A Virtual Human Agent Model with Behaviour Based on Feeling Exhaustion

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Abstract. A computational agent model for monitoring and control of a virtual human agent's resources and exhaustion is presented. It models a physically grounded intelligent decision making process within the agent model for physical effort to be spent. Simulation results are discussed, and a formal analysis is presented on conditions under which the agent model functions properly, for example, such that it can be used to avoid running out of resources. Finally, the model is related to a model for monitoring or simulating a person's heart rate.

1 Introduction

To generate intelligent agent behaviour, it is more and more recognized that in addition to the brain, often the body plays a crucial role as well, and thus contributes to the intelligence. Some authors argue that also the design of artificial intelligent systems could gain benefit of such analyses by incorporating relevant physiological aspects in models developed; e.g., [2], [3], [5]. An example is the intelligence with which a person manages exhaustion (or fatigue). When the body would only give a signal when complete exhaustion occurs, no intelligent management would be possible. Fortunately, by gradually getting a feeling of becoming fatigued, more information is available before a total breakdown occurs. In this paper a computational model and analysis is made of the role that this physiological aspect (as described in more detail in the literature on physical exercise and sport) plays in monitoring and intelligent control of resources.

For certain types of artificial or virtual human agents, monitoring and control of the own resources may be of importance, for example, for human-like characters in a virtual reality context. When such a virtual human agent is equipped with the capability to get a feeling of becoming fatigued, it may show a more realistic intelligent behaviour. This behaviour can be based on a context-sensitive type of decision making to manage limitations of resources, incorporating the intelligence related to aspects of the body. Another application area is formed by ambient intelligence used in physical exercise and sport: devices that monitor human functioning, are able to analyse this functioning, and respond accordingly. When more sophisticated agent models are used, more advanced ambient intelligent agent applications can be developed.

In this paper literature on physical exercise and sport (e.g., [14]) is taken as a point of departure. One of the issues addressed is how the generated effort is controlled and what is the role of feeling fatigue in this process; e.g., [4], [6], [7], [8], [9], [10], [11], [12]. The interplay of mind and body in this process is considered a crucial factor. The classical perspective on fatigue is based on the assumption that fatigue occurs either when the muscles run out of resources, or they are in a sense poisoned by waste material produced; e.g., [8], [9]. Resources may involve oxygen, glycogen (which fuels the muscles), or ATP (adenosine triphosphate, the molecule that muscles use to store energy) that are lacking. Waste material concerns by-products of exercises, such as lactic acid. In this perspective the body reaches some states in which its functioning is disturbed so that only limited effort is possible, and this co-occurs with (or is expressed by) feeling fatigue. Noakes and his colleagues (e.g., [11], [12]) emphasize the notion of homeostasis: the property of a system (for example, a living organism), to regulate its internal environment in such a way that stable, more or less constant, conditions are maintained. In this view the mind keeps the body in physical conditions that are better prepared for expected or possible future efforts. A cognitive or neural decision making mechanism is assumed that incorporates information on the extent of exhaustion of the agent's body.

In Section 2 the computational agent model for monitoring and management of resources is presented and formalised. Section 3 discusses simulation results for the domain of cycling, which is used as a case study. In Section 4 a formal analysis is presented, which identifies the conditions and parameter values for which the model will function properly. Section 5 briefly analyses how the agent model can be related to a person's heart rate, which is sometimes used as monitor information in sports. Section 6 is a discussion.

2 The Computational Agent Model

This section describes the agent model for monitoring and control of resources. A central concept used is power that is generated. The basic idea behind the model is that it is easier to monitor the generated power at any time point, than the store of resources left. When based on monitor data on the generated power, the brain performs some form of accumulation or integration, then this can be used as a faithful indicator for the resources used. Within the literature on exercise and sports the notion of *critical power CP* plays an important role. This is the maximal level of power that can be generated and sustained over longer periods without becoming exhausted, assuming no prior exercising. It is an asymptote of the hyperbolic power-duration curve defined by $(GP - CP).t = W'$ that (as shown in various experiments) models the relationship between a constantly generated power GP (above the critical power CP) and the time t that this can be sustained; e.g., [6], [7], [8], [9]. Here W' is the total amount of work that can be spent above the critical power (the availble stored resources). Often it is (implicitly or explicitly) assumed that this critical power CP is a constant, that is not affected by prior exercising, and is a capacity to provide power based on aerobic processes. Power generated above this critical power is assumed to be based on anaerobic processes, that exploit an available fixed reservoir of stored basic resources BR, which is one of the parameters of the hyperbolic power-duration curve (often indicated by W'). Experiments show that after highly intensive prior exercising leading to exhaustion of the basic resources *BR*, for example, power at 90% of the critical power *CP* cannot be sustained; e.g., [4]. Therefore a main assumption made for the model developed below is that a *basic critical power BCP* (applicable when no prior exercising took place) can be distinguished from a *dynamic critical power DCP* (applicable when prior exercising has taken place). It is assumed that the dynamic critical power can vary between an upper bound (the basic critical power *BCP*) and a lower bound (*lowest critical power LCP*).

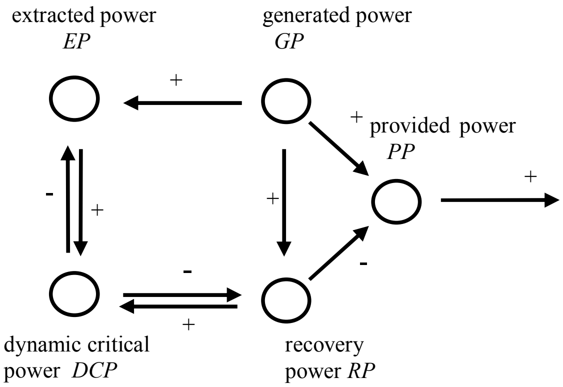

Fig. 1. Overview of the variables and their dependencies

Part of the dynamics of the model concerns how this dynamic critical power is affected by the effort spent above the critical power in the preceeding time interval (i.e., the extent to which the basic resources *BR* were already used), and how recovery can take place when the effort spent is below the critical power. Because the dynamic critical power has a direct relationship with the history of effort spent, it can be compared to the feeling of becoming fatigued, which indicates the extent of exhaustion, or the state of the (remaining) resources. In this way the dynamic critical power can be taken as a monitoring instrument to maintain homeostasis. Based on this indicator, decisions can be made on generated power. A possible decision model, for example, takes care that the dynamic critical power always is kept above a certain lower bound. When it is assumed that the dynamic critical power relates to fatigue (the lower the dynamic critical power, the more fatigue), this means that fatigue is kept limited. To obtain a formal model, the concepts used are formalised by numerical variables. Power is the amount of energy spent per time unit (expressed in Watt). Different types of power are distinguished; see Fig. 1 for the global effects they have on each other. Note that here the state *DCP* has a temporal relation to the other nodes, whereas the others have state relations; below more specific formulae are discussed.

generated power and dynamic critical power → extracted power

The *generated power* is the power level chosen by the person. If this is more than can be produced in a direct manner (by the aerobic system), part of this power is *extracted* from the (longer term) resources (the anaerobic system). When generated power is above the dynamic critical power, then the difference is extracted from the resources, indicated by:

$$EP(t) = Pos(GP(t) - DCP(t))$$

where the operator *Pos* for the positive part is defined by $Pos(x) = (x + |x|)/2$, or, alternatively: $Pos(x) = x$ if $x \geq 0$ and 0 else.

generated power and dynamic critical power → recovery power

When the generated power is lower than the dynamic critical power, a fraction of the generated power is contributed as recovery power. It is assumed that this recovery power is proportional to the difference between generated power and dynamic critical power, and also proportional to the difference between dynamic critical power and basic critical power. The proportion factor is β.

$$RP(t) = \beta\, GP(t)\, Pos(DCP(t) - GP(t)) \frac{BCP - DCP(t)}{BCP}$$

extracted power and recovery power → dynamic critical power

Extracted power decreases the dynamic critical power. Recovery power increases the dynamic critical power, bounded upward by the basic critical power *BCP*. The adjustment of the dynamic critical power after a time interval from t to $t+\Delta t$ is assumed proportional to the recovery power (factor γ_1), respectively the extracted power (factor γ_2).

$$DCP(t+\Delta t) = DCP(t) + (\gamma_1\, RP(t) - \gamma_2\, EP(t))\, \Delta t$$

The continuous model can be described by a differential equation:

$$\frac{dDCP(t)}{dt} = \gamma_1\, RP(t) - \gamma_2\, EP(t)$$

$$= \gamma_1\, \beta\, GP(t)\, Pos(DCP(t) - GP(t)) \frac{BCP - DCP(t)}{BCP} - \gamma_2\, Pos(GP(t) - DCP(t))$$

recovery power and generated power → provided power

The provided power is the difference between generated power and recovery power.

$$PP(t) = GP(t) - RP(t)$$

dynamic critical power → dynamic maximal power

The notion of maximal power models a limitation on the choice of of the generated power. The higher dynamic critical power, the higher the dynamic maximal power. Extracted power can only be positive as long as the dynamic critical power is above its minimum value *LCP*. The maximum power possible is assumed to be the dynamic critical power plus a constant *C*, as long as $DCP > LCP$ (no complete exhaustion), and equal to *LCP* when $DCP = LCP$ (complete exhaustion).

$$DMP(t) = C + DCP(t) \text{ when } DCP > LCP$$
$$DMP(t) = LCP(t) \text{ when } DCP = LCP$$
$$BMP = C + BCP$$

Note that before reaching complete exhaustion, maximal power is substantially above the dynamic critical power, but upon reaching complete exhaustion, the maximal power drops to the level of the critical power, in accordance with the experiments reported, for example, in [4]. The agent model as described provides possibilities to make decisions based on the dynamic critical power as an indicator. When it is assumed that the dynamic critical power represents the feeling of fatigue, that feeling is in fact the indicator.

3 An Example Simulation

First a model for a cycling case study is described. In this case study the *provided power* is used to move a bike with a certain speed, depending on the resistance. Mechanical resistance can be taken into account in a *cycling efficiency factor*, for the process of generating *actual power* to move the bike. Further resistance is mainly based on air resistance, and if the road is ascending or descending on gravitation resistance. Given these resistances, *velocities* can be determined, and from them *distances*. For the sake of simplicity no gravitation resistance is considered; air resistance depends on a parameter called *air resistance coefficient*. To obtain a formal model, numerical variables are used: cycling efficiency factor *CEF*, air resistance coefficient *ARC,* actual cycling power *ACP*, and velocity v. The actual cycling power is the cycling efficiency factor times the provided power: $ACP(t) = CEF*PP(t)$. It is assumed that power exerted for movement is used to work against air resistance. For air resistance it is assumed that it is a force proportional to the square of velocity, with resistance coefficient *ARC*. Actual cycling power is the work performed per time unit, which is equal to this resistance force times the distance covered divided by the time (which is the velocity); therefore $ACP(t) = ARC*v(t)^3$ or:

$$v(t) = \sqrt[3]{\frac{ACP(t)}{ARC}}$$

Based on the model described in Section 2 and the cycling model described above, a number of simulation experiments have been performed, using existing numerical simulation tools. In Figure 2 results are shown of one of them, with time scale displayed in minutes; the step size Δt was taken one minute. The fixed parameter settings are: $\beta = 0.02$, $\gamma_1 = \gamma_2 = 0.4$, $\rho = 0.8$, $BCP = 400$, $LCP = 300$, and $C = 100$. The story goes as follows. First the cyclist generates power a bit above (*403*) the dynamic critical power (initially *400*), riding alone (air resistance coefficient *0.3*). The dynamic critical power slightly decreases. Then she joins a group of cyclists that passes by, and hence has less air resistance (coefficient *0.25*). The generated power is now lower than the dynamic critical power, while the speed is higher. Some recovery takes place. After a while in the group she is persuaded that it is her turn to take the front position. Now she has higher air resistance again, upon which she generates higher power (but lower speed); this brings the dynamic critical power down to near *350*. After some time she leaves the front position to somebody else, and while still being in the group she has less air resistance. However, as the dynamic critical power was decreased, the generated power now is still above the dynamic critical power; the dynamic critical power continues to decrease and approaches *300*: she cannot maintain the speed of this group after her effort at the front. She decides to leave the group and take some recovery time, riding on her own, with much lower speed, with more air resistance. The dynamic critical power increases; when another group passes by, she joins this group, and has lower air resistance again. Now the generated power

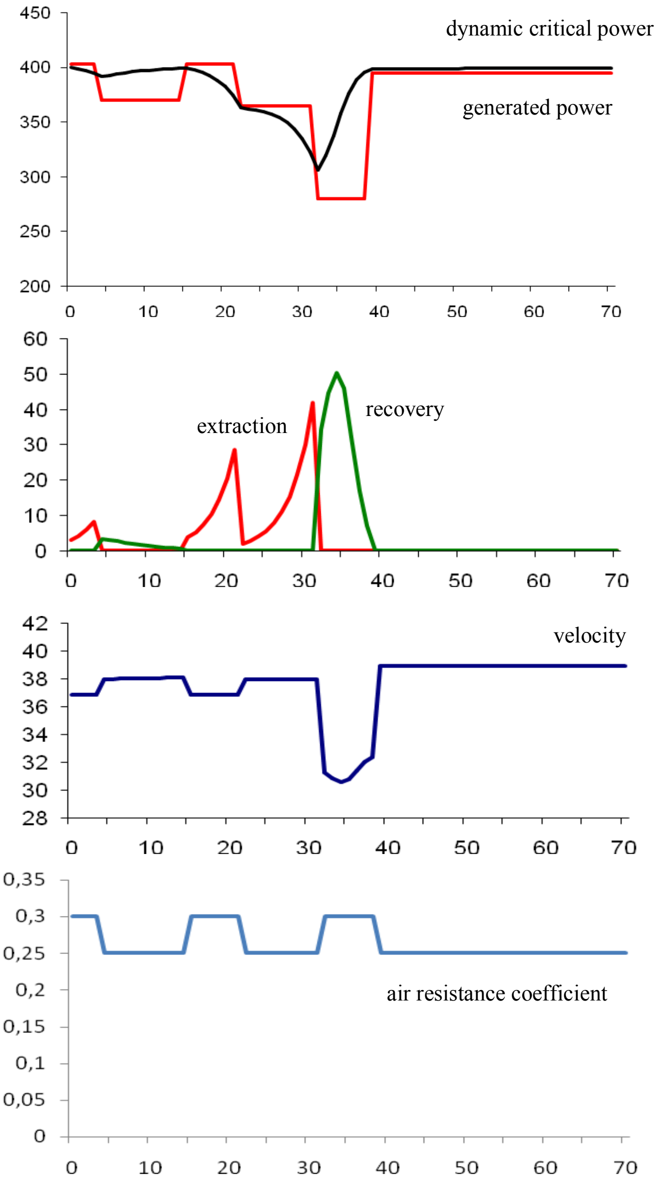

Fig. 2. Example simulation trace for the cycling case

stays slightly under the dynamic critical power, although this group has a higher speed. A state occurs with (almost) constant dynamic critical power.

Note that after 30 minutes a critical situation occurred. Taking the front position in the (first) group took so much of the resources that after leaving the front position, still the generated power to stay within the group was higher than the dynamic critical power. Therefore the cyclist could not recover and instead had to extract more and more from the resources, bringing the dynamic critical power further down. If no decision would have been made to decrease the generated power, within a very short time the decreasing trend of the dynamic critical power would have led to the lower bound *LCP* of the dynamic critical power, after which the maximal generated power would drop to the level of *LCP*.

The decisions of the cyclist have been modelled by the generated power levels over time, set by hand (see the lowest graph in Fig. 2). Note that in the second picture in Fig. 2, the areas under the extraction graph and the recovery graph are more or less the same, which means that the dynamic critical power after 40 minutes is almost equal to the one at the start, which also can be seen in the first graph.

4 Formal Analysis

A main question addressed in the formal analysis is whether the introduced agent model allows the person to monitor and control its resources in a proper manner. For proof sketches, see Appendix A.

Maintaining a steady state

Enduration sporters often try to maintain what they call a steady state: a state in which the main parameters are kept constant. In particular, it can be analysed how $DCP(t)$ can be kept constant. From the differential equation for $DCP(t)$ it follows:

$$\frac{dDCP(t)}{dt} = \gamma_1\, \beta\, GP(t)\, Pos(DCP(t) - GP(t))\, \frac{BCP - DCP(t)}{BCP} - \gamma_2\, Pos(GP(t) - DCP(t))$$

Since never both $Pos(x)$ and $Pos(-x)$ are nonzero, $\frac{dDCP(t)}{dt} = 0$ is equivalent to

$$GP(t)\, Pos(DCP(t) - GP(t))\, \frac{BCP - DCP(t)}{BCP} = 0 \quad \text{and} \quad Pos(GP(t) - DCP(t)) = 0$$

This is logically equivalent to

either $GP(t) = 0$ and $Pos(GP(t) - DCP(t)) = 0$

or $Pos(DCP(t) - GP(t)) = 0$ and $Pos(GP(t) - DCP(t)) = 0$

or $\frac{BCP - DCP(t)}{BCP} = 0$ and $Pos(GP(t) - DCP(t)) = 0$

which can be rewritten to

either $GP(t) = 0$ and $DCP(t) \geq 0$

or $GP(t) = DCP(t)$

or $DCP(t) = BCP$ and $GP(t) \leq DCP(t)$

This analysis is summarised in the following theorem expressing how a steady state for $DCP(t)$ can be characterised.

Theorem 1 (Characterising a Steady State)

For any time point *t*, when $GP(t) > 0$, the following are equivalent:

(i) $\frac{dDCP(t)}{dt} = 0$ (dynamic critical power equilibrium)

(ii) Either $GP(t) = DCP(t)$ or $GP(t) \leq DCP(t) = BCP$

How the indicator relates to the resources

A next issues to address in the analysis is the question how the dynamic critical power relates to the real resources. The level of resources $R(t)$ is assumed to be based on losses per time unit that are proportional to the extracted power (factor α_2) and gains proportional to the recovery power (factor α_1):

$$\frac{dR(t)}{dt} = \alpha_1\, RP(t) - \alpha_2\, EP(t)$$

In a different form this can be expressed by.

$$R(t_2) = R(t_1) + \alpha_1 \int_{t1}^{t2} RP(t)dt \quad \alpha_2 \int_{t1}^{t2} EP(t)dt$$

Recall that by BR the basic (additional) resources are denoted (in the literature often called W'). When at time point t_0 the resources are the basic resources, then the model can be described as:

$$R(t) = BR + \alpha_1 \int_{t0}^{t} RP(t)dt - \alpha_2 \int_{t0}^{t} EP(t)dt$$

To analyse the relationships between the indicators and the resource level, two special extreme cases are considered first: the case that the values are equal to the basic values, as in a situation at rest (addressed in Proposition 1), and the case that the resource level is 0, as in a situation with completely exhausted resources (addressed in Proposition 2). For proper functioning in both special cases certain conditions on the parameters are identified in the two propositions. These conditions also turn out sufficient for proper functioning for the general case, as covered by Theorem 2.

In Proposition 1 it is investigated under which conditions the dynamic critical power is a proper indicator for the basic resources. It turns out that this is the case under a certain condition on the parameters; the same condition implies that the resource level is a linear function of the dynamic critical power. The proposition makes use of Lemma 1 which related the dynamic critical power power to the resource levels.

Lemma 1 (Critical Power vs Resource Level)

For all time points t_1 and t_2 it holds:

$$\gamma_1\, (R(t_2) - R(t_1)) = \alpha_1\, (DCP(t_2) - DCP(t_1)) + (\alpha_1\gamma_2 - \gamma_1\alpha_2) \int_{t1}^{t2} EP(t)dt$$

Proposition 1 (Indicating Basic Resource Levels)

The following are equivalent:

(i) The dynamic critical power is a proper indicator for basic resources:

$$\forall t\ [\, DCP(t) = BCP \Leftrightarrow R(t) = BR\,]$$

(ii) $\gamma_1 / \alpha_1 = \gamma_2 / \alpha_2$

(iii) For all time points t_1 and t_2 it holds:

$$\alpha_1\, (DCP(t_2) - DCP(t_1)) = \gamma_1\, (R(t_2) - R(t_1))$$

(iv) The expression $\gamma_1\, R(t) - \alpha_1\, DCP(t)$ is invariant over time: for all time points t_1 and t_2 it holds:

$$\gamma_1\, R(t_2) - \alpha_1\, DCP(t_2) = \gamma_1 R(t_1) - \alpha_1\, DCP(t_1)$$

(v) The dynamic critical power is a linear function of the resource level: for a given point t_0, for all time points t it holds:

$$\alpha_1\, DCP(t) = \gamma_1\, R(t) + \alpha_1\, DCP(t_0) - \gamma_1 R(t_0)$$

In Proposition 2 it is investigated under which conditions the dynamic critical power is a proper indicator for running out of resources. This is the case under a certain further condition on the parameters; this condition implies that the resource level is proportional to the dynamic critical power, as expressed in Theorem 2. Below it is assumed that at the initial time point t_0 the resources $BR(t_0)$ are the basic resources BR and the dynamic critical power $DCP(t_0)$ is the basic critical power BCP.

Proposition 2 (Indicating Running Out of Resources)
Suppose $\gamma_1/\alpha_1 = \gamma_2/\alpha_2$ and let $\eta = BCP/BR$. Then the following are equivalent:

(i) The dynamic critical power is a proper indicator for running out of resources:

$$\forall t \ [\, DCP(t) = LCP \ \Leftrightarrow \ R(t) = 0 \,]$$

(ii) At any point of time the dynamic critical power as an indicator faithfully (proportionally) reflects the resources left:

$DCP(t) \ / \ R(t)$ is constant over time

(iii) $\gamma_1/\alpha_1 = \gamma_2/\alpha_2 = \eta$

Theorem 2 (Proper Indicator for Resources)
In case $\gamma_1/\alpha_1 = \gamma_2/\alpha_2 = \eta$, with $\eta = BCP/BR$, the following hold:

a) The dynamic critical power is a proper general indicator for the resource level. More specifically, $DCP(t)$ is proportional to $R(t)$ over time with η as factor:

$$\forall t \quad DCP(t) = \eta \ R(t)$$

b) Suppose for some lower bound $L>LCP$ the basic power power $DCP(t)$ is always kept above L i.e., $\forall t \ DCP(t) \geq L$, then the person will never run out of resources.

c) When the person has nonzero resources and $DCP(t)$ is kept constant, then the person will never run out of resources.

In this theorem a) and b) immediately follow from Proposition 2. Concerning c), when the person keeps $DCP(t)$ at a constant value above LCP, then b) applies. Therefore the person can only run out of resources when $DCP(t) = LCP$, but as $DCP(t)$ is kept constant, then by Proposition 2 there already were no resources, which is not the case.

Corollary
In case $\gamma_1/\alpha_1 = \gamma_2/\alpha_2 = \eta$, with $\eta = BCP/BR$, the resource level $R(t)$ over time satisfies the following differential equation

$$\frac{dR(t)}{dt} = \alpha_1 \, \beta \, GP(t) \, Pos(\eta \, R(t) - GP(t)) \, \frac{BR - R(t)}{BR} \quad - \alpha_2 \, Pos(GP(t) - \eta \, R(t))$$

This corollary follows from the differential equation

$$\frac{dR(t)}{dt} = \alpha_1 \, RP(t) - \alpha_2 \, EP(t)$$

for $R(t)$ by expressing $EP(t)$ and $RP(t)$ in $GP(t)$ and $DCP(t)$ and then, according to Theorem 2, replacing $DCP(t)$ by $\eta \ R(t)$. An alternative derivation is by taking the differential equation for $DCP(t)$ and replacing $DCP(t)$ by $\eta \, R(t)$.

Notice that the parameters α_1, α_2 are physiological parameters related to mechanisms for extraction from and recovery of resources. In contrast the parameters γ_1, γ_2 are neural or cognitive parameters, assumed to be set by the brain. To obtain a faithful indicator system, the relationships between the physiological and neural parameters as expressed in Theorem 2 are needed. It is assumed that within the brain the neural parameters stand in these relationships to the physiological parameters.

5 Relation to Heart Rate

In exercising and sport practice sometimes not (only) the feeling of fatigue, but (also) the heart rate is used to monitor the extent of exhaustion; here often some estimations are used, assuming a linear relation of heart rate to generated power above the critical power point. For example, if it is assumed that the heart rate for generated power at the level of the critical power is a constant *BHR*, and the maximal heart rate is a constant *MHR*, then for cases where $GP(t)$ is at least $DCP(t)$ the heart rate can be estimated as:

$$\begin{aligned} HR(t) &= BHR + (MHR-BHR).(GP(t)-DCP(t))/(DMP(t)-DCP(t)) \\ &= BHR + (MHR-BHR)/(GP(t)-DCP(t))/C = BHR + \gamma\,(GP(t)-DCP(t)) \end{aligned}$$

with $\gamma = (MHR-BHR)/C$. From this relation it follows that

$$\frac{dHR(t)}{dt} = \gamma\,\frac{dGP(t)}{dt} - \gamma\,\frac{dDCP(t)}{dt}$$

Therefore in case of generated power in equilibrium (for example, constant), the following can be derived.

Theorem 3 (Heart Rate as Indicator)

a) If $GP(t) \geq DCP(t)$ and it holds $\frac{dGP(t)}{dt} = 0$ (generated power equilibrium) then the following are equivalent:

(i) $\frac{dHR(t)}{dt} = 0$ (heart rate equilibrium)

(ii) $\frac{dDCP(t)}{dt} = 0$ (critical power equilibrium)

b) When both the generated power and heart rate are constant, then no running out of resources will occur.

6 Discussion

In this paper a virtual human agent model was introduced that addresses the notion of critical power which plays a central role in scientific and practical literature on exercising and sport. It performs integration of monitoring data on generated power over time and uses this to determine in a dynamic manner the critical power as an indicator for the amount of stored resources left. The model realises homeostasis under the assumption that the human uses this indicator to make the proper decisions about its generated power. Decision criteria for generated power are, for example, keeping this indicator constant (achieving a steady state), or keeping the indicator between certain bounds, thus avoiding running out of resources. It has also been discussed how a person's heart rate can be related to the dynamic critical power and used as an indicator. The model has been used to perform a number of simulations, one of which was presented above. Moreover, a formal analysis has been undertaken that shows under which conditions the critical power indicator indeed correlates to what it is expected to indicate, and that some strategies often used are guaranteed to work out well. A first test of the model against empirical data has shown some preliminary results which were positive.

The agent model may be useful in a number of application areas for intelligent agents. In the first place it may by useful as a basis for virtual characters with a realistic appearance (which, for example, may show a heart rate depending on efforts made). Furthermore, the agent model may be applied as part of an intelligent ambient agent device interacting with humans in physical exercise or sport, or in other demanding circumstances. Supporting devices mostly they concentrate on the sensoring and do not possess much intelligence to perform analysis of sensor data. Incorporating a dynamical model may provide a basis for more intelligent ambient agents. As an example of this development, some of the ideas reported in the current paper already were the inspiration for a part of the functional state model described in [1]. Finally, the model may be used for social simulations, to investigate how persons can cooperate in order to manage their resources more economically. For example, in cycling sports, based on riding behind each other sophisticated cooperation strategies between competing teams are followed, including negotiation processes.

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Appendix A Some Further Details

This Appendix gives proof sketches for the lemma and propositions.

Proof of Lemma 1 Multiplying

$$R(t_2) = R(t_1) + \alpha_1 \int_{t1}^{t2} RP(t)dt - \alpha_2 \int_{t1}^{t2} EP(t)dt$$

$$DCP(t_2) = DCP(t_1) + \gamma_1 \int_{t1}^{t2} RP(t)dt - \gamma_2 \int_{t1}^{t2} EP(t)dt$$

by γ_1 respectively α_1 provides:

$$\gamma_1 R(t_2) = \gamma_1 R(t_1) + \gamma_1\alpha_1 \int_{t1}^{t2} RP(t)dt - \gamma_1\alpha_2 \int_{t1}^{t2} EP(t)dt$$

$$\alpha_1 DCP(t_2) = \alpha_1 DCP(t_1) + \alpha_1\gamma_1 \int_{t1}^{t2} RP(t)dt - \alpha_1\gamma_2 \int_{t1}^{t2} EP(t)dt$$

From subtracting, the statement of Lemma 1 follows. ■

Proof of Proposition 1 (i) ⇒ (ii) Suppose for two time points t_1 and t_2 it holds:

$R(t_2) = R(t_1) = BR$ and $DCP(t_2) = DCP(t_1) = BCP$

whereas between these time points resources have been extracted, after which full recovery took place, i.e.,

$$\int_{t1}^{t2} EP(t)dt \neq 0$$

By applying Lemma 1 it follows that $\alpha_1\gamma_2 - \gamma_1\alpha_2 = 0$.

(ii) ⇒ (i) This follows from Lemma 1, applied to the time interval between initial time point t_0 and t.

(ii) ⇔ (iii) ⇔ (iv) ⇔ (v) This also follows from Lemma 1. ■

Proof of Proposition 2 (i) ⇒ (iii) Apply Proposition 1 (iii) to a time interval from the initial time point t_0 and a time point t, to obtain that the expression $\alpha_1 DCP(t) - \gamma_1 R(t)$ is invariant over time. From this it follows that for all t

$$\alpha_1 DCP(t) - \gamma_1 R(t) = c$$

with

$$c = \alpha_1 DCP(t_0) - \gamma_1 R(t_0) = \alpha_1 BCP - \gamma_1 BR$$

From this it follows that

$$[\forall t\ DCP(t) = LCP \Leftrightarrow R(t) = 0] \Leftrightarrow c = \alpha_1 LCP \Leftrightarrow \alpha_1 (BCP-LCP) = \gamma_1 BR$$
$$\Leftrightarrow \gamma_1/\alpha_1 = \eta \Leftrightarrow \forall t\quad DCP(t) = \eta R(t)$$

So under these conditions the dynamic critical power and the resource level are proportional over time with factor $\gamma_1/\alpha_1 = \gamma_2/\alpha_2 = \eta$. This proves Proposition 2. ■