

# Dynamic Allocation of a Domestic Heating Task to Gas-Based and Heatpump-Based Heating Agents

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**Abstract.** In this paper a multi-agent model for a domestic heating task is introduced and analysed. The model includes two alternative heating agents (for gas-based heating and for heatpump-based heating), and a third allocation agent which determines the most economic allocation of the heating task to these heating agents over the days in a year. For allocation decisions it is analysed how the performance of a heatpump depends on the outdoor temperature. One method discussed is a what-if analysis method using agent-based simulation, another method is by mathematical analysis to derive more precise knowledge about the most optimal allocation choice. These methods can be used by the allocation agent to determine in a dynamic, adaptive manner per day a most economic allocation, depending on the (predicted) outdoor temperature.

## 1 Introduction

Especially in more northern countries, a substantial amount of domestic energy use during the winter season concerns heating. Often the heating systems used are based on not renewable resources such as gas and oil, which over the years are rapidly becoming more expensive. Moreover, as a byproduct, serious negative effects on the climate are obtained. For these reasons more and more often, alternative domestic heating systems are considered. An often considered alternative is the use of a heatpump (e.g., [1, 7, 9, 12, 15], which takes thermal energy from the environment (from air, water or soil) and uses this to heat the water of a central heating system in the house. However, there are some drawbacks.

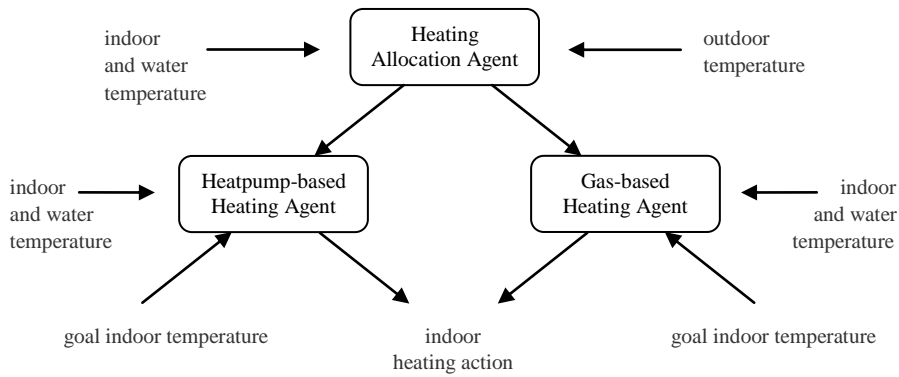
One issue is that water is often not available in the direct neighbourhood, so then this source is excluded. Another issue is that to use thermal energy from the soil often a serious financial investment is needed, whereas a heatpump by itself is usually already a much more expensive investment than a gas- or oil-based heating system. Therefore often a heatpump is considered which takes thermal energy from the air (air to water heatpump). Another main issue here is that at the coldest days, when heating needs most energy, the air temperature is low, and due to that an air to water heatpump becomes less economic in use, or even lacks capacity to achieve the heating of the water of the heating system to the required level.

Due to these drawbacks in many cases a more feasible option considered is to combine different heating systems and to allocate the heating task in a dynamic manner to one of these systems, depending on the circumstances, in particular the outdoor temperature. For example, a gas- or oil-based system can be allocated the heating task only on the coldest days (their efficiency does not depend on the outdoor temperature), whereas for all other days a heatpump is allocated. This already can save money and reduce the negative effects on the climate. It is such a hybrid configuration that is considered here.

In a hybrid system it is crucial to make the right allocation at the right moment in a dynamic, adaptive manner. In this paper this is modelled and analysed from an agent-based perspective. Two alternative heating systems are considered as heating agents, and a third allocation agent determines the most economic allocation of the heating task to these agents over the days in a year; this setup will be introduced in Section 2. To be able to make good allocation decisions a number of aspects have to be modelled. First, it has to be known how the performance of a heatpump depends on the outdoor temperature; this is addressed in Section 3. In Section 4 a model is introduced and simulations with this model over a year period are analysed, based on realistic data from 2012. In Section 5 by a mathematical analysis more precise knowledge is obtained about the most optimal allocation choice.

## 2 General Setup

The modelling setup addressed in this paper follows what is often done in hybrid heating systems in practice, and has the form of an agent-based model based on three agents: the Gas-based Heating Agent (GHA), the Heatpump-based Heating Agent (HHA), and the Heating Allocation Agent (HAA); see Fig. 1. Each of the first two agents can take care of the heating via control of the water temperature of the central heating system. They are responsive for input communicated by the Heating Allocation Agent.



**Fig. 1** Overview of the setup with the three agents

The allocation agent has as its goal to generate smart decisions (on the fly) about to which of the other agents to allocate the task of heating, and to communicate these decisions to both agents. Its main source of information is acquired by sensing and monitoring the outdoor temperature and the indoor air and water temperature. It uses knowledge about how economic the two heating agents are for different outdoor temperatures. Especially the efficiency (performance factor) of heating by a heatpump strongly depends on the outdoor temperature. For each given (predicted) outdoor temperature the allocation agent allocates the most economic heating agent to the heating task. So, more specifically its goal is to obtain a most economic performance of the heating task,

Each of the heating agents HHA and GHA receives the allocation information communicated by the allocation agent. Each of them responds to that communication by actually performing the heating when the communicated information indicates that it is allocated, and by being idle when this information indicates it is not allocated. As the heating agents are purely responsive, most of the intelligence concerning optimisation of the heating by this 3-agent system lies in the allocation agent HAA. To achieve its goal it needs detailed domain knowledge. In subsequent sections the required domain knowledge is discussed and it is analysed how the overall system works based on this knowledge.

The design of the three agents is based on the component-based Generic Agent Model (GAM) presented in [6], designed according to the component-based agent system design method DESIRE [4, 5]. Within this model GAM the component *World Interaction Management* takes care of interaction with the world, the component *Agent Interaction Management* of communication with other agents. Moreover, the component *Maintenance of World Information* maintains information about the world, and the component *Maintenance of Agent Information* information about other agents. The processes involved in controlling the agent and of maintaining a self model are the task of the component *Own Process Control*. In the component *Agent Specific Task*, specific tasks of the agents can be modelled.

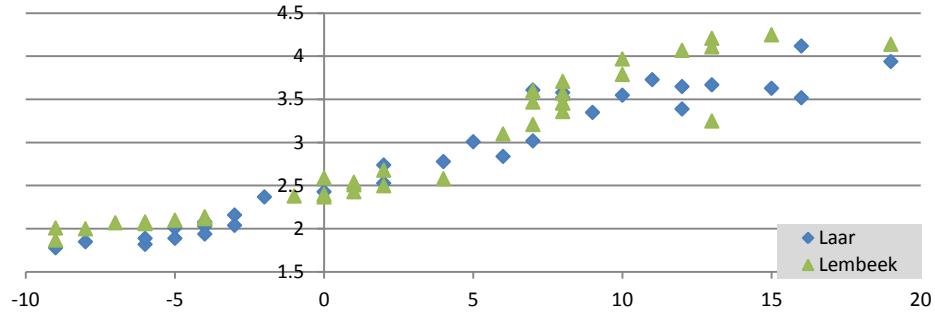
For the two heating agents GHA and HHA the component Agent Interaction Management handles the communication with the Heating Allocation Agent AA and with the human(s) in the house (via the thermostat as a communication mean). The received allocation information is stored as self information in Own Process Control and as the heating agents are purely responsive, from there immediately this information flows to World Interaction Management (if the allocation information expresses that the agent has been allocated the task), resulting in generation of the heating action. The heating action itself also depends on information the heating agent perceives about the indoor temperature and the water temperature. This is received in World Interaction Management and as an intermediate step stored in Maintenance of World Information. Moreover, the information about the goal indoor temperature communicated by the humans in the house is taken into account. This communication is handled via Agent Interaction Management; as an intermediate step the received

goal information is stored in Maintenance of Agent Information. The Heating Allocation Agent HAA involves more complex processing in its Agent Specific Task (which is heating allocation). This task will be addressed in more detail in subsequent sections. The other components function in a way similar to how they function in the heating agents. World Interaction Management and Maintenance of World Information take care of receiving and storing world information about indoor and outdoor temperature and water temperature of the heating system. Agent Interaction Management and Maintenance of Agent Information take care of communication with the heating agents and storing the allocation information involved.

For the Agent Specific Task heating allocation the agent HAA needs to perform an analysis of the expected costs of the two heating agents. First of all, in order to be able to compare the two heating systems on efficiency it needs a way of estimating the seasonal performance factor of the heatpump-based heating system for given circumstances. This is addressed in Section 3. Next it needs methods to assess in a comparative manner how economic the two heating systems are. This can be done in (at least) two different ways. The first method, discussed in Section 4 is by an agent-based what-if analysis (simulation). The Heating Allocation Agent could incorporate such a what-if analysis in its Agent Specific Task component in order to make allocation decisions. This provides a more elaborate variant of this agent. The second method, discussed in Section 5 is by mathematical analysis. Results of such a mathematical analysis can be used as a form of compiled knowledge in the Agent Specific Task component of the Heating Allocation Agent. This provides a more concise variant of this agent.

### **3 Estimation of Seasonal Performance Factors for a Heatpump**

The seasonal performance factor SPF strongly depends on the water temperature of the heating system and the outdoor temperature. Manufacturers often give indications of these performance factors for just a few water and outdoor temperatures. However, to determine the electricity use of a heatpump over a year, it is needed to have an estimation of SPF for the given water temperature and each possible outdoor temperature, as this outdoor temperature shows much variation over the year. To obtain a reasonable estimation of how for a given water temperature the performance factor depends on the outdoor temperature, theoretical analyses can be made. However, such theoretical analyses are often not guaranteed to provide values that occur in reality. A different route is to take empirical data as a point of departure and make an approximation of them by a mathematical function. A useful source of such data can be found at [16]. In Fig. 2 a graph is shown with values from this Website for the average day temperature on the horizontal axis and the performance factor on the vertical axis (for water temperature approximately 50°C).



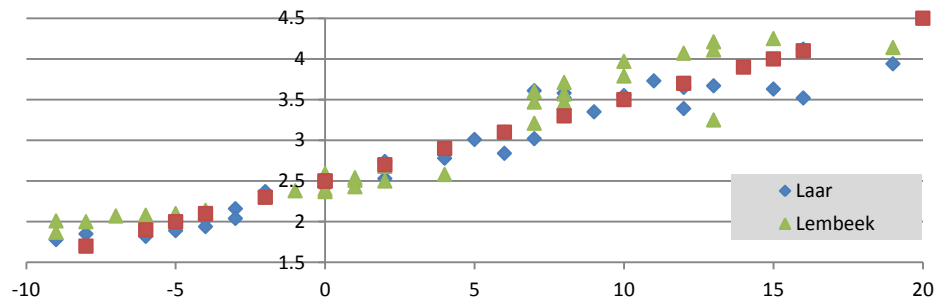
**Fig. 2.** Empirical data on seasonal performance factors in relation to outdoor temperature over 2012 for sites in Lembeek and Laar (water temperature 50°C)

More specifically, this has been done for the sites at Lembeek and Laar, where the General Waterstage HT heatpump combination WH16/WOH16 is used. In Fig. 3 a linear approximation of SPF for the interval from -10°C to +20°C is drawn; this is assumed of the form

$$SPF(T_{od}) = 7.5 - 0.1 * (T_{water} - T_{od}) \quad \text{with } T_{water} = 50$$

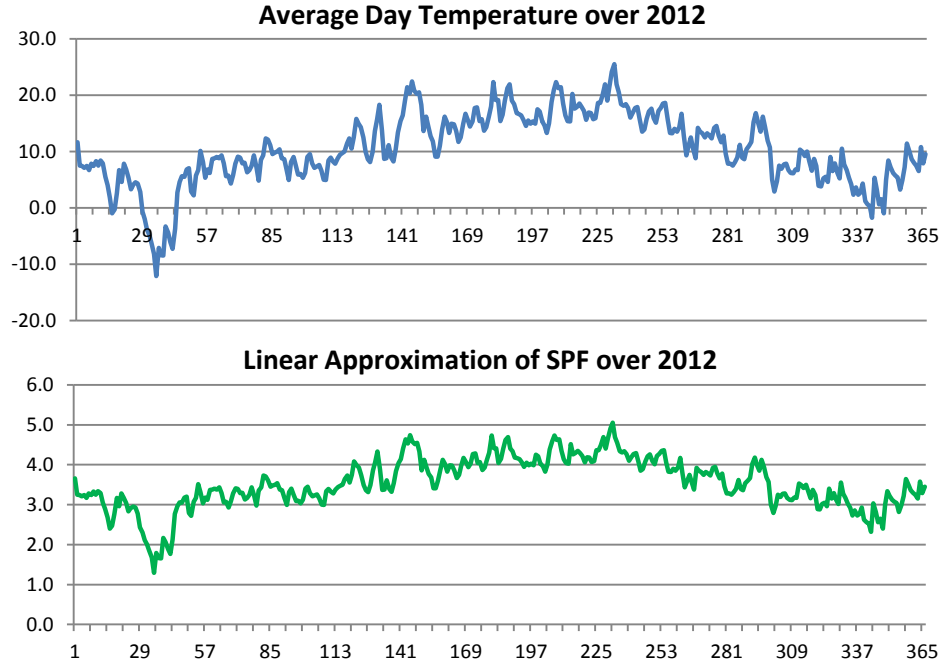
This linear approximation suggests a *rule of thumb* stating:

*Every degree Celsius lower in outdoor temperature makes the performance factor SPF drop by 0.1.*



**Fig. 3.** Linear approximation of seasonal performance factors in relation to outdoor temperature compared to empirical data over 2012 for sites in Lembeek and Laar (water temperature 50°C)

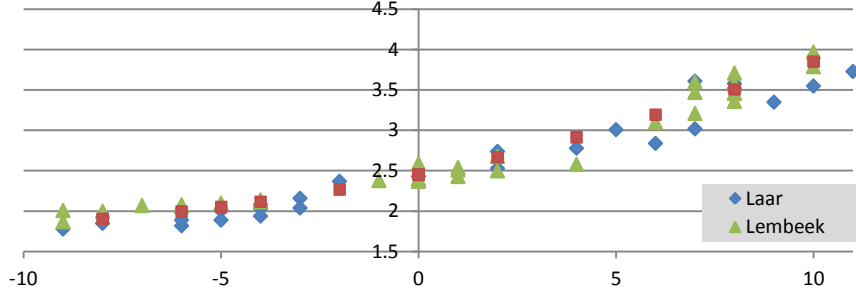
Using this approximation, the seasonal performance factor can be estimated throughout a year, when the day temperatures are given. For example, in Fig. 4 in the upper graph the (average) day temperatures (in De Bilt, The Netherlands) of all days of 2012 are given, and in the lower graph the seasonal performance factor is estimated for all these days based on the linear approximation.



**Fig. 4.** Empirical data on average day temperature in De Bilt, the Netherlands (upper graph) and estimation of seasonal performance factors over 2012 based on the linear approximation (lower graph)

From Fig. 4 it can be seen that for outdoor temperatures from 11°C and higher, the values of SPF have much more variation than for the lower outdoor temperatures. Therefore any approximation, including the linear approximation introduced above, may show relatively high deviations in the interval above 10°C. If the values for outdoor temperatures from 11°C and higher are neglected, a more close inspection of the remaining interval from -10°C to +10°C reveals a pattern of a slightly bended upward curve with empirical values closer to -10°C to +10°C that are higher than the linear approximation (which fits best in the middle area of this interval, say from -5°C to +5°C). This curve can be described in the interval from -10°C to +10°C by a quadratic pattern as a more accurate approximation than the linear one, as is shown in Fig. 5. The quadratic pattern shown is described by

$$SPF(T_{od}) = 7.45 - 0.1 * (T_{water} - T_{od}) + 0.004 T_{od}^2 \quad \text{with } T_{water} = 50$$



**Fig. 5.** Quadratic approximation of seasonal performance factors in relation to outdoor temperature compared to empirical data over 2012 for sites in Lembeek and Laar (water temperature 50°C)

#### 4 What-if Analysis Using an Agent-Based Simulation Model

As a first method for the heating allocation agent to analyse the costs of energy use for heating for the two heating agents, what-if analysis by agent-based simulation is used. The simulations make use of the three agents introduced in Section 2: the gas-based heating agent GHA, the heatpump-based heating agent HHA, and the heating allocation agent HAA. Fig. 6 shows the variables used in the model used and the dependencies between them, and Table 1 summarizes them. The heating to be performed by the agents is responsive for the circumstances in the environment, in particular, for the average outdoor day temperature  $T_{od}$ . This is assumed given (either obtained by sensing, or as a prediction from a weather forecast). A general format to determine how much energy is to be provided to the heating system makes use of the concept of *degree day*, denoted by  $dd$ . This concept is based on the assumption that the amount of energy needed to maintain a difference in temperature (between indoor and outdoor) is proportional to this difference (e.g., [13]). The number of degree days for a given day  $t$  directly relates to the difference between the outdoor and the (average) indoor day temperature  $T_{id}$  (when the latter is higher than the former), and else is 0:

$$dd(t) = \begin{cases} \sigma(t) * (T_{id}(t) - T_{od}(t)) & \text{when } T_{id}(t) > T_{od}(t) \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

Here  $\sigma(t)$  is as seasonal correction weight factor which is 1.1 for the months November, December, January and February, 1 for the months March and October, and 0.8 for the months April, May, June, July, August and September.

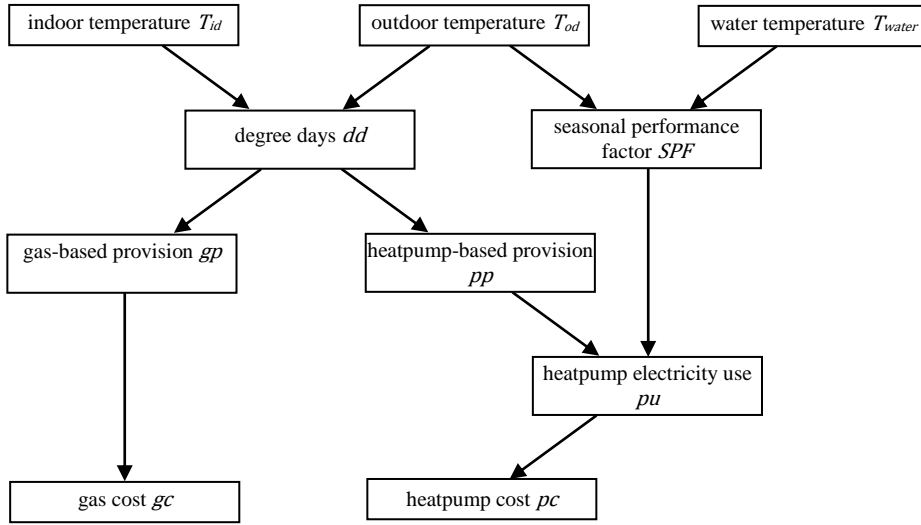
For a period consisting of a number of days these degree days are simply added. For a gas-based heating system the total cost of heating, for example, during a year is proportional to the number of degree days in the following manner. First, for each

degree day an amount  $\eta$  of gas (in  $m^3$ ) is needed. Therefore the gas-based provision  $gp(t)$  ( $m^3$  gas used for day  $t$ ) is determined as (see also Fig. 6, left hand side):

$$gp(t) = \eta dd(t)$$

Next, assuming that every  $m^3$  gas costs  $\pi_{gas}$  euro, the costs  $gc(t)$  of gas for day  $t$  is given by

$$gc(t) = \pi_{gas} gp(t) = \pi_{gas} \eta dd(t) \quad (2)$$



**Fig. 6.** Dependencies of the variables in the model

For the costs of heating based on a heatpump a similar but slightly more complex model is used. First, for each degree day an amount  $\varepsilon$  of electricity (in  $kWh$ ) has to be provided. Therefore the heatpump-based provision  $pp(t)$  ( $kWh$  provided for heating at day  $t$ ) is determined as (see also Fig. 6, right hand side):

$$pp(t) = \varepsilon dd(t)$$

Note that this is the amount provided but not the amount  $pu(t)$  used by the heatpump itself, as part of the provided energy  $pp(t)$  comes from the environment. This is expressed using the seasonal performance factor  $SPF$  as follows:

$$pu(t) = pp(t) / SPF(T_{od}(t)) = \varepsilon dd(t) / SPF(T_{od}(t))$$

Finally, assuming that one  $kWh$  electricity costs  $\pi_{el}$  euro, the costs  $pc(t)$  of heating for day  $t$  by the heatpump is given by

$$pc(t) = pu(t) * \pi_{el} = (\pi_{el} \varepsilon / SPF(T_{od}(t))) dd(t) \quad (3)$$



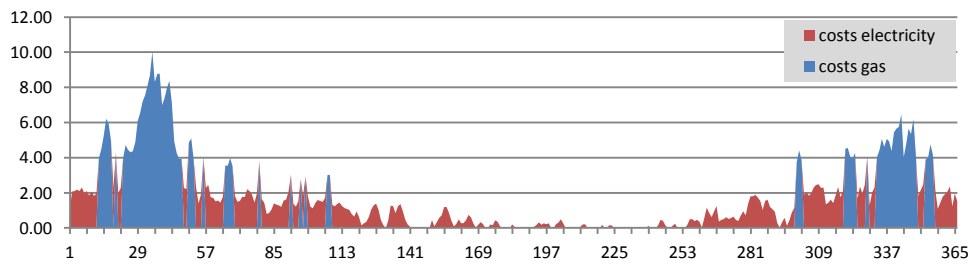
**Table 1** Main concepts

notation	description		unit	values used
$SPF$	seasonal performance factor			
$T_{od}(t)$	average outdoor temperature at day t		$^{\circ}C$	
$T_{id}(t)$	average indoor temperature at day t		$^{\circ}C$	
$T_{water}(t)$	average water temperature of the heating system at day t		$^{\circ}C$	
$\sigma(t)$	seasonal weight factor	0.8 for months 4-9, 1 for months 3 and 10, 1.1 for months 11-2		
$dd(t)$	degree days at day t		$dd$	
$gp(t)$	gas provision for day t		$m^3$	
$pp(t)$	heat pump energy provision for day t		$kWh$	
$pu(t)$	heat pump energy use for day t		$kWh$	
$\eta$	gas needed per degree day		$m^3/dd$	0.39
$\varepsilon$	electricity needed per degree day		$kWh/dd$	4.0
$\pi_{gas}$	price of gas		$euro/m^3$	0.8
$\pi_{el}$	price of electricity		$euro/kWh$	0.16

The model as described by (1), (2), and (3) above has been used to perform a number of simulations over 2012, thereby using the temperature data from Section 3 and the linear approximation of SPF shown in Fig. 3. Here for each day, depending on the outdoor temperature, a choice was made by the allocation agent to allocate either the gas-based agent or the heatpump-based agent to the heating task. The choice criterion concerns how the outdoor temperature  $T_{od}(t)$  compares to some fixed *threshold value*  $T_{th}$  for the outdoor temperature:

$$\begin{aligned}
 T_{od}(t) &\geq T_{th} &\Rightarrow &\text{allocation to heatpump agent} \\
 T_{od}(t) &< T_{th} &\Rightarrow &\text{allocation to gas-based heating agent}
 \end{aligned}$$

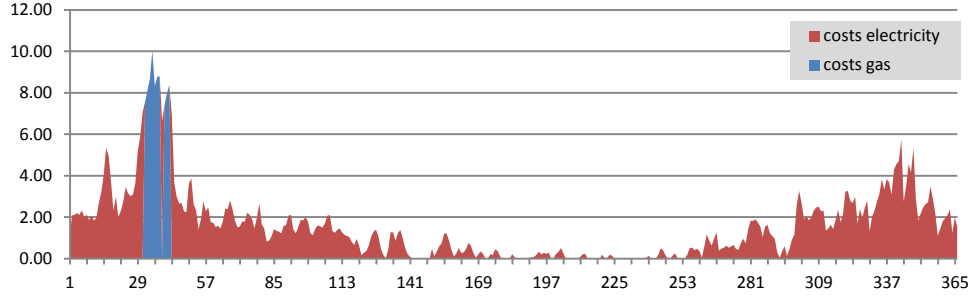
First, in Fig. 7 results from a simulation are shown in which the allocation agent has allocated the gas-based agent to all days with average outdoor temperature below  $T_{th} = +6^{\circ}C$  and the heatpump-based agent for the other days. In this case the overall costs are € 388 for gas and € 260 for electricity for the heatpump (€ 648 in total).



**Fig. 7.** Example simulation with allocation threshold temperature  $+6^{\circ}C$ . Overall gas cost € 388, overall heatpump cost € 260, total costs € 648.

Fig. 8 shows a similar simulation, but this time the allocation agent has allocated the gas-based agent to all days with average outdoor temperature below  $T_{th} = -4^{\circ}C$  and

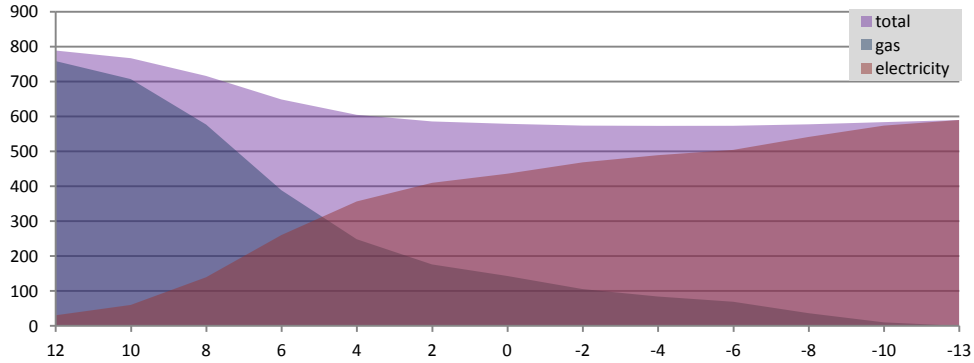
the heatpump-based agent for the other days. In this case the gas costs over 2012 are € 84, the electricity costs € 489, in total € 573.



**Fig. 8.** Example simulation with allocation threshold temperature  $-4^{\circ}\text{C}$ . Overall gas cost € 84, overall heatpump cost € 489, total costs € 573.

For the extreme cases in which only one agent is used for the whole year the total costs are € 815 (only gas) and € 590 (only heatpump). Note that both are higher than the combined simulation shown in Fig. 8.

An overview of the year costs for several what-if simulations for which the threshold value  $T_{th}$  for allocation varied from  $-13^{\circ}\text{C}$  to  $+12^{\circ}\text{C}$  is shown in Fig. 9 below. It is shown that there is a minimum somewhere between  $0^{\circ}\text{C}$  and  $-8^{\circ}\text{C}$ ; this minimum represents the most economic choice for the threshold temperature  $T_{th}$ . In the next section, in a more general setting this minimum will be determined more precisely by mathematical analysis.



**Fig. 9.** Overview of costs for different simulations over 2012 for varying allocation threshold temperatures from  $12^{\circ}\text{C}$  to  $-13^{\circ}\text{C}$ . The vertical axis represents the costs for the whole year.

This method based on what-if analysis using simulation can be used by the allocation agent HAA in its Agent-Specific Task. However, this makes the model rather complex. In next section, based on mathematical analysis a more compiled form of knowledge will be found that will provide a more concise model for agent HAA.

## 5 Mathematical Analysis

In this section by mathematical analysis some conclusions are derived from the model introduced in Section 4. These conclusions will provide more adequate knowledge for the allocation agent HAA. First the outdoor temperature threshold value is considered that is used by the allocation agent to decide which agent should get the heating task allocated. It will be determined what this threshold value should be to obtain minimal overall costs (the lowest point in the graph for total costs in Fig. 9). Recall the two expressions (2) and (3) for gas costs  $gc(t)$  and heatpump costs  $pc(t)$  in Section 4:

$$gc(t) = \pi_{gas} \eta dd(t) \quad (2)$$

$$pc(t) = (\pi_{el} \varepsilon / SPF(T_{od}(t))) dd(t) \quad (3)$$

By comparing these expressions it follows that the following hold:

- Heatpump more economic than gas-based system:  
 $\pi_{el} \varepsilon / SPF(T_{od}(t)) < \eta \pi_{gas} \Leftrightarrow SPF(T_{od}(t)) > \pi_{el} \varepsilon / (\eta \pi_{gas})$
- Heatpump and gas-based system equally economic:  
 $\pi_{el} \varepsilon / SPF(T_{od}(t)) = \eta \pi_{gas} \Leftrightarrow SPF(T_{od}(t)) = \pi_{el} \varepsilon / (\eta \pi_{gas})$
- Heatpump less economic than gas-based system:  
 $\pi_{el} \varepsilon / SPF(T_{od}(t)) > \eta \pi_{gas} \Leftrightarrow SPF(T_{od}(t)) < \pi_{el} \varepsilon / (\eta \pi_{gas})$

It turns out that the value of  $\pi_{el} * \varepsilon / (\eta * \pi_{gas})$  serves as a the *optimal threshold value for the performance factor*, denoted by  $SPF_{oth}$ :

$$SPF_{oth} = \pi_{el} \varepsilon / (\eta \pi_{gas})$$

For values for  $\pi_{el}$ ,  $\varepsilon$ ,  $\eta$ , and  $\pi_{gas}$  as indicated in Table 1 it holds  $SPF_{oth} = 2.05$ . So, for this case from the mathematical analysis it follows that for the circumstances modelled in the simulations, for a given average outdoor temperature  $T_{od}(t)$  at day  $t$  the heatpump needs to have an SPF value of 2.05 or more to be at least as (or more) economic compared to a gas-based system. Therefore, a *rule of thumb* can be used stating:

*As long as the performance factor SPF does not drop below 2, it is more economic to allocate the heatpump instead of a gas-based installation for the heating system*

As  $SPF$  (strictly) monotonically depends on  $T_{od}$ , associated to  $SPF_{oth}$ , (as long as  $SPF_{oth}$  is in the range of  $SPF$ ), there is a (unique) *optimal threshold temperature*  $T_{oth}$  for  $T_{od}$  such that  $SPF(T_{oth}) = SPF_{oth}$ . Then it holds:

Heatpump more economic than gas-based  $\Leftrightarrow SPF(T_{od}(t)) > SPF_{oth} \Leftrightarrow T_{od}(t) > T_{oth}$

Heatpump and gas-based equally economic  $\Leftrightarrow SPF(T_{od}(t)) = SPF_{oth} \Leftrightarrow T_{od}(t) = T_{oth}$

Heatpump less economic than gas-based  $\Leftrightarrow SPF(T_{od}(t)) < SPF_{oth} \Leftrightarrow T_{od}(t) < T_{oth}$

Assuming the linear approximation of  $SPF$ , for  $T_{water} = 50^\circ\text{C}$  in the following way the optimal threshold value  $T_{oth}$  can be expressed in  $SPF(T_{oth})$ :

$$SPF(T_{oth}) = 7.5 - 0.1 * (50 - T_{oth}) \Leftrightarrow$$

$$\begin{aligned}
SPF(T_{oth}) &= 7.5 - 0.1*(50-T_{oth}) \Leftrightarrow \\
SPF(T_{oth}) &= 2.5 + 0.1 T_{oth} \Leftrightarrow \\
T_{oth} &= 10 SPF(T_{oth}) - 25
\end{aligned}$$

This relation can be used as a form of compiled knowledge by the Agent Specific Task component of the Heating Allocation Agent HAA. For values for  $\pi_{el}$ ,  $\varepsilon$ ,  $\eta$ , and  $\pi_{gas}$  as indicated in Table 1 it holds  $SPF(T_{oth}) = SPF_{oth} = \pi_{el} \varepsilon / (\eta \pi_{gas})$ ; therefore:

$$T_{oth} = 10 * 2.05 - 25 = -4.5^\circ\text{C}$$

Similarly assuming the quadratic approximation of  $SPF$ , for  $T_{water} = 50^\circ\text{C}$  the value of  $T_{oth}$  can be determined:

$$\begin{aligned}
SPF(T_{oth}) &= 7.45 - 0.1*(50-T_{oth}) + 0.004 T_{oth}^2 \Leftrightarrow \\
0 &= 2.45 - SPF(T_{oth}) + 0.1 T_{th} + 0.004 T_{oth}^2 \Leftrightarrow \\
T_{oth}^2 + 25 T_{oth} + (2450 - 1000 SPF(T_{oth}))/4 &= 0 \Leftrightarrow \\
T_{oth} &= (-25 \pm \sqrt{(25^2 - 2450 + 1000 SPF(T_{oth}))})/2 \Leftrightarrow \\
T_{oth} &= (-25 \pm \sqrt{(1000 SPF(T_{oth}) - 1825)})/2
\end{aligned}$$

For the values from Table 1 it holds  $SPF(T_{oth}) = SPF_{oth} = \pi_{el} \varepsilon / (\eta \pi_{gas}) = 2.05$ . Then this becomes

$$\begin{aligned}
T_{oth} &= (-25 \pm \sqrt{225})/2 = (-25 \pm 15)/2 \\
T_{oth} &= -20 \text{ or } -5
\end{aligned}$$

As -20 falls outside the range from -10 to 10 of the approximation, the only relevant value is  $T_{oth} = -5^\circ\text{C}$ . This is close to the value -4.5 that came out using the linear approximation of  $SPF$ .

The mathematical analysis can also be used to determine  $T_{water}$  for which the heatpump is always most economic to allocate, in the following manner. The minimal average day temperature in 2012 was  $-13^\circ\text{C}$ . Therefore, when  $SPF$  is at least  $SPF(T_{oth})$  for temperatures as low as  $-13^\circ\text{C}$ , then the heatpump is always most economic. Then, assuming that the linear approximation for  $SPF$  also holds for values of  $T_{water}$  lower than  $50^\circ\text{C}$ , the value of  $T_{water}$  can be determined as follows:

$$\begin{aligned}
SPF(T_{oth}) &= SPF(-13) = 7.5 - 0.1*(T_{water} - (-13)) \Leftrightarrow \\
0.1 * T_{water} &= 7.5 - 1.3 - SPF(T_{oth}) \Leftrightarrow \\
0.1 * T_{water} &= 6.2 - SPF(T_{oth}) \Leftrightarrow \\
T_{water} &= 62 - 10 SPF(T_{oth}) \Leftrightarrow
\end{aligned}$$

For the values from Table 1 it holds  $SPF(T_{oth}) = 2.05$ ; then this becomes

$$T_{water} = 41.5^\circ\text{C}$$

The same can be done using the quadratic approximation of  $SPF$ .

$$\begin{aligned}
SPF(T_{oth}) &= 7.45 - 0.1*(T_{water} - (-13)) + 0.004 (-13)^2 \Leftrightarrow \\
SPF(T_{oth}) &= 7.45 - 0.1*T_{water} - 1.3 + 0.004*13^2 \Leftrightarrow
\end{aligned}$$

$$\begin{aligned}
0.1 \ T_{water} &= 6.15 + 0.004 * 13^2 - SPF(T_{oth}) && \Leftrightarrow \\
T_{water} &= 61.5 + 6.8 - 10SPF(T_{oth}) && \Leftrightarrow \\
T_{water} &= 68.3 - 10SPF(T_{oth})
\end{aligned}$$

For the values from Table 1 it holds  $SPF(T_{oth}) = 2.05$ ; then this becomes

$$T_{water} = 47.8^\circ\text{C}$$

So, an estimation based on the linear and quadratic approximation may be that when the water in the heating system is kept around  $45^\circ\text{C}$ , then always allocation of the heatpump is more economic. However, the two estimations differ by  $6^\circ\text{C}$ , so they may not be very accurate.

## 6 Discussion

In this paper a multi-agent model for a domestic heating task was introduced and analysed. The aim was to model a hybrid heating system as often used in practice by an agent-based modelling perspective. The model includes two alternative heating agents (for gas-based heating and for heatpump-based heating), and a third allocation agent which determines the most economic allocation of the heating task to these heating agents over the days in a year. To be able to make good allocation decisions first it was analysed how the performance of a heatpump depends on the outdoor temperature. One method discussed was a what-if analysis method using agent-based simulation, and tried out for realistic data from 2012. Another method discussed was by mathematical analysis deriving more precise knowledge about the most optimal allocation choice. These methods can be used by the allocation agent to determine in a dynamic, adaptive manner per day or per hour a most economic allocation, depending on the (predicted) outdoor temperature.

Note that for the sake of simplicity heating costs were assumed proportional to the energy used. In practice, often a distinction is made in fixed costs and variable costs, where fixed costs have to be paid even if no energy is used at all. The methods described in the paper can easily be adapted to include such cost models. Although the paper has focused on optimal decisions from a financial perspective, the methods introduced can also be applied to other aspects, for example,  $\text{CO}_2$  emission. It is also easy to extend the model by using more than two heating agents, and to incorporate environmental dynamical models within the allocation agent, for example, to predict the outdoor temperature (e.g., [14]). Another extension of the presented approach is to allow for partial allocation, in which the allocation is not exclusive, but also have the form of allocation both agents at the same time, but each for a certain fraction of the heating task. Finally, another interesting extension is to analyse the current approach when it is combined with electricity production by solar panels (e.g., [2, 8, 10, 11, 15]).

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