A Formal Analysis of Complexity Monotonicity

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1 Introduction

Behaviour of organisms can occur in different types and complexities, varying from very simple behaviour to more sophisticated forms. Depending on the complexity of the externally observable behaviour, the internal mental representations and capabilities required to generate the behaviour also show a large variety in complexity. From an evolutionary viewpoint, for example, Wilson [1992] and Darwin [1871] point out how the development of behaviour relates to the development of more complex cognitive capabilities. Godfrey-Smith [1996, p. 3] assumes a relationship between the complexity of the environment and the development of mental representations and capabilities. He formulates the main theme of his book in condensed form as follows: 'The function of cognition is to enable the agent to deal with environmental complexity' (the *Environmental Complexity Thesis*). In this paper, this thesis is refined as follows:
- the more complex the environment, the more sophisticated is the behaviour required to deal with this environment,
- the more sophisticated the behaviour, the more complex are the mental representations and capabilities needed

This refined thesis will be called the *Complexity Monotonicity Thesis*. The idea is that to deal with the physical environment, the evolution process has generated and still generates a variety of organisms that show new forms of behaviour. These new forms of behaviour are the result of new architectures of organisms, including cognitive systems with mental representations and capabilities of various degrees of complexity. The occurrence of such more complex architectures for organisms and the induced more complex behaviour itself increases the complexity of the environment during the evolution process. New organisms that have to deal with the behaviour of such already occurring organisms live in a more complex environment, and therefore need more complex behaviour to deal with this environment, (to be) realised by an architecture with again more complex mental capabilities. In particular, more complex environments often ask for taking into account more complex histories, which requires more complex internal cognitive representations and dynamics, by which more complex behaviour is generated.

This perspective generates a number of questions. First, how can the Complexity Monotonicity Thesis be formalised, and in particular how can the 'more complex' relation be formalised for (1) the environment, (2) externally observable agent

behaviour and (3) internal cognitive dynamics? Second, connecting the three items, how to formalise (a) when does a behaviour fit an environment: which types of externally observable behaviours are sufficient to cope with which types of environments, and (b) when does a cognitive system generate a certain behaviour: which types of internal cognitive dynamics are sufficient to generate which types of externally observable agent behaviour?

In this paper these questions are addressed from a dynamics perspective, and formalised by a temporal logical approach. Complexity of the dynamics of environment, externally observable agent behaviour and internal cognitive system are formalised in terms of structure of the formalised temporal specifications describing them, thus answering (1) to (3). Moreover, (a) and (b) are addressed by establishing formalised logical (entailment) relations between the respective temporal specifications. Furthermore, four cases of an environment, suitable behaviour and realising cognitive system are analysed and compared with respect to complexity, thus testing the Complexity Monotonicity Thesis. More details of this study can be found in [Bosse et al. 2008].

2 The Complexity Monotonicity Thesis

The environment imposes certain requirements that an organism's behaviour needs to satisfy; these requirements change due to changing environmental circumstances. The general pattern is as follows. Suppose a certain goal G for an organism (e.g., sufficient food uptake over time) is reached under certain environmental conditions ES1 (Environmental Specification 1), due to its Behavioural Specification BS1, realised by its internal (architecture) CS1 (Cognitive Specification 1). In other words, the behavioural properties BS1 are sufficient to guarantee G under environmental conditions ES1, formally $ES1 \& BS1 \Rightarrow G$, and the internal dynamics CS1 are sufficient to guarantee BS1, formally $CS1 \Rightarrow BS1$.

In other environmental circumstances, described by environmental specification ES2 (for example, more complex) the old circumstances ES1 may no longer hold, so that the goal G may no longer be reached by behavioural properties BS1. An environmental change from ES1 to ES2 may entail that behaviour BS1 becomes insufficient. It has to be replaced by new behavioural properties BS2 (also more complex) which express how under environment ES2 goal G can be achieved, i.e., $ES2 \& BS2 \Rightarrow G$.

Thus, a population is challenged to realise such behaviour BS2 by changing its internal architecture and its dynamics, and as a consequence fulfill goal G again. This challenge expresses a redesign problem: the given architecture of the organism as described by CS1 (which entails the old behavioural specification BS1) is insufficient to entail the new behavioural requirements BS2 imposed by the new environmental circumstances ES2; the evolution process has to redesign the architecture into one with internal dynamics described by some CS2 (also more complex), with $CS2 \Rightarrow BS2$, to realise the new requirements on behaviour.

The Complexity Monotonicity Thesis can be formalised in the following manner. Suppose $< E_1, B_1, C_1 >$ and $< E_2, B_2, C_2 >$ are triples of environment, behaviour and

cognitive system, respectively, such that the behaviours B_i are adequate for the respective environment E_i and realised by the cognitive system C_i. Then the Complexity Monotonicity Thesis states that

$$E_1 \leq_c E_2 \Rightarrow B_1 \leq_c B_2 \quad \& \quad B_1 \leq_c B_2 \Rightarrow C_1 \leq_c C_2$$

Here $\leq_c$ is a partial ordering in complexity, where $X \leq_c Y$ indicates that Y is more complex than X. A special case is when the complexity ordering is assumed to be a total ordering where for every two elements X, Y either $X \leq_c Y$ or $Y \leq_c X$ (i.e., they are comparable), and when some complexity measure cm is available, assigning degrees of complexity to environments, behaviours and cognitive systems, such that

$$X \leq_c Y \Leftrightarrow cm(X) \leq cm(Y)$$

where $\leq$ is the standard ordering relation on (real or natural) numbers. In this case the Complexity Monotonicity Thesis can be reformulated as

$$cm(E_1) \leq cm(E_2) \Rightarrow cm(B_1) \leq cm(B_2) \quad \&$$
$$cm(B_1) \leq cm(B_2) \Rightarrow cm(C_1) \leq cm(C_2)$$

The Temporal Complexity Monotonicity Thesis can be used to explain increase of complexity during evolution in the following manner. Make the following assumption on Addition of Environmental Complexity by Adaptation: "adaptation of a species to an environment adds complexity to this environment".

Suppose an initial environment is described by ES0, and the adapted species by BS0. Then this transforms ES0 into a more complex environmental description ES1. Based on ES1, the adapted species will have description BS1. As ES1 is more complex than ES0, by the Complexity Monotonicity Thesis it follows that this BS1 is more complex than BS0: ES0 $\leq$ ES1 $\Rightarrow$ BS0 $\leq$ BS1. Therefore BS1 again adds complexity to the environment, leading to ES2, which is more complex than ES1, et cetera. This argument shows that the increase of complexity during evolution can be related to and explained by two assumptions: the Complexity Monotonicity Thesis, and the Addition of Environmental Complexity by Adaptation assumption. This paper focuses on the former assumption.

3 Variations in Behaviour and Environment

To evaluate the approach put forward, a number of cases of increasing complexity are analysed, starting from very simple *stimulus-response behaviour* solely depending on stimuli the agent gets as input at a given point in time. This can be described by a very simple temporal structure: direct associations between the input state at one time point and the (behavioural) output state at a next time point. A next class of behaviours, with slightly higher complexity, analysed is *delayed response behaviour*: behaviour that not only depends on the current stimuli, but also may depend on input of the agent in the past. This pattern of behaviour cannot be described by direct functional associations between one input state and one output state; it increases temporal complexity compared to stimulus-response behaviour. For this case, the description relating input states and output states necessarily needs a reference to inputs received in the past. Viewed from an internal perspective, to describe mental capabilities generating such a behaviour, often it is assumed that it involves a

memory in the form of an internal model of the world state. Elements of this world state model mediate between the agent's input and output states.

Other types of behaviour go beyond the types of reactive behaviour sketched above. For example, behaviour that depends in a more indirect manner on the agent's input in the present or in the past. Observed from the outside, this behaviour seems to come from within the agent itself, since no direct relation to current inputs is recognised. It may suggest that the agent is motivated by itself or acts in a goal-directed manner. For a study in *goal-directed behaviour* and foraging, see, for example, [Hill 2006]. Goal-directed behaviour to search for invisible food is a next case of behaviour analysed. In this case the temporal description of the externally observable behavioural dynamics may become still more complex, as it has to take into account more complex temporal relations to (more) events in the past, such as the positions already visited during a search process. Also the internal dynamics may become more complex. To describe mental capabilities generating such a type of behaviour from an internal perspective, a mental state property *goal* can be used. A goal may depend on a history of inputs. Finally, a fourth class of behaviour analysed, which also goes beyond reactive behaviour, is *learning behaviour* (e.g., conditioning). In this case, depending on its history comprising a (possibly large) number of events, the agent's externally observable behaviour is tuned. As this history of events may relate to several time points during the learning process, this again adds temporal complexity to the specifications of the behaviour and of the internal dynamics.

To analyse these four different types of behaviour in more detail, four cases of a food supplying environment are considered in which suitable food gathering behaviours are needed. These cases are chosen in such a way that they correspond to the types of behaviour mentioned above. For example, in case 1 it is expected that stimulus-response behaviour is sufficient to cope with the environment, whilst in case 2, 3 and 4, respectively, delayed response behaviour, goal-directed behaviour, and learning behaviour is needed). The basic setup is inspired by experimental literature in animal behaviour such as [Tinklepaugh 1932]. The world consists of a number of positions which have distances to each other. The agent can walk over these positions. Time is partitioned in fixed periods (days) of a duration of d time units (hours). Every day the environment generates food at certain positions, but this food may or may not be visible, accessible and persistent at given points in time. The different types of environment with increasing temporal complexity considered are:

(1) Food is always visible and accessible. It persists until it is taken.
(2) Food is visible at least at one point in time and accessible at least at one later time point. It persists until it is taken.
(3) Food either is visible at least at one point in time and accessible at least at one later time point, or it is invisible and accessible the whole day. It persists until it is taken.
(4) One of the following cases holds:
  a) Food is visible at least at one point in time and accessible at least at one later time point. It persists until it is taken.
  b) Food is invisible and accessible the whole day. It persists until it is taken.

c) Food pieces can disappear, and later new pieces can appear, possibly at different positions. For every position where food appears, there are at least three different pieces in one day. Each piece that is present is visible. Each position will be accessible at least after the second food piece disappeared.

4 Modelling Approach

For describing different variations in behaviour of an agent and environment, a formal modelling approach is needed. The simplest type of behaviour, stimulus-response behaviour, can be formalised by a functional input-output association, i.e., a (mathematical) function F : Input_states $\rightarrow$ Output_states of the set of possible *input states* to the set of possible *output states*. A state at a certain point in time as it is used here is an indication of which of the state properties of the system and its environment are true (hold) at that time point.

Note that according to this formalisation, stimulus-response behaviour is deterministic. Behaviour of this type does not depend on earlier processes, nor does it on (not observable) internal states. If also non-deterministic behaviour is taken into account, the function in the definition above can be replaced by a relation between input states and output states, which relates each input state to a number of alternatives of behaviour, i.e., R: Input_states $\times$ Output_states. For example, a simple behaviour of an animal that after seeing food at the position p goes to this position on condition that no obstacles are present, can be formalised using a functional association between an input state where it sees food at p and no obstacles, and an output state in which it goes to p.

As opposed to stimulus-response behaviour, in less simple cases an agent's behaviour often takes into account previous processes in which it was involved; for example, an agent that observed food in the past at position p may still go to p, although it does not observe it in the present. Instead of a description as a function or relation from the set of possible input states to the set of possible output states, in more general cases, a more appropriate description of behaviour by an *input-output correlation* is given in the following definition:

a) A *trace* (or trajectory) is defined as a time-indexed sequence of states, where time points can be expressed, for example, by real or integer values. If these states are input states, such a trace is called an *input trace*. Similarly for an *output trace*. An *interaction trace* is a trace of (combined) states consisting of an input part and an output part.

b) An *input-output correlation* is defined as a binary relation C : Input_traces $\times$ Output_traces between the set of possible *input traces* and the set of possible *output traces*.

c) A *behavioural specification* S is a *set of dynamic properties* in the form of temporal statements on interaction traces.

d) A given interaction trace $\mathcal{T}$ *fulfils* or *satisfies* a behavioural specification S if all dynamic properties in S are true for the interaction trace $\mathcal{T}$.

e) A behavioural specification S is a *specification of an input-output correlation* C if and only if for all interaction traces $\mathcal{T}$ input-output correlation C holds for $\mathcal{T}$ if and only if $\mathcal{T}$ fulfils S.

To express formal specifications for environmental, behavioural and cognitive dynamics for agents, the Temporal Trace Language (TTL, see [Bosse et al. 2006]) is used. This language is a variant of order-sorted predicate logic. In dynamic property

expressions, TTL allows explicit references to time points and traces. If a is a state property, then, for example state(γ, t, input(agent)) |= a denotes that this state property holds in trace γ at time point t in the input state of the agent. Based on such building blocks, dynamic properties can be formulated. For example, a dynamic property that describes stimulus-response behaviour of an agent that goes to food, observed in the past can be formalised as follows:

∀t ∀x ∀p ∀p'
[state(γ, t, input(agent)) |= observed(at(agent, p)) ∧
  observed(at(food(x), p')) ∧ observed(accessible(p'))
  ⇒ state(γ, t+1, output(agent)) |= performing_action(goto(p'))]

Using this approach, the four variations in behaviour and environment have been formalised in detail. The results can be found in [Bosse et al. 2008].

5 Formalisation of Temporal Complexity

The Complexity Monotonicity Thesis discussed earlier involves environmental, behavioural and cognitive dynamics of living systems. In an earlier section it was shown that based on a given complexity measure cm this thesis can be formalised in the following manner:

$$cm(E_1) \leq cm(E_2) \Rightarrow cm(B_1) \leq cm(B_2) \ \&$$
$$cm(B_1) \leq cm(B_2) \Rightarrow cm(C_1) \leq cm(C_2)$$

where $< E_1, B_1, C_1 >$ and $< E_2, B_2, C_2 >$ are triples of environments, behaviours and cognitive systems, respectively, such that the behaviours B_i are adequate for the respective environment E_i and realised by the cognitive system C_i. What remains is the existence or choice of the complexity measure function cm. To measure degrees of complexity for the three aspects considered, a temporal perspective is chosen: complexity in terms of the temporal relationships describing them. For example, if references have to be made to a larger number of events that happened at different time points in the past, the temporal complexity is higher. The temporal relationships have been formalised in the temporal language TTL based on predicate logic. This translates the question how to measure complexity to the question how to define complexity of syntactical expressions in such a language. In the literature an approach is available to define complexity of expressions in predicate logic in general by defining a function that assigns to every expression a size: [Huth and Ryan 2000]. To measure complexity, this approach was adopted and specialised to the case of the temporal language TTL. Roughly spoken, the complexity (or size) of an expression is (recursively) calculated as the sum of the complexities of its components plus 1 for the composing operator. In more details it runs as follows.

Similarly to standard predicate logic, predicates in TTL are defined as relations on terms. The size of a TTL-term t is a natural number s(t) recursively defined as:

(1) s(x)=1, for all variables x.
(2) s(c)=1, for all constant symbols c.
(3) $s(f(t_1,..., t_n))= s(t_1) + ... + s(t_n) + 1$, for all function symbols f.

For example, the size of the term observed(not(at(food(x), p))) from the property BP1 (see [Bosse et al. 2008]) is equal to 6.

Furthermore, the size of a TTL-formula ψ is a positive natural number $s(\psi)$ recursively defined as follows:

(1) $s(p(t_1,\ldots, t_n)) = s(t_1) + \ldots + s(t_n) + 1$, for all predicate symbols p.
(2) $s(\neg\varphi) = s((\forall x)\ \varphi) = s((\exists x)\ \varphi) = s(\varphi) + 1$, for all TTL-formulae φ and variables x.
(3) $s(\varphi\&\chi) = s(\varphi|\chi) = s(\varphi\Rightarrow\chi) = s(\varphi) + s(\chi) + 1$, for all TTL-formulae φ, χ.

In this way, for example, the complexity of behavioural property BP1 amounts to 53, and the complexity of behavioural property BP2 is 32. As a result, the complexity of the complete behavioural specification for the stimulus-response case (which is determined by BP1 & BP2) is 85 (see [Bosse et al. 2008] for the properties).

Using this formalisation of a complexity measure, the complexity measures for environmental, internal cognitive, and behavioural dynamics for the considered cases of stimulus-response, delayed response, goal-directed and learning behaviours have been determined. Table 1 provides the results.

Table. 1. Temporal complexity of environmental, behavioural and cognitive dynamics.

Case	Environmental dynamics	Behavioural dynamics	Cognitive dynamics
Stimulus-response	262	85	85
Delayed response	345	119	152
Goal-directed	387	234	352
Learning	661	476	562

The data given in Table 1 confirm the Complexity Monotonicity Thesis put forward in this paper, that the more complex the environmental dynamics, the more complex the types of behaviour an organism needs to deal with the environmental complexity, and the more complex the behaviour, the more complex the internal cognitive dynamics.

6 Discussion

In this paper, the temporal complexity of environmental, behavioural, and cognitive dynamics, and their mutual dependencies, were explored. As a refinement of Godfrey-Smith [1996]'s Environmental Complexity Thesis, the Complexity Monotonicity Thesis was formulated: for more complex environments, more complex behaviours are needed, and more complex behaviours need more complex internal cognitive dynamics. A number of example scenarios were formalised in a temporal language, and the complexity of the different formalisations was measured. Complexity of environment, behaviour and cognition was taken as temporal complexity of dynamics of these three aspects, and the formalisation of the measurement of this temporal complexity was based on the complexity of the syntactic expressions to characterise these dynamics in a predicate logic language, as known from, e.g., [Huth and Ryan 2000]. The outcome of this approach is that the results confirm the Complexity Monotonicity Thesis.

In [Godfrey-Smith 1996], in particular in chapters 7 and 8, mathematical models are discussed to support his Environmental Complexity Thesis, following, among others, [Sober 1994]. These models are made at an abstract level, abstracting from the

temporal dimension of the behaviour and the underlying cognitive architectures and processes. Therefore, the more detailed temporal complexity as addressed in this paper is not covered. Based on the model considered, Godfrey-Smith [1996, Ch 7, p. 216, see also p. 118] concludes that the flexibility to accommodate behaviour to environmental conditions, as offered by cognition, is favoured when the environment shows (i) unpredictability in distal conditions of importance to the organism, and (ii) predictability in the links between (observable) proximal and distal. This conclusion has been confirmed to a large extent by the formal analysis described in this paper. Comparable claims on the evolutionary development of learning capabilities in animals are made by authors such as Stephens [1991]. According to these authors, learning is an adaptation to environmental change. All these are conclusions at a global level, compared to the more detailed types of temporal complexity considered here, where cognitive processes and behaviour extend over time, and their complexity can be measured in a detailed manner as temporal complexity of their dynamics.

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