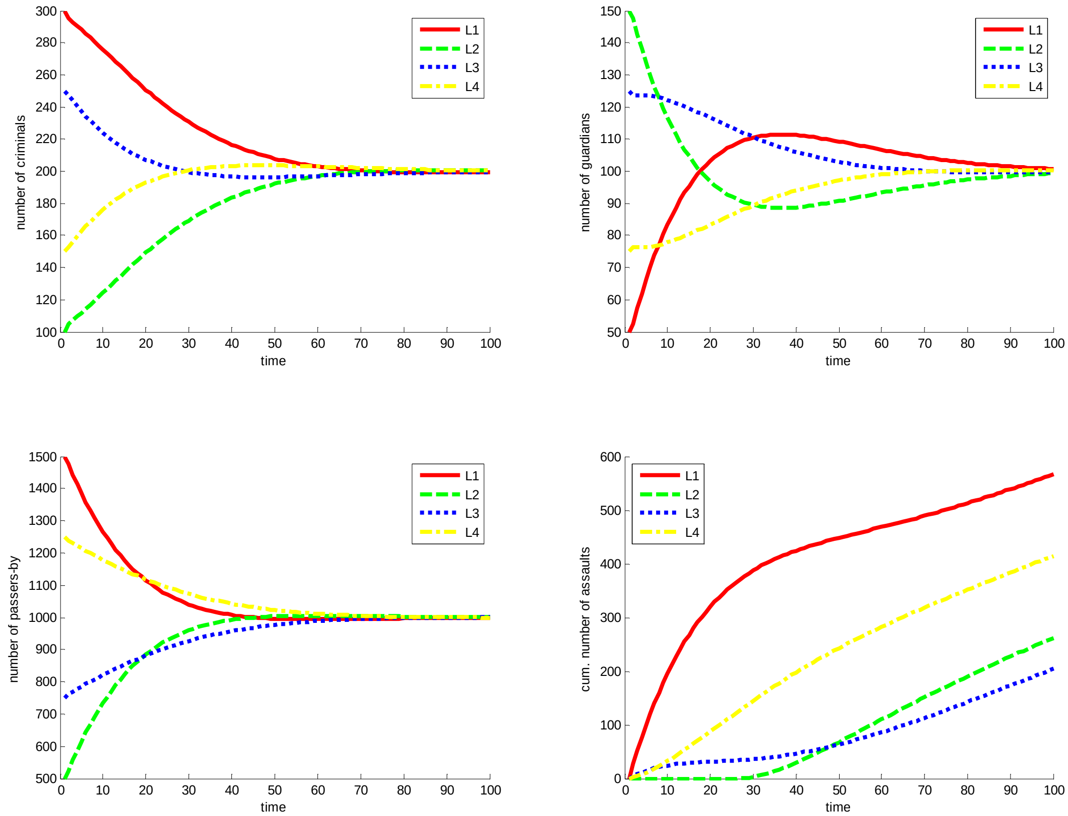

Figure 2. Population-based model - simulation results with complex function

5. An agent-based model

In this section the agent-based model is defined and simulation results thereof are presented. Hereby, the same variable names are used as shown in Table 1.

For the agent-based model, the following algorithm is used (implemented in C++):

1. initialise all agents on locations
2. for each time step repeat the following
   a. calculate the density of each type of agent *p(L, t)*, *c(L, t)*, *g(L, t)* at all locations and communicate it to all agents.
   b. each agent calculates the attractiveness of a location depending on its type (passers-by, criminals, and guardians) for all locations using the following formulae (i.e. similar to those used in the population-based model):

      $\beta(L, c, t) = p(L, t) / p$ for criminals

      $\beta(L, g, t) = c(L, t) / c$ for guardians

      $\beta(L, p, t) = g(L, t) / g$ for passers-by
   c. η % of the agents of each type is selected at random to decide whether the agent moves to a new location or stay at the old one
   d. the selected agents move to a location with a probability proportional to the attractiveness of the specific location (i.e. a selected agent has a higher probability of moving to a relative attractive location than to a non-attractive one).

5.1. Simple attractiveness function

The results using this agent-based model with the same parameters as the population based model with simple attractiveness function are shown in Figure 3. The figure shows the averages over 100 runs of the agent based model. Hereby, the agent-based model requires a total computation time of 16.39 seconds. It can be seen that the trends and even the number of agents at the various locations are very closely related. A maximum deviation between the number of agents of around 2% is seen. These differences are the result of the fact that agents can only move as a whole, whereas in the population based model real numbers are used to represent the densities of agents.

5.2. Complex attractiveness function

The results using the more complex attractiveness function with the same parameter settings as used in Section 4.2 are shown in Figure 4. The results are in accordance with those of the population-based model.

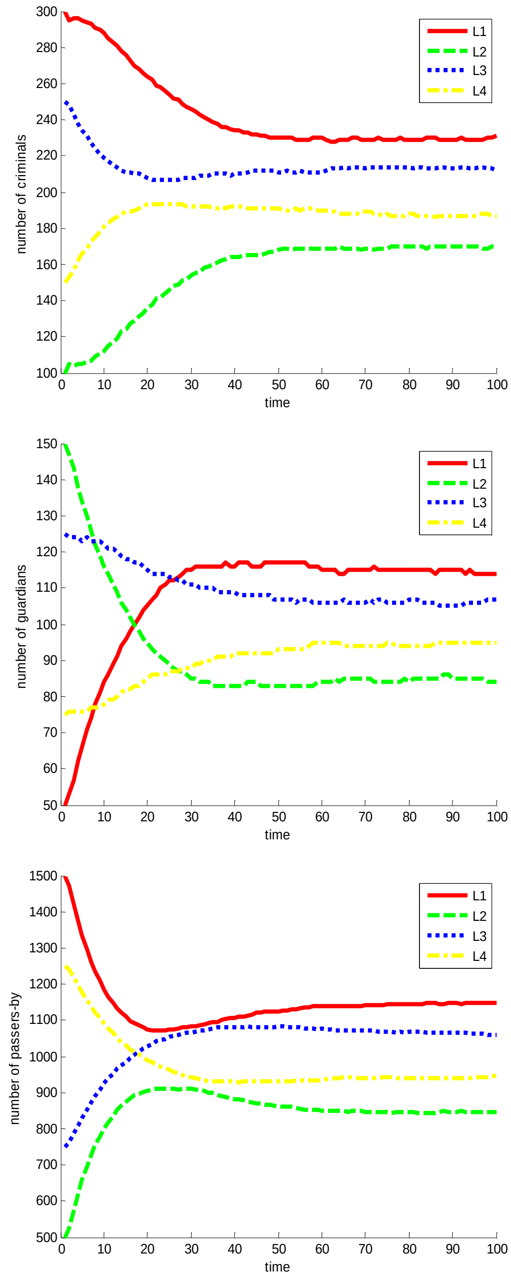

Figure 3. Agent-based model - simulation results with simple function

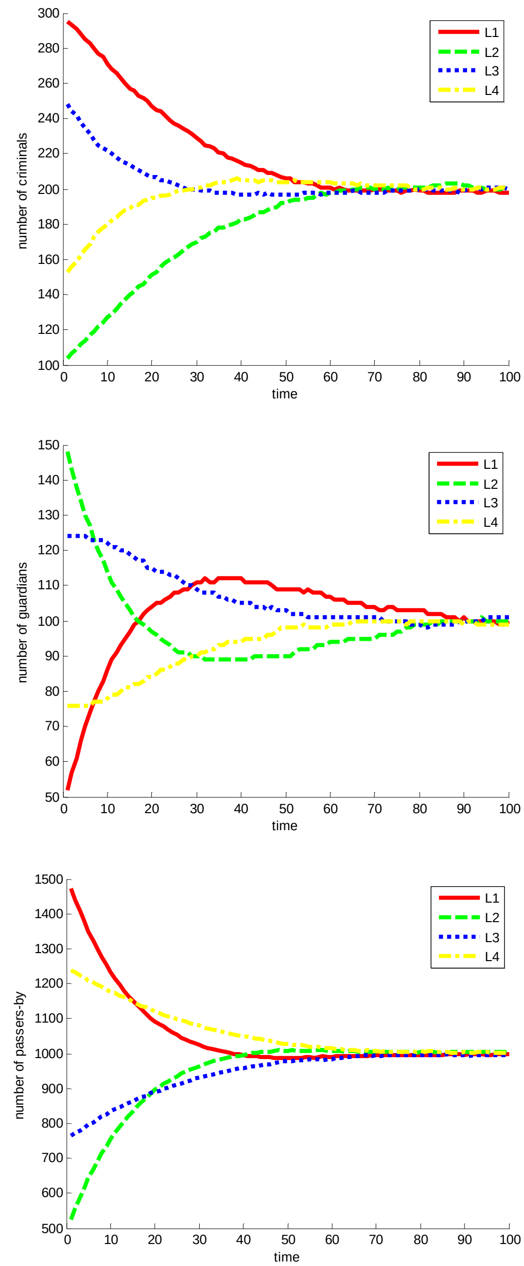

Figure 4. Agent-based model - simulation results with complex function

6. Formal evaluation

In this section, a number of *dynamic properties* of the displacement of crime are formalised in the Temporal Trace Language TTL [3], and checked against the simulation traces. This predicate logical temporal language supports formal specification and analysis of dynamic properties, covering both qualitative and quantitative aspects. TTL is built on atoms referring to *states* of the world, *time points* and *traces*, i.e. trajectories of states over time. In addition, *dynamic properties* are temporal statements that can be formulated with respect to traces based on the state ontology Ont in the following manner. Given a trace γ over state ontology Ont, the state in γ at time point t is denoted by state(γ, t). These states can be related to state properties via the formally defined satisfaction relation denoted by the infix predicate |=, comparable to the Holds-predicate in the Situation Calculus: state(γ, t) |= p denotes that state property p holds in trace γ at time t. Based on these statements, dynamic properties can be formulated in a formal manner in a sorted first-order predicate logic, using quantifiers over time and traces and the usual first-order logical connectives such as ¬, ∧, ∨, ⇒, ∀, ∃. A dedicated software environment has been developed for TTL, featuring both a Property Editor for building and editing TTL properties and a Checking Tool that enables formal verification of such properties against a set of (simulated or empirical) traces.

For the current domain, a number of hypotheses have been expressed as dynamic properties in TTL. For example, consider the following dynamic property (P1a), which expresses that the number of criminals at a certain location is persistent.

P1(Criminals) -
Stable number of criminals at locations

There is a time point t such that for each time point t1 and t2 after t and for all locations l, if at t1 there are x criminals at location l and at t2 there are x2 criminals at location l, then the difference between x and x2 is smaller than α% of the total amount of criminals.

∃t:TIME ∀t1,t2:TIME ∀l:location ∀x,x2:real
[t1>t & t2>t &
state(γ, t1) |= agents_of_type_at_location(x, criminal, l) &
state(γ, t2) |= agents_of_type_of_location(x2, criminal, l)
⇒ abs(x-x2) ≤ c*α/100]

This property (as well as the properties below) has been checked against the traces generated by both simulation models. In particular, they have been checked them against four traces: trace1 (i.e., the population-based trace that was shown in Section 4), trace2 (which is an average trace over 100 simulation runs of the agent-based model of Section 5), trace3 (i.e., the population-based trace based on the complex attractiveness function), and trace4 (an average trace over 100 simulation runs of the agent-based model based on the complex attractiveness function). Some results of this check are shown in Table 4. It was found, among others, that for an α of 1.0 (i.e. 1%) stabilisation of criminals occurs at time point 35 in trace1, at time point 65 in trace2, at t.p. 51 in trace3, and at t.p. 54 in trace4 (see first column).Similar properties have been checked for passers-by and guardians. Thus, in all traces eventually a stable situation occurs, but the moment at which this occurs is a bit later in the agent-based traces. This is due to the fact that the agent-based model works with natural numbers instead of real numbers, which causes a rounding error (as explained in the Section 5.1).

Table 4. Checking results of property P1.

	Criminals	Passers-by	Guardians
trace1	35	38	28
trace2	65	56	50
trace3	51	41	68
trace4	54	47	76

Besides checking whether the number of agents is persistent per location, also other properties can be verified. For example, it can be checked what the point of equilibrium is. To analyse this, properties like the following have been established:

P2 -
Equal percentage of different agents per location

For each location l, for the three different agent types, the number of agents of that type at the location divided by the overall population of that agent type is the same, namely r.

∀l:location ∃r:REAL ∀x1,x2,x3:real
[state(γ, last_time) |= agents_of_type_at_location(x1, criminal, l) &
state(γ, last_time) |= agents_of_type_at_location(x2, passer-by, l) &
state(γ, last_time) |= agents_of_type_at_location(x3, guardian, l)
⇒ r = x1/c±β = x2/p±β = x3/g±β]

For a β of 0.01 this property indeed turned out to be true. Table 5 indicates the values for r that were found for the different locations, for all four traces. Note the small differences between trace 1 and 2, which is due to the rounding error mentioned above.

Table 5. Checking results of property P2.

	Location 1	Location 2	Location 3	Location 4
trace1	0.29	0.21	0.27	0.23
trace2	0.31	0.21	0.27	0.21
trace3	0.25	0.25	0.25	0.25
trace4	0.25	0.25	0.25	0.25

Finally, a number of properties have been specified to investigate whether the spread of agents of a certain kind over the locations is equal (illustrated here for criminals):

P3(Criminals) -
Equal spread of criminals over locations

There is a time point t such that for all time points t1 after t for all locations l, the amount of criminals at l is within a range of δ of the total amount of criminals c divided by the number of locations NL.

∃t:TIME ∀t1:TIME ∀l:location ∀x:real
[t1>t & state(γ, t1) |= agents_of_type_at_location(x, criminal, l)
⇒ c/NL = x±c*δ]

As was already clear from the table above, this property generally does not hold, since the agents do not equally spread over the locations. The property only holds for a very high δ. In addition to the checks mentioned above, these properties have been checked against a number of other simulation traces under different parameter settings. Due to space limitations, the results are not shown here. All in all, these checks pointed out that in all of the cases roughly the same pattern was found. For all traces, eventually the numbers of agents of the different groups (e.g., criminals, passers-by and guardians) at the different locations more or less stabilise. Moreover, per location, eventually the same percentage of the overall population is present for the three different agent types. Finally, it turns out that the agents (per type) are not really spread equally over the locations, but this depends very much on the initial distribution.

7. Discussion

In this paper two models have been introduced to investigate the criminological phenomenon of the displacement of crime. Hereby, a population-based model has been introduced as well as an agent-based model. These models have been presented in a generic format to allow for an investigation of a variety of different functions representing aspects such as the attractiveness of locations. Using mathematical analysis, and confirmed by simulation results, the population-based model was shown to end up in an equilibrium for one variant of the model. The parameter settings for these simulations have been determined in cooperation with criminologists. The simulation results for the agent-based model using the same parameter settings show an identical trend to the population-based model except for some minor deviations that can be attributed to the fact that the agent-based model is discrete, as confirmed by the formal evaluation. The computation time of the populations-based model was shown to be much lower than the computation time of the agent-based model.

Note that the results reported in this paper are not completely in accordance with the results reported in [4]. In the results using an agent-based model reported in that paper, cyclic patterns were observed whereby there is a continuous movement of so called hot-spots (i.e. places where a lot of crime takes place). As already stated before, this paper shows that the population of agents at the various locations stabilises over time. The difference can be attributed to the fact that in [4] all agents decide where to move to based upon the attractiveness of locations, whereas in the case of the models presented in this paper only a subset of the agents move. The results can however be reproduced using the model presented in this paper as well by using an $\eta = 1$ and $\Delta t = 1$ (see the Appendix [17], Figure C and D). Determining what settings are most realistic in real life is future work.

The idea that population-based models approximate agent-based models for larger populations is indeed confirmed by the simulation results reported in this paper. Future work is to introduce a general framework to make a comparison between the models possible. Furthermore, in future work, also agent-based models will be studied where the agents have bounded rationality (e.g., are able to sense just their local surroundings and to communicate with a limited number of other agents).

8. References

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